Tools 2020 - COFFE in brief

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What is COFFE?

- COFFE (COrrelation Function Full-sky Estimator) $\to$ a cosmology code, developed at the University of Geneva, which can compute the following quantities:
- the full-sky and flat-sky redshift-space 2-point correlation function (2PCF) of galaxy number counts, $\xi(\bar{z}, r, \mu)$, and its angular equivalent, $\xi(\bar{z}, \theta)$
- multipoles of the 2PCF, defined as:

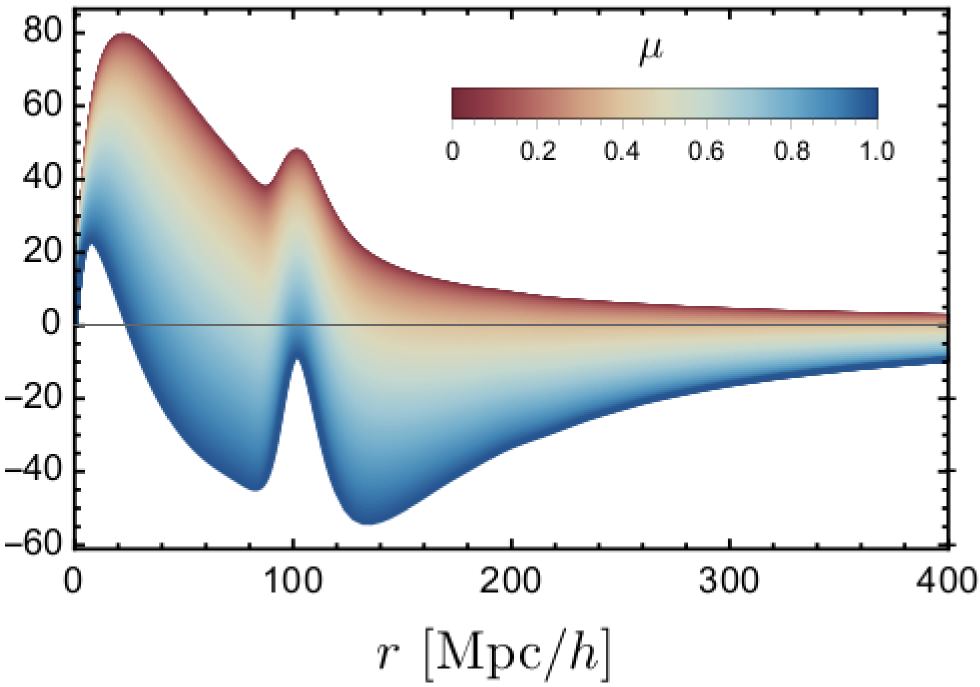
$$\xi_\ell(\bar{z}, r) \equiv \frac{2\ell + 1}{2} \int_{-1}^{1} \mathrm{d}\mu \, \xi(\bar{z}, r, \mu) \, P_\ell(\mu)$$

where $P_\ell(\mu)$ are the Legendre polynomials.

- the covariance of the even multipoles of the 2PCF, $\mathrm{cov}(\xi_\ell, \xi_{\ell'})(\bar{z}, r, r')$
- the redshift-averaged multipoles of the 2PCF, defined as:

$$\Xi_\ell(r, z_1, z_2) = \frac{\mathcal{H}_0}{z_2(r) - z_1(r)} \int\limits_{z_1(r)}^{z_2(r)} dz \, \frac{\xi_\ell(r, z)}{\mathcal{H}(z)(1 + z)}$$

A possible output from COFFE can look something like this:



Example output from COFFE for the flat-sky 2PCF at $\bar{z} = 0.1$ for various values of μ

Theory - the 2PCF

In brief:

- when we count galaxies in a survey, we observe them in a given direction and at a given redshift, then split the survey into multiple redshift bins with mean redshift $\bar{z}$ and some bin width Δz
- the expression from linear perturbation theory for the over-density $\Delta(\mathbf{n}, z) \equiv N(\mathbf{n}, z)/\langle N(\mathbf{n}, z)\rangle - 1$ of galaxies at a redshift z and in direction $\mathbf{n}$ is given by:

$$\Delta(\mathbf{n}, z) = \Delta^{\mathrm{den}} + \Delta^{\mathrm{rsd}} + \Delta^{\mathrm{len}} + \Delta^{\mathrm{d1}} + \Delta^{\mathrm{d2}} + \Delta^{\mathrm{g1}} + \Delta^{\mathrm{g2}} + \Delta^{\mathrm{g3}} + \Delta^{\mathrm{g4}} + \Delta^{\mathrm{g5}}$$

where the various superscripts denote the contributions, such as the real-space galaxy overdensity, redshift-space distortions (RSD), gravitational lensing, etc., while $\langle \cdots \rangle$ denotes an ensemble (angular) average

- the 2PCF of the above Δ is defined as:

$$\xi(z_1, z_2, \mathbf{n}_1, \mathbf{n}_2) \equiv \langle \Delta(z_1, \mathbf{n}_1)\Delta(z_2, \mathbf{n}_2)\rangle$$

- due to isotropy and homogeneity, the above is just a function of three variables, usually taken to be $\{z_1, z_2, \cos\theta = \mathbf{n}_1 \cdot \mathbf{n}_2\}$
- we can re-express it in any other three variables; in particular, the ones used by COFFE are the following:

$$\bar{z} = \frac{z_1 + z_2}{2}$$
$$r = \sqrt{\chi^2(z_1) + \chi^2(z_2) - 2\chi(z_1)\,\chi(z_2)\,\cos\theta}$$
$$\mu = \frac{\chi(z_1) - \chi(z_2)}{r} = \frac{r_\parallel}{r} = \cos\alpha$$

where $\chi(z)$ is the comoving distance to redshift z. r denotes just the separation between the two points.

