



Photo Detection Efficiency of SPAD

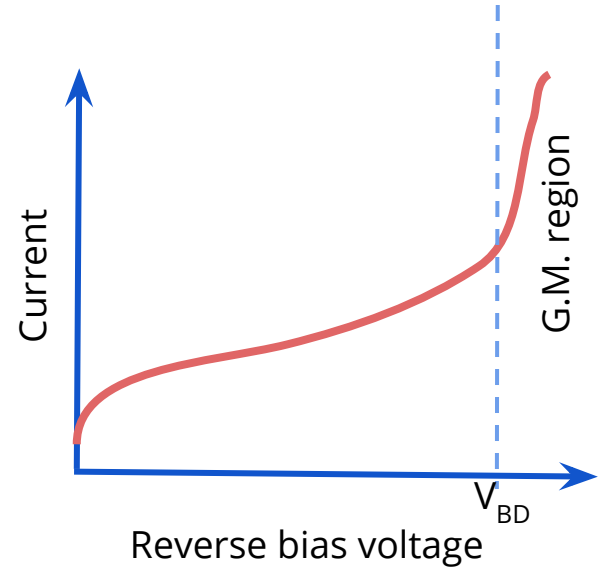
HighRR Biweekly Seminar
11-11-2020



Basic Terms

Impact Ionization

- Process of acceleration of free charge carrier with enough kinetic energy - creation of e-h pairs by ionization process.
- Geiger mode operation - multiplication of e-h pairs goes infinite.



Basic Terms

- If electrons or holes collide with equal effective masses - -
 - According to law of conservation of momentum,

$$\boxed{\text{Initial Momentum}} \quad k_0 = k_1 + k_e + k_h \quad \boxed{\text{Final Momentum}}$$

$$\hbar^2 k_0^2 / 2m^* = E_i = E_g + \hbar^2 k_1^2 / 2m^* + \hbar^2 k_e^2 / 2m^* + \hbar^2 k_h^2 / 2m^*$$

Momentum of secondary charge carriers are equal after collision ; $k_1 = k_e = k_h = \frac{1}{3} k_0$

=>

$$\boxed{\text{Threshold energy}(E_i) = 1.5 E_g}$$

- Carrier energy > Threshold energy; **No. of carriers (n) ~ exp(-E_i/k_BT_e).**

Basic Terms

- From energy balance equation,

$$eFv_d\tau = 3/2 k_B(T_e - T) \approx k_B T_e$$

- At high fields, drift velocity(v_d) = constant and with constant relaxation time (τ); carrier mean free path, $\lambda = v_d\tau$.
- **ionization rate or coefficient** can be determined by $\alpha = (1/n) dn/dx$;

$$\alpha \propto \exp(-E_i/k_B T_e) = \exp(-E_i/eF\lambda)$$

=> Average no. of e-h pairs produced by single carrier per unit length.

What is SPAD?

- **SINGLE PHOTON AVALANCHE DIODE**

- Reverse-biased p-n junction
- $V_b > V_{bd}$; Geiger-mode operation
- High electric field ($>3 \times 10^5$ V/cm)
- Self sustained avalanche
- Quenching with resistor in series.

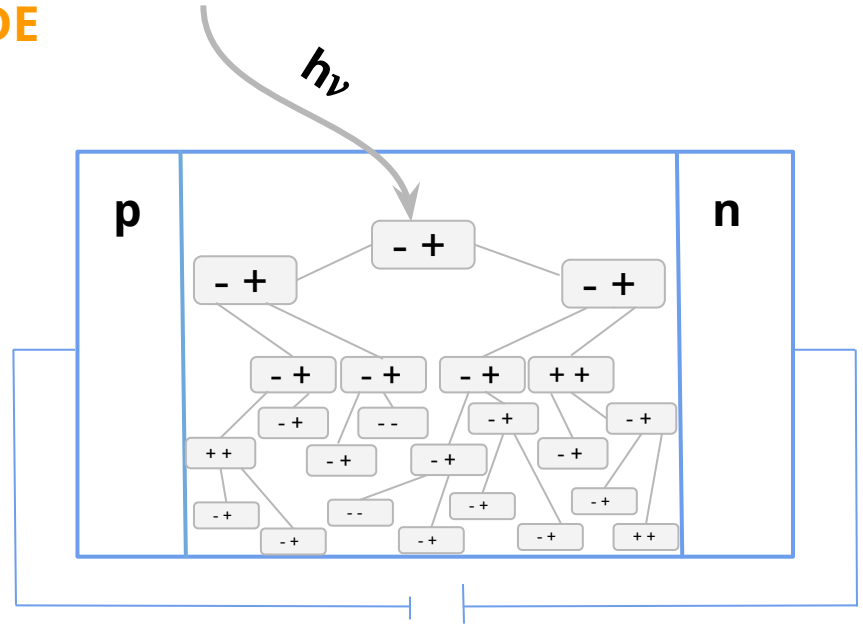


Photo Detection Efficiency

$$\text{PDE}(\lambda) = (1-R(\lambda)) \int_{z_p}^{z_n} \alpha(\lambda) e^{-\alpha(\lambda)z} P_t(z) dz + \eta_{e,ph}(\lambda, z_p) P_e(z_p) + \eta_{h,ph}(\lambda, z_n) P_h(z_n)$$

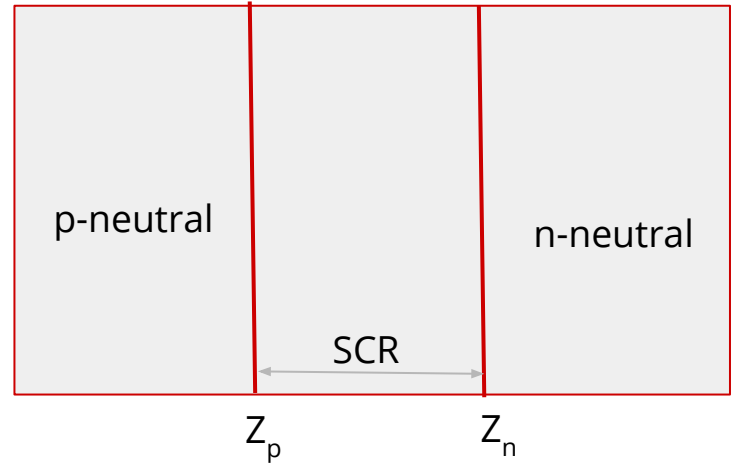
$R(\lambda)$ = Reflectivity ; $\alpha(\lambda)$ = Absorption Coefficient ;

η = Quantum Efficiency , z = Depth;

