Small-x Physics at the LHC and future DIS experiments
Work Package 13: small x activity reports

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Report for the NA2: small x working group,
STRONG2020, Sep 16^{th}, 2020

1. Multi-Regge limit with Monte Carlo event generator `BFKLex`
2. Beyond the Multi-Regge limit: Collinear double logs
3. More exclusive observables in multi-jet production at LHC
4. Odderon exchange and high energy complexity
5. Unintegrated gluon density beyond NLO
6. High energy effective action for electroweak processes

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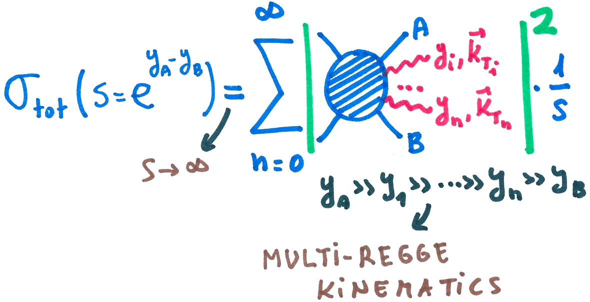
Regge theory preludes QCD. Pomeron in terms of quarks & gluons?

Perturbation theory with large scale $Q > \Lambda_{\mathrm{QCD}} \rightarrow \alpha_s(Q) \ll 1$.

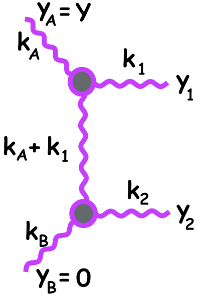
$s \gg t, Q^2 \rightarrow \alpha_s(Q) \log\left(\frac{s}{t}\right) \sim \mathcal{O}(1)$. Resummation needed.



$$\sigma_{\mathrm{tot}}^{\mathrm{LL}} = \sum_{n=0}^{\infty} \mathcal{C}_n^{\mathrm{LL}} \alpha_s^n \int_{y_B}^{y_A} dy_1 \int_{y_B}^{y_1} dy_2 \cdots \int_{y_B}^{y_{n-1}} dy_n = \sum_{n=0}^{\infty} \frac{\mathcal{C}_n^{\mathrm{LL}}}{n!} \underbrace{\alpha_s^n \left(y_A - y_B\right)^n}_{\mathrm{LL}}$$

Effective Feynman rules: basis of Lipatov's High Energy effective action

Gluon Regge trajectory: $\omega(\vec{q}) = -\frac{\alpha_s N_c}{\pi} \log \frac{q^2}{\lambda^2}$
Non-IR finite



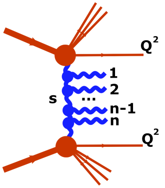
Modified propagators in the t-channel:

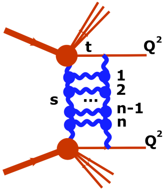
$$\left(\frac{s_i}{s_0}\right)^{\omega(t_i)} = e^{\omega(t_i)(y_i - y_{i+1})}$$

$$\left(\frac{\alpha_s N_c}{\pi}\right)^2 \int d^2\vec{k}_1 \frac{\theta\left(k_1^2 - \lambda^2\right)}{\pi k_1^2} \int d^2\vec{k}_2 \frac{\theta\left(k_2^2 - \lambda^2\right)}{\pi k_2^2} \delta^{(2)}\left(\vec{k}_A + \vec{k}_1 + \vec{k}_2 - \vec{k}_B\right)$$

$$\times \int_0^Y dy_1 \int_0^{y_1} dy_2 e^{\omega\left(\vec{k}_A\right)(Y - y_1)} e^{\omega\left(\vec{k}_A + \vec{k}_1\right)(y_1 - y_2)} e^{\omega\left(\vec{k}_A + \vec{k}_1 + \vec{k}_2\right) y_2}$$

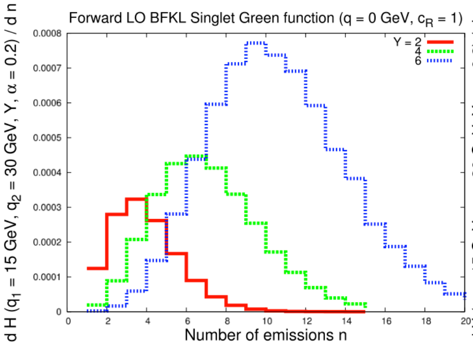
Each diagram is Non-IR finite when $\lambda \to 0$.
Only after summation over all possible final states we get IR finiteness

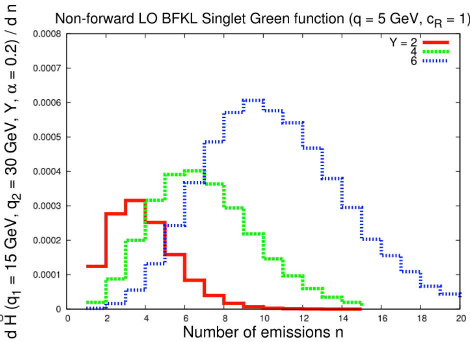




Cut Pomeron: Number of emissions?

