

QED Factorization in Non-Leptonic Charmless *B* Meson Decays

Philipp Böer & Keri Vos

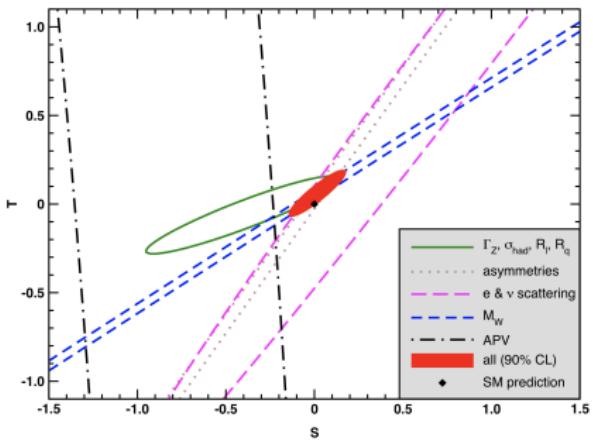
in collaboration with M. Beneke and J-N. Toelstede

arXiv:2008.10615

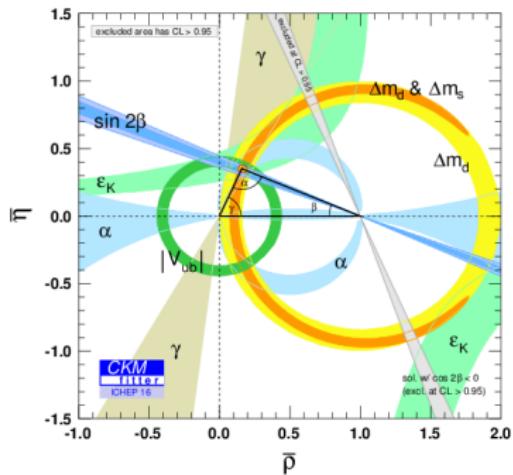


Technische Universität München

Testing the Standard Model



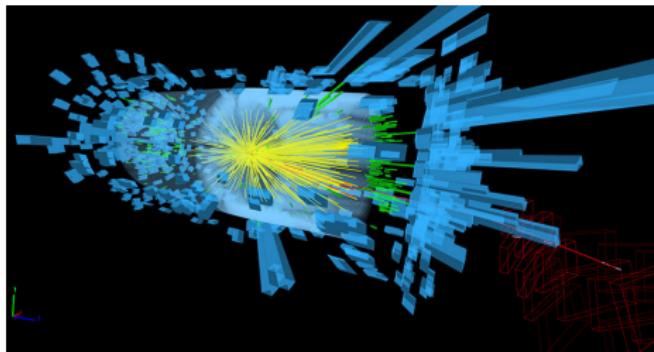
Gauge Structure



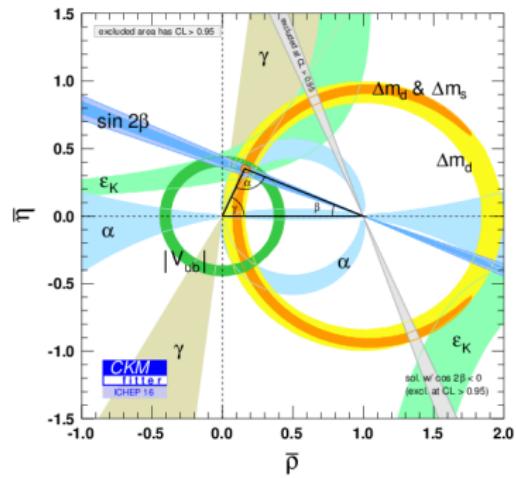
Flavour Structure

No significant deviation found (yet)!

Testing the Standard Model



Energy/Direct



Precision/Indirect

Precision frontier

Tiny deviations from SM predictions constrain effects of New Physics

Precision Frontier in B Decays

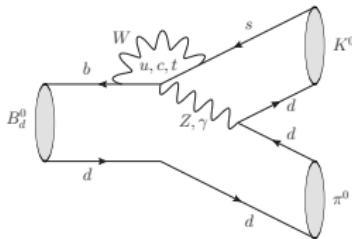
- Next step: systematically include electromagnetic effects $\sim \mathcal{O}(\alpha_{\text{em}})$
- QED corrections in $B_s \rightarrow \mu^+ \mu^-$ already studied
→ power-enhanced QED effects [Beneke, Bobeth, Szafron]
- QED in non-leptonic decays
 - can mimic isospin-breaking electroweak penguin effects
 - perturbative and non-perturbative dynamics disentangled using Heavy-Quark Expansion
- QED more complicated than QCD due to charged external states

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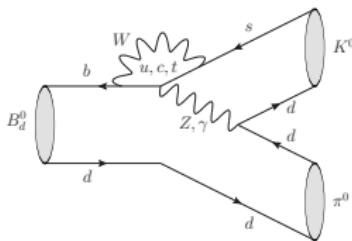
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Non-leptonic B decays

see Buras, Buchalla, Lautenbacher [1995]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{j=u,c} V_{jq}^* V_{jb} \left(\sum_{i=1,2} C_i(\mu) Q_i^{jq}(\mu) + \sum_{i=3}^{10} C_i(\mu) Q_i^q \right)$$

$C_i(\mu)$ real short-distance coefficient

$\langle Q_i \rangle$ long-distance physics

$$Q_1 = [\bar{u} \gamma^\mu T^a (1 - \gamma_5) b] [\bar{D} \gamma_\mu T^a (1 - \gamma_5) u] \quad Q_2 = [\bar{u} \gamma^\mu (1 - \gamma_5) b] [\bar{D} \gamma_\mu (1 - \gamma_5) u]$$

Current-current operators

- Decoupling of W,Z bosons at $\mu_W \sim 80$ GeV
- RG evolution from $\mu_W \rightarrow \mu_b$

→ $\langle f | Q_i(\mu) | B \rangle$ at leading power in $1/m_b$ expansion: QCD Factorization

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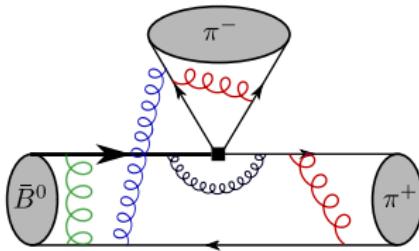
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QCD Factorization



QCD Factorization Formula

[Beneke, Buchalla, Neubert, Sachrajda '99]

$$\langle M_1 M_2 | Q_i | B \rangle = F^{B \rightarrow M_1}(q^2 = 0) \int_0^1 du T_i^I(u) f_{M_2} \phi_{M_2}(u) \\ + \int_0^\infty d\omega \int_0^1 du dv T_i^{II}(u, v, \omega) f_{M_1} \phi_{M_1}(v) f_{M_2} \phi_{M_2}(u) f_B \phi_B(\omega)$$

- **hard & hard-collinear** scattering kernels $T_i^{I, II}$ (perturbative, process dependent)
- light-meson LCDAs ϕ_{M_i} contain (anti-)collinear dynamics (non-perturbative, universal)
- B -meson LCDA ϕ_B contains soft dynamics (non-perturbative, universal)
- $B \rightarrow M_1$ form factor $F^{B \rightarrow M_1}$ absorbs endpoint divergent convolutions

