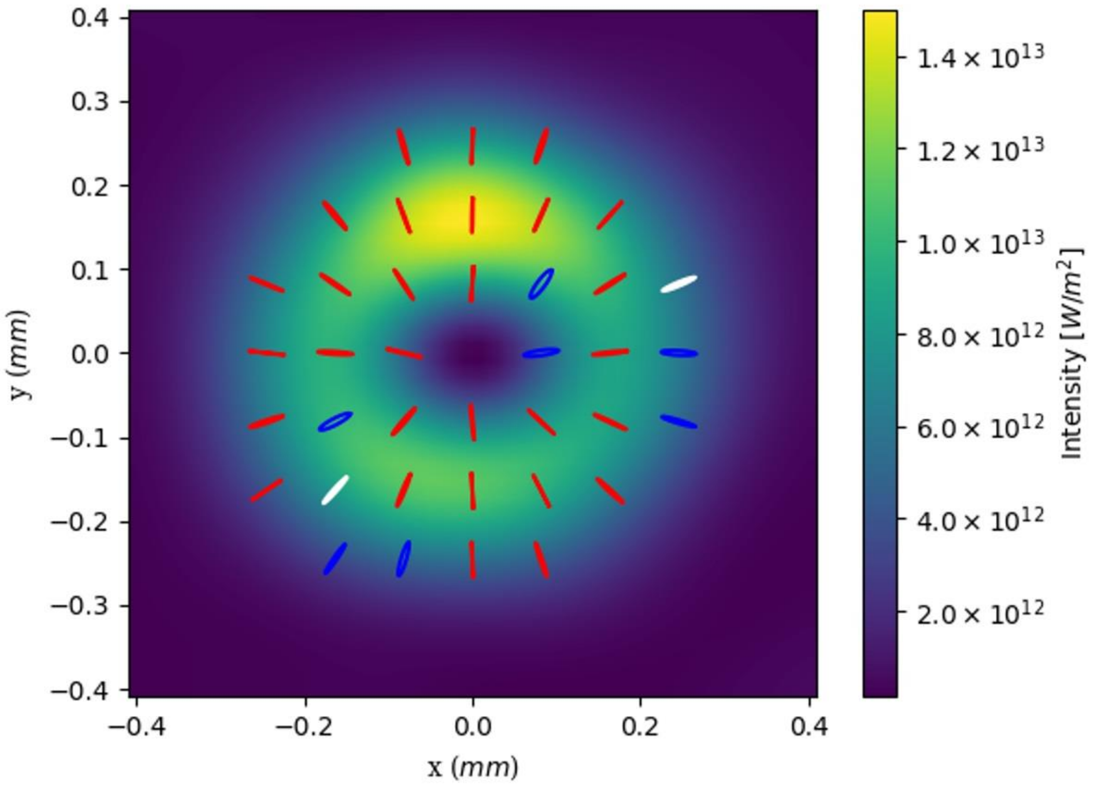
Afterburner configurations to control polarisation of FEL output



Jenny Morgan
University of Strathclyde

Computational Nonlinear and Quantum Optics

The Cockcroft Institute of Accelerator Science and Technology

University of Strathclyde Science

SUPA

department of physics at the university of strathclyde

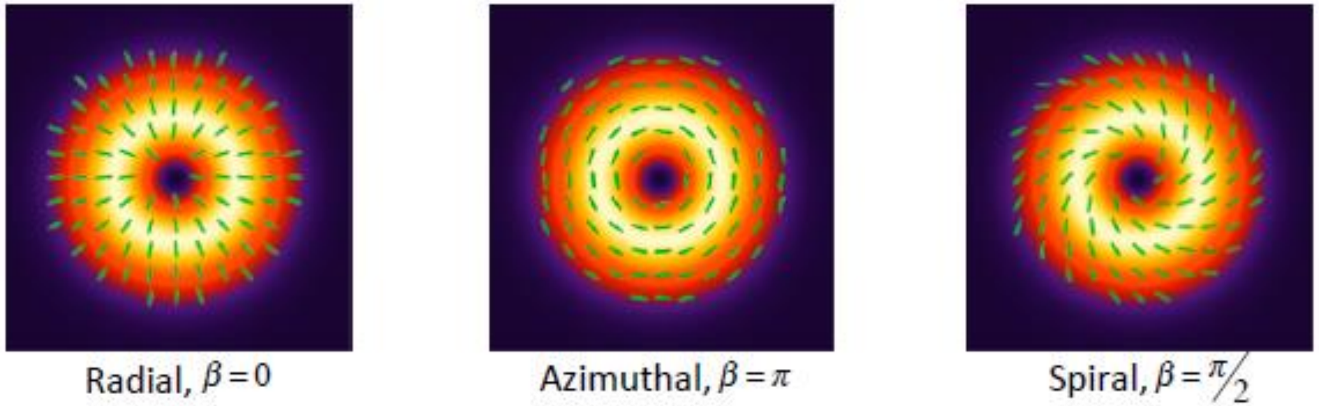
Overview

- What are Poincaré beams?
- Transverse mode generation in a FEL
- Poincaré beam generation in a FEL
- Results
- Other polarisation variation