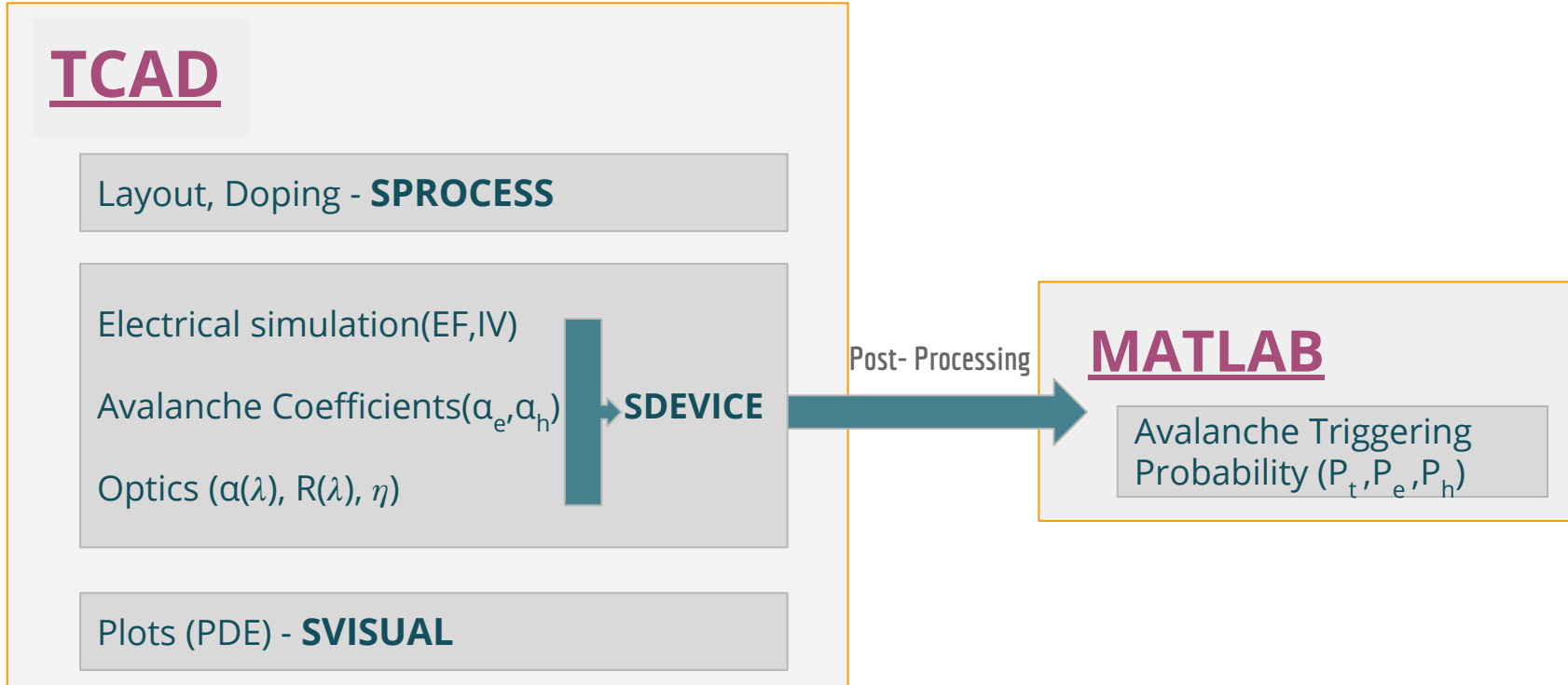
P_t = Total Triggering Probability;

P_e = Electron Triggering Probability;

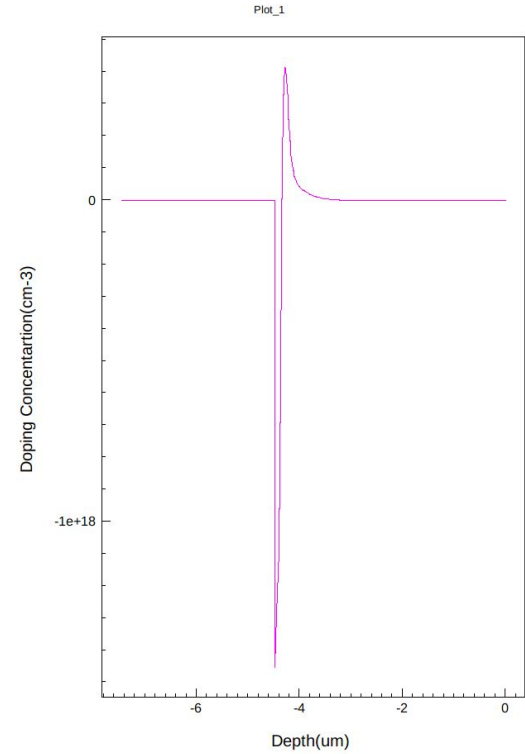
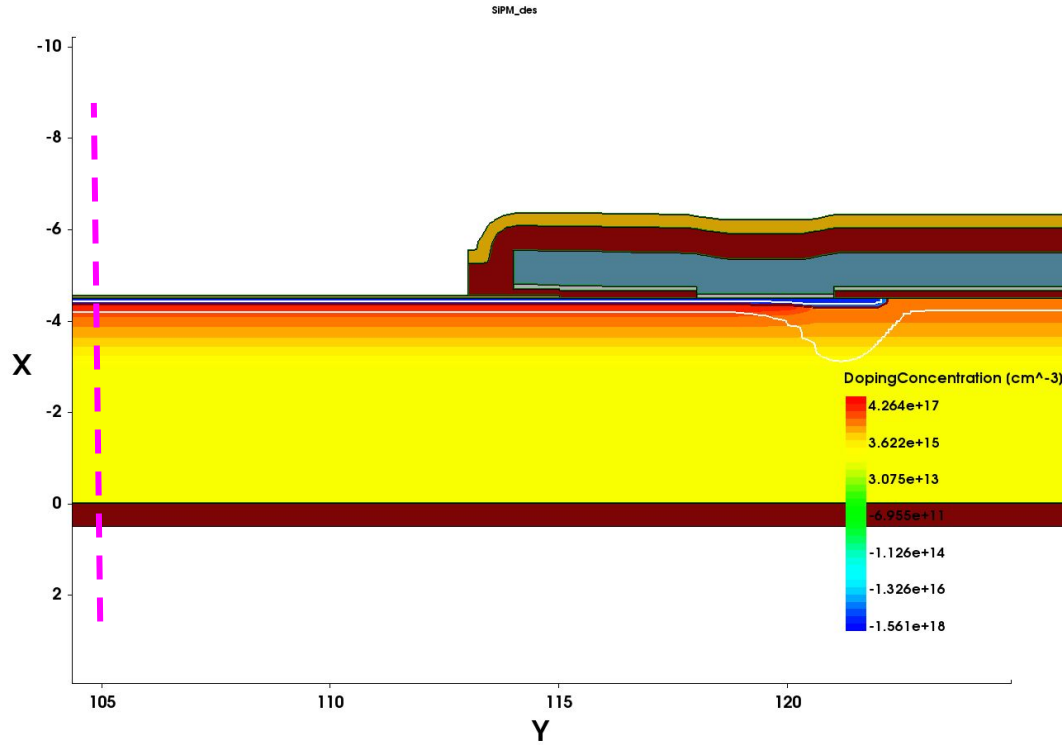
P_h = Hole Triggering Probability



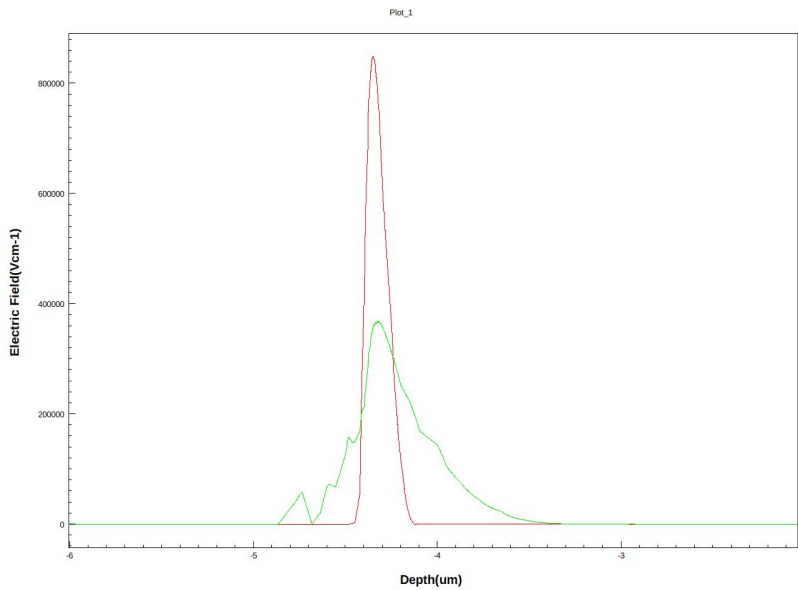
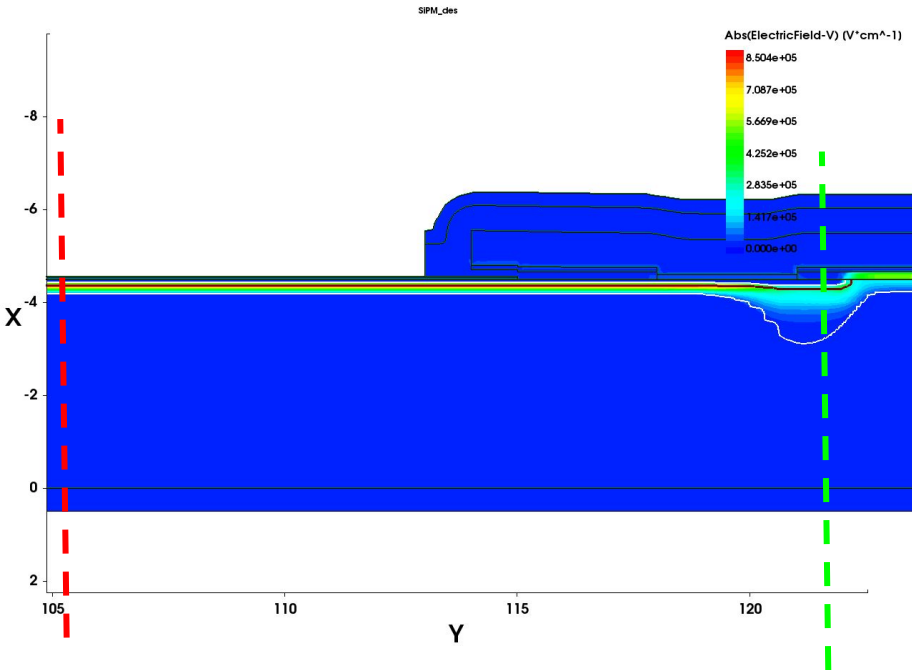
PDE Modelling



