An introduction to DDCalc

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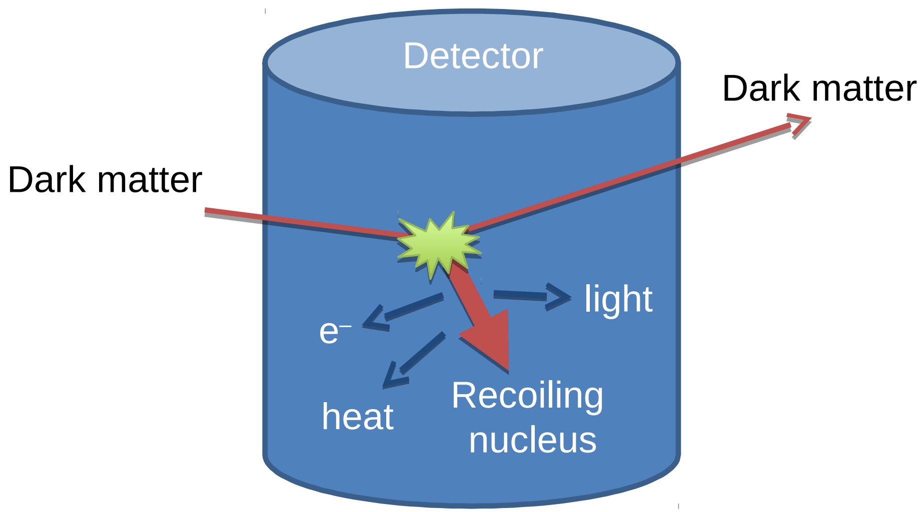
Online

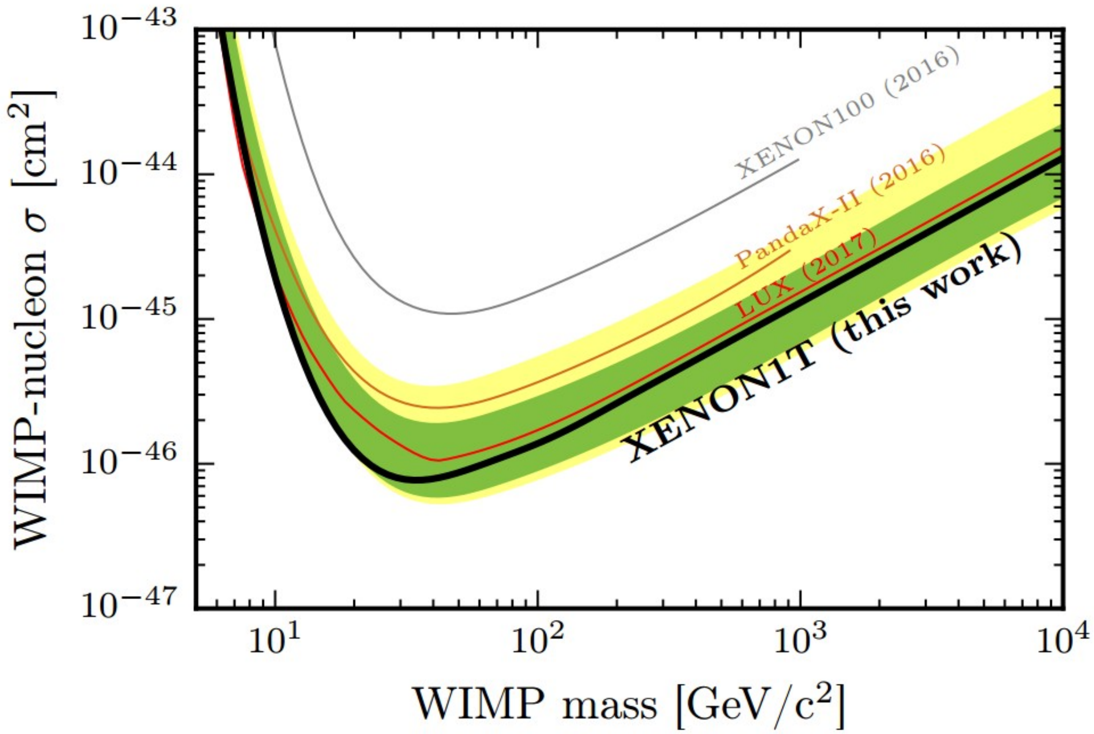
Dark matter direct detection

- Dark matter particles from the Milky Way halo constantly **pass through the Earth**
- Even though these particles have tiny interactions, they are predicted to occasionally **scatter off ordinary nuclei**
- If such a scattering occurs in a dedicated low-background detector, we can observe the **nuclear recoil energy E_R** deposited in heat, light or ionisation.