Non-leptonic B decays

$$A_{M_1 M_2} \equiv i \frac{G_F}{\sqrt{2}} m_B^2 F_0^{BM_1} f_{M_2}$$

Amplitude parametrization

[Beneke, Neubert [2003]]

$$\begin{aligned} A_{B^- \rightarrow \pi^- \bar{K}^0} &= A_{\pi K} \hat{\alpha}_4^p, \\ \sqrt{2} A_{B^- \rightarrow \pi^0 K^-} &= A_{\pi K} [\delta_{pu} \color{blue}{\alpha_1} + \hat{\alpha}_4^p] + A_{K\pi} \left[\delta_{pu} \color{blue}{\alpha_2} + \delta_{pc} \frac{3}{2} \alpha_{3,\text{EW}}^c \right], \\ A_{\bar{B}^0 \rightarrow \pi^+ K^-} &= A_{\pi K} [\delta_{pu} \color{blue}{\alpha_1} + \hat{\alpha}_4^p], \\ \sqrt{2} A_{\bar{B}^0 \rightarrow \pi^0 \bar{K}^0} &= A_{\pi K} [-\hat{\alpha}_4^p] + A_{K\pi} \left[\delta_{pu} \color{blue}{\alpha_2} + \delta_{pc} \frac{3}{2} \alpha_{3,\text{EW}}^c \right], \end{aligned}$$

- $\color{blue}{\alpha_1}$ and $\color{blue}{\alpha_2}$ color-allowed and color-suppressed tree coefficients
- α_4 and $\alpha_{3,\text{EW}}$ penguin and electromagnetic penguin coefficients
- contain all perturbative effects up to NNLO (α_s^2)
e.g. [Bell, Beneke, Huber, Li]

QED effects in B decays

QED effects in \bar{B} decays

$$\Gamma[\bar{B} \rightarrow M_1 M_2](\Delta E) \equiv \Gamma[\bar{B} \rightarrow M_1 M_2 + X_s] \Big|_{E_{X_s} \leq \Delta E},$$

- IR finite observable (width) must include **ultra-soft photon** radiation
- X_s are soft photons with total energy less than **ultrasoft scale** ΔE
- Factorizes in **non-radiative** amplitude and **ultrasoft** function

$$\Gamma[\bar{B} \rightarrow M_1 M_2](\Delta E) = |\mathcal{A}(\bar{B} \rightarrow M_1 M_2)|^2 \sum_{X_s} |\langle X_s | (\bar{S}_v^{(Q_B)} S_{\nu_1}^{\dagger(Q_{M_1})} S_{\nu_2}^{\dagger(Q_{M_2})}) | 0 \rangle|^2 \theta(\Delta E - E_{X_s})$$

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Simple classification:

- Ultra-soft photons: eikonal approximation, well understood

$$\Delta E \ll \Lambda_{\text{QCD}}$$

- Non-universal, structure dependent corrections

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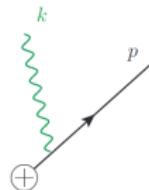
Both effects important: virtual photons can resolve the structure of the meson!

QED effects in B decays

Eikonal approximation:

- Only works for point-like coupling

$$\epsilon_\mu(k) \bar{u}(p) \gamma^\mu \frac{\not{p} + \not{k} + m}{(k+p)^2 - m^2} \rightarrow \epsilon_\mu(k) \frac{p^\mu}{p \cdot k} \bar{u}(p)$$



- Ultra-soft corrections exponentiate and dress the pure-QCD amplitude

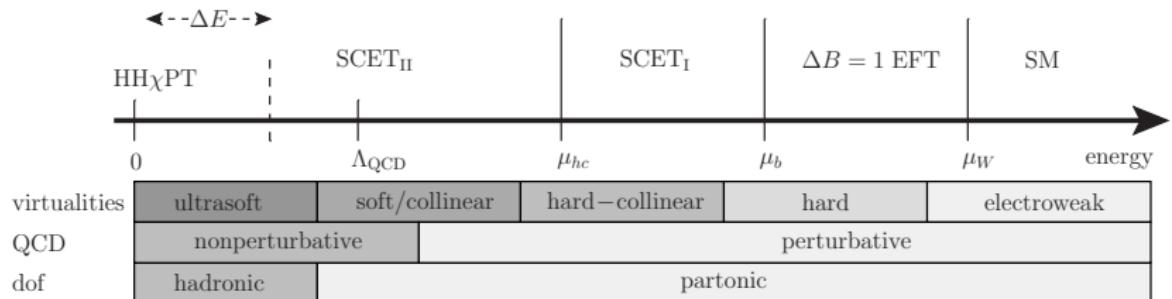
$$\sum_{X_s} |\langle X_s | (\bar{S}_\nu^{(Q_B)} S_{\nu_1}^{\dagger(Q_{M_1})} S_{\nu_2}^{\dagger(Q_{M_2})}) | 0 \rangle|^2 \sim \left(\frac{\Delta E}{\Lambda} \right)^{A(\alpha \rightarrow \beta)}$$

- $\Lambda \ll \Lambda_{\text{QCD}}$ cut-off!
- Often done: Assume pointlike approximation up to the scale m_B [Baracchini, Isidori]
 - fails to account for all large logarithms (and scales)!
 - photons with energy $\gtrsim \Lambda_{\text{QCD}}$ probe the partonic structure of the mesons

Multi-Scale Problem: Tower of EFTs

Hierarchy of energy scales:

$$M_W \sim 80 \text{ GeV} \gg m_b \sim 4.2 \text{ GeV} \gg \text{few times } \Lambda_{\text{QCD}} \gg \Delta E \sim 60 \text{ MeV}$$



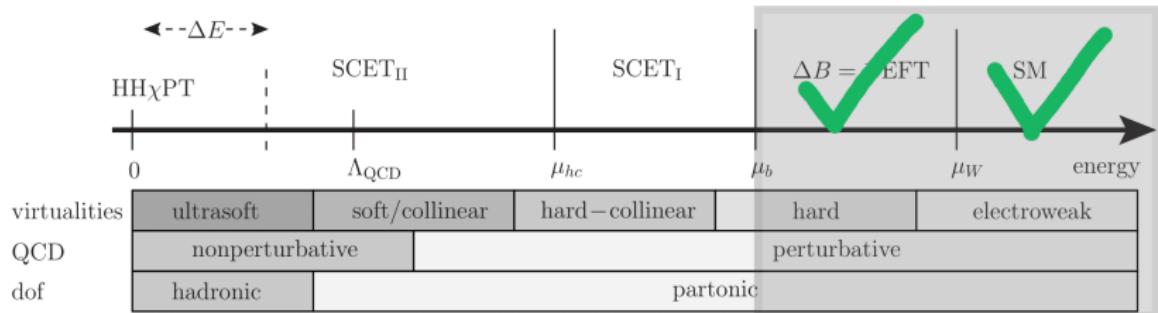
(figure from [Beneke, Bobeth, Szafron '19])

- Decoupling of W, Z bosons at $\mu_W \sim 80 \text{ GeV}$: RG evolution from $\mu_W \rightarrow \mu_b$
- Soft-collinear effective field theory (SCET): $\mu_b \sim m_b \sim 4.2 \text{ GeV} \gg \text{few times } \Lambda_{\text{QCD}}$
 - NEW: include **structure dependent**
- Ultrasoft region $\mu_{\text{us}} \ll \Lambda_{\text{QCD}}$
 - Point-like meson approximation well understood

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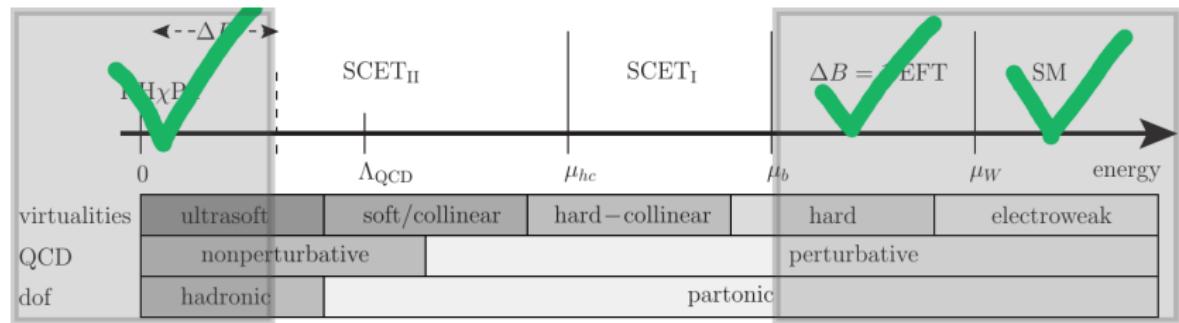
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Our work:

