Selection rules of scattering amplitudes in EFTs

–Based on work with J. Shu, M.-L. Xiao, Y.-H. Zheng, arXiv:2001.04481

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Outline

- Introduction:
  - Spinor helicity formalism for scattering amplitudes
  - Amplitude basis of EFTs
- Partial wave amplitude basis
- (Non)-renormalization from angular momentum conservation.
- Vanishing loops from angular momentum conservation.
- Summary and outlook.

Intro: spinor helicity formalism

Spinor helicity formalism is a natural way in studying on-shell helicity amplitudes of massless particles.

$$p^2 = 0 \to p_{\alpha\dot{\alpha}} \equiv p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} = |p\rangle[p| \tag{1}$$

Under little group transformation, a massless particle with helicity h transforms as

$$|h\rangle \to e^{2ih\theta}|h\rangle \tag{2}$$

which is represented through the transformtion of the spinors:

$$|p\rangle \to e^{-i\theta}|p\rangle, [p| \to e^{i\theta}[p| \tag{3}$$

Scattering amplitudes are then Lorentz-invariant functions of λ and $\tilde{\lambda}$ with the correct little-group helicity weights:

$$\mathcal{M}(\omega^{-1}\lambda_i, \omega\tilde{\lambda}_i) = \Pi_i \omega^{2h_i} \mathcal{M}(\lambda_i, \tilde{\lambda}_i) \tag{4}$$

Intro: spinor helicity formalism

Three point amplitudes:

$$\mathcal{M}(h_1, h_2, h_3) = g \begin{cases} \langle 12 \rangle^{h_3 - h_1 - h_2} \langle 23 \rangle^{h_1 - h_2 - h_3} \langle 31 \rangle^{h_2 - h_1 - h_3}, \sum_i h_i \leq 0 \\ [12]^{h_1 + h_2 - h_3} [23]^{h_2 + h_3 - h_1} [31]^{h_1 + h_3 - h_2}, \sum_i h_i \geq 0 \end{cases} \tag{5}$$

In renormalizable theory with only dimensionless couplings:

$$[g] = 0 \rightarrow \sum_i h_i = \pm 1 \tag{6}$$

or the non-zero three point amplitudes are

$$\mathcal{M}(\psi^{\pm}, \psi^{\pm}, \phi), \mathcal{M}(\psi^{+}, \psi^{-}, V^{\pm}), \mathcal{M}(\phi, \phi, V^{\pm}) \tag{7}$$

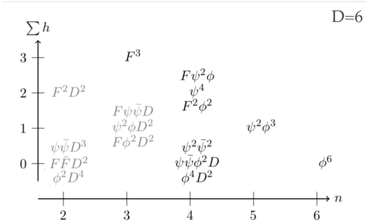
Intro: Amplitude basis for EFTs

Enumerating high dimensional operators in a massless theory:

field	ϕ	ψ_α	$\bar{\psi}_{\dot{\alpha}}$	$F_{\alpha\beta}$[1]	$\bar{F}_{\dot{\alpha}\dot{\beta}}$
representation	$(0,0)$	$(\frac{1}{2},0)$	$(0,\frac{1}{2})$	$(1,0)$	$(0,1)$
helicity	0	$\frac{1}{2}$	$-\frac{1}{2}$	1	-1

Table: Fields in irreducible representations of Lorentz group and the helicity of outgoing particles they excite.

- Lorentz invariance;
- Remove total derivatives and EoM redundancy.