Dark matter direct detection

- Leading direct detection experiments are sensitive to **less than one event per kg per year** with nuclear recoil energy of a **few keV**
- The non-observation of such events places some of the **strongest bounds on the interactions** between dark matter and Standard Model particles



Two key equations

The **differential event rate** is given by

$$\frac{\mathrm{d}R}{\mathrm{d}E_\mathrm{R}} = \frac{\rho}{m_T\, m_\chi} \int_{v_\mathrm{min}}^{\infty} v f(\boldsymbol{v} + \boldsymbol{v}_\mathrm{E}(t)) \frac{\mathrm{d}\sigma}{\mathrm{d}E_\mathrm{R}} \mathrm{d}^3 v$$

ρ:	Local dark matter density
m_χ:	Dark matter mass
m_T:	Target nucleus mass
v:	Dark matter velocity
$v_\mathrm{E}(t)$:	Earth velocity
$f(v)$:	Velocity distribution
$\mathrm{d}\sigma/\mathrm{d}E_\mathrm{R}$:	Differential scattering cross section

$$v_\mathrm{min}(E_\mathrm{R}) = \sqrt{\frac{m_T E_\mathrm{R}}{2\,\mu_{T\chi}^2}}$$

The **number of events** is given by

$$N_\mathrm{p} = MT \int_0^\infty \phi(E) \frac{dR}{dE}(E)\, dE$$

M:	Total target mass
T:	Total run time
E:	Detected energy
$\Phi(E)$:	Detector acceptance

The conversion from true nuclear recoil energy E_R to detected energy E depends on the energy resolution and quenching factors

Additional complications

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- The velocity distribution is often assumed to be a Maxwell-Boltzmann distribution with dispersion v_0 cut off at the Galactic escape velocity v_esc
- Numerical simulations confirm this to be a reasonable approximation, but there are many possible deviations
  - Anisotropy
  - Substructure (e.g. streams)
- Many studies of the impact of astrophysical uncertainties

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- For most dark matter models the cross section is independent of E_R
  - Spin-independent scattering
  - Spin-dependent scattering
- More complicated cross sections arise in various well-motivated models
  - Light mediator exchange
  - Dipole interactions
  - Pseudoscalar exchange
- Can be described by a general non-relativistic effective theory

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- Essential to exploit difference in signal and background distributions
- Cut-and-count analysis typically insufficient
  - Need to calculate expected signal in several different signal regions (bins)
  - Perform unbinned analysis
- Background distributions not always published → challenging to construct accurate likelihoods

DDCalc

- **DDCalc v1** created by Chris Savage
  - Partial release as LUXCalc (Savage et al., arXiv:1502.02667)
  - First fully released together with GAMBIT v1.0 (Bringmann et al., arXiv:1705.07920)
- **DDCalc v2** developed by FK and Sebastian Wild
  - Release together with GAMBIT v1.2 (Athron et al., arXiv:1808.10465)
- **DDCalc v3** under development by Jonathan Cornell and Lauren Street
  - Implementation of rate and likelihood calculations for annual modulations

- Further contributions: Gonzalo Herrera, Lauren Hsu, Andre Scaffidi and Pat Scott

Available via https://ddcalc.hepforge.org/ and https://github.com/patscott/DDCalc

DDCalc: Main features

- Support for **wide range of dark matter models**
- Large **database of experiments**
  - Most recent additions: PICO-60, DarkSide-50, CRESST-III and XENON1T
  - Abstract detection interface allows easy implementation of new experiments
- Various **statistical methods** for likelihood calculations and/or limit setting
  - (Binned) Poisson, Feldman-Cousins, maximum gap/interval
- Possibility to **vary astrophysical parameters** or load tabulated velocity distributions
- Highly optimised for **computational speed**
- User interfaces in **Fortran, C/C++ and Python** (example files provided for all of these)

