Feynrules for the SMEFT in the background field gauge

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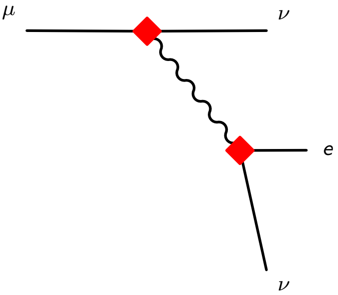
Based on: TC arXiv:2010.15852 & TC, Trott arXiv: 2010.08451

The Fermi-theory example

In the SM



$$\mathcal{M} \sim \frac{g_{\mathrm{W}}^2}{2} \frac{(\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_e)}{k^2 - M_W^2}$$

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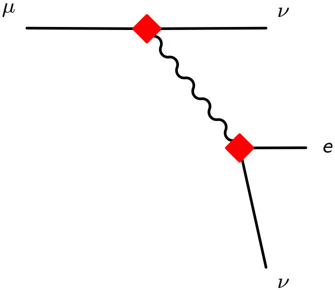
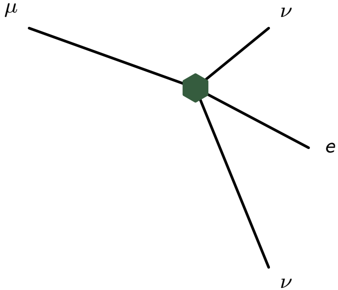


In the Fermi theory



$$\mathcal{M} \sim \frac{g_{\rm W}^2}{2}\frac{(\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_e)}{k^2 - M_W^2}$$

$$\frac{1}{M_W^2}(\bar{\psi}\gamma^\mu P_L \psi)^2$$

$$\mathcal{M} \sim -\frac{g_{\rm W}^2}{2M_W^2}(\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_e) + \cdots$$

The SMEFT at dimension-six

D6 operators from SM field content $\Rightarrow$ SMEFT @ D6

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Type I: X^3		Type II, III: H^6, $H^4 D^2$		Type V: $\Psi^2 H^3$ + h.c.	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_H	$(H^\dagger H)^3$	Q_{eH}	$(H^\dagger H)(\bar{L}eH)$
$Q_{\tilde{G}}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	Q_{uH}	$(H^\dagger H)(\bar{Q}u\tilde{H})$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	Q_{HD}	$(H^\dagger D^\mu H)^* (H^\dagger D^\mu H)$	Q_{dH}	$(H^\dagger H)(\bar{Q}dH)$
$Q_{\tilde{W}}$	$\epsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

Type IV: $X^2\Phi^2$		Type VI: $\Psi^2 H^3$		Type VII: $\Psi^2 H^2 D$	
Q_{HG}	$(H^\dagger H) G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar{L}\sigma^{\mu\nu} e)\tau^I H W^I_{\mu\nu}$	$Q_{HL}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{L}\gamma^\mu L)$
$Q_{H\tilde{G}}$	$(H^\dagger H) \tilde{G}^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar{L}\sigma^{\mu\nu} e)\tau^I H B_{\mu\nu}$	$Q_{HL}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{L}\tau^I\gamma^\mu L)$
Q_{HW}	$(H^\dagger H) W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar{Q}\sigma^{\mu\nu} T^A u)\tilde{H} G^A_{\mu\nu}$	Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}\gamma^\mu e)$
$Q_{H\tilde{W}}$	$(H^\dagger H) \tilde{W}^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar{Q}\sigma^{\mu\nu} u)\tau^I \tilde{H} W^I_{\mu\nu}$	$Q_{HQ}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q)$
Q_{HB}	$(H^\dagger H) B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{Q}\sigma^{\mu\nu} u)\tilde{H} B_{\mu\nu}$	$Q_{HQ}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}\tau^I\gamma^\mu q)$
$Q_{H\tilde{B}}$	$(H^\dagger H) \tilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{Q}\sigma^{\mu\nu} T^A d) H G^A_{\mu\nu}$	Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu u)$
Q_{HWB}	$(H^\dagger \tau^I H) W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar{Q}\sigma^{\mu\nu} d)\tau^I H W^I_{\mu\nu}$	Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}\gamma^\mu d)$
$Q_{H\tilde{W}B}$	$(H^\dagger \tau^I H) \tilde{W}^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar{Q}\sigma^{\mu\nu} d)\tilde{H} B_{\mu\nu}$	Q_{Hud}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu d)$