[1] $F_{\mu\nu}\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma^{\nu}_{\beta\dot{\beta}} = F_{\alpha\beta}\bar{\epsilon}_{\dot{\alpha}\dot{\beta}} + \bar{F}_{\dot{\alpha}\dot{\beta}}\epsilon_{\alpha\beta}$

Intro: Amplitude basis for EFTs

The on-shell construction of the EFTs:

- Using contact on-shell amplitudes as basic "building blocks" instead of operators.
- The size of the amplitude basis is exactly the same as that of the operator basis!
- Full amplitudes are constructed with these basic amplitudes from unitarity.
- Example:

$$
\begin{aligned}
O_{HB} = H^\dagger H B^{\mu\nu} B_{\mu\nu}; \, O_{H\tilde{B}} &= H^\dagger H B^{\mu\nu} \tilde{B}_{\mu\nu} \\
O_{HB} - iO_{H\tilde{B}} \leftrightarrow \mathcal{M}(B^+, B^+, H_\alpha, H^\dagger_{\dot\alpha}) &= [12]^2 \delta_{\alpha\dot\alpha} \\
O_{HB} + iO_{H\tilde{B}} \leftrightarrow \mathcal{M}(B^-, B^-, H_\alpha, H^\dagger_{\dot\alpha}) &= \langle 12\rangle^2 \delta_{\alpha\dot\alpha}
\end{aligned} \tag{8}
$$

Partial Wave Amplitude Basis

- Scattering amplitudes are usually expressed with incoming and outgoing multi-particle states with definite momentum.
- We can also decompose the momentum eigenstate into angular momentum eigenstate

$$
{}_{\otimes}\langle\Psi_N|\Psi_N\rangle_j \equiv C_N^{j,j_z}(P;\psi_{i=1,\dots,N})\delta(P-\sum p_i). \tag{9}
$$

- And decompose the amplitude into the "partial wave amplitude basis" $\mathcal{B}^j$.

$$
\begin{aligned}
\mathcal{A}_{N\to M} &\equiv {}_{\otimes}\langle\Psi_M|\mathcal{M}|\Psi_N\rangle_{\otimes} && (10)\\
&= \sum_j {}_j\langle\Psi_M|\mathcal{M}|\Psi_N\rangle_j \sum_{j_z} C_M^{j,j_z}(C_N^{j,j_z})^* && (11)\\
&\equiv \sum_{j,a} \mathcal{M}^{j,a}(s)\mathcal{B}^{j,a}_{N\to M}, && (12)
\end{aligned}
$$

Partial Wave Amplitude Basis

- For amplitudes expressed with spinor helicity formalism, we derive Pauli-Lubanski operator acting on function of spinors:

$$M_{\mathcal{I},\alpha\beta} = i\sum_{i\in\mathcal{I}}\left(\lambda_{i\alpha}\frac{\partial}{\partial\lambda_i^{\beta}} + \lambda_{i\beta}\frac{\partial}{\partial\lambda_i^{\alpha}}\right), \tag{13}$$

$$\tilde{M}_{\mathcal{I},\dot{\alpha}\dot{\beta}} = i\sum_{i\in\mathcal{I}}\left(\tilde{\lambda}_{i\dot{\alpha}}\frac{\partial}{\partial\tilde{\lambda}_i^{\dot{\beta}}} + \tilde{\lambda}_{i\dot{\beta}}\frac{\partial}{\partial\tilde{\lambda}_i^{\dot{\alpha}}}\right), \tag{14}$$

$$W_{\alpha\dot{\alpha}} = \frac{i}{2}\left(P_{\alpha\dot{\beta}}\bar{M}^{\dot{\beta}}{}_{\dot{\alpha}} - M_{\alpha}{}^{\beta}P_{\beta\dot{\alpha}}\right) \tag{15}$$

- Partial wave amplitude basis at some specified channel are eigenfunctions of the operator W^2:

$$W^2\mathcal{B}^j = -P^2 j(j+1)\mathcal{B}^j \tag{16}$$

- Given some amplitude basis, we can obtain the partial wave amplitude basis by diagonalizing W^2.

Partial Wave Amplitude Basis

- Examples: consider $2 \to 2$ scattering, acting W^2 to the following amplitudes at $(1,2)$ channel

$$W^2\langle 12\rangle = 0 \to j = 0, \tag{17}$$

$$W^2\langle 13\rangle\langle 23\rangle = -2s_{12}\langle 13\rangle\langle 23\rangle \to j = 1, \tag{18}$$

$$W^2(s_{13} - s_{23}) = -2s_{12}(s_{13} - s_{23}) \to j = 1. \tag{19}$$

- The point is that amplitudes directly generated by effective operators, or amplitude basis, can be deposed into superposition of finite number of "partial wave amplitude basis".