The make it easier to visualize what's going on, the geometry is displayed in the schematic below.

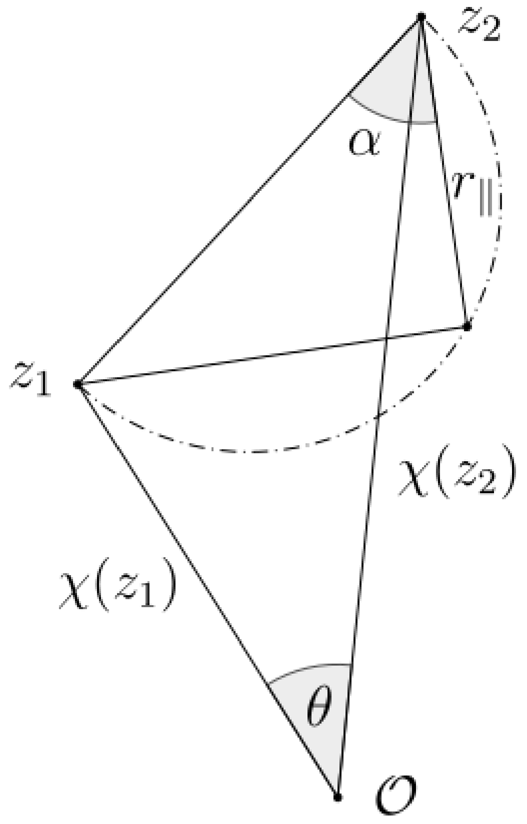


Diagram of all of the relevant variables for the 2PCF

Hence, COFFE can compute the 2PCF with contributions from all of the Δs, and the other various quantities defined in the first section.

Side note: there's a one-to-one correspondence with the *angular power spectrum*, that is, the standard C_ℓs (as computed by CLASS or CAMB):

$$\xi(\theta, z_1, z_2) = \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell(z_1, z_2) \mathcal{P}_\ell(\cos\theta)$$

Side note 2: if you're used to simulations, those usually output the comoving-space matter power spectrum, and the 2PCF is just the Fourier transform of that (caveat: can only define a Fourier transform on the equal-redshift hypersurface!):

$$\xi(r, z) = \frac{1}{(2\pi)^3} \int \mathrm{d}^3k \, e^{i\mathbf{k}\cdot\mathbf{r}} P(k, z)$$

Motivation for another code

Future galaxy surveys (such as SKA2, Euclid, DESI...) will go high redshifts, and will explore very large volumes, i.e. will have large redshift bin widths Δz, with large values of the angle θ. There are some problems and/or shortcomings when applying the standard estimators on such large scales:

- an analysis with the matter power spectrum is fast, but only works for small redshift bins, and effects such as gravitational lensing are ignored (or cannot be consistently included)

- an analysis with the C_ℓs can include all of the effects, but for spectroscopic surveys needs many (n) thin redshift bins, so we need roughly $\sim n^2$ C_ℓs. This is not feasible if we need to do an MCMC, which requires $\mathcal{O}(10^5)$ or so evaluations, while simultaneously including almost all of the effects (especially lensing).
- observers usually use the multipoles of the redshift-space 2PCF to extract relevant quantities such as the galaxy bias b and the growth rate f

The takeaway is that we need a code that consistently takes into account all of the effects, while still being fast and accurate $\to$ COFFE

Code structure

COFFE is made up of several main modules:

- `parser` - parses the settings file containing the cosmology and precision parameters
- `background` - computes the various background quantities as a function od redshift, such as the scale factor a, the comoving distance χ, etc.
- `integrals` - computes the spherical Bessel transforms of the matter power spectrum, i.e. the integrals:

$$I_\ell^n(r) = \frac{1}{2\pi^2} \int_{-\infty}^{\infty} \mathrm{d}k \, P(k) \, \frac{j_\ell(kr)}{(kr)^n}$$

- `functions` - contains the definitions of the actual terms $\langle \Delta^{\mathrm{den}} \Delta^{\mathrm{den}} \rangle$, $\langle \Delta^{\mathrm{rsd}} \Delta^{\mathrm{den}} \rangle$ etc. For any terms integrated along the line of sight, such as lensing-lensing, or density-lensing, contains the *integrands* only.
- `signal` - if needed, integrates the `functions` module along the line of sight (for the 2PCF), and/or over the angle μ (for the multipoles), and/or over the redshift range $[z_1, z_2]$ (for redshift-averaged multipoles)
- `corrfunc`, `multipoles`, `average_multipoles`, `covariance` - respectively computes the 2PCF, multipoles, the redshift-averaged multipoles, or the covariance, using all of the above modules, for all of the input data
- `output` - writes the output files

Future outlook

Some of the features we plan to implement include, but are not limited to the following:

- covariance for multiple populations of galaxies
- C++ compatibility
- Python wrapper

References

- The full-sky relativistic correlation function and power spectrum of galaxy number counts: I. Theoretical aspects
- COFFE: a code for the full-sky relativistic galaxy correlation function
- The flat-sky approximation to galaxy number counts - redshift space correlation function