Type VIII: $5 \times (\bar{L}L)(\bar{L}L) + 7 \times (\bar{R}R)(\bar{R}R) + 8 \times (\bar{L}L)(\bar{R}R)$
$+(\bar{L}R)(\bar{R}L) + 4[(\bar{L}R)(\bar{L}R) + \text{h.c.}] = 25(\bar{\Psi}\Psi)(\bar{\Psi}\Psi)$

The SMEFT at dimension-six

D6 operators from SM field content $\Rightarrow$ SMEFT @ D6

Type I: X^3		Type II, III: $H^6,\ H^4D^2$		Type V: $\Psi^2H^3+\mathrm{h.c.}$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_H	$(H^\dagger H)^3$	Q_{eH}	$(H^\dagger H)(\bar{L}eH)$
$Q_{\tilde{G}}$					$(H^\dagger H)(\bar{Q}u\tilde{H})$
Q_W					$(H^\dagger H)(\bar{Q}dH)$
$Q_{\tilde{W}}$					

In this talk:

1. 3 key features of Feynrules package
2. Comment on hurdles in creating package
3. Validation and Ward Identities
4. Conclusions

				ype VII: Ψ^2H^2D	
Q_{HG}					$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{L}\gamma^\mu L)$
$Q_{H\tilde{G}}$					$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{L}\tau^I\gamma^\mu L)$
Q_{HW}					$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}\gamma^\mu e)$
$Q_{H\tilde{W}}$					$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q)$
Q_{HB}					$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}\tau^I\gamma^\mu q)$
$Q_{H\tilde{B}}$	$(H^\dagger H)\tilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{Q}\sigma^{\mu\nu}T^Ad)HG^A_{\mu\nu}$	Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu u)$
Q_{HWB}	$(H^\dagger\tau^I H)W^I_{\mu\nu}B^{\mu\nu}$	Q_{dW}	$(\bar{Q}\sigma^{\mu\nu}d)\tau^I HW^I_{\mu\nu}$	Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}\gamma^\mu d)$
$Q_{H\tilde{W}B}$	$(H^\dagger\tau^I H)\tilde{W}^I_{\mu\nu}B^{\mu\nu}$	Q_{dB}	$(\bar{Q}\sigma^{\mu\nu}d)\tilde{H}B_{\mu\nu}$	Q_{Hud}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu d)$

$$\text{Type VIII: } 5\times(\bar{L}L)(\bar{L}L)+7\times(\bar{R}R)(\bar{R}R)+8\times(\bar{L}L)(\bar{R}R)$$
$$+(\bar{L}R)(\bar{R}L)+4[(\bar{L}R)(\bar{L}R)+\mathrm{h.c.}]=25(\bar{\Psi}\Psi)(\bar{\Psi}\Psi)$$

Inclusion of Cl 3 & 4 operators

Following geosmeft (Helset & Trott 1803.08001, + Martin 2001.01453) rewrite the doublet:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{pmatrix} \Rightarrow \phi_I = \{\phi_1, \phi_2, \phi_3, \phi_4\}$$

Then defining the field-space metric, h:

$$h_{IJ} \equiv \delta_{IJ} - 2c_{H\Box}\phi_I\phi_J + \frac{1}{4}c_{HD}\Gamma^A_{IJ}\Gamma^A_{KL}\phi_K\phi_L \qquad (\Gamma \text{ are matrices})$$

Then the Class 3 operators are written in simple form:

$$\begin{aligned} \mathcal{L}_{\text{cl3}} &= h_{IJ}(D^\mu\phi)^I(D_\mu\phi)^J \\ &= (D^\mu H)^\dagger(D_\mu H) + c_{H\Box}(H^\dagger H)\Box(H^\dagger H) + c_{HD}(H^\dagger D^\mu H)^*(H^\dagger D_\mu H) \end{aligned}$$