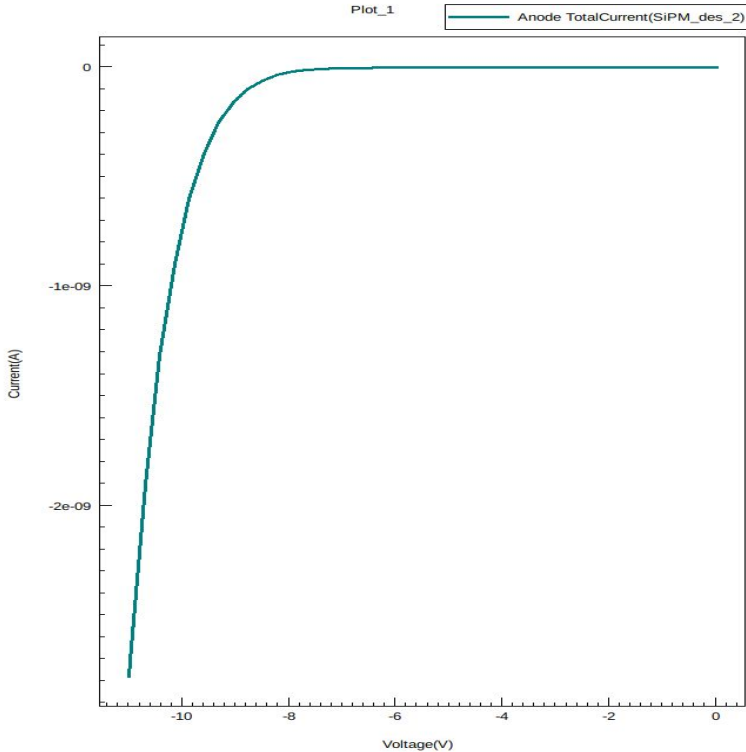
Doping Profile



Electric Field Distribution

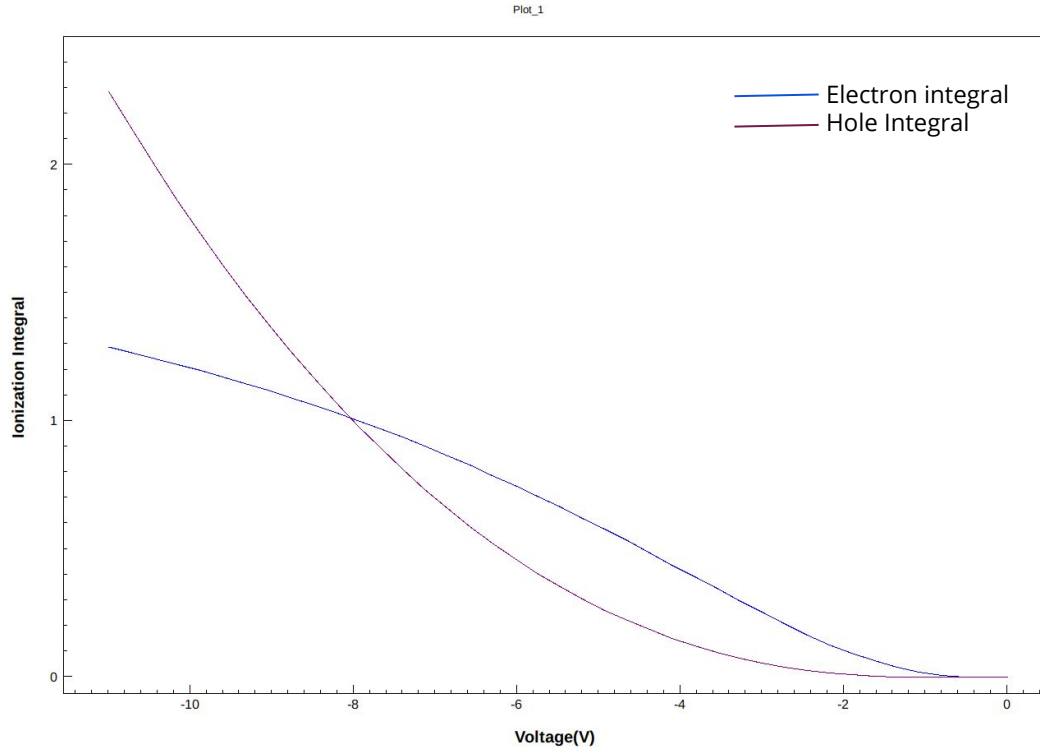


IV Characteristics



- TCAD Breakdown Model : Ionization integral with carrier analysis
- **Breakdown Voltage = 8V**

Ionization Integral



$$1 - 1/M_h = \int \alpha_h \exp(-\int (\alpha_h - \alpha_n) dz) dz = \phi_h$$

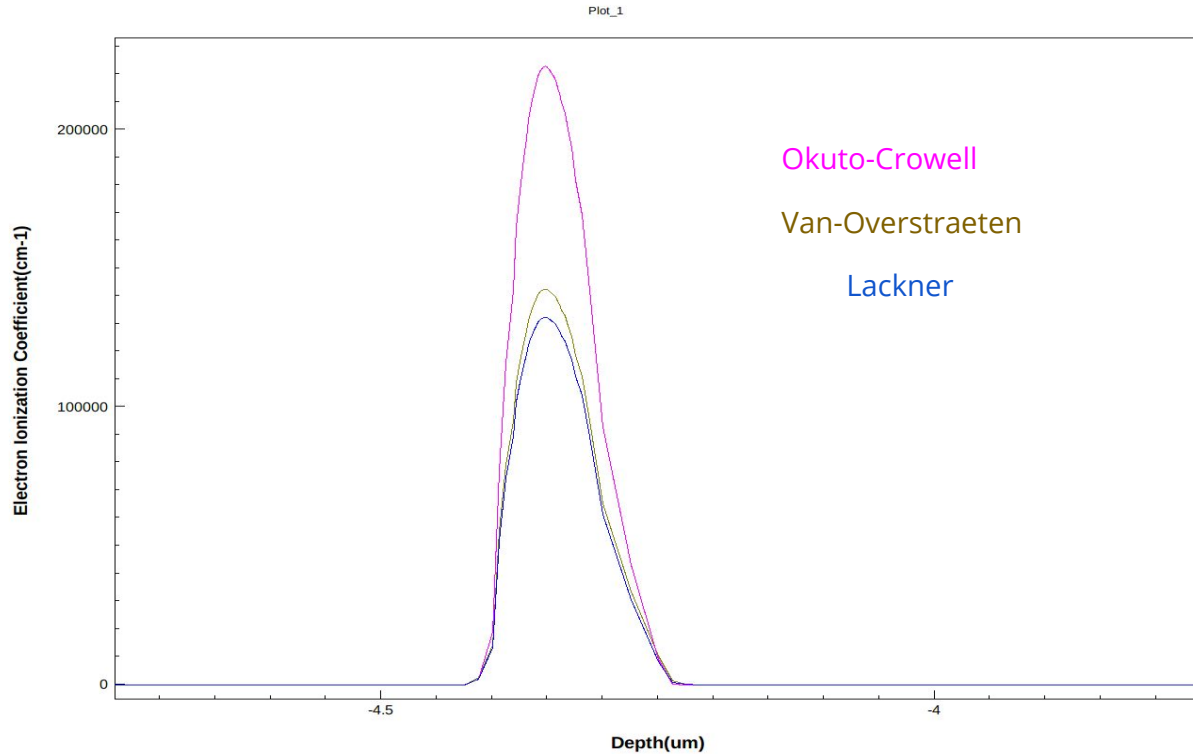
- $M_h = \infty$, when avalanche occurs $\Rightarrow \phi_h = 1$
- Breakdown occurs when one of the ionization integrals reach one.