Derive all-order factorization formula in QCD×QED for *non-radiative amplitude*

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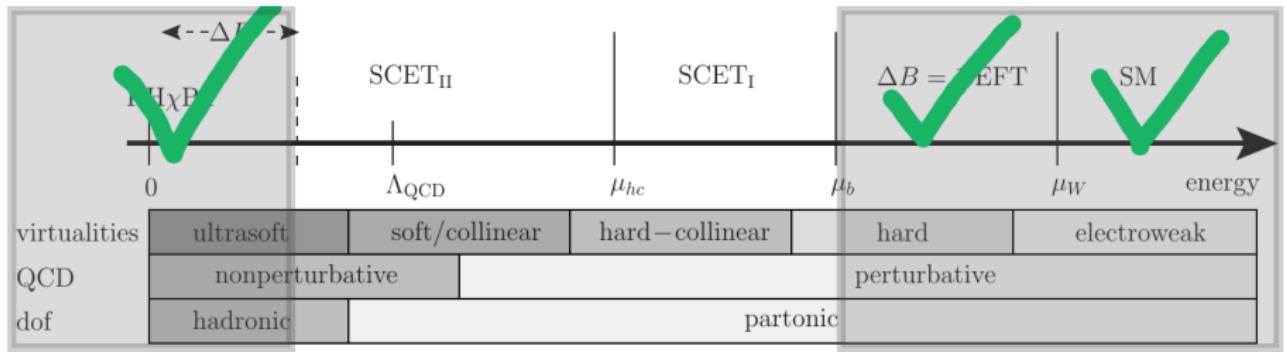
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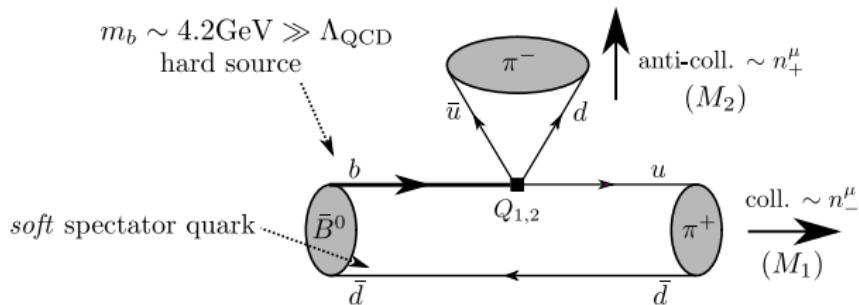
Phenomenological implications on $B \rightarrow \pi K$ decays

- include effects above m_b
- structure dependent contributions between m_b and a few times Λ_{QCD}
- include ultrasoft effects below Λ_{QCD}

Theoretical Framework

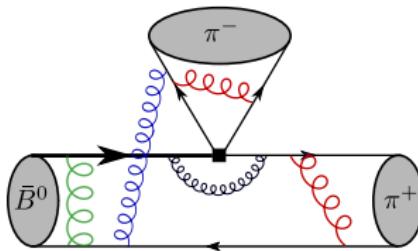


Kinematics



- in the B rest frame the final-state pions share large energy: $E_\pi = m_B/2 \gg \Lambda_{\text{QCD}} \sim m_\pi$
- we are interested in the amplitude in the **heavy-quark limit** $m_b \sim E_\pi \rightarrow \infty$
 - corrections are of order $\Lambda_{\text{QCD}}/m_b \sim 1/10$
- pion momenta define light-cone vectors:
 - **collinear direction:** $n_-^\mu = (1, 0, 0, -1)$
 - **anti-collinear direction:** $n_+^\mu = (1, 0, 0, +1)$
 - four-velocity of the B meson: $v^\mu = (1, 0, 0, 0) = \frac{1}{2}(n_+^\mu + n_-^\mu)$(up to corrections $\sim m_\pi^2/m_B^2$)

Energy Scales

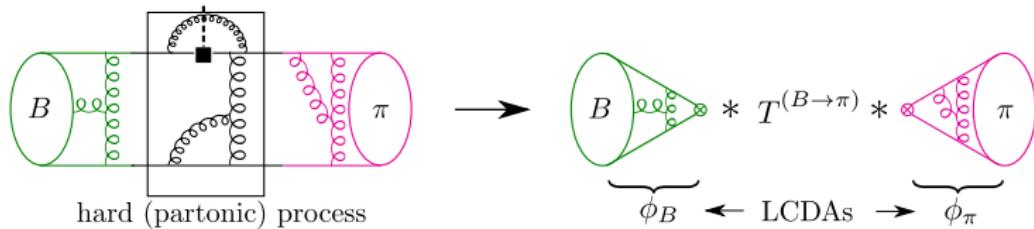


- QCD at different energy scales
 - **hard** gluons with virtualities $\sim m_b^2$ ⇒ perturbative ✓
 - **soft** gluons with virtualities $\sim \Lambda_{\text{QCD}}^2$ ⇒ non-perturbative ↴
 - intermediate scales ?
- Effective field theory approach
 - disentangle energy scales in an **expansion in** $\lambda \equiv \Lambda_{\text{QCD}}/m_b \sim \Lambda_{\text{QCD}}/E_\pi$
 - light/energetic final states: the formalism of **Soft-Collinear Effective Theory** (SCET) allows us to derive **factorization formulas** in the heavy-quark limit
 - resummation of large logarithms $\sim \log \Lambda_{\text{QCD}}/m_b$ through RG evolution

Heuristic Discussion of QCDF

Factorization = separation of energy/length/time scales

- we are interested in a B decay into energetic pions: $E_\pi \sim m_b \gg \Lambda_{\text{QCD}}$



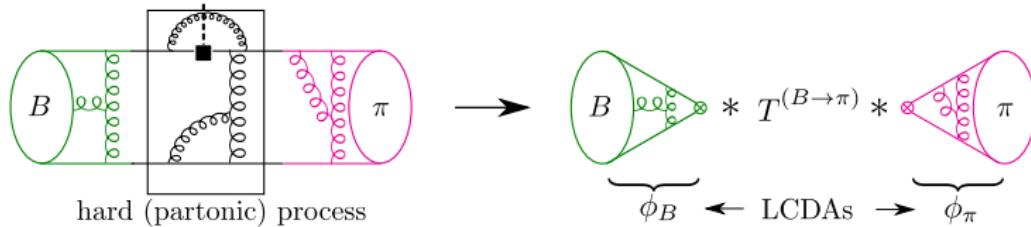
$$A \simeq \int_0^\infty d\omega \int_0^1 du \phi_B^+(\omega) T^{(B \rightarrow \pi)}(u, \omega; E_\pi) \phi_\pi(u) + \mathcal{O}(\Lambda_{\text{QCD}}/m_b)$$

- color-neutral mesons \Rightarrow decoupling of soft gluons
- low-energy dynamics contained in **Light-Cone Distribution Amplitudes** (LCDAs)
 - $\phi_B^+(\omega)$: describes **soft** dynamics in the B meson
 - $\phi_\pi(u)$: describes **collinear** dynamics in the pion
- process-specific but perturbative hard scattering kernel T