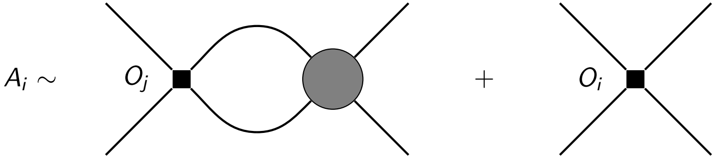
Anomalous dimension matrix at one loop

Considering a process that has a direct tree level contribution from an operator $\mathcal{O}_i$ and one-loop contribution from another operator $\mathcal{O}_j$

$$\mathcal{A}_i \sim c_i(\mu) - \gamma_{ij}\frac{1}{16\pi^2}c_j(\mu)(\frac{1}{2\epsilon} + \log\mu + \ldots), \tag{20}$$

$\frac{1}{2\epsilon} + \log\mu$ come from the UV divergence of the one loop contribution. Demanding the amplitude being independent of the scale μ:

$$\frac{\mathrm{d}c_i(\mu)}{\mathrm{d}\log\mu} = \sum_j \frac{1}{16\pi^2}\gamma_{ij}c_j. \tag{21}$$



Zeros in $d = 6$ γ_{ij} and non-renormalization theorem:

	$(w, \bar{w})$	F^3 $(0,6)$	$F^2\phi^2$ $(2,6)$	$F\psi^2\phi$ $(2,6)$	ψ^4 $(2,6)$	$\psi^2\phi^3$ $(4,6)$	$\bar{F}^3$ $(6,0)$	$\bar{F}^2\phi^2$ $(6,2)$	$\bar{F}\bar{\psi}^2\phi$ $(6,2)$	$\bar{\psi}^4$ $(6,2)$	$\bar{\psi}^2\phi^3$ $(6,4)$	$\bar{\psi}^2\psi^2$ $(4,4)$	$\bar{\psi}\psi\phi^2 D$ $(4,4)$	$\phi^4 D^2$ $(4,4)$	ϕ^6 $(6,6)$
F^3	$(0,6)$			$\times$	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$F^2\phi^2$	$(2,6)$				$\times$	$\times$				$\times$	$\times$	$\times$			$\times$
$F\psi^2\phi$	$(2,6)$									$\times$				$\times$	$\times$
ψ^4	$(2,6)$	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	y^2		$\times$	$\times$
$\psi^2\phi^3$	$(4,6)$	$\times^*$									y^2				$\times$
$\bar{F}^3$	$(6,0)$			$\times$	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$\bar{F}^2\phi^2$	$(6,2)$				$\times$	$\times$				$\times$	$\times$	$\times$			$\times$
$\bar{F}\bar{\psi}^2\phi$	$(6,2)$				$\times$									$\times$	$\times$
$\bar{\psi}^4$	$(6,2)$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$			$\times$	$\bar{y}^2$		$\times$	$\times$
$\bar{\psi}^2\phi^3$	$(6,4)$					$\bar{y}^2$	$\times^*$								$\times$
$\bar{\psi}^2\psi^2$	$(4,4)$		$\times$		$\bar{y}^2$	$\times$		$\times$		y^2	$\times$			$\times$	$\times$
$\bar{\psi}\psi\phi^2 D$	$(4,4)$														$\times$
$\phi^4 D^2$	$(4,4)$				$\times$					$\times$		$\times$			$\times$
ϕ^6	$(6,6)$	$\times^*$		$\times$	$\times$		$\times^*$		$\times$	$\times$		$\times$			

Table describing the renormalization of the operators in the column by those in the row.
C. Cheung, C.H. Shen [arxiv:1505.01844]