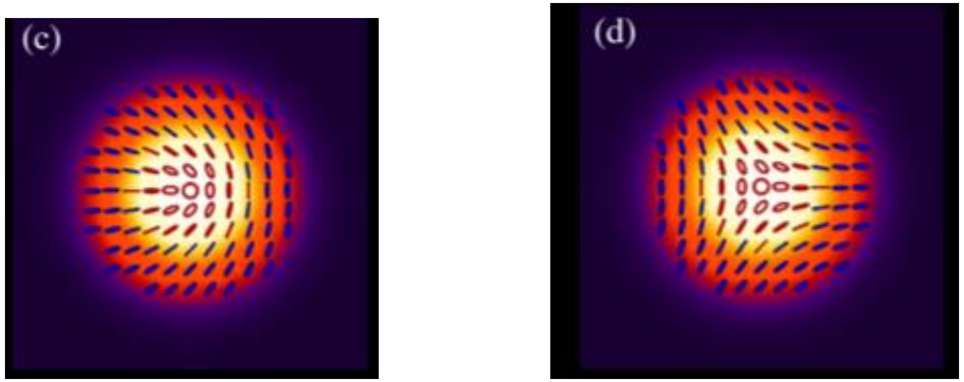
Transverse Polarisation

Poincaré beams – polarisation which varies transversely across the beam.

Cylindrical Vector



Full Poincaré beams



Mapping Polarisation

From the Stokes Vectors,

$$S_0 = |E_x|^2 + |E_y|^2 \,, \qquad S_1 = |E_x|^2 - |E_y|^2$$

$$S_2 = \mathrm{Re}\left(E_x^* E_y\right) \,, \qquad S_3 = \mathrm{Im}(E_x^* E_y)$$

Calculate the ellipticity, χ , and orientation, ψ, of the polarisation ellipse,

$$\chi = \frac{1}{2}\sin^{-1}\left(\frac{S_3}{S_0}\right), \qquad \psi = \frac{1}{2}\tan^{-1}\left(\frac{S_2}{S_1}\right)$$

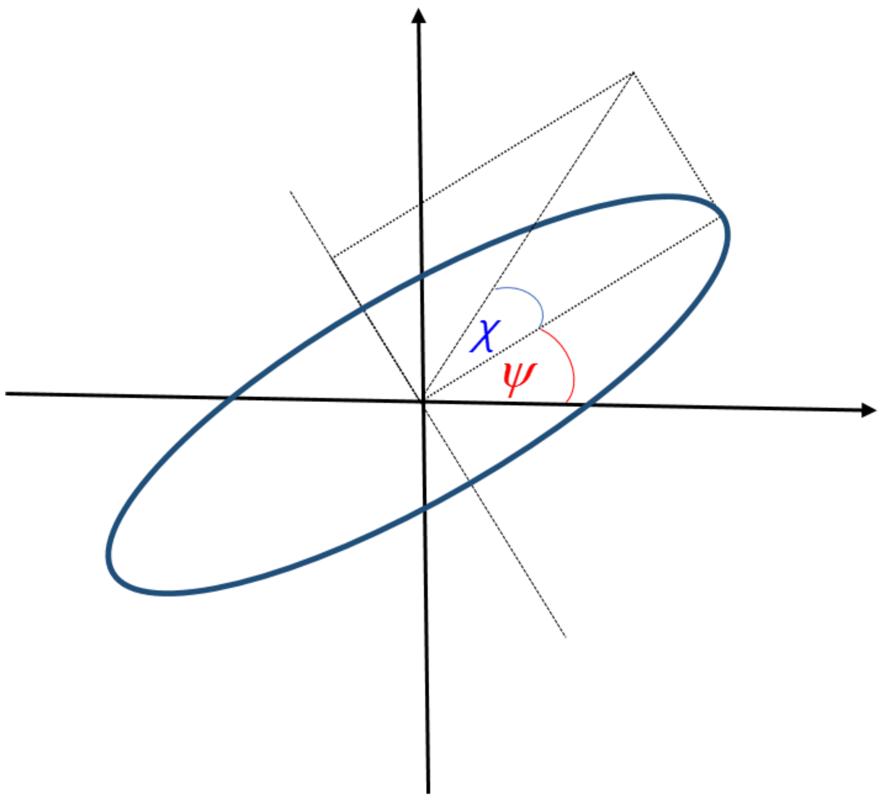


Figure: Polarisation ellipse

Transverse Polarisation

Poincaré Beams:

$$E(\boldsymbol{r},\phi) = E_1(\boldsymbol{r},\phi)\hat{\boldsymbol{e}}_{\boldsymbol{1}} + e^{i\beta}E_2(\boldsymbol{r},\phi)\hat{\boldsymbol{e}}_{\boldsymbol{2}}$$