TCAD Avalanche Models

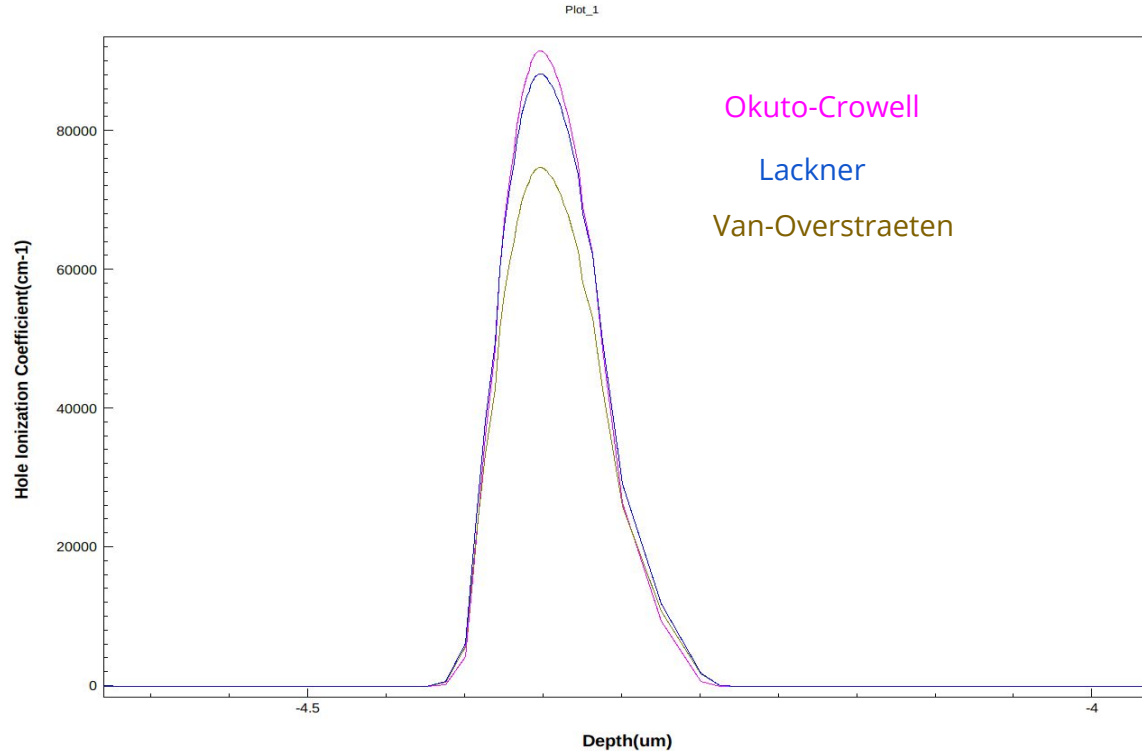
Models	Temperature	Electric Field (Vcm^{-1})
Van Overstraeten	300K	$1.75 \times 10^5 - 6 \times 10^5$
Okuto-Crowell	300K	$10^5 - 10^6$
Lackner	300K	$2 \times 10^5 - 6 \times 10^5$
UniBo	300 - 675K	$4 \times 10^4 - 5 \times 10^5$
UniBo2	300 - 773K	$4 \times 10^4 - 5 \times 10^5$

**EMPIRICAL
MODEL(USED)**

Comparison of Avalanche Models



Comparison of Avalanche Models



Avalanche Triggering Probability

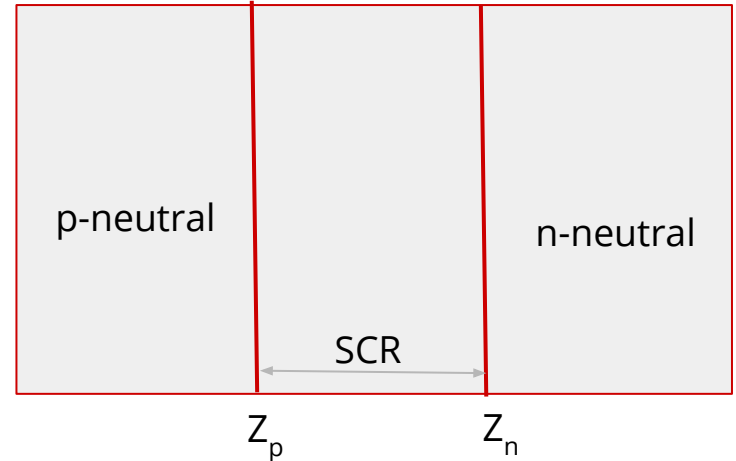
$$dP_e/dz = (1 - P_e) \alpha_e [P_e + P_h - P_e P_h]$$

$$dP_h/dz = -(1 - P_h) \alpha_h [P_e + P_h - P_e P_h]$$

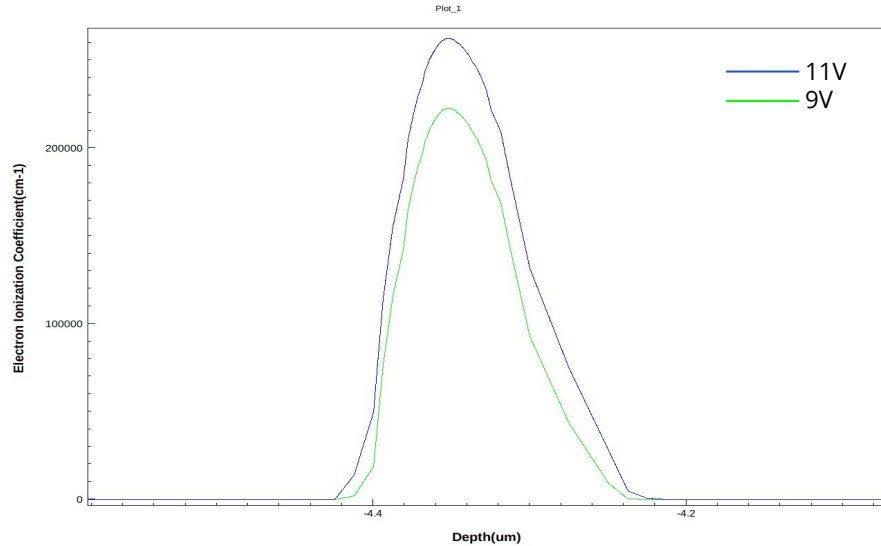
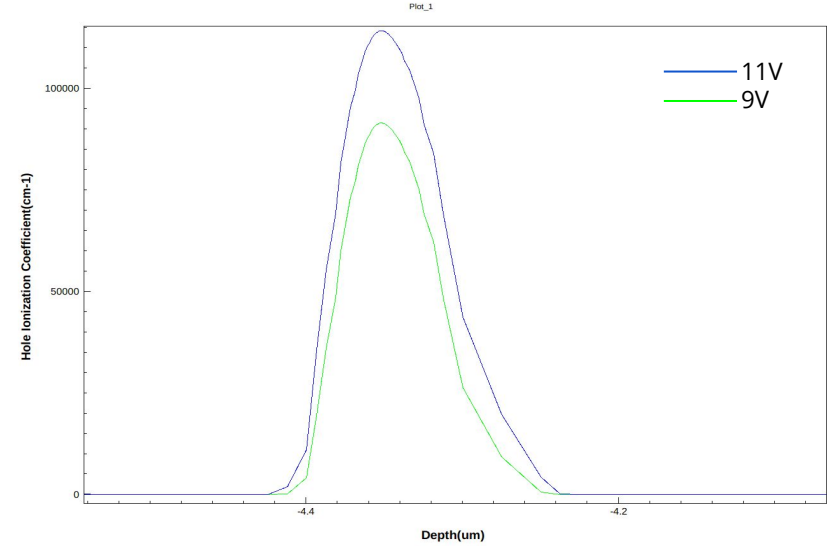
$$P_t = P_e + P_h - P_e P_h$$

