Small-x Physics at the LHC and in new DIS experiments

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A. Kovner, E. Levin, M. Li and M. Lublinsky,
"The JIMWLK evolution and the s-channel unitarity," arXiv:2006.15126 [hep-ph], to appear in JHEP

"Reggeon Field Theory and Self Duality: Making Ends Meet," arXiv:2007.12132 [hep-ph], submitted to JHEP

The JIMWLK evolution and the s-channel unitarity

We discuss QCD Reggeon Field Theory (RFT) and formulate restrictions imposed on its Hamiltonian by the unitarity of underlying QCD. We identify explicitly the QCD RFT Hilbert space, provide algebra of the basic degrees of freedom (Wilson lines and their duals) and the algorithm for calculating the scattering amplitudes. We formulate conditions imposed on the "Fock states" of RFT by unitary nature of QCD, and explain how these constraints appear as unitarity constraints on possible RFT hamiltonians that generate energy evolution of scattering amplitudes. We study the realization of these constraints in the dense-dilute limit of RFT where the appropriate Hamiltonian is the JIMWLK Hamiltonian H_{JIMWLK}. We find that the action of H_{JIMWLK} on the dilute projectile states is unitary, but acting on dense "target" states it violates unitarity and generates states with negative probabilities through energy evolution.

Unitarity in zero dimensional toy model A. Kovner, E. Levin and M. Lublinsky, JHEP 08, 031 (2016)

1. The commutation relations of P and $\bar{P}$ are based on their perturbative identification:

$$[P, \bar{P}] = -\gamma; \qquad \gamma \sim O(\alpha_s^2)$$

where γ is the zero dimensional proxy for the dipole-dipole scattering amplitude.

2. The S-matrix element for the scattering of $\bar{n}$ dipoles of the projectile on m dipoles of the target

$$\langle m|\bar{n}\rangle = \int d\bar{P}\delta(\bar{P})(1-P)^m(1-\bar{P})^{\bar{n}}$$

This can be conveniently represented in an alternative form

$$\langle m|\bar{n}\rangle = \langle 0|(1-P)^m(1-\bar{P})^{\bar{n}}|0\rangle$$

where the left and right Pomeron "Fock space vacua" are defined by

$$\langle 0|\bar{P} = P|0\rangle = 0; \qquad \langle 0|\bar{d} = \langle 0|; \qquad d|0\rangle = |0\rangle$$

3. The S matrix element is evolved in rapidity according to

$$\langle m|\bar{n}\rangle_Y = \langle 0|(1-P)^m e^{H(P,\bar{P})Y}(1-\bar{P})^{\bar{n}}|0\rangle$$

Fan diagram (BK) Hamiltonian

$$H_{BK} = -\frac{1}{\gamma}\left[P\bar{P} - P\bar{P}^2\right]$$

The unitarity condition

$$\langle m|e^{HY} = \sum_i a_i(Y)\langle i|; \qquad 1 \geq a_i \geq 0; \quad \sum_i a_i = 1$$

$$e^{HY}|\bar{n}\rangle = \sum_i \bar{a}_i(Y)|i\rangle; \qquad 1 \geq \bar{a}_i \geq 0; \quad \sum_i \bar{a}_i = 1$$

Here a_i and $\bar{a}_i$ have the meaning of probabilities.

Action on the dilute projectile (unitarity is preserved)

$$e^{\Delta H}|\bar{n}\rangle \approx (1-\Delta\bar{n})|\bar{n}\rangle + \Delta\bar{n}|\bar{n}+1\rangle$$

Action on the dense target (unitarity is violated)

$$\langle m|e^{\Delta H} = (1+\Delta m)\langle m| - \Delta m[1+\gamma(m-1)]\langle m-1| + \Delta\gamma m(m-1)\langle m-2|$$

Same story for JIMWLK (dense-dilute limit of RFT)

Wilson line operator and its dual

$$\bar{U}(\mathbf{x}) = e^{T^a \frac{\delta}{\delta\rho^a(\mathbf{x})}}\;; \qquad\qquad U(\mathbf{x}) = e^{igT^a \int_y \phi(\mathbf{x}-\mathbf{y})\rho^a(\mathbf{y})}$$

The algebra encodes a gluon-gluon scattering and is very complicated

$$[\bar{U}(\mathbf{x}), U(\mathbf{x})] \;=\; 1 \;+\; O(g)$$

Left and rRight Fock vacuum states

$$\langle L|\bar{U}_{ab} = \delta_{ab}\langle L|; \qquad\quad U_{ab}|R\rangle = \delta_{ab}|R\rangle$$