DDCalc: General structure

- To calculate predicted event numbers, DDCalc needs three objects (structures):
  - **WIMPStruct:** Carries information on DM (mass/spin) and its interactions
  - **HaloStruct:** Provides information on the DM density and velocity distribution
  - **DetectorStruct:** Stores experimental details, such as efficiencies and exposure

- The function **DDCalc_CalcRates(Detector,WIMP,Halo)** then calculates differential event rates and stores them in the DetectorStruct

- The DetectorStruct can then be passed to various analysis functions, such as **DDCalc_LogLikelihood(Detector)**

Available dark matter models

- In addition to spin-independent and spin-dependent interactions, DDCalc supports the **general basis* of non-relativistic effective interactions**

$$\mathcal{L}_{\text{eff}} = \sum_{\lambda=1}^{15} \sum_{\tau=0}^{1} c_\lambda^{(\tau)} \mathcal{O}_\lambda^{(\tau)}$$

and performs an **automatic matching** onto the corresponding response functions and nuclear form factors

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_\mathrm{R}} = \frac{m_T}{2\pi v^2} \sum_{\alpha=1}^{8} \sum_{\tau=0}^{1} \sum_{\tau'=0}^{1} S_\alpha^{(\tau,\tau')}\left(\mathbf{z}, v^2, q^2\right) \, \widetilde{W}_\alpha^{(\tau,\tau')}(q^2)$$

All common **nuclear form factors** are provided in tabulated form.

* In fact, two inequivalent sets of operators are used in the literature. Dent et al. (arXiv:1505.03117) include additional operators for spin-1 dark matter, while Bishara et al. (arXiv:1611.00368) include meson pole contributions. DDCalc supports both of these sets.

Defining a dark matter model

- If the **coefficients** for the various effective operators are known, they can be directly set within DDCalc:

```
#include "DDCalc.hpp"
int WIMP;
WIMP = DDCalc::InitWIMP();
DDCalc::SetWIMP_NREffectiveTheory(WIMP,50,0.5);
DDCalc::SetNRCoefficient(WIMP,3,0,0.1);
DDCalc::SetNRCoefficient(WIMP,3,1,0.2);
```

- Alternatively, DDCalc provides a **direct interface with DirectDM** (see Fady's talk), which makes it possible to automatically calculate all relevant coefficients for a given set of dark matter interactions (specified at some higher energy scale)

- Additional dark matter models not captured by the non-relativistic effective theory (e.g. **light mediators** or **inelastic scattering**) will be included in future versions

Statistical analyses

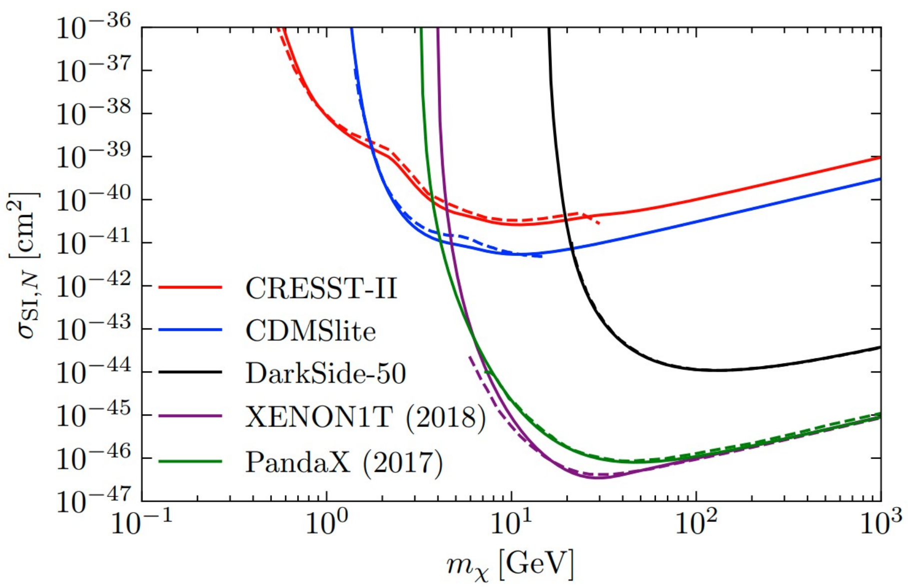
- DDCalc calculates exclusion limits at a given confidence level using a range of different methods:
  - **Known background** (few observed events): Feldman-Cousins
  - **Known background** (many observed events): Binned Poisson
  - **Unknown background** (few observed events): Maximum gap / maximum interval
  - **Unknown background** (many observed events): Binned Poisson (with nuisances)
- The (binned) Poisson method is also used to calculate the likelihood for a given experiment