Pomeron: Number of rungs?





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We can extend the formalism to include collinear regions

$$f = e^{\omega\left(\vec{k}_A\right)Y}\left\{\delta^{(2)}\left(\vec{k}_A-\vec{k}_B\right)+\sum_{n=1}^{\infty}\prod_{i=1}^{n}\frac{\alpha_s N_c}{\pi}\int d^2\vec{k}_i\frac{\theta\left(k_i^2-\lambda^2\right)}{\pi k_i^2}\right.$$
$$\left.\times\int_0^{y_{i-1}}dy_i e^{\left(\omega\left(\vec{k}_A+\sum_{l=1}^{i}\vec{k}_l\right)-\omega\left(\vec{k}_A+\sum_{l=1}^{i-1}\vec{k}_l\right)\right)y_i}\delta^{(2)}\left(\vec{k}_A+\sum_{l=1}^{n}\vec{k}_l-\vec{k}_B\right)\right\}$$

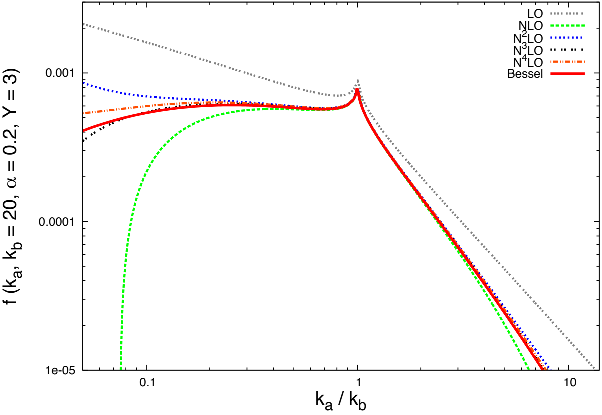
Key at NLL: $\theta\left(k_i^2-\lambda^2\right)\to\theta\left(k_i^2-\lambda^2\right)\underbrace{-\frac{\bar{\alpha}_s}{4}\ln^2\left(\frac{\vec{k}_A^2}{\left(\vec{k}_A+\vec{k}_i\right)^2}\right)}_{\text{NLL}}$

Resum it to all orders (SV)[2005]:

$$\theta\left(k_i^2-\lambda^2\right)\to\theta\left(k_i^2-\lambda^2\right)+\sum_{n=1}^{\infty}\frac{\left(-\bar{\alpha}_s\right)^n}{2^n n!(n+1)!}\ln^{2n}\left(\frac{\vec{k}_A^2}{\left(\vec{k}_A+\vec{k}_i\right)^2}\right)$$

It corresponds to a Bessel function $J_1\left(\sqrt{2\bar{\alpha}_s\ln^2\left(\frac{\vec{k}_A^2}{\left(\vec{k}_A+\vec{k}_i\right)^2}\right)}\right)$

$$\sigma(Q_1, Q_2, Y) \quad = \quad \int d^2\mathbf{k}_a d^2\mathbf{k}_b \, \phi_A(Q_1, \mathbf{k}_a) \, \phi_B(Q_2, \mathbf{k}_b) \, f(\mathbf{k}_a, \mathbf{k}_b, Y)$$



Important to go beyond the MRK limit (Ciafaloni-Colferai-Salam-Stasto).
For original BFKL we need "δ-like" impact factors $\phi_{A,B}$ & $Q_1 \simeq Q_2$.

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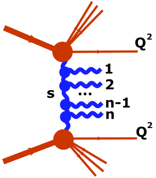
LHC observable proposed as ideal to pin down original BFKL, without collinear contamination: remove χ_0