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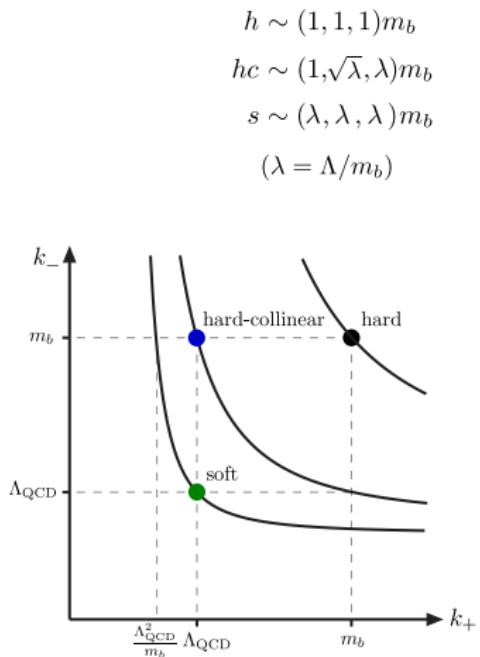
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- factorization formula holds to all orders in pert. theory in the heavy-quark limit $m_b \rightarrow \infty$
- can be formally derived in SCET
- Convolution can be divergent !
→ endpoint-singularities still an unsolved problem (→ see talk by M. Neubert)

Soft-Collinear Effective Theory

- Soft-Collinear Effective Theory (SCET) is designed to describe the long-distance physics in processes with energetic particles (jets)
- What are the relevant degrees of freedom (momentum regions)?
 - “hard” scale: $m_b = 4.2\text{GeV}$
 - “soft”/“collinear” scale: $\Lambda \sim 0.5\text{GeV}$
 - “hard-collinear” scale: $\sqrt{m_b \Lambda} \sim 1.5\text{GeV}$ (soft-collinear momentum transfer)
- light-cone vectors: $n_{\pm}^{\mu} = (1, 0, 0, \pm 1)$
- two-step matching

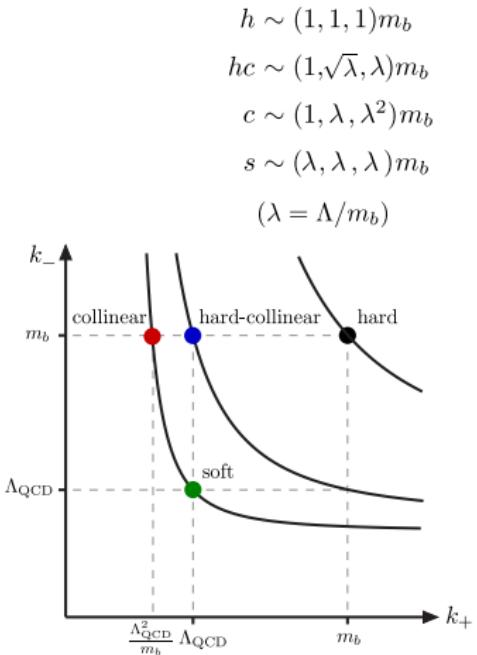
$$\text{QCD} \xrightarrow{m_b \rightarrow \infty} \text{SCET}_I$$



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$$\text{QCD} \xrightarrow{m_b \rightarrow \infty} \text{SCET}_I \xrightarrow{\sqrt{m_b \Lambda} \rightarrow \infty} \text{SCET}_{II}$$



From the EFT to Factorization Theorems

Matching of weak current onto SCET

$$\bar{u}\Gamma b \rightarrow \int ds dt C(s,t) \mathcal{O}_C(s) \mathcal{O}_S(t)$$

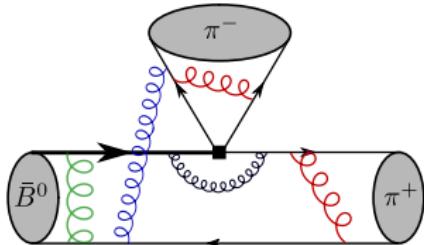
- C : matching coefficient (contains short-distance fluctuations, perturbative)
- \mathcal{O} : SCET operators build from soft or collinear fields

- field operators have manifest power-counting in λ
 - systematic expansion of amplitudes around high-energy limit
- operators in SCET are non-local along the light-cone
 - convolution with matching coefficients
 - dress fields with Wilson-lines to construct gauge-invariant building blocks

$$\xi_C = \frac{\not{p}_- \not{p}_+}{4} \psi_C \rightarrow \chi_C \equiv W_C^\dagger \xi_C, \quad \text{with} \quad W_C(x) = \exp \left\{ ig_s \int_{-\infty}^0 ds n_+ G_C(x + sn_+) \right\}$$

- Interaction terms between soft and collinear sectors can be removed from the SCET Lagrangian by a field redefinition (“decoupling transformation”).
 - Factorization into universal (process-independent) low-energy matrix elements

QCD Factorization Formula



SCET_I operators:

$$\mathcal{O}^I \sim [\bar{x}_C h_\nu] \quad [\bar{x}_{\bar{C}}(tn_-) \not{p}_- \gamma_5 \chi_{\bar{C}}]$$

$$\mathcal{O}^{II} \sim [\bar{x}_C \mathcal{A}_{C,\perp}(sn_+) h_\nu] [\bar{x}_{\bar{C}}(tn_-) \not{p}_- \gamma_5 \chi_{\bar{C}}]$$

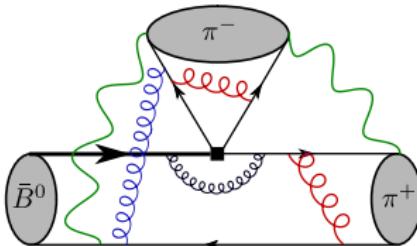
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[Beneke, Buchalla, Neubert, Sachrajda '99]

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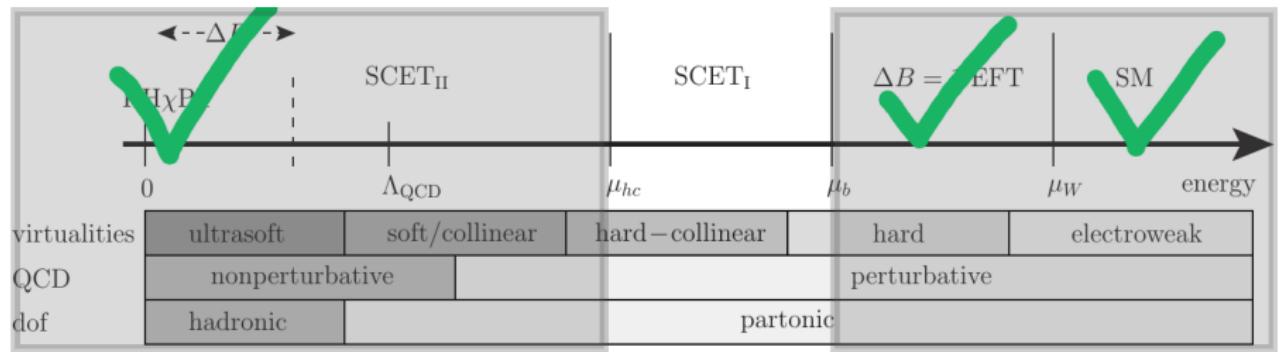
$\text{QCD} \times \text{QED}$ Factorization Formula



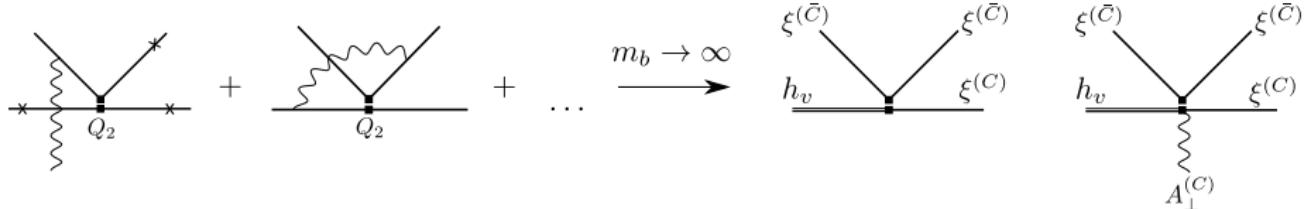
$\text{QCD} \times \text{QED}$ Factorization Formula

$$\begin{aligned} \langle M_1 M_2 | Q_i | \bar{B} \rangle \Big|_{\text{non-rad.}} &= \mathcal{F}_{Q_2}^{B \rightarrow M_1}(q^2 = 0) \int_0^1 du \mathbf{T}_{i, Q_2}^I(u) \mathcal{F}_{M_2} \Phi_{M_2}(u) \\ &+ \int d\omega \int_0^1 du dv \mathbf{T}_{i, \otimes}^{II}(u, v, \omega) \mathcal{F}_{M_1} \Phi_{M_1}(v) \mathcal{F}_{M_2} \Phi_{M_2}(u) \mathcal{F}_{B, \otimes} \Phi_{B, \otimes}(\omega) \end{aligned}$$