Inclusion of Cl 3 & 4 operators

Then the Class 3 and 4 operators are written in simple form:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{cl3}} &= h_{IJ}(D^{\mu}\phi)^{I}(D_{\mu}\phi)^{J} \\
\mathcal{L}_{\mathrm{cl4}} &= -\frac{1}{4}g_{AB}W_{A\mu\nu}W_{B\mu\nu}
\end{aligned}
$$

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\end{aligned}
$$

Can trivially include D8 ops. of Cl. 3 & 4 by updating h and g:

$$
\begin{aligned}
h_{IJ} &= \left[1+\frac{\phi^4}{4}\left(c_{HD}^{(8)}+c_{HD2}^{(8)}\right)\right]\delta_{IJ}-2c_{H\Box}\phi_I\phi_J \\
&\quad +\frac{1}{4}\left(c_{HD}+\phi^2 c_{HD2}^{(8)}\right)\Gamma^A_{IJ}\Gamma^A_{KL}\phi_K\phi_L \\
g_{AB} &= \delta_{AB}-4\left[c_{HW}(1-\delta_{A4})+c_{HB}\delta_{A4}\right]\frac{\phi^2}{2}\delta_{AB} \\
&\quad -4\left[c_{HW}^{(8)}(1-\delta_{A4})+c_{HB}^{(8)}\delta_{A4}\right]\frac{\phi^4}{4}\delta_{AB} \\
&\quad +\left(c_{HWB}+c_{HWB}^{(8)}\frac{\phi^2}{2}\right)\left[(\phi_I\Gamma^A_{IJ}\phi_J)(1-\delta_{A4})\delta_{B4}+A\leftrightarrow B\right] \\
&\quad -c_{HW2}^{(8)}(\phi_I\Gamma^A_{IJ}\phi_J)(\phi_L\Gamma^B_{LK}\phi_K)(1-\delta_{A4})(1-\delta_{B4})
\end{aligned}
$$

Inclusion of Cl 3 & 4 operators

Then the Class 3 and 4 operators are written in simple form:

$$\mathcal{L}_{\mathrm{cl3}} = h_{IJ}(D^\mu\phi)^I(D_\mu\phi)^J$$
$$\mathcal{L}_{\mathrm{cl4}} = -\tfrac{1}{4}g_{AB}W_{A\mu\nu}W_{B\mu\nu}$$

Ca

Inclusion of class 3,4 dimension 8 ops

- geosmeft notation trivializes inclusion of D8 cl3 & 4 ops
- the cl2 operator at D8 is also included, $c_H^{(8)}(H^\dagger H)^4$
- (very) longterm goal to include all D8 ops

$$\begin{aligned} g_{AB} = \;& \delta_{AB} - 4\left[c_{HW}(1-\delta_{A4}) + c_{HB}\delta_{A4}\right]\frac{\phi^2}{2}\delta_{AB} \\ & -4\left[c_{HW}^{(8)}(1-\delta_{A4}) + c_{HB}^{(8)}\delta_{A4}\right]\frac{\phi^4}{4}\delta_{AB} \\ & +\left(c_{HWB} + c_{HWB}^{(8)}\frac{\phi^2}{2}\right)\left[(\phi_I\Gamma^A_{IJ}\phi_J)(1-\delta_{A4})\delta_{B4} + A\leftrightarrow B\right] \\ & -c_{HW2}^{(8)}(\phi_I\Gamma^A_{IJ}\phi_J)(\phi_L\Gamma^B_{LK}\phi_K)(1-\delta_{A4})(1-\delta_{B4}) \end{aligned}$$

Shifts in kinetic terms

CI3 & 4 operators $\rightarrow$ finite shifts in kinetic terms in the $\mathcal{L}$:

$$\begin{aligned} \mathcal{L} &= (D^\mu H)^\dagger (D_\mu H) + c_{H\Box}(H^\dagger H)\Box(H^\dagger H) + Q(H^n) \\ &\rightarrow \tfrac{1}{2}(1 - 2c_{H\Box}v^2)(\partial^\mu h)^2 + Q(h^n) + \cdots \end{aligned}$$