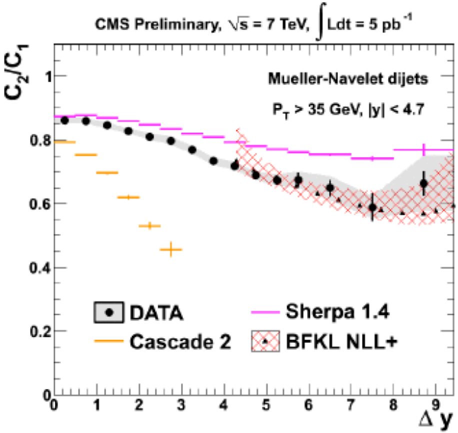


(Del Duca-Stirling)$^{\text{Tevatron}}$

$< \cos(m\,\theta) > \simeq e^{\alpha Y(\chi_m - \chi_0)}$

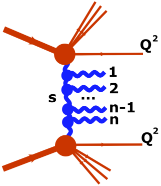
(SV)^{2006} (SV-Schwennsen)^{2007}

$$\mathcal{R}_{m,n} = \frac{<\cos(m\,\theta)>}{<\cos(n\,\theta)>} \simeq e^{\alpha Y(\chi_m - \chi_n)}$$

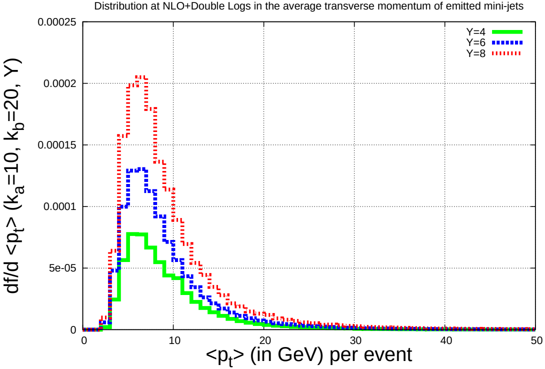


Confirmed in 2013 (Wallon et al) (Colferai et al) (Papa et al)

Implementation in BFKLex with Greg Chachamis
Average transverse momentum of emitted mini-jets?



$$\langle p_t \rangle = \frac{1}{n}\sum_{i=1}^{n} |k_i|$$

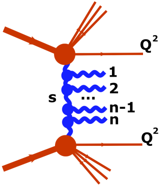


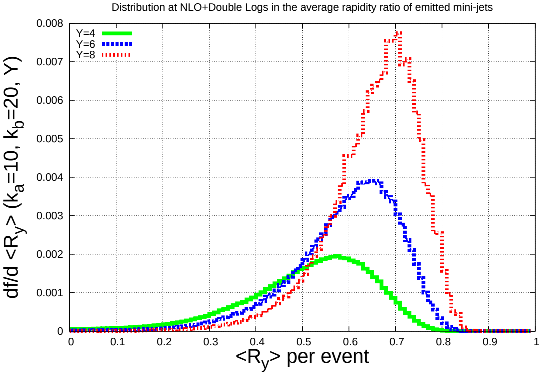
Mini-jet $\langle p_t \rangle_{\max}$ independent of rapidity separation of tagged forward jets.

Average rapidity separation among emitted mini-jets?

$$\langle \mathcal{R}_y \rangle = \frac{1}{n+1} \sum_{i=1}^{n+1} \frac{y_i}{y_{i-1}} \quad \simeq 1 + \frac{\Delta}{Y} \ln \frac{\Delta}{Y}$$

if $Y \simeq N\Delta$ in MRK and $Y \gg \Delta$





Higher $\langle \mathcal{R}_y \rangle_{\max}$ for higher energies: $\Delta_{\rm LO} \simeq 0.62$, $\Delta_{\rm LO+DLs} \simeq 0.81$

Lower mini-jet multiplicity when including higher order corrections

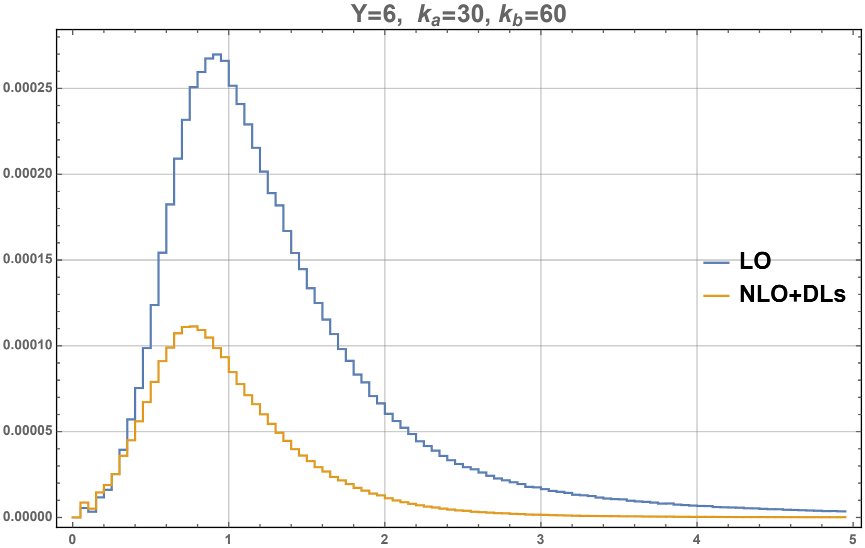
A new observable

$$y_b(=0) \ll y_n \ll \cdots \ll y_2 \ll y_1 \ll y_a$$

$$|k_{b\perp}| \simeq |k_{n\perp}| \simeq \cdots \simeq |k_{2\perp}| \simeq |k_{1\perp}| \simeq |k_{a\perp}|$$