Not all zeros are explained! R. Alonso, E. Jenkins, A. Manohar, M. Trott

$$\dot{C}_{\substack{eW\\rs}} = -2g_2 N_c C^{(3)}_{\substack{lequ\\rspt}}[Y_u]_{tp} + C_{\substack{eW\\rt}}[Y_e Y_e^\dagger]_{ts} + \cdot$$

$$\dot{C}_{\substack{eB\\rs}} = 4g_1 N_c (\mathsf{y}_u + \mathsf{y}_q) C^{(3)}_{\substack{lequ\\rspt}}[Y_u]_{tp} + C_{\substack{eB\\rt}}[Y_e Y_e^\dagger]_{ts} +$$

$$\begin{aligned}\dot{C}_{\substack{eH\\rs}} &= 2(\eta_1+\eta_2+i\eta_5)[Y_e^\dagger]_{rs} + [Y_e^\dagger Y_e Y_e^\dagger]_{rs}(C_{HD}-6C_{H\Box}) + 2C^{(1)}_{\substack{Hl\\rt}}[Y_e^\dagger Y_e Y_e^\dagger]_{ts} - 2[Y_e^\dagger Y_e Y_e^\dagger]_{rt} C_{\substack{He\\ts}} \\ &+ 8C_{\substack{le\\rpts}}[Y_e^\dagger Y_e Y_e^\dagger]_{pt} - 4C_{\substack{ledq\\rspt}} N_c [Y_d^\dagger Y_d Y_d^\dagger]_{tp} + 4C^{(1)}_{\substack{lequ\\rstp}} N_c [Y_u Y_u^\dagger Y_u]_{pt} + 4C_{\substack{eH\\rt}}[Y_e Y_e^\dagger]_{ts} \\ &+ 5[Y_e^\dagger Y_e]_{rt} C_{\substack{eH\\ts}} + 3\gamma_H^{(Y)} C_{\substack{eH\\rs}} + \gamma^{(Y)}_{\substack{l\\rv}} C_{\substack{eH\\vs}} + C_{\substack{eH\\rv}} \gamma^{(Y)}_{\substack{e\\vs}} \end{aligned} \tag{A.18}$$

$$\begin{aligned} O^1_{lequ} &= (\bar{l}e)\epsilon_{jk}(\bar{q}u);\ O^3_{lequ} = (\bar{l}\sigma_{\mu\nu}e)\epsilon_{jk}(\bar{q}\sigma^{\mu\nu}u) \\ O_{eW} &= (\bar{l}\sigma^{\mu\nu}e)\tau^I H W^I_{\mu\nu};\ O_{eB} = (\bar{l}\sigma^{\mu\nu}e)HB_{\mu\nu};\ O_{eH} = H^\dagger H(\bar{l}eH). \end{aligned}$$

(Non)-renormalization from angular momentum

From the angular momentum consideration, at (l, e) channel:

$$
\begin{aligned}
J &= 0 : O^1_{lequ} \sim \langle 12\rangle\langle 3'4'\rangle, O_{eH} = H^\dagger H(\bar{l}eH) \sim \langle 12\rangle; \\
J &= 1 : O^3_{lequ} \sim \langle 13'\rangle\langle 24'\rangle, O_{eW}/O_{eB} \sim \langle 13\rangle\langle 23\rangle
\end{aligned} \tag{22}
$$

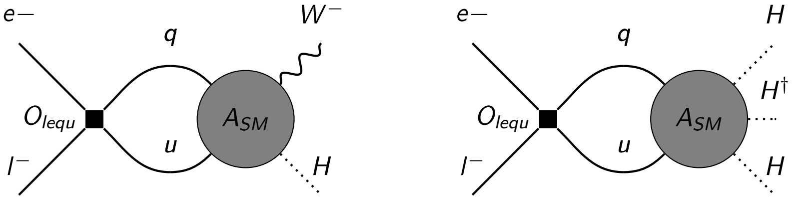


Figure: Contribution from O_{lequ} to the running of O_{eW} and O_{eH}

.