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We redefine h to get a canonical kinetic term:

$$h \rightarrow \frac{1}{\sqrt{1 - 2c_{H\Box}v^2}} h' \sim h' + c_{H\Box}v^2 h' + 2c_{H\Box}^2 v^2 + \cdots$$

$$\begin{aligned}\mathcal{L} &= (D^\mu H)^\dagger (D_\mu H) + c_{H\Box}(H^\dagger H)\Box(H^\dagger H) + Q(H^n) \\ &\rightarrow \tfrac{1}{2}(\partial^\mu h')^2 + \left(\tfrac{1}{1-2c_{H\Box}v^2}\right)^{n/2} Q(h'^n) + \cdots\end{aligned}$$

Shifts in kinetic terms

More formally:

$$\mathcal{L}_{\mathrm{cl3}} = h_{IJ}(D^\mu\phi)^I(D_\mu\phi)^J \qquad H = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{pmatrix}$$

$$h_{IJ} \equiv \langle h_{IJ}\rangle + \text{field dependent part}$$

$$\langle h\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 + \frac{c_{HD}v^2}{2} & 0 \\ 0 & 0 & 0 & 1 - 2c_{H\Box}v^2 + \frac{c_{HD}v^2}{2} \end{pmatrix}$$

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We can rotate (shift) to the physical Φ basis:

$$\Phi = \begin{pmatrix}\phi^+ \\ \phi^- \\ \chi \\ h\end{pmatrix} = \langle h_{IJ}\rangle^{1/2} V^{-1}_{JK}\phi_K \qquad V_{JK} \equiv \begin{pmatrix} -\frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Shifts in kinetic terms

More formally:

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We can rotate to the physical Φ basis:

$$\Phi = V^{-1}\langle h\rangle^{1/2}\phi \qquad \Leftrightarrow \qquad \phi = \langle h\rangle^{-1/2}V\Phi$$

Shifts in kinetic terms

More formally:

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$$\begin{aligned} h_{IJ}(D\phi)_I(D\phi)_J &\to \langle h\rangle(D\phi)(D\phi) + (\text{fields})(D\phi)(D\phi) \\ &= \langle h\rangle\langle h\rangle^{-1}(DV\Phi)(DV\Phi) + (\text{fields})(D\phi)(D\phi) \end{aligned}$$

Shifts in kinetic terms

More formally:

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Define $h^{-1,1/2}$ perturbatively:

$$\begin{aligned} h^{-1} &\equiv 1 - dh + dh^2 + \cdots \\ h^{1/2} &\equiv 1 + \frac{dh}{2} - \frac{dh^2}{8} + \cdots \end{aligned}$$

Shifts in kinetic terms

More formally:

$$\mathcal{L}_{\text{cl3}} = h_{IJ}(D^\mu \phi)^I (D_\mu \phi)^J \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{pmatrix}$$

Inclusion of D6^2 and D8 shifts

We ca

- Scalar field shifts are implemented in this way up to and including D6^2 and D8
- Similarly the shifts in vector fields are included $\mathcal{W}^\mu = \{W^+, W^-, Z, A\} = \langle g \rangle^{1/2} U^{-1} W^\mu$ up to and including D6^2 and D8
- This is the only tool to include shifts to $\mathcal{O}(1/\Lambda^4)$

h_{IJ} $\phi)(D\phi)$

Define $h^{-1,1/2}$ perturbatively:

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The background field method

For QFT fundamentals and QCD: L. Abbott, Acta Phys. Polon. B 13 (1982) 33.