$E_1(\boldsymbol{r},\phi), E_2(\boldsymbol{r},\phi)$ are spatial modes

$\hat{\boldsymbol{e}}_{\boldsymbol{1}}$, $\hat{\boldsymbol{e}}_{\boldsymbol{2}}$ are orthogonal polarisation vectors

β is the phase between the two modes

Laguerre-Gaussian beams

Laguerre-Gaussian modes make up a complete orthonormal basis set:

$$LG_p^l(r,\phi) = \sqrt{\frac{2p!}{\pi(p+|l|!)}}\frac{1}{w(z)}\left(\frac{r\sqrt{2}}{w(z)}\right)^{|l|}\exp\left(\frac{-r^2}{w(z)^2}\right)L_p^l\left(\frac{2r^2}{w(z)^2}\right)\exp(il\phi)$$

l is the OAM index:

$r = \sqrt{x^2 + y^2}$ is the radial coordinate

$\phi = \tan^{-1}(y/x)$ is the azimuthal coordinate

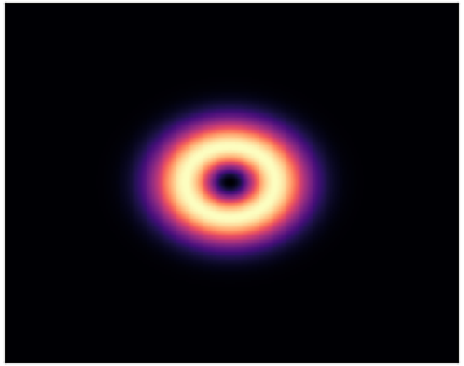


Figure: Intensity profile of the $l = 1$ mode

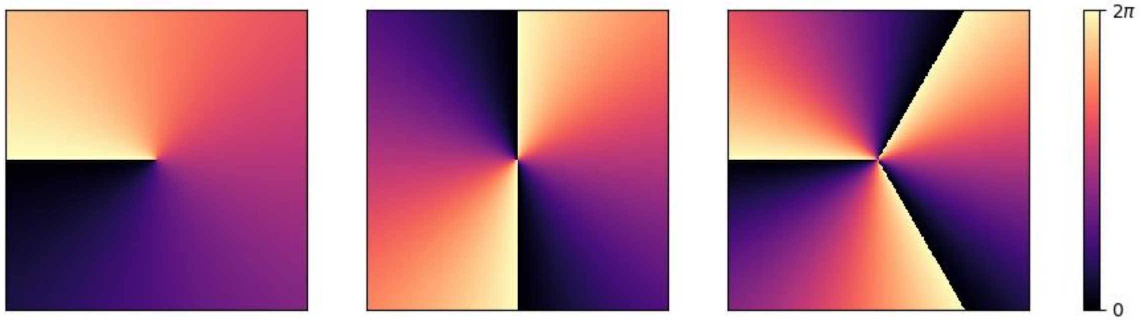
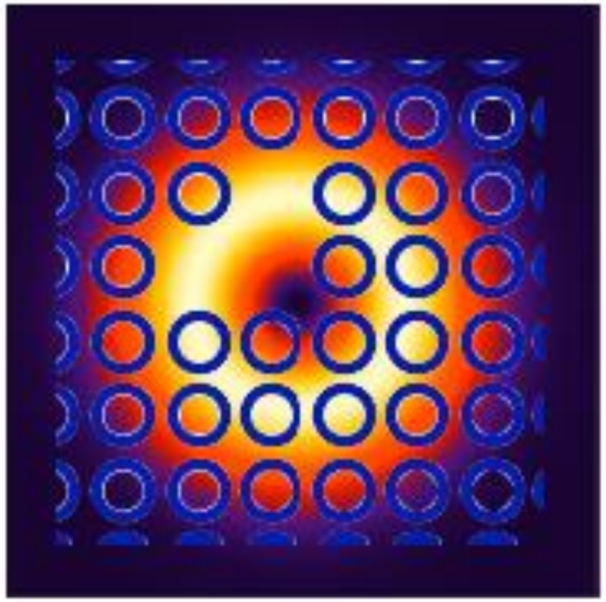


Figure: Transverse phase distributions due of different Laguerre - Gaussian modes: $l = 1,2,3$ respectively.