(Non)-renormalization from angular momentum

The other dim-6 example: $H^4 D^2 \to \psi^2 H^2 D$

$$
\begin{aligned}
O_{H\Box} &= H^\dagger H \Box (H^\dagger H), O_{HD} = (H^\dagger D_\mu H)^* (H^\dagger D^\mu H); \\
O^1_{Hq} &= (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q), O_{Hu} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu u) ... (J=1, I=0) \\
O^3_{Hl} &= (H^\dagger i \overleftrightarrow{D}_\mu \tau^a H)(\bar{l}\gamma^\mu \tau^a l), O^3_{Hq} = (H^\dagger i \overleftrightarrow{D}_\mu \tau^a H)(\bar{q}\gamma^\mu \tau^a q) ... (J=1, I=1)
\end{aligned}
$$

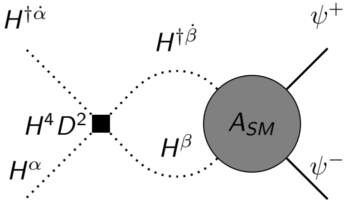


Figure: Contribution from $H^4 D^2$ to the running of $\psi^2 H^2 D$

(Non)-renormalization from angular momentum

$$
\begin{aligned}
M(H^\alpha, H^{\dagger\dot\alpha}, H^\beta, H^{\dagger\dot\beta}) &= 2C_{H\Box}(s_{12}\delta^{\alpha\dot\alpha}\delta^{\beta\dot\beta} + s_{23}\delta^{\alpha\dot\beta}\delta^{\beta\dot\alpha}) \\
&+ C_{HD}(s_{12}\delta^{\alpha\dot\beta}\delta^{\beta\dot\alpha} + s_{23}\delta^{\alpha\dot\alpha}\delta^{\beta\dot\beta})
\end{aligned}
$$

Decompose into s channel angular momentum and isospin eigenfunctions: s_{12} for $j = 0$ and $s_{13} - s_{23}$ for $j = 1$; $T^1 = \delta^{\alpha\dot\alpha}\delta^{\beta\dot\beta}$, $T^3 = \frac{1}{2}(\sigma^a)_{\alpha\dot\alpha}(\sigma^a)_{\beta\dot\beta}$.

$$
\begin{aligned}
M(H^\alpha, H^{\dagger\dot\alpha}, H^\beta, H^{\dagger\dot\beta}) &= [3C_{H\Box}T^1 + (C_{HD} - C_{H\Box})T^3]s_{12} \\
&+ [(2C_{H\Box} + 2C_{HD})T^1 + 2C_{H\Box}T^3](s_{13} - s_{23}).
\end{aligned}
$$

then make the right prediction!

$$
\dot C^{(1)}_{\substack{Hq\\pr}} = \frac{1}{2}[Y_u^\dagger Y_u - Y_d^\dagger Y_d]_{pr}\,(C_{H\Box} + C_{HD})
$$

$$
\dot C^{(3)}_{\substack{Hq\\pr}} = -\frac{1}{2}[Y_u^\dagger Y_u + Y_d^\dagger Y_d]_{pr} C_{H\Box}
$$

(Non)-renormalization from angular momentum: dim-6

Channels	$j=0$	$j=1/2$	$j=1$
F^+F^+	$F^2\phi^2(2,6)$		
$F^+\psi^+$		$F\psi^2\phi(2,6)$	
$F^+\phi$			$F\psi^2\phi(2,6)$, $F^2\phi^2(2,6)$
$\psi^+\psi^+$	$\psi^4(2,6)$, $\psi^2\bar{\psi}^2(4,4)$, $\psi^2\phi^3(4,6)$		$\psi^4(2,6)$, $F\psi^2\phi(2,6)$
$\psi^+\psi^-$			$\psi\bar{\psi}\phi^2 D(4,4)$, $\psi^2\bar{\psi}^2(4,4)$
$\psi^+\phi$		$\psi^2\phi^3(4,6)$, $F\psi^2\phi(2,6)$, $\psi\bar{\psi}\phi^2 D(4,4)$	
$\phi\phi$	$\phi^4 D^2(4,4)$, $\psi^2\phi^3(4,6)$, $\phi^6(6,6)$		$\psi\bar{\psi}\phi^2 D(4,4)$, $\phi^4 D^2(4,4)$

Table: Dimension 6 operators classified by their angular momentum in the specified channel. Numbers in the bracket are the (anti-)holomorphic weights.