For EW SM: Denner, Weiglein, Dittmaier, arXiv:9410338

1. Take your fields, ϕ, and double them: $\phi \to \phi + \hat{\phi}$
2. Gauge fix only the quantum fields, ϕ, not the background fields $\hat{\phi}$
3. BG fields can't propagate, source the quantum fields
4. Quantum fields transform as $\Delta\phi \to \phi\Delta\alpha - \partial(\Delta\alpha)$
   Background fields transform as $\delta\phi \to \phi\delta\alpha - \partial(\delta\alpha)$
5. The effective action, Γ in traditional gauges $= \Gamma_{\mathrm{BGFM}}$
   $\Gamma[\phi] = \Gamma[\phi, \hat{\phi}]\Big|_{\phi\to 0}$

In the SMEFT

Trott, Helset, Paraskevas, arXiv:1803.08001

$$\mathcal{G}^X = \partial_\mu W^X_\mu - g_2 \epsilon^{XCD} \hat{W}^C_\mu W^D_\mu + \frac{\xi}{2} \hat{g}^{XC} \phi^I \hat{h}_{IK} \tilde{\gamma}^K_{C,J} \hat{\phi}^J$$

$$\mathcal{L}_{GF} = -\frac{\hat{g}_{AB}}{2\xi} \mathcal{G}^A \mathcal{G}^B$$

In the SMEFT

Trott, Helset, Paraskevas, arXiv:1803.08001

$$
\begin{aligned}
\mathcal{G}^X &= \partial_\mu W^X_\mu - g_2 \epsilon^{XCD} \hat{W}^C_\mu W^D_\mu + \frac{\xi}{2} \hat{g}^{XC} \phi^I \hat{h}_{IK} \tilde{\gamma}^K_{C,J} \hat{\phi}^J \\
\mathcal{L}_{GF} &= -\frac{\hat{g}_{AB}}{2\xi} \mathcal{G}^A \mathcal{G}^B
\end{aligned}
$$

hatted quantities, $\hat{W}$, $\hat{h}$, etc include only BG fields
here (and only here) unhatted quantities = Q fields

In the SMEFT

Trott, Helset, Paraskevas, arXiv:1803.08001

$$
\begin{aligned}
\mathcal{G}^X &= \partial_\mu W^X_\mu - g_2 \epsilon^{XCD} \hat{W}^C_\mu W^D_\mu + \tfrac{\xi}{2} \hat{g}^{XC} \phi^I \hat{h}_{IK} \tilde{\gamma}^K_{C,J} \hat{\phi}^J \\
\mathcal{L}_{GF} &= -\tfrac{\hat{g}_{AB}}{2\xi} \mathcal{G}^A \mathcal{G}^B
\end{aligned}
$$

Ghost terms:

$$
\begin{aligned}
\mathcal{L}_{\rm ghost} = \hat{g}_{AB} \bar{u}^B \Big[& -\partial^2 \delta^A_C - \overleftarrow{\partial} \tilde{\epsilon}^A_{DC} (W^{D\mu} + \hat{W}^{D\mu}) \\
& + \tilde{\epsilon}^A_{DC} \hat{W}^D_\mu \partial^\mu - \tilde{\epsilon}^A_{DE} \tilde{\epsilon}^E_{FC} \hat{W}^D_\mu (W^{F\mu} + \hat{W}^{F\mu}) \\
& - \tfrac{\xi}{4} \hat{g}^{AD} (\phi^J + \hat{\phi}^J) \tilde{\gamma}^I_{CJ} \hat{h}_{IK} \tilde{\gamma}^K_{DL} \hat{\phi}^L \Big] u^C
\end{aligned}
$$

In the SMEFT

Trott, Helset, Paraskevas, arXiv:1803.08001

Gauge fixed to $\mathcal{O}(1/\Lambda^4)$

- BG Gauge fixing included
- field shifts consistent to $\mathcal{O}(1/\Lambda^4)$
  $\Rightarrow$ GF consistent to $\mathcal{O}(1/\Lambda^4)$

$\hat{\phi}^J$

Ghost

$$\begin{aligned}\mathcal{L}_{\mathrm{ghost}} = \hat{g}_{AB}\bar{u}^B \Big[& -\partial^2 \delta^A_C - \overleftarrow{\partial}\, \tilde{\epsilon}^A_{DC}(W^{D\mu} + \hat{W}^{D\mu}) \\ & + \tilde{\epsilon}^A_{DC}\hat{W}^D_\mu \partial^\mu - \tilde{\epsilon}^A_{DE}\tilde{\epsilon}^E_{FC}\hat{W}^D_\mu (W^{F\mu} + \hat{W}^{F\mu}) \\ & - \tfrac{\xi}{4}\hat{g}^{AD}(\phi^J + \hat{\phi}^J)\tilde{\gamma}^I_{CJ}\hat{h}_{IK}\tilde{\gamma}^K_{DL}\hat{\phi}^L \Big] u^C\end{aligned}$$