Boundary Conditions : $P_e(Z_n) = 0$

$$P_h(Z_p) = 0$$

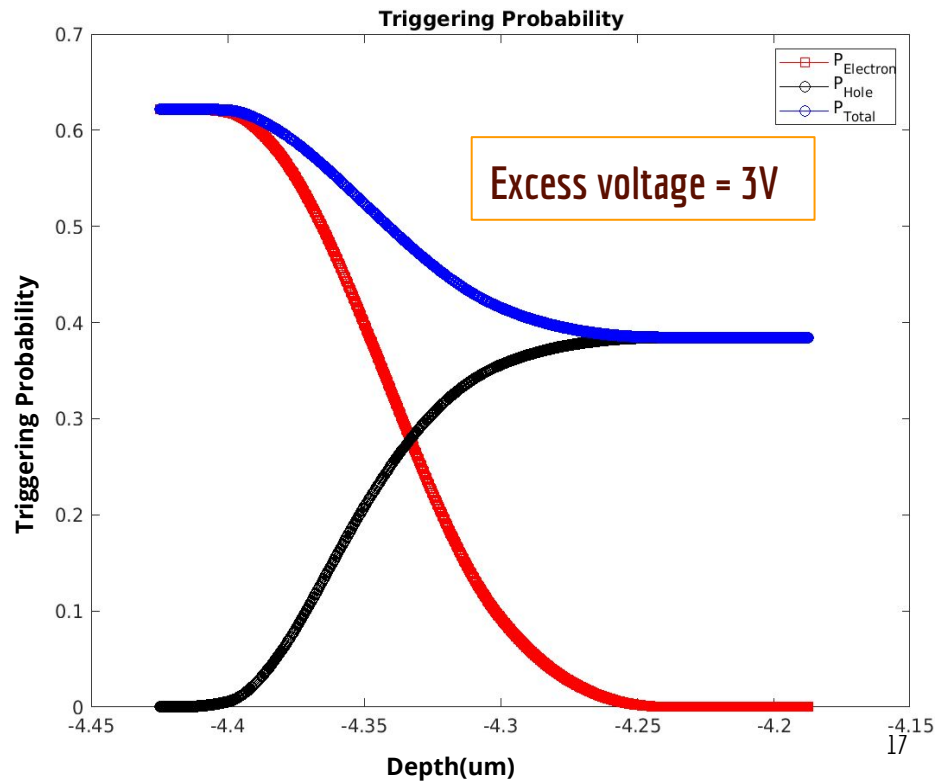
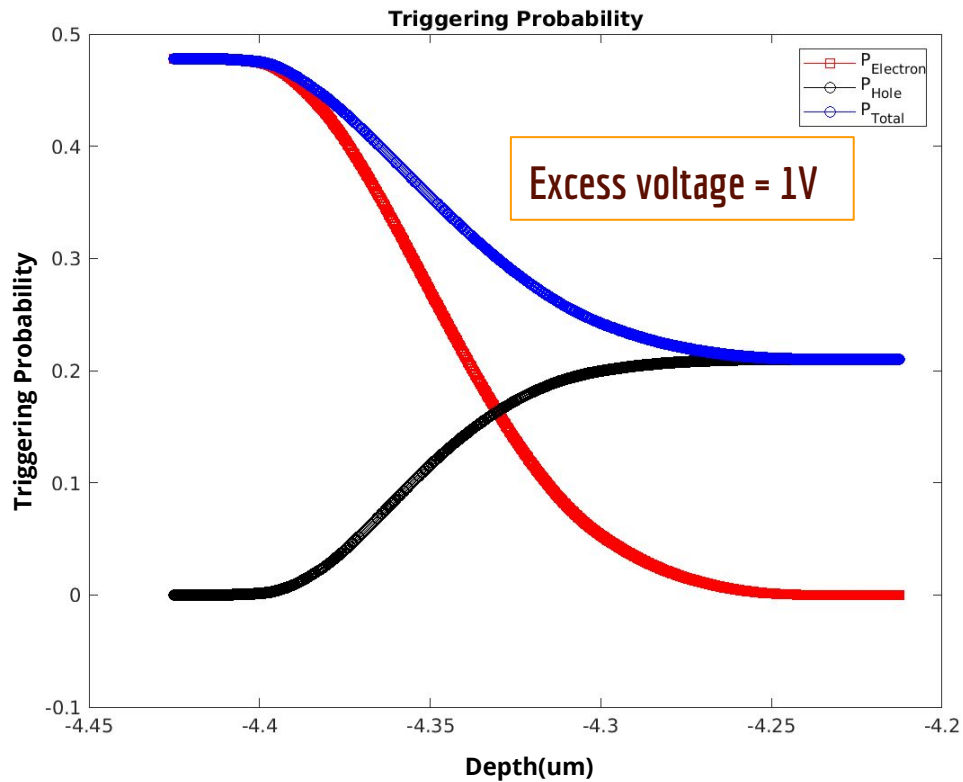


Ionization Coefficient

 α_e  α_h 

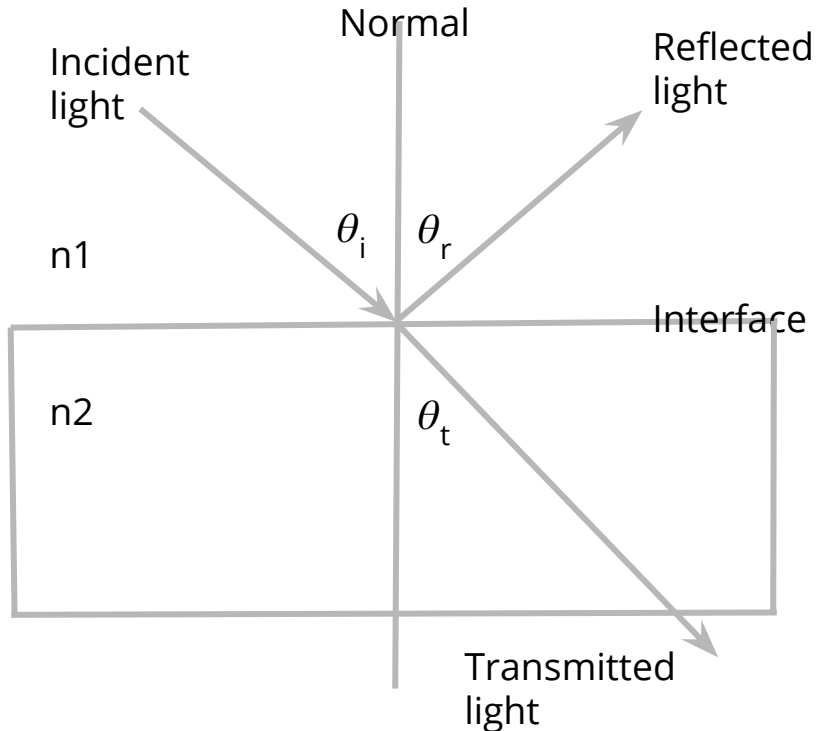
α_e is larger than α_h .