Vanishing loops from angular momentum

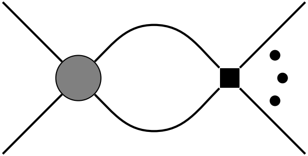
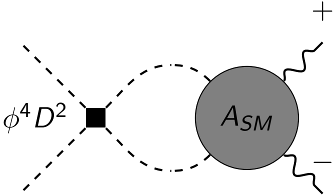


Figure: One-loop diagram for $2 \to N$ scattering in EFT. The solid rectangle on the right hand side represents a contanct interaction.

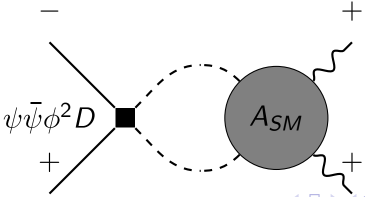
We see that the external state connected to the effective operator at RHS have limited j. If the state at LHS does not satisfy this constraint, this amplitude vanish.

Vanishing loops from angular momentum

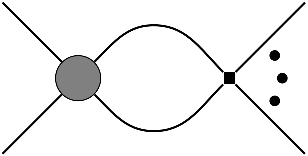
- For two particle with opposite helicity: $j \geq |\Delta h|$
  (In the CoM $j \geq j_z = |\Delta h|$)



- For two identical particle and same helicity: $j =$even
  (particle of odd spin can't decay into identical particles with same helicity)



Vanishing loops from angular momentum



LHS (Δh)	RHS	Operators at RHS
F^+F^- (2)	$\phi\phi$	$\phi^4(0)$, $\phi^4 D^2(0,1)$, $\phi^4 D^4(0,1,2)$, $\psi\bar{\psi}\phi^2 D(1)$, $\psi\bar{\psi}\phi^2 D^3(1,2)$
	$\psi^+\psi^-$	$\psi\bar{\psi}\phi^2 D^3(1)$, $\psi\bar{\psi}\phi^2 D^3(1,2)$, $\bar{\psi}^2\psi^2(1)$, $\bar{\psi}^2\psi^2 D^2(1,2)$
	F^+F^+	$F^2\phi^2(0)$, $F^2\phi^2 D^2(0,1)$, $F^2\psi\bar{\psi}D(1)$
$F^+\psi^-$ (3/2)	$\psi^-\phi$	$\psi\bar{\psi}\phi^2 D(1/2)$, $\psi\bar{\psi}\phi^2 D^3(1/2,3/2)$
	$F^+\psi^+$	$F\psi^2\phi(1/2)$, $F\psi^2\phi D^2(1/2,3/2)$
$F^+\phi$ (1)	$\psi^\pm\psi^\pm$	$\bar{\psi}^2\psi^2(0)$,$\psi^4(0,1)$, $\bar{\psi}^2\psi^2 D^2(0,1)$,$\psi^4 D^2(0,1,2)$
$\psi^+\psi^-$ (1)	$F^\pm F^\pm$	$F^2\phi^2(0)$,$F^2\phi^2 D^2(0,1)$, $F^2\bar{F}^2(0)$, $F^4(0,1,2)$
	$\phi\phi$	$F^2\phi^2(0)$, $F^2\phi^2 D^2(0,1)$

Table: Vanishing one-loop amplitudes from contribution of specific operators.

Summary and outlook

Summary:

- The multi-particle states from an effective operator usually have limited angular momentum.
- The amplitude basis with definite angular momentum can be obtained using W^2.
- Angular momentum conservation tells us non-trivial information in EFTs.

Outlook:

- massive case; higher loops...
- Calculate anomalous dimension with unitarity method combined with partial wave expansion. P. Baratella, C. Fernandez, B. Harling, A. Pomarol