- retains its form but non-perturbative objects need to be generalized $\otimes = (Q_1, Q_2)$
- endpoint divergences are still absorbed in $\mathcal{F}_{Q_2}^{B \rightarrow M_1}$
 - $\mathcal{F}_0^{B \rightarrow M_1} = F^{B \rightarrow M_1}$ for neutral M_2
 - $\mathcal{F}_{-}^{B \rightarrow M_1} \sim A^{B \rightarrow M_1 \ell^- \bar{\nu}_\ell} \Big|_{E_\ell = m_B/2}$ for charged M_2
- soft photons sensitive to charge and direction of final-state mesons
 - soft function becomes process dependent!



$\mathcal{O}(\alpha_{\text{em}})$ Matching onto SCET_I



SCET_I operators

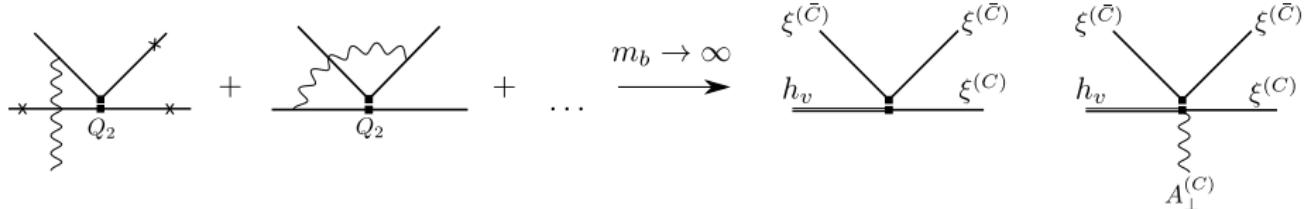
$$\begin{aligned}\mathcal{O}_-^I(t) &= [\bar{\chi}_C^{(u)} \not{p}_+ h_\nu S_{n_+}^{\dagger(Q_2)}] & [\bar{\chi}_{\bar{C}}^{(D)}(tn_-) \frac{\not{p}_-}{2} \gamma_5 \chi_{\bar{C}}^{(u)}] \\ \mathcal{O}_-^{II\gamma}(t, s) &= \frac{1}{m_b} [\bar{\chi}_C^{(u)} \frac{\not{p}_+}{2} \mathcal{A}_{C,\perp}(sn_+) h_\nu S_{n_+}^{\dagger(Q_2)}] & [\bar{\chi}_{\bar{C}}^{(D)}(tn_-) \frac{\not{p}_-}{2} \gamma_5 \chi_{\bar{C}}^{(u)}]\end{aligned}$$

(and \mathcal{O}_-^{IIf} by replacing $\mathcal{A} \rightarrow \mathcal{G}$)

QED-QCD Wilson lines

$$\begin{aligned}W_C^{(q)}(x) &= \exp \left\{ iQ_q e \int_{-\infty}^0 ds n_+ A_C(x + sn_+) \right\} \mathbf{P} \exp \left\{ ig_s \int_{-\infty}^0 ds' n_+ G_C(x + s'n_+) \right\} \\ S_{n_+}^{(q)}(x) &= \exp \left\{ -iQ_q e \int_0^\infty ds n_+ \cdot A_s(x + sn_+) \right\}\end{aligned}$$

$\mathcal{O}(\alpha_{\text{em}})$ Matching onto SCET_I



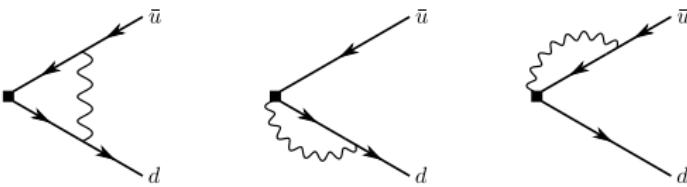
SCET_I Operators

$$\begin{aligned}\mathcal{O}_-^I(t) &= [\bar{\chi}_C^{(u)} \not{p}_+ h_v S_{n_+}^{\dagger(Q_2)}] & [\bar{\chi}_{\bar{C}}^{(D)}(tn_-) \frac{\not{p}_-}{2} \gamma_5 \chi_{\bar{C}}^{(u)}] \\ \mathcal{O}_-^{II\gamma}(t,s) &= \frac{1}{m_b} [\bar{\chi}_C^{(u)} \frac{\not{p}_+}{2} \mathcal{A}_{C,\perp}(sn_+) h_v S_{n_+}^{\dagger(Q_2)}] & [\bar{\chi}_{\bar{C}}^{(D)}(tn_-) \frac{\not{p}_-}{2} \gamma_5 \chi_{\bar{C}}^{(u)}]\end{aligned}$$

(and \mathcal{O}_-^{IIG} by replacing $\mathcal{A} \rightarrow \mathcal{G}$)

- want to **factorize** anti-collinear sector
 - light-cone distribution amplitude for charged π^-
 - $\bar{B}^0 \rightarrow \pi^+$ form factors in SCET_I inherit soft QED Wilson lines

Charged-Pion LCDA in QED



- AD of anti-collinear operator $\bar{\chi}_{\bar{c}}^{(d)}(tn_-) \frac{\not{n}_-}{2} \gamma_5 \chi_{\bar{c}}^{(u)}(0)$ using off-shell regularization

$$\begin{aligned}\Gamma(u, v) = & -\frac{\alpha_{\text{em}} Q_2}{\pi} \delta(u - v) \left(\frac{3Q_2}{4} + Q_d \log \frac{\mu^2}{-k_d^2} - Q_u \log \frac{\mu^2}{-k_u^2} \right) \\ & - \left(\frac{\alpha_s C_F}{\pi} + \frac{\alpha_{\text{em}}}{\pi} Q_u Q_d \right) \left[\left(1 + \frac{1}{v-u} \right) \frac{u}{v} \theta(v-u) + \left(1 + \frac{1}{u-v} \right) \frac{1-u}{1-v} \theta(u-v) \right]_+\end{aligned}$$

- AD depends on IR regulator ↴

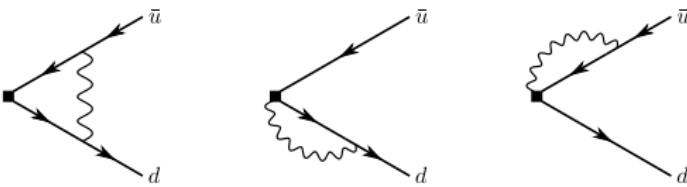
→ remove soft overlap by a “soft re-arrangement”

cf. [Beneke, Bobeth, Szafron]

→ leave all re-scattering phases in soft function

$$\mathcal{O} = \mathcal{O}_{\bar{c}} \times \mathcal{O}_{s,C} \rightarrow (\mathcal{O}_{\bar{c}} R_{\bar{c}}) \times \left(\frac{\mathcal{O}_{s,C}}{R_{\bar{c}}} \right) \quad \text{with} \quad \left| \langle 0 | S_{n+}^{\dagger(Q_2)} S_{n-}^{(Q_2)} | 0 \rangle \right| \equiv R_{\bar{c}} R_c$$