$$\langle \mathcal{R}_{kY} \rangle = \frac{1}{n+1} \sum_{i=1}^{n+1} \frac{k_i \, e^{y_i}}{k_{i-1} \, e^{y_{i-1}}}$$

The new observable still probes the rapidity ordering in Multi-Regge kinematics but with the added feature that it also encodes the dependence on the transverse size of the emitted jets.



Y=6, k_a=30, k_b=60
0.00025
0.00020
0.00015
0.00010
0.00005
0.00000
0
1
2
3
4
5
LO
NLO+DLs

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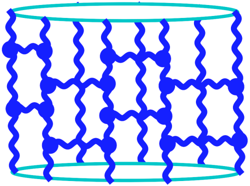
Amplitudes in the Generalized Leading Logarithmic Approximation (GLLA)



This is an old standing problem in High Energy QCD

Mapped onto a Closed Spin Chain
(Lipatov, Faddeev, Korchemsky,
Janik, Wosiek, Kotanski, Derkachov, Manashov ...)

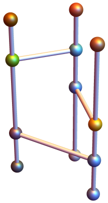
Monte Carlo integration can be applied in this case (GC-SV)[2016]

Let us consider singlet exchange in t-channel with 3 Reggeized gluons:

ODDERON

Solution of the IR-finite Bartels-Kwiecinski-Praszalowicz (BKP) equation:

$$\begin{aligned}(\omega - \omega(\mathbf{p}_1) - \omega(\mathbf{p}_2) - \omega(\mathbf{p}_3))\, f_\omega(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \; = \\ \delta^{(2)}(\mathbf{p}_1 - \mathbf{p}_4)\, \delta^{(2)}(\mathbf{p}_2 - \mathbf{p}_5)\, \delta^{(2)}(\mathbf{p}_3 - \mathbf{p}_6) \\ + \int d^2\mathbf{k}\, \xi(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{k})\, f_\omega(\mathbf{p}_1 + \mathbf{k}, \mathbf{p}_2 - \mathbf{k}, \mathbf{p}_3) \\ + \int d^2\mathbf{k}\, \xi(\mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_1, \mathbf{k})\, f_\omega(\mathbf{p}_1, \mathbf{p}_2 + \mathbf{k}, \mathbf{p}_3 - \mathbf{k}) \\ + \int d^2\mathbf{k}\, \xi(\mathbf{p}_1, \mathbf{p}_3, \mathbf{p}_2, \mathbf{k})\, f_\omega(\mathbf{p}_1 + \mathbf{k}, \mathbf{p}_2, \mathbf{p}_3 - \mathbf{k})\end{aligned}$$

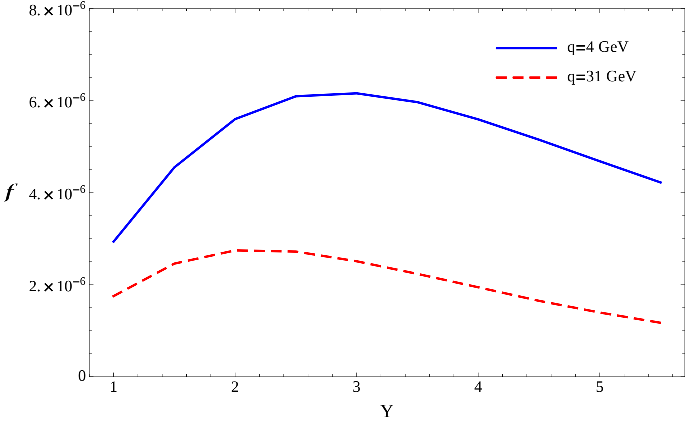


Square of Lipatov's emission vertex:

$$\xi(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{k}) \;=\; \frac{\alpha_s N_c}{4} \frac{\theta(\mathbf{k}^2 - \lambda^2)}{\pi^2 \mathbf{k}^2} \left(1 + \frac{(\mathbf{p}_1 + \mathbf{k})^2 \mathbf{p}_2^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 \mathbf{k}^2}{\mathbf{p}_1^2 (\mathbf{k} - \mathbf{p}_2)^2} \right)$$

Gluon Regge trajectory: $\omega(\mathbf{p}) = -\frac{\bar{\alpha}_s}{2} \ln \frac{\mathbf{p}^2}{\lambda^2}$

Our solution must contain previous solutions in the literature.
They are singled out by particular impact factors (work with Greg).