Initial state

$$|\Psi_i\rangle \;=\; |\mathbf{x}_1, a_1; ...; \mathbf{x}_N, a_N\rangle_T |\mathbf{y}_1, c_1; ...; \mathbf{y}_M, c_M\rangle_P$$

final state

$$|\Psi_f\rangle \;=\; |\mathbf{x}_1, b_1; ...; \mathbf{x}_N, b_N\rangle_T |\mathbf{y}_1, d_1; ...; \mathbf{y}_M, d_M\rangle_P$$

S-matrix

$$S_{if} \;=\; \langle L|U^{a_1 b_1}(\mathbf{x}_1)\ldots U^{a_N b_N}(\mathbf{x}_N)\bar{U}^{c_1 d_1}(\mathbf{y}_1)\ldots \bar{U}^{c_M d_M}(\mathbf{y}_M)|R\rangle$$

Energy evolution with RFT Hamiltonian $H_{RFT}[U,\bar{U}]$.

$$S_{if}(Y) = \langle L|U^{a_1 b_1}(\mathbf{x}_1)\dots U^{a_N b_N}(\mathbf{x}_N) e^{Y H_{RFT}[U,\bar{U}]} \bar{U}^{c_1 d_1}(\mathbf{y}_1)\dots \bar{U}^{c_M d_M}(\mathbf{y}_M)|R\rangle$$

Projectile evolution

$$\bar{U}^{c_1 d_1}(\mathbf{y}_1)\dots \bar{U}^{c_M d_M}(\mathbf{y}_M)|R\rangle \to e^{Y H_{RFT}[U,\bar{U}]} \bar{U}^{c_1 d_1}(\mathbf{y}_1)\dots \bar{U}^{c_M d_M}(\mathbf{y}_M)|R\rangle$$

$$= \sum_{m,\{\bar{c},\bar{d};\bar{\mathbf{y}}\}} G^m_M(Y,\{c,d,\mathbf{y};\ \bar{c},\bar{d},\bar{\mathbf{y}}\}) \prod_{i=1}^{m} [\bar{U}^{\bar{c}_i \bar{d}_i}(\bar{\mathbf{y}}_i)]|R\rangle$$

Projectile unitarity condition:

$$1 > G^m_M(Y,\{c,c,\mathbf{y};\ \bar{c},\bar{c},\bar{\mathbf{y}}\}) > 0; \qquad \sum_m \sum_{\bar{c}} \int \{d\bar{\mathbf{y}}\} G^m_M(Y,\{c,c,\mathbf{y};\ \bar{c},\bar{c},\bar{\mathbf{y}}\}) = 1$$

Projectile unitarity condition is satisfied by H_{JIMWLK}

Target evolution

$$\langle L|U^{a_1b_1}(\mathbf{x}_1)\dots U^{a_Nb_N}(\mathbf{x}_N) \to \langle L|U^{a_1b_1}(\mathbf{x}_1)\dots U^{a_Nb_N}(\mathbf{x}_N)e^{YH_{RFT}}$$
$$= \sum_{n,\{\bar a,\bar b;\bar{\mathbf{x}}\}} F^n_N(Y,\{a,b,\mathbf{x};\ \bar a,\bar b,\bar{\mathbf{x}}\})\langle L|\prod_{i=1}^{n}[U^{\bar a_i\bar b_i}(\bar{\mathbf{x}}_i)]$$

Target unitarity condition:

$$1 > F^n_N(Y,\{a,a,\mathbf{x};\ \bar a,\bar a,\bar{\mathbf{x}}\}) > 0; \qquad \sum_n\sum_{\bar a}\int\{d\bar{\mathbf{x}}\}F^n_N(Y,\{a,a,\mathbf{x};\ \bar a,\bar a,\bar{\mathbf{x}}\}) = 1$$

Target unitarity condition is violated by H_{JIMWLK}!

Reggeon Field Theory and Self Duality: Making Ends Meet

Motivated by the question of unitarity of Reggeon Field Theory, we use the effective field theory philosophy to find possible Reggeon Field Theory Hamiltonians H_{RFT} . We require that H_{RFT} is self dual, reproduce all known limits (dilute-dense and dilute-dilute) and exhibits all the symmetries of the JIMWLK Hamiltonian. We find a family of Hamiltonians which satisfy all the above requirements. One of these is identical in form to the so called "diamond action" . However we show by explicit calculation that the so called "diamond condition" is not satisfied beyond leading perturbative order.