Charged-Pion LCDA in QED

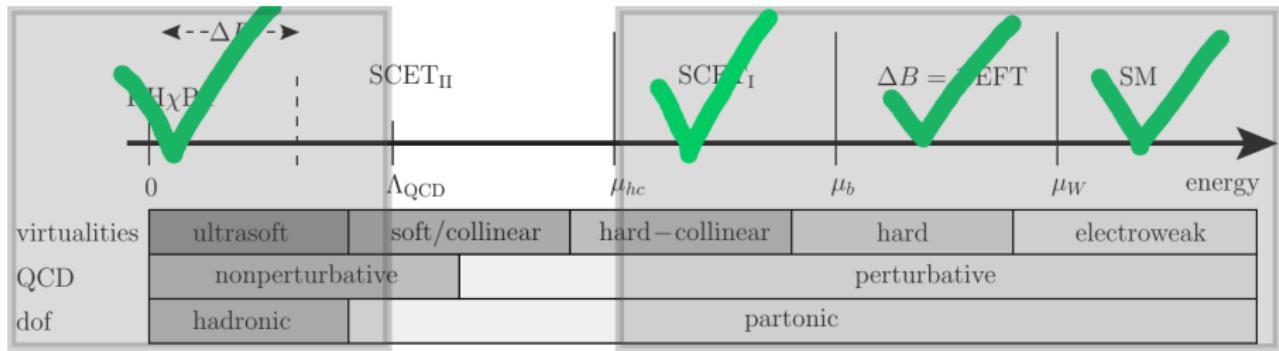


- AD of anti-collinear operator $\bar{\chi}_{\bar{c}}^{(q)}(tn_-) \frac{\not{n}_-}{2} \gamma_5 \chi_{\bar{c}}^{(u)}(0)$ after soft re-arrangement

$$\begin{aligned}\Gamma(u, v) = & -\frac{\alpha_{\text{em}} Q_2}{\pi} \delta(u - v) \left(Q_d \ln \frac{\mu}{2E u} - Q_u \ln \frac{\mu}{2E(1-u)} + \frac{3Q_2}{4} \right) \\ & - \left(\frac{\alpha_s C_F}{\pi} + \frac{\alpha_{\text{em}}}{\pi} Q_u Q_d \right) \left[\left(1 + \frac{1}{v-u} \right) \frac{u}{v} \theta(v-u) + \left(1 + \frac{1}{u-v} \right) \frac{1-u}{1-v} \theta(u-v) \right]_+\end{aligned}$$

- depends on the large energy $2E = m_B$ and on $\log(u)$ and $\log(1-u)$ terms
 - Gegenbauer expansion does no longer diagonalize evolution kernel
 - LCDA becomes **asymmetric** due to different quark charges

$$R_{\bar{c}} \langle \pi^- | \bar{\chi}_{\bar{c}}^{(d)}(tn_-) \frac{\not{n}_-}{2} \gamma_5 \chi_{\bar{c}}^{(u)}(0) | 0 \rangle = -iE \int_0^1 du e^{iut} \mathcal{F}_{\pi^-} \Phi_{\pi^-}(u)$$



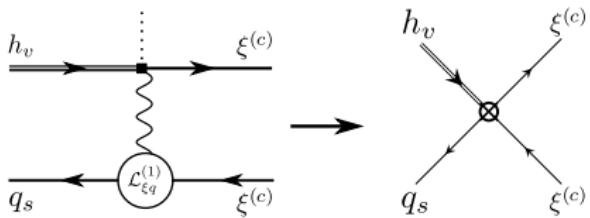
Soft Form Factor and Matching onto SCET_{II}

generalized SCET_I form factors

$$\frac{1}{R_{\bar{c}}} \langle \pi^+ | (\bar{\chi}_C^{(u)} \not{p}_+ h_v S_{n_+}^{\dagger(Q_2)}) | \bar{B}^0 \rangle = 4 E_{M_1} \zeta^{\bar{B}^0 \pi^+}(q^2)$$

$$\frac{1}{R_{\bar{c}}} \langle \pi^+ | \frac{1}{m_b} \bar{\chi}_C^{(u)} \frac{\not{p}_+}{2} \mathcal{A}_{c,\perp}(sn_+) h_v S_{n_+}^{\dagger(Q_2)} | \bar{B}^0 \rangle = -2 E_{M_1} \int_0^1 d\tau e^{i\tau \hat{s}} \gamma^{\bar{B}^0 \pi^+}(\tau; q^2)$$

- $\zeta^{\bar{B}^0 \pi^+}$ contains endpoint divergences after SCET_{II} matching (and $\Sigma \leftrightarrow \mathcal{O}_-^{\text{IIG}}$)
 - only information from the charged M_2 is the charge and direction
 - replace $\zeta^{\bar{B}^0 \pi^+}$ with semi-leptonic amplitude $\bar{B}^0 \rightarrow \pi^+ \ell^- \bar{\nu}_\ell$ amplitude
- $\gamma^{\bar{B}^0 \pi^+}$ can be matched onto the final theory SCET_{II}



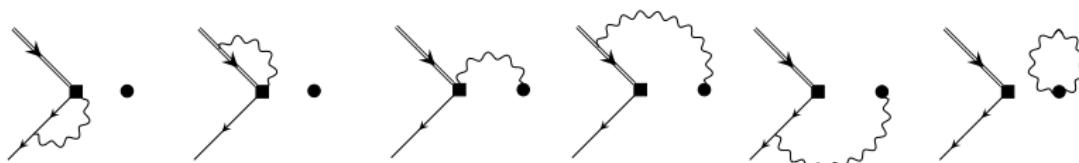
Soft Function

Soft Function for $\bar{B}^0 \rightarrow M_1^+ M_2^-$

cf. [Beneke, Bobeth, Szafron]

$$im_B \int d\omega e^{-i\omega t} \mathcal{F}_{B,+-} \Phi_{B,+-}(\omega) = \frac{1}{R_c R_{\bar{c}}} \langle 0 | \bar{q}_s^{(d)}(tn_-) [tn_-, 0]_{n_-}^{(d)} \frac{\not{n}_-}{2} h_\nu (S_{n_+}^{\dagger, Q_2} S_{n_-}^{Q_2}) | \bar{B}^0 \rangle$$

- $B \rightarrow M_1 M_2$ decays: four different soft functions for various charge assignments
- different objects compared to standard B meson LCDA in QCD
 - final-state rescattering, different support properties, ...



- coupling of soft photon/gluon to incoming b quark with $n_- p_b = m_b \rightarrow \infty$
 - $\omega \in [0, \infty)$
- coupling of soft photon to outgoing anti-coll. π^- with $n_- q = m_b \rightarrow \infty$
 - QED B LCDA has support $\omega \in (-\infty, \infty)$ if anti-coll. meson is charged