Avalanche Triggering Probability



Optics

Fresnel Reflection Coefficient

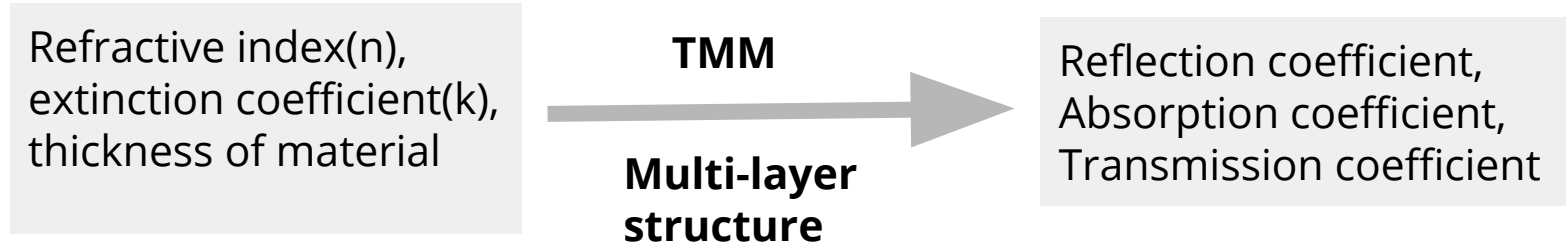


At normal incidence,
 $r = (n_1 - n_2) / (n_1 + n_2)$

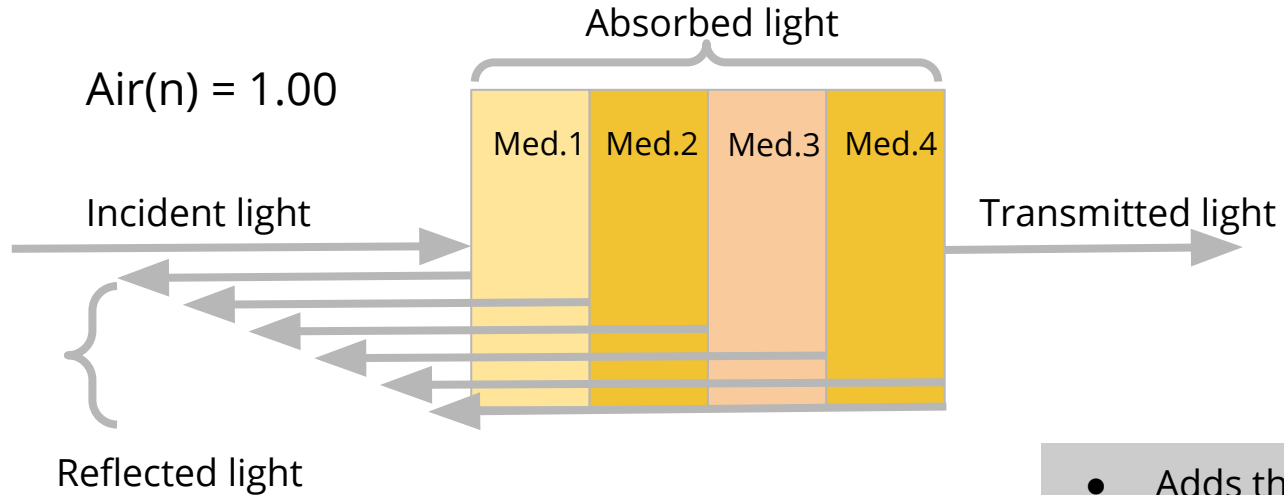
- If $n_1 < n_2$, $r = -ve$
 - Phase change π or 180°
 - External reflection
 - Destructive interference
- If $n_1 > n_2$, $r = +ve$
 - Phase change 0°
 - Internal reflection
 - Constructive interference

Reflectance, $R = r^2 = [(n_1 - n_2) / (n_1 + n_2)]^2$

Transfer Matrix Method



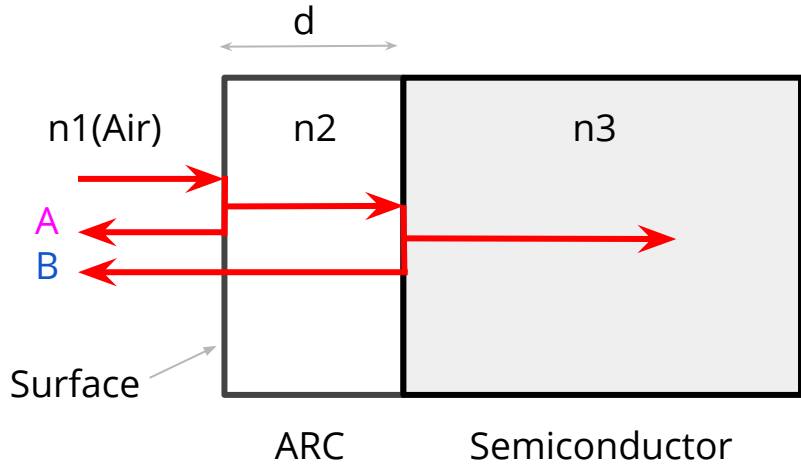
Transfer Matrix Method



- Adds the reflections from every interface in matrix form.

Reflectance+Absorption+Transmission = 100%

Anti-Reflection Coating



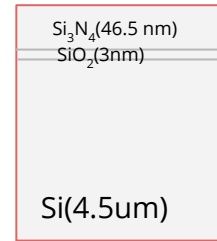
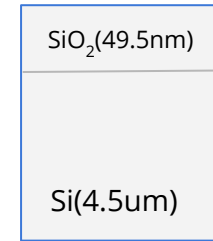
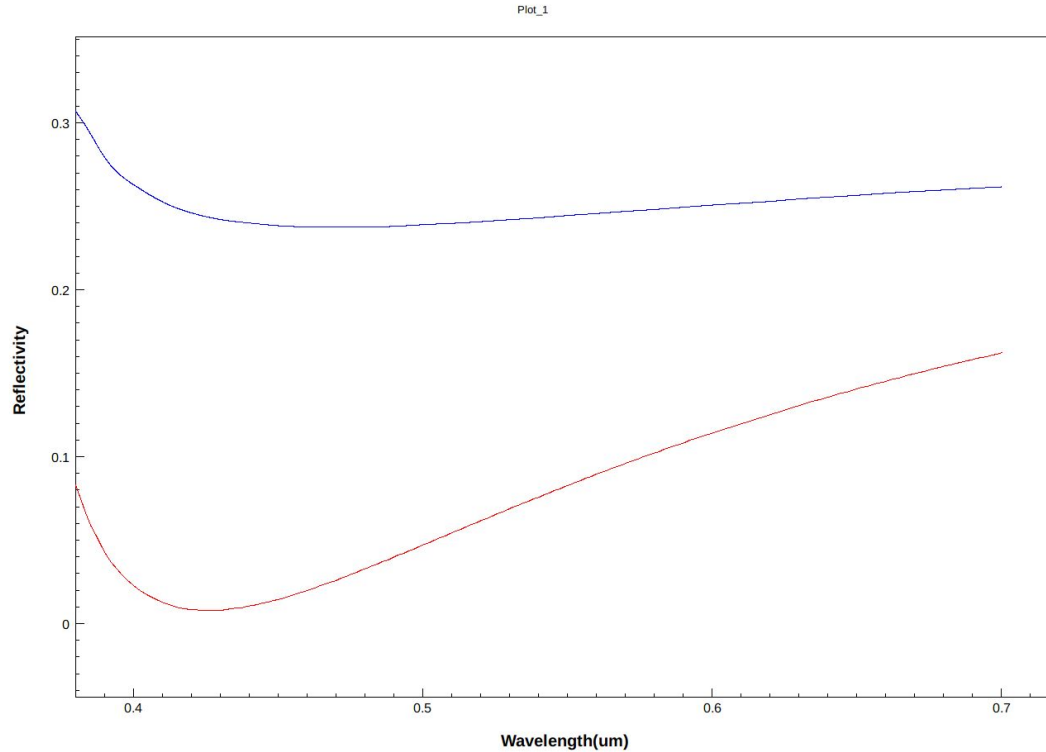
ARC reduces the reflected light intensity