$$H_{JIMWLK} = \frac{\alpha_s}{2\pi^2}\int_{\mathbf{x},\mathbf{y},\mathbf{z}} \frac{(\mathbf{x}-\mathbf{z})\cdot(\mathbf{y}-\mathbf{z})}{(\mathbf{x}-\mathbf{z})^2(\mathbf{y}-\mathbf{z})^2}\left[2\mathcal{J}_L^a(\mathbf{x})\mathcal{J}_R^b(\mathbf{y})\bar{U}^{ab}(\mathbf{z}) - \mathcal{J}_L^a(\mathbf{x})\mathcal{J}_L^a(\mathbf{y}) - \mathcal{J}_R^a(\mathbf{x})\mathcal{J}_R^a(\mathbf{y})\right]$$

$$[\mathcal{J}_L^a(\mathbf{x}), \bar{U}^{mn}(\mathbf{y})] = -(T^a\bar{U}(\mathbf{y}))^{mn}\delta(\mathbf{x}-\mathbf{y}) \, ,$$
$$[\mathcal{J}_R^a(\mathbf{x}), \bar{U}^{mn}(\mathbf{y})] = -(\bar{U}(\mathbf{y})T^a)^{mn}\delta(\mathbf{x}-\mathbf{y}) \, .$$

$SU(2)\times SU(2)$:

$$[\mathcal{J}_L^a(\mathbf{x}), \mathcal{J}_L^b(\mathbf{y})] = if^{abc}\mathcal{J}_L^c(\mathbf{x})\delta(\mathbf{x}-\mathbf{y}) \, ,$$

$$[\mathcal{J}_R^a(\mathbf{x}), \mathcal{J}_R^b(\mathbf{y})] = -if^{abc}\mathcal{J}_R^c(\mathbf{x})\delta(\mathbf{x}-\mathbf{y})$$
$$[\mathcal{J}_L^a(\mathbf{x}), \mathcal{J}_R^a(\mathbf{y})] = 0$$

H_{JIMWLK} **is obviously not symmetric (self-dual) under** $U \leftrightarrow \bar{U}$ **(interchange of projectile and target)**

Self-dual H_{RFT}?

$$[U^{mn}(\mathbf{y}), \mathcal{I}_L^a(\mathbf{x})] = -(T^a U(\mathbf{y}))^{mn} \delta(\mathbf{x}-\mathbf{y}) \,,$$
$$[U^{mn}(\mathbf{y}), \mathcal{I}_R^a(\mathbf{x})] = -(U(\mathbf{y}) T^a)^{mn} \delta(\mathbf{x}-\mathbf{y}) \,.$$

Another $SU(2) \times SU(2)$:

$$[\mathcal{I}_L^a(\mathbf{x}), \mathcal{I}_L^b(\mathbf{y})] = -if^{abc}\mathcal{I}_L^c(\mathbf{x})\delta(\mathbf{x}-\mathbf{y}) \,,$$

$$[\mathcal{I}_R^a(\mathbf{x}), \mathcal{I}_R^b(\mathbf{y})] = if^{abc}\mathcal{I}_R^c(\mathbf{x})\delta(\mathbf{x}-\mathbf{y})$$
$$[\mathcal{I}_L^a(\mathbf{x}), \mathcal{I}_R^a(\mathbf{y})] = 0$$

Wilson line like operators in the *fundamental representation*

$$V_L(\mathbf{x}) = \mathrm{Exp}\left\{ i\int_{\mathbf{y}} g\phi(\mathbf{x}-\mathbf{y}) t^e \mathcal{J}_L^e(\mathbf{y}) \right\}$$

$$V_R(\mathbf{x}) = \mathrm{Exp}\left\{ -i\int_{\mathbf{y}} g\phi(\mathbf{x}-\mathbf{y}) t^e \mathcal{J}_R^e(\mathbf{y}) \right\}$$

$$\bar{V}_L(\mathbf{x}) = \mathrm{Exp}\left\{ i\int_{\mathbf{y}} g\phi(\mathbf{x}-\mathbf{y}) t^e \mathcal{I}_L^e(\mathbf{y}) \right\}$$

$$\bar{V}_R(\mathbf{x}) = \mathrm{Exp}\left\{ -i\int_{\mathbf{y}} g\phi(\mathbf{x}-\mathbf{y}) t^e \mathcal{I}_R^e(\mathbf{y}) \right\}.$$

Self-dual Hamiltonian:

$$H_{RFT} = \frac{1}{\pi g^2}\int d^2\mathbf{x}\, \partial^2[\bar{V}_L^{\beta\gamma}(\mathbf{x})\bar{V}_R^{\delta\alpha}(\mathbf{x})] V_L^{\alpha\beta}(\mathbf{x}) V_R^{\gamma\delta}$$

Reduces to JIMWLK in the dense-dilute limit