## Theory Needs for FCC-ee Part I: Towards high precision EWPO calculations

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## Abstract

The FCC-ee is at the High Energy and Precision Frontier and will provide a set of ground-breaking measurements of a large number of new-physics sensitive observables, with improvement by one to two orders of magnitude in experimental precision. The full exploitation of the significantly increased experimental precision in Z-pole observables, W boson and top quark masses, and a broad array of Higgs observables, necessitates SM predictions accurate at a level commensurate with this precision. In this submission we outline the numerous opportunities for significant theoretical and experimental impact through the furtherance of EWPO calculations in the SM, both for electroweak and QCD sectors. The FCC-ee is a multi-decade project offering theoretical challenges on a comparable timescale.

In this LoI we summarize outcome of the last two years of activity on precision needs for FCC-ee, discussed in detail during two workshops [1,2] with published material [3,4], supplemented with the ESPPU input [5] and the recent work [6]. The state of art and open issues for analysis in the context of FCC-ee near Z-boson resonance has been reported in [3]. We list below the main goals and tasks discussed, adapted from the executive summaries given in [3,4].

- 1. Tera-Z will deliver the highest luminosity. To meet this experimental precision for EWPOs, 3-loop and partial 4-loop calculations of the  $Zf\bar{f}$ -vertex will be needed. The leading 3-loop corrections are  $\mathcal{O}(\alpha\alpha_s^2)$ ,  $\mathcal{O}(N_f\alpha^2\alpha_s)$ ,  $\mathcal{O}(N_f^2\alpha^3)$ , where  $\alpha$  denotes an electroweak loop, and  $N_f^n$  denotes n or more closed internal fermion loops. Sub-leading corrections may also be relevant, depending on insights gained from computing the leading terms.
- 2. Full 2-loop corrections to the  $Zf\bar{f}$ -vertex have been completed recently. The principal techniques for the electroweak loop calculations have been and will likely be numerical, due to the large number of scales involved. A few digits of internal precision will be sufficient at the 3-loop level.
- 3. The  $Zf\bar{f}$ -vertex corrections are embedded in the hard scattering process  $e^+e^- \to f\bar{f}$ , based on matrix elements in the form of a Laurent series around the Z pole. Here, additional non-trivial contributions must be implemented properly.
- 4. Numerically dominant effects due multi-photon emission constitute another key problem as their complexity is often comparable to that of the electroweak loop calculations. The methodology of joint treatment of electroweak and QCD loop corrections with the photonic corrections is essentially at hand. In practice, however, construction of new Monte Carlo programs that can handle efficiently the above problems will require dedicated efforts.
- 5. To decrease the  $\alpha_{\rm QED}$  uncertainty by a factor of five to ten, to the level (3–5)  $\times$  10<sup>-5</sup>, will require improvements in low-energy experiments. Alongside this, the perturbative QCD (pQCD) prediction of the Adler function must be improved by a factor of two, accomplished with better uncertainty estimates for  $m_{\rm c}$  and  $m_{\rm b}$ . The next required improvements are: (i) four-loop massive pQCD calculation of the Adler function; (ii) improved  $\alpha_{\rm s}$  in the low  $Q^2$  region above the  $\tau$  mass; (iii) a better control and understanding of  $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ , in terms of R data; (iv) different methods for directly accessing  $\alpha(M_Z^2)$ , e.g. the muon forward–backward asymmetry, or for calculating  $\alpha_{\rm QED}$ , either based on a radiative return experiment, e.g. at the FCC-ee Tera-Z, or using lattice QCD methods.
- 6. FCC-ee precision measurements require many improvements on the theoretical QCD side. Including: (i) higher-order pQCD fixed-order calculations; (ii) higher-order logarithmic resummations; (iii) per-mille-precision extractions of the  $\alpha_s$  coupling; and (iv) accurate control of non-perturbative QCD effects (such as colour reconnection, hadronization), both analytically and as implemented in the Monte Carlo generators.