- $n_1 < n_2 < n_3$; Phase change = π
- B travels twice of A
- Phase diff. between A & B : $n_2 k(2d)$

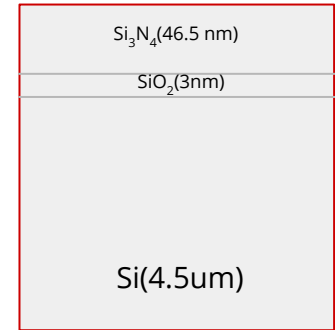
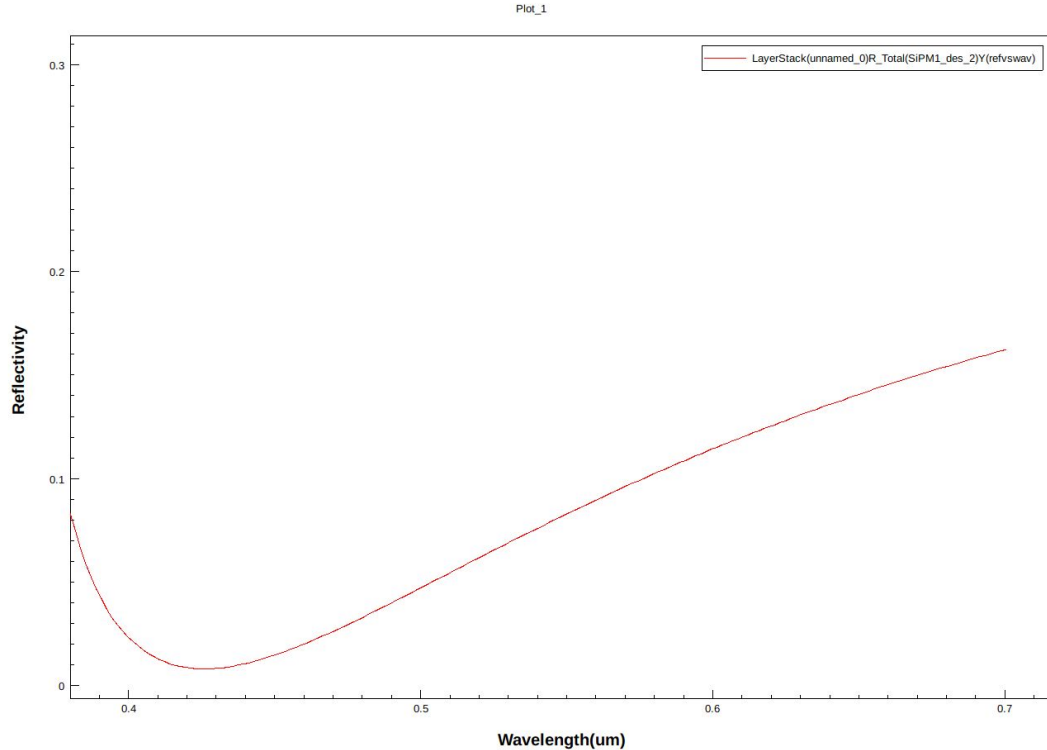
How to adjust the thickness of ARC?