Some hurdles

1. Field multiplicity slows FRs to a crawl
   (Can restrict to n-point vertices only)

2. Formcalc can't handle the complexity of rules
   simple Mathematica nb simplifies rules, e.g. h^3 vertex:
   $3igc_{60}[10c_H v^2+\lambda(-4-12c_{H\Box}v^2+3c_{HD}v^2)]+2i(4c_{H\Box}-c_{HD})gc_{60}p_1\cdot p_2+\cdots$
   $\Rightarrow r_1 + r_2 p_1 \cdot p_2 + cdots$

Validation

FRs presented in arXiv:2010.15852
Ward Identities confirmed at one-loop D6 in arXiv:2010.08451
Ward Identities confirmed at one-loop for CI2,3,4 $D6^2$ in 2010.15852

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$$\Sigma_L^{AA}(k^2) = 0$$

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$$\Sigma_L^{AA}(k^2) = 0$$

Consider c_{HW}^2 contribution:

$$\Sigma_L^{AA} = -c_{HW}^2 \frac{g_1^2 g_2^4 v^6}{128\pi^2\epsilon(g_1^2+g_2^2)\xi}(3+9\xi-7\xi^2+3\xi^3)(1-\Delta) \to 0$$

Validation

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Ward Identity	Wilson coefficient combination violating WId
$\Sigma_L^{AA}(k^2) = 0$	c_{HW}^2
$\Sigma_L^{AZ}(k^2) = 0$	$c_{HB}^2,\ c_{HW}^2,\ c_{HB}c_{HWB},\ c_{HW}c_{HWB}$
$\Sigma_L^{\mathcal{W}\mathcal{W}} + \bar{M}_W\Sigma^{\phi W} = 0$	$c_{H\Box}^2,\ c_{HD}^2,\ c_{HW}^2,\ c_{HWB}^2,\ c_{H\Box}c_{HD}$
$k^2\Sigma^{\mathcal{W}\phi} + \bar{M}_W\Sigma^{\phi\phi} + \bar{g}_2(\cdots)T^H = 0$	$c_{H\Box}^2,\ c_{HD}^2,\ c_{HB}^2,\ c_{HW}^2,\ c_{HWB}^2,\ c_{H\Box}c_{HD},\ c_{HB}c_{HWB}$
$\Sigma_L^{\mathcal{Z}\mathcal{Z}} - i\bar{M}_Z\Sigma^{\chi Z} = 0$	$c_{H\Box}^2,\ c_{HD}^2,\ c_{HB}^2,\ c_{HW}^2,\ c_{HWB}^2,\ c_{H\Box}c_{HD},$ $c_{HB}c_{HWB},\ c_{HW}c_{HWB}$
$k^2\Sigma^{\mathcal{Z}\chi} - i\bar{M}_Z\Sigma^{\chi\chi} + i\bar{g}_Z(\cdots)T^H = 0$	$c_{H\Box}^2,\ c_{HD}^2,\ c_{HB}^2,\ c_{HW}^2,\ c_{HWB}^2,\ c_{H\Box}c_{HD},$ $c_{HB}c_{HWB},\ c_{HW}c_{HWB}$

Summary

Feynrules package for SMEFT in the background field formalism exists!
https://feynrules.irmp.ucl.ac.be/wiki/SMEFT_BGFM

1. SMEFT @ D6 all ops + D8 cl2,3,4 ops
2. consistent to $\mathcal{O}(1/\Lambda^4)$
3. BG gauge fixed $\rightarrow$ many benefits at one loop
4. Validated against the Ward Identities at one loop
   D6+D6^2
5. Anc files on arXiv and Feynrules webpage include the rules and notebooks necessary to confirm Ward Ids
6. Analytic understanding of processes at one loop is invaluable
   $\rightarrow$ cross checks of automated efforts
   $\rightarrow$ better understanding of the underlying physics