On-going work on phenomenological applications.

Number of Rungs = 4

We evaluate the average contribution to the GGF per Complexity Class:

Complexity	# diagrams	Average weight in GGF
15	4	2.6×10^{-8}
16	2	3.3×10^{-8}
19	4	2.6×10^{-8}
23	2	3.2×10^{-8}
24	2	3.3×10^{-8}
56	2	0

A "Complexity Democracy" emerges ...

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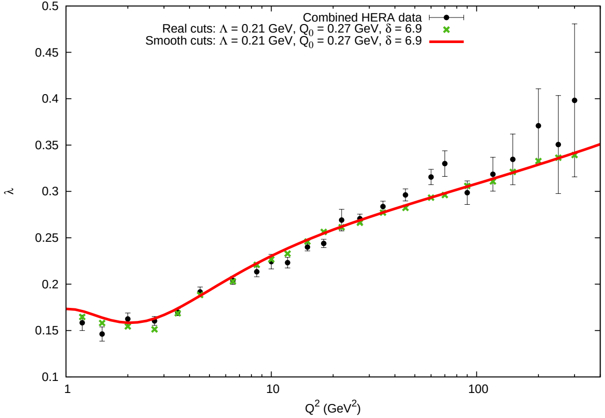
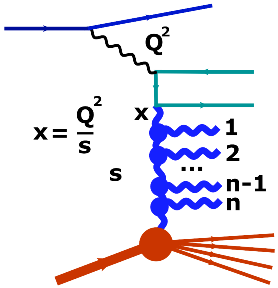
DIS data: $F_2(x, Q^2) \simeq x^{-\lambda(Q^2)}$

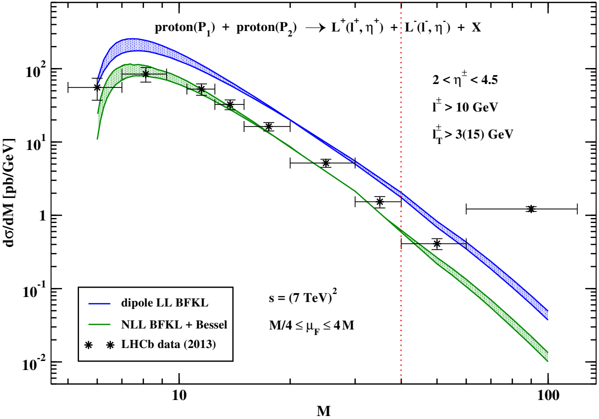
A NLL Multi-Regge approach fits data well (Hentschinski-Salas-SV)^{2012}

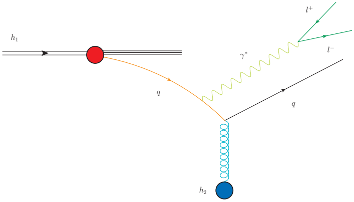




Transition from a perturbative to a non-perturbative Pomeron not well understood. Need more exclusive observables: LHC is the playground now.

Forward Drell-Yan production at LHC (Celiberto-Gordo-SV)^{2018}
The same unintegrated gluon density works well for current data





Previous analysis by (Brzeminski-Motyka-Sadzikowski-Stebel)
We work with BFKL at NLL plus collinear corrections

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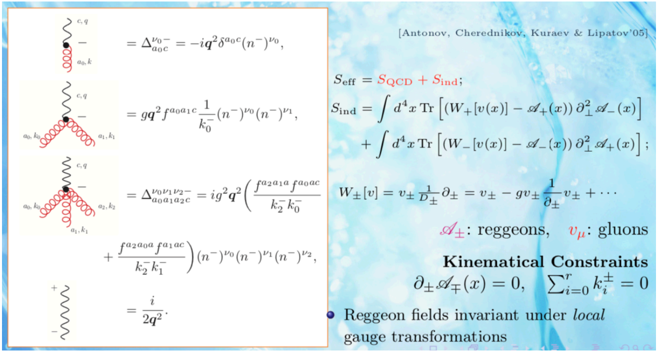
Construction of High Energy Effective action for the electroweak sector of the Standard Model with Martin Hentschinski.



Valid to evaluate NLO and beyond corrections.

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