	$\delta\Gamma_Z [{ m MeV}]$	$\delta R_l [10^{-4}]$	$\delta R_b \ [10^{-5}]$	$\delta \sin_{eff}^{2,l} \theta \ [10^{-6}]$
Present EWPO uncertainties				
EXP-2018	2.3	250	66	160
TH-2018	0.4	60	10	45
Future EWPO uncertainties				
EXP-FCC-ee	0.1	10	$2 \div 6$	6
TH-FCC-ee	0.07	7	3	7

Table 1: Comparison for selected precision observables of present experimental measurements (EXP-2018), current theory errors (TH-2018), FCC-ee precision goals at the end of the Tera-Z run (EXP-FCC-ee) and rough estimates of the theory errors assuming that electroweak 3-loop corrections and the dominant 4-loop EW-QCD corrections are available at the start of FCC-ee (TH-FCC-ee). Based on the discussion in [3].

- 7. Experience in high precision calculations for physics near the Z pole will provide a strong basis for a more advanced treatment of the W mass and width measurements. The reduction of the theoretical uncertainty of the total W pair production cross-section to the level of  $\sim 0.01\%$  at the FCC-ee-W requires the calculation of  $\mathcal{O}(\alpha^2)$  and dominant  $\mathcal{O}(\alpha^3)$  corrections to double-resonant diagrams. Effective field theory (EFT) estimates show that the theory-induced systematic uncertainty of the mass measurement from a threshold scan can be at the level of  $\Delta M_W = (0.15-0.60)$  MeV. The lower value results from assuming that the non-resonant corrections are under control. In addition, it is also essential to reduce the uncertainty from initial-state radiation (ISR) corrections and QCD corrections for hadronic final states to the required accuracy.
- 8. In spite of the fact that the experimental precision is less demanding, (permil rather than a few permillion), more precise calculations will also be required for Higgs and top physics. Predictions for Higgs properties are known with sufficient accuracy for the LHC. At the FCC-ee, the Higgs mass can be measured with a precision below 0.05 GeV. The dependence of EWPOs on  $M_{\rm H}$  is mild,  $\propto \alpha \log(M_{\rm H}/M_{\rm W})$ , and an accuracy of 0.05 GeV of  $M_{\rm H}$  will not affect their determination. The main improvements in Higgs boson studies will be connected with a better determination of branching ratios and self-couplings.
- 9. The top pair line shape for centre-of-mass energies close to the  $t\bar{t}$  production threshold is highly sensitive to the mass of the top quark, allowing its determination with unprecedented precision. The statistical uncertainty of the measurement ( $\sim 20$  MeV) is projected to be significantly less than the current theoretical uncertainty. It is thus crucial to improve the theoretical prediction. The most sensitive observable is the total production cross-section for  $b\bar{b}W^+W^-X$  final states near the top pair production threshold. A very precise knowledge of the strong coupling constant from other sources will be required in order to meaningfully constrain the top Yukawa coupling.

The understanding of all sources of possible theoretical uncertainties will be fundamental for success of the FCC-ee data analysis [6]. Just for illustration, the complexity of future perturbative calculations is detailed in Table 1 showing the current experimental and theoretical errors (EXP-2018, TH-2018) for some basic Z-physics EWPOs, and the prospective measurement errors at FCC-ee (EXP-FCC-ee) together with the corresponding estimate for theoretical uncertainties after the leading 3/4-loop results become available (TH-FCC-ee). The entry TH-2018 takes into account recent completion of the 2-loop electroweak calculations [7], so the error estimate comes solely from an estimate of magnitudes of missing 3-loop and 4-loop EW and mixed EW-QCD corrections. The estimated TH-FCC-ee error stems from remaining 4-loop and higher effects. These are rather difficult to estimate presently, however, a rough conservative upper bound on them has been provided in [3]. They are denoted in Tab. 1 as TH-FCC-ee.

## References

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