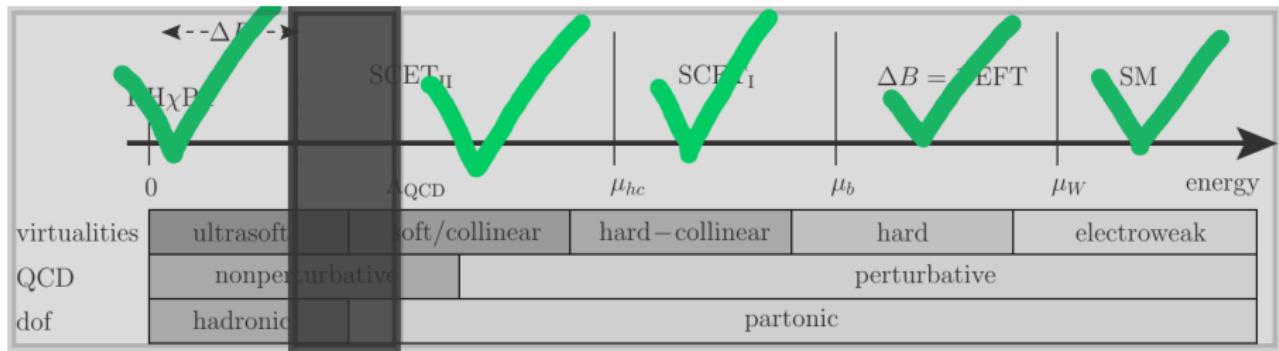
Soft Function: Anomalous Dimension

Anomalous Dimension for Φ_{\pm}

$$\begin{aligned}\Gamma_>(\omega, \omega'; \mu) &= \left(\frac{\alpha_{\text{em}}}{4\pi} Q_d^2 + \frac{\alpha_s C_F}{4\pi} \right) \left\{ \delta(\omega - \omega') \left(2 \log \frac{\mu^2}{\omega^2} - 5 \right) - 4 F_>(\omega, \omega') \right\} \\ &\quad - \frac{\alpha_{\text{em}}}{4\pi} 2 Q_d Q_2 \left\{ \delta(\omega - \omega') 2 \log \frac{\mu^2}{\omega^2} - 2 G_>(\omega, \omega') \right\} - \frac{\alpha_{\text{em}}}{\pi} Q_2^2 \delta(\omega - \omega') i\pi \\ \Gamma_<(\omega, \omega'; \mu) &= \left(\frac{\alpha_{\text{em}}}{4\pi} Q_d^2 + \frac{\alpha_s C_F}{4\pi} \right) \left\{ \delta(\omega - \omega') \left(2 \log \frac{\mu^2}{\omega^2} - 5 \right) - 4 F_<(\omega, \omega') \right\} \\ &\quad - \frac{\alpha_{\text{em}}}{4\pi} 2 Q_d Q_2 \left\{ \delta(\omega - \omega') 2 \log \frac{\mu^2}{-\omega^2} - 2 G_<(\omega, \omega') \right\} - \frac{\alpha_{\text{em}}}{\pi} Q_2^2 \delta(\omega - \omega') i\pi\end{aligned}$$

- contains plus-distributions and generalized plus-distributions, e.g.

$$G_> = \omega \left[\frac{\theta(\omega' - \omega) \theta(\omega)}{\omega'(\omega' - \omega)} \right]_+ + \left[\frac{\theta(\omega' - \omega)}{\omega' - \omega} \right]_{\otimes} \quad \text{with} \quad [\dots]_{\otimes} f(\omega) \rightarrow [\dots] (f(\omega) - \theta(\omega) f(\omega'))$$



Phenomenological Implications

Different QED effects

$$\mathcal{A}(M_1 M_2) \equiv i \frac{G_F}{\sqrt{2}} m_B^2 \mathcal{F}_{Q_2}^{BM_1}(0) \mathcal{F}_{M_2}$$

$$\langle M_1 M_2 | Q_i | B \rangle = \mathcal{A}(M_1 M_2) \alpha_i(M_1 M_2) = \mathcal{A}_{M_1 M_2} \left(\alpha_i^{\text{QCD}}(M_1 M_2) + \delta \alpha_i(M_1 M_2) \right)$$

- Electroweak scale to m_B : QED corrections to the Wilson coefficients
- m_B to μ_c : QED corrections to the hard-scattering kernels, form factors and decay constants
- below Λ_{QCD} : Ultrasoft QED effects (for the rate!)

$$\delta \alpha_i(M_1 M_2) \equiv \delta \alpha_i^{\text{WC}}(M_1 M_2) + \delta \alpha_i^K(M_1 M_2) + \delta \alpha_i^{\text{F,V}}(M_1 M_2) + \delta \alpha_i^{\text{F,sp}}(M_1 M_2).$$

$$\rightarrow \delta \alpha_i^{\text{WC}} = \mathcal{O}(10^{-3})$$

[Huber, Lunghi, Misiak, Wyler [2006]]

$$\rightarrow \delta \alpha_i^K = \mathcal{O}(10^{-3})$$

$$\rightarrow \delta \alpha_i^{\text{F,V}} = ??$$

$$\rightarrow \delta \alpha_i^{\text{F,sp}} = ?? \text{ but } \mathcal{O}(\alpha_{\text{em}} \alpha_s)$$

Ultrasoft Contribution

Ultrasoft effects enter at the level of the rate

Ultrasoft factor

$$U(M_1 M_2) = \left(\frac{2\Delta E}{m_B} \right)^{-\frac{\alpha_{\text{em}}}{\pi}} \left(Q_B^2 + Q_{M_1}^2 \left[1 + \ln \frac{m_{M_1}^2}{m_B^2} \right] + Q_{M_2}^2 \left[1 + \ln \frac{m_{M_2}^2}{m_B^2} \right] \right)$$

- ΔE is the window of the πK invariant mass around m_B
- Theory requires $\Delta E \ll \Lambda_{\text{QCD}} \rightarrow \Delta E \sim 60 \text{ MeV}$
- Large effects:
 - $U(\pi^+ K^-) = 0.914$, $U(\pi^0 K^-) = U(K^- \pi^0) = 0.976$ and $U(\pi^- \bar{K}^0) = 0.954$
- Experimentally ulsoft effects included using PHOTOS
- Challenging to compare theory with experiment!

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Ratios and isospin sumrules

- QED gives sub-percent corrections to Branching ratios
- Beneficial to consider ratios in which QCD is suppressed

$$R_L = \frac{2\text{Br}(\pi^0 K^0) + 2\text{Br}(\pi^0 K^-)}{\text{Br}(\pi^- K^0) + \text{Br}(\pi^+ K^-)} = R_L + \cos \gamma \text{Re } \delta_E + \delta_U$$

- QED corrections to kernels enter linearly, QCD only quadratically

$$\delta_E = (-1.12 + 0.16i) \cdot 10^{-3}$$

- Ultrasoft effects dominant

$$\delta_U \equiv \frac{1 + U(\pi^0 K^-)}{U(\pi^- \bar{K}^0) + U(\pi^+ K^-)} - 1 = 5.8\%$$

- Combined QED effect larger than QCD uncertainty!

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Isospin sumrule

Isospin Sumrule

$$\begin{aligned}\Delta(\pi K) &\equiv A_{\text{CP}}(\pi^+ K^-) + \frac{\Gamma(\pi^- \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\text{CP}}(\pi^- \bar{K}^0) - \frac{2\Gamma(\pi^0 K^-)}{\Gamma(\pi^+ K^-)} A_{\text{CP}}(\pi^0 K^-) \\ &\quad - \frac{2\Gamma(\pi^0 \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\text{CP}}(\pi^0 \bar{K}^0) \equiv \Delta(\pi K)^{\text{QCD}} + \delta\Delta(\pi K)\end{aligned}$$

- Sensitive to new physics effects: $\Delta(\pi K)^{\text{QCD}} = (0.5 \pm 1.1)\%$ [Bell, Beneke, Huber, Li]
- QED contribution $\delta\Delta(\pi K) = -0.42\%$ [Beneke, PB, Toelstede, KKV]
- Isospin sumrule also robust against QED effects!