- ARC thickness, d should be $\lambda/4n_2$

Effect of ARC

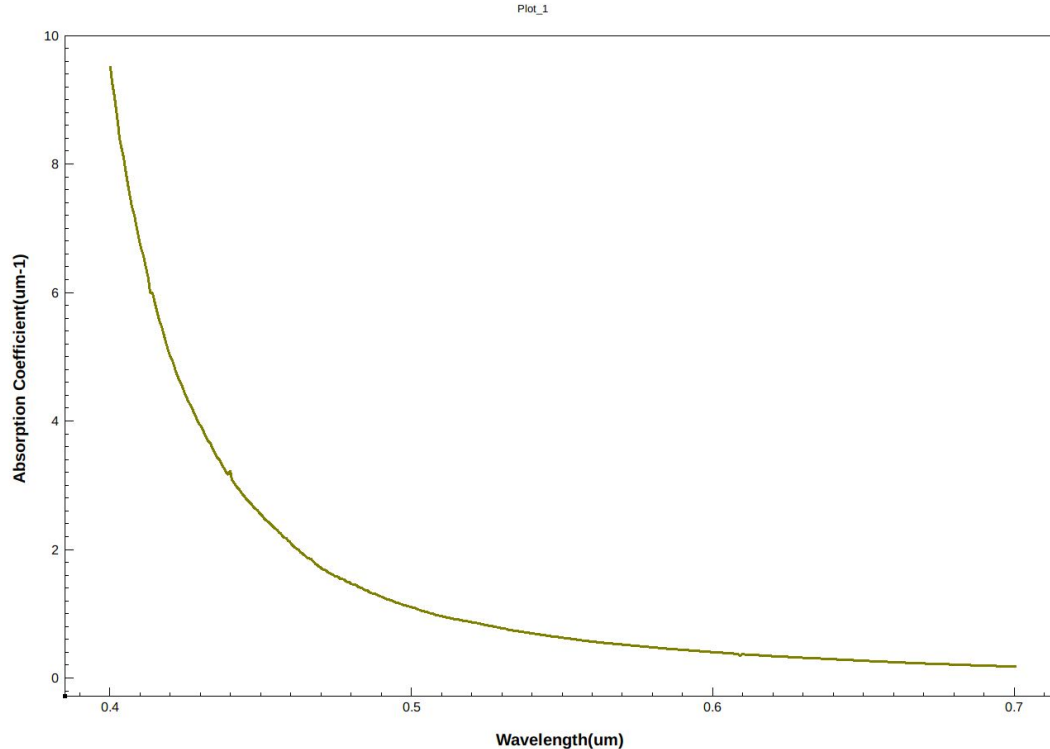


Tuned ARC at 420 nm



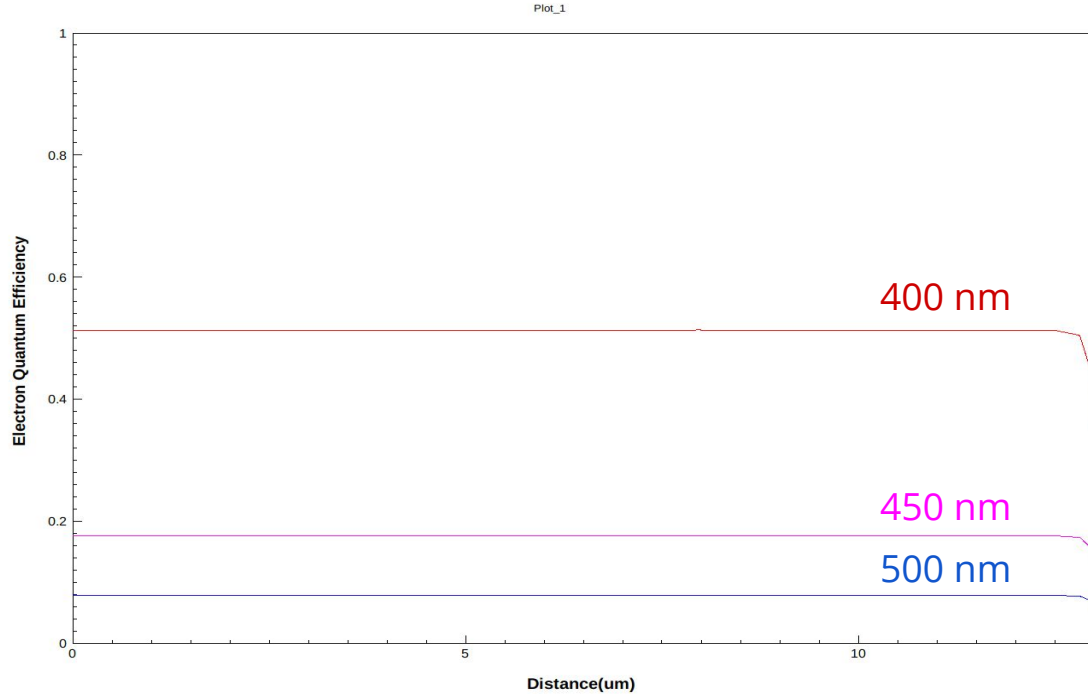
- ~1% reflection

Absorption Coefficient ($\alpha(\lambda)$) of Si



- Strongly depends on wavelength.
- $\alpha(\lambda) = 4\pi k/\lambda$

Quantum Efficiency



Drops at depletion edge

$$\eta = (hc/q\lambda) * J/P$$

$$P = 0.05W/cm^2$$

Quantum Efficiency

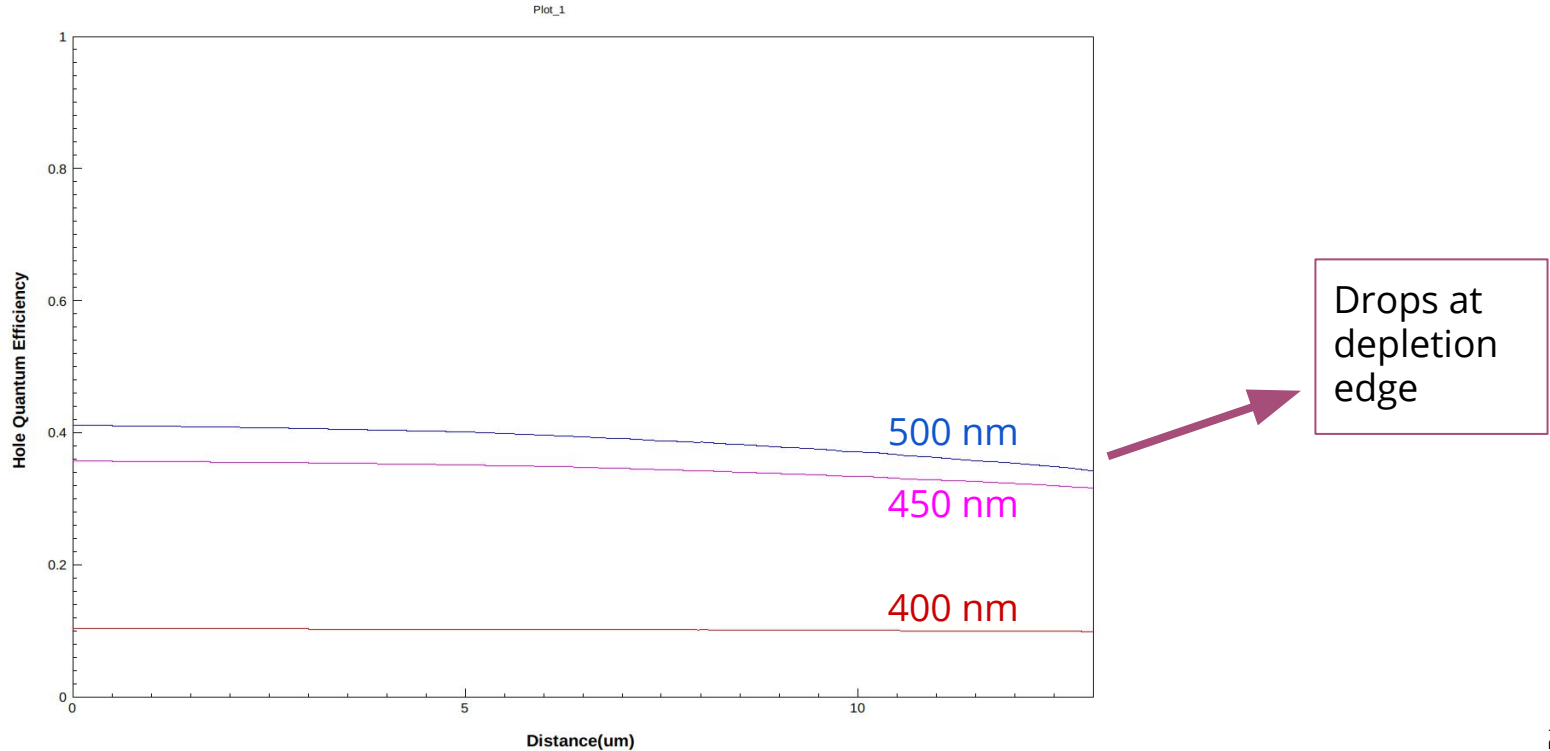
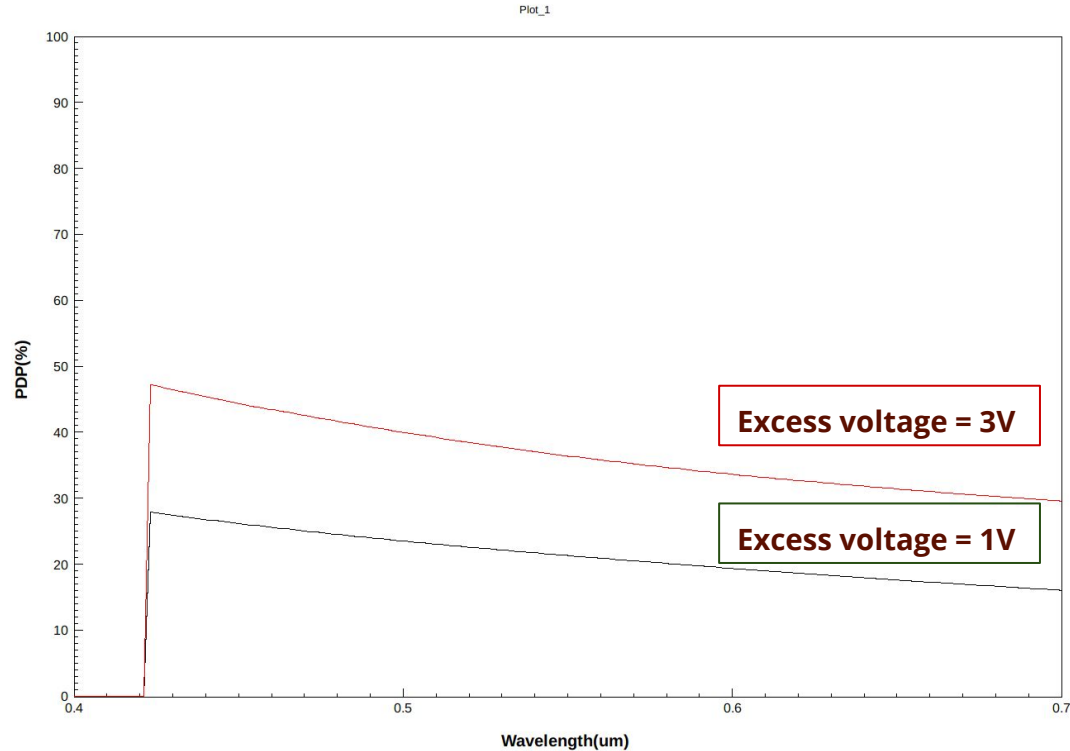


Photo Detection Probability



- 28% for $V_{ex} = 1V$
- 48% for $V_{ex} = 3V$

Conclusions

- Anti-reflection coating can be adjusted by $\lambda/4n_2$.
- Reflectivity $\sim 1\%$ at 420 nm of wavelength.
- PDP = 28% for $1V_{ex}$; 48% for $3V_{ex}$ at 420 nm.
- PDP increases with the increase of excess voltage.