  → Can be used to **combine several different experiments** with each other (and with complementary information)

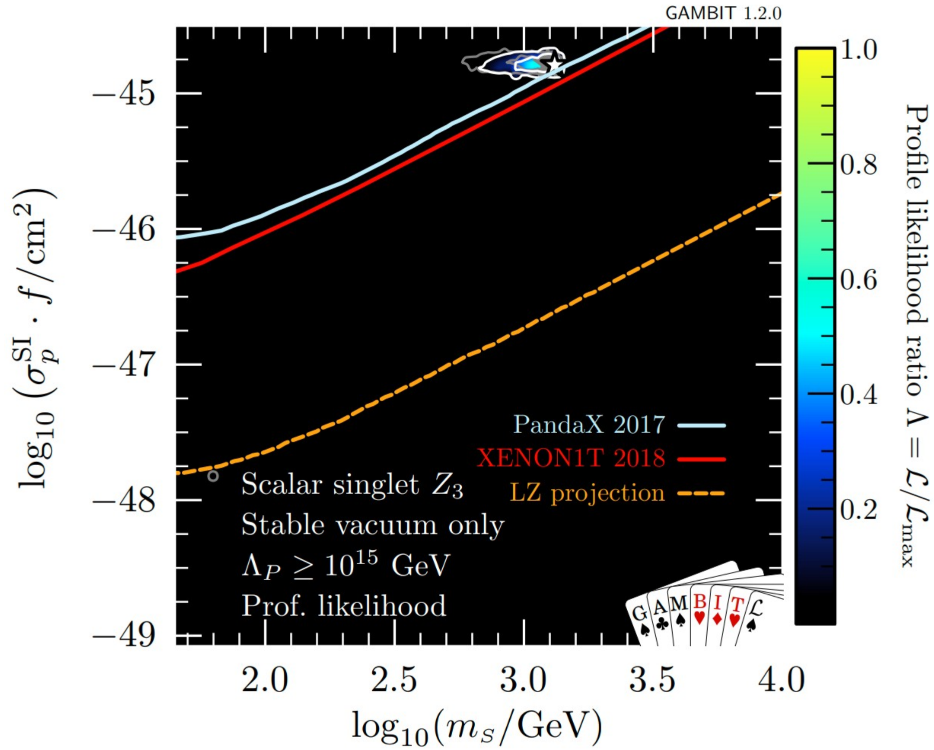
- **Note:** It is not always possible to recover the (exact) exclusion limit from the likelihood, e.g. if the assumption of the asymptotic limit does not hold

DDCalc: Results

- DDCalc accurately **reproduces exclusion bounds** from existing experiments for standard assumptions



- The resulting likelihoods can be **included in global fits** of a wide range of dark matter models



The GAMBIT community is currently using DDCalc to carry out a **comprehensive global analysis** of a general set of dark matter effective operators (Bloor et al., in preparation)

Current limitations and plans for the future

- Not all relevant **nuclear form factors** are currently available in the literature
  - → Additional theory work needed to provide this information
- It is currently not possible to perform **unbinned profile likelihood** analyses
  - → No conceptual difficulty, but experimental information typically not available
- Currently DDCalc offers only a limited number of **nuisance parameters**
  - → Desirable to extend user interface in order to study uncertainties in nuclear form factors and background estimates
- DDCalc does not currently calculate **projected sensitivities** and **expected limits**
  - → Desirable to provide a routine for simulating pseudo-experiments
- DDCalc currently only considers the time-independent event rate
  - → **Annual modulations** will be included in DDCalc v3 (under development)

Conclusions

- DDCalc is a **fast and easy tool** to calculate direct detection bounds and likelihoods
  - Large database of the most recent experiments (continuously updated)
  - Support for general effective dark matter interactions (more models to come)
  - Flexible interface (can be hooked up to DirectDM)

- DDCalc is **ideally suited** for being used as part of a larger analysis or in a global fit

- DDCalc v3 with **many new features** is currently under development