CP asymmetries:

- QED effects smaller than QCD uncertainties

$$\delta(\pi K) \equiv A_{\text{CP}}(\pi^0 K^-) - A_{\text{CP}}(\pi^+ K^-) = \delta^{\text{QCD}}(\pi K) + 0.02\%$$

- QED factorization more complicated than in QCD due to charged external states
- QCD×QED factorization formula ...
 - same structure as in QCD but with generalized non-pert. functions
 - new objects also show up in other processes
 - endpoint-divergences still universal
 - soft rearrangement allows factorization of soft and coll. sectors
- Soft functions ...
 - become process dependent
 - develop support for $-\infty < \omega < +\infty$ [Beneke, PB, Toelstede, KKV [in progress!]]
 - soft physics qualitatively different from standard hard-scattering picture
- Structure dependent QED effects small
 - can compete with QCD uncertainty
- Theoretical versus experimental branching ratio

Backup-Slides

Kernel Contribution

$$\delta\alpha_i^K(M_1 M_2) = \frac{\alpha_{\text{em}}(\mu)}{4\pi} \sum_{j=1,2} C_j \left[\mathcal{V}_j^{(1)}(M_2) + H_{j,Q_{M_2}}^{\text{em}}(M_1 M_2) \right]$$

$$\mathcal{V}_i(M_2) = \int_0^1 du \ T_{i,Q_2}^1(u) \phi_{M_2}(u) \quad H_{2,-}^{\text{em}}(M_1 M_2) \equiv \frac{4\pi^2 Q_{sp} Q_u}{N_c} \frac{r_{\text{sp}}(M_1)}{9} \int_0^1 du \ dv \ \frac{\phi_{M_2}(u)\phi_{M_1}(v)}{u\bar{v}}$$

- Vertex contributions only depend on charge M_2
- Spectator-scattering depends on both charges

Kernel Contribution

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Kernel Contribution

$$\delta\alpha_i^K(M_1 M_2) = \frac{\alpha_{\text{em}}(\mu)}{4\pi} \sum_{j=1,2} C_j \left[\gamma_j^{(1)}(M_2) + H_{j,Q_{M_2}}^{\text{em}}(M_1 M_2) \right] \sim \mathcal{O}(10^{-3})$$

- Vertex contributions only depend on charge M_2
 - Spectator-scattering depends on both charges
- Unique relation between color-suppression and charge configuration
- Isospin-breaking effects enters via spectator-scattering only

$$\delta\alpha_1^K(\pi^0 K^-) = \delta\alpha_1^K(\pi^+ K^-) + \frac{\alpha_{\text{em}}(\mu)}{4\pi} \Delta_1^K,$$

$$\delta\alpha_2^K(K^- \pi^0) = \delta\alpha_2^K(K^0 \pi^0) + \frac{\alpha_{\text{em}}(\mu)}{4\pi} \Delta_2^K.$$

Details on Soft Subtraction

- regularize Wilson line propagators by the corresponding off-shell momenta of the original hard-collinear fields

$$[n_+ k - i0^+] \rightarrow [n_+ k - \delta - i0^+] , \quad \delta \equiv k_q^2 / (n_- k_q) , \quad (q = \bar{u}, d)$$

- use same δ for both anti-collinear fields to ensure

$$S_{n_+}^{\dagger(d)} S_{n_+}^{(u)} = S_{n_+}^{\dagger, Q_2} \Rightarrow k_d^2 / (2E u) = k_{\bar{u}}^2 / (2E(1-u))$$

- similarly for the collinear sector: $[n_- k + i0^+] \rightarrow [n_- k + \delta_c + i0^+]$

- then the vacuum ME yields

$$\langle 0 | S_{n_+}^{\dagger(Q_2)} S_{n_-}^{(Q_2)} | 0 \rangle = 1 - \frac{\alpha_{\text{em}}}{4\pi} 2Q_2^2 \left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln \left(\frac{\mu^2}{-\delta \delta_c} \right) + \mathcal{O}(\epsilon^0) \right]$$

- split symmetrically to obtain (choose $\delta, \delta_c < 0$ to avoid spurious imag. parts)

$$R_c R_{\bar{c}} = \left| \langle 0 | S_{n_+}^{\dagger(Q_2)} S_{n_-}^{(Q_2)} | 0 \rangle \right| , \quad R_{\bar{c}} = 1 - \frac{\alpha_{\text{em}}}{4\pi} Q_2^2 \left[\frac{1}{\epsilon^2} + \frac{2}{\epsilon} \ln \left(\frac{\mu}{-\delta} \right) \right]$$

- consistent with other IR-regulators, e.g. photon mass m_γ
 - requires further non-dimensional (analytic) regulators ⇒ rapidity RGE
 - soft subtraction removes ν dependence, but $\log \mu / (2E)$ remain from large rapidity separation

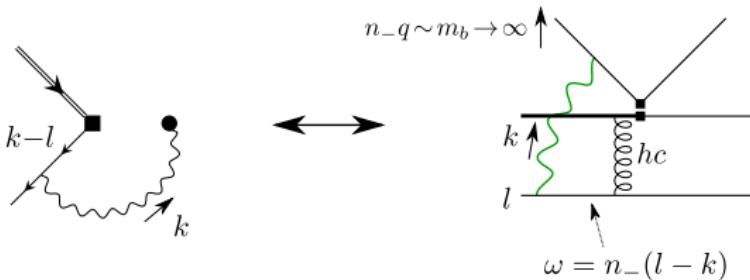
Various B Meson LCDAs

The four different soft functions relevant to $B \rightarrow M_1 M_2$ decays are defined as

$$\begin{aligned} & i m_B \int_0^\infty d\omega e^{-i\omega t} \mathcal{F}_{B,\bar{Q}} \Phi_{B,\bar{Q}}(\omega) \\ &= \frac{1}{R_{\bar{c}}^{(Q_2)} R_c^{(Q_1)}} \langle 0 | \bar{q}_s^{(q)}(tn_-) [tn_-, 0]_{n_-}^{(q)} \frac{\not{h}_-}{2} h_\nu(0) (S_{n_+}^{\dagger, Q_2} S_{n_-}^{Q_2})(0) | \bar{B}_q \rangle \end{aligned}$$

- $\bar{B}^0 \rightarrow M_1^0 M_2^0$: $\bar{q}_s^{(d)}(tn_-) [tn_-, 0]_{n_-}^{(d)} \frac{\not{h}_-}{2} h_\nu(0)$
- $\bar{B}^0 \rightarrow M_1^+ M_2^-$: $\frac{1}{R_{\bar{c}}^{(Q_2)} R_c^{(Q_1)}} \bar{q}_s^{(d)}(tn_-) [tn_-, 0]_{n_-}^{(d)} \frac{\not{h}_-}{2} h_\nu(0) (S_{n_+}^{\dagger, Q_2} S_{n_-}^{Q_2})(0)$
- $B^- \rightarrow M_1^0 M_2^-$: $\frac{1}{R_{\bar{c}}^{(Q_2)}} \bar{q}_s^{(u)}(tn_-) [tn_-, 0]_{n_-}^{(q)} \frac{\not{h}_-}{2} h_\nu(0) (S_{n_+}^{\dagger, Q_2})(0)$
- $B^- \rightarrow M_1^- M_2^0$: $\frac{1}{R_c^{(Q_1)}} \bar{q}_s^{(u)}(tn_-) [tn_-, 0]_{n_-}^{(q)} \frac{\not{h}_-}{2} h_\nu(0) (S_{n_-}^{Q_2})(0)$

On the Support of QED B LCDAs



- even for on-shell massive partons with $\Phi^{(0)}(\omega) = \delta(\omega - m)$ the one-loop soft photon exchange with the anti-coll. π^- generates a support for $\omega < 0$
- diagram has e.g. the following contribution

$$\int d^d k \frac{\delta(\omega - n_- \ell + n_- k)}{(k^2 + i0)[(k - \ell)^2 - m^2 + i0] (n_+ k - i0)} \quad \left| \text{pick up residues in } (n_+ k) \right.$$
$$\sim \Gamma(\epsilon) \int_{n_- \ell}^{\infty} d(n_- k) (n_- k)^{-1-\epsilon} \delta(\omega - n_- \ell + n_- k) = \Gamma(\epsilon) (n_- \ell - \omega)^{-1-\epsilon} \theta(-\omega)$$

- QED B LCDA no longer linear in ω as $\omega \rightarrow 0$ but rather const.
→ no endpoint singularity in first inverse moment

$$\int_{-\infty}^{+\infty} \frac{d\omega}{\omega - i0} \Phi_{B,+-}(\omega)$$