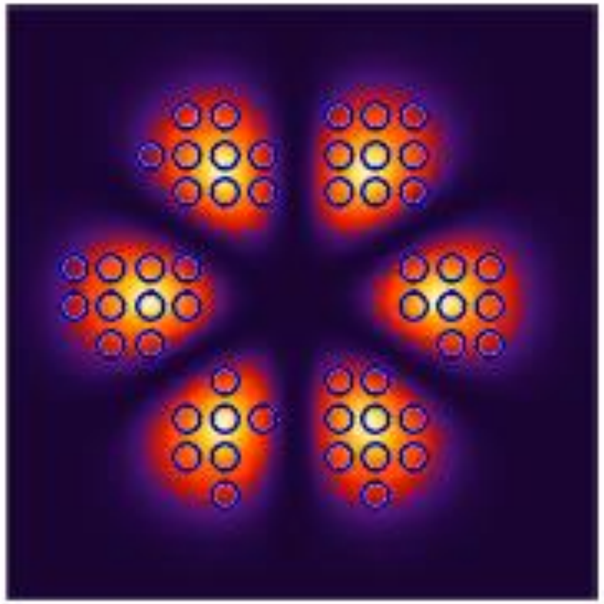
[1] Alison M. Yao and Miles J. Padgett, "*Orbital angular momentum: origins, behavior and applications*," Adv. Opt. Photon. 3, 161-204 (2011)

Combining Laguerre-Gaussian beams

Single right or left hand polarisation: Scalar combination of spin and angular momentum:



$$E = LG_0^1 \hat{\boldsymbol{e}}_{\boldsymbol{L}}$$



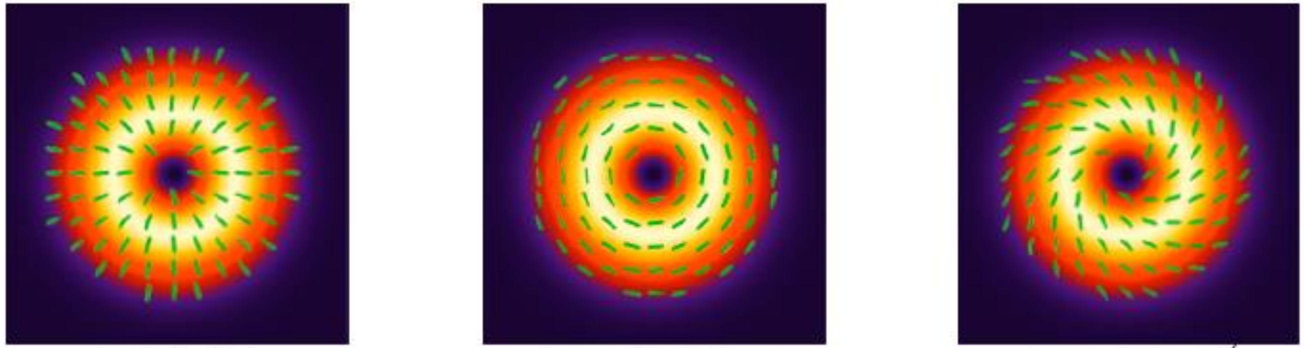
$$E = (LG_0^3 + LG_0^{-3})\hat{\boldsymbol{e}}_{\boldsymbol{L}}$$

Combining Laguerre-Gaussian beams

Vector combination of spin and OAM: $E = LG_p^{l_L}\hat{\boldsymbol{e}}_{\boldsymbol{L}} + e^{i\beta} LG_p^{l_R}\hat{\boldsymbol{e}}_{\boldsymbol{R}}$

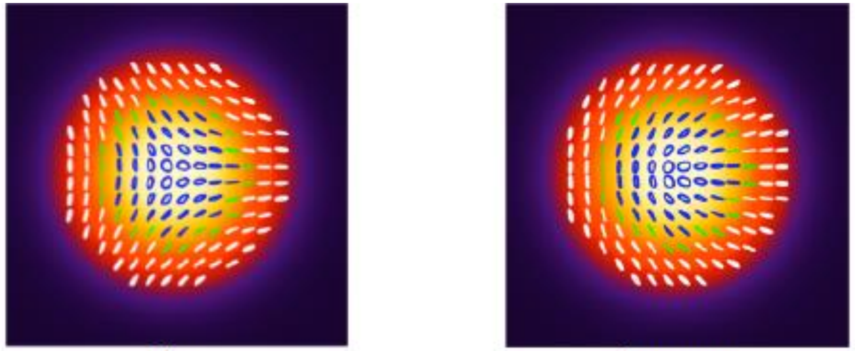
Cylindrical vector beams l_L=-l_R

l_L=1



Radial, $\beta = 0$ Azimuthal, $\beta = \pi$ Spiral, $\beta = \pi/2$

Full Poincaré Beams $l_L \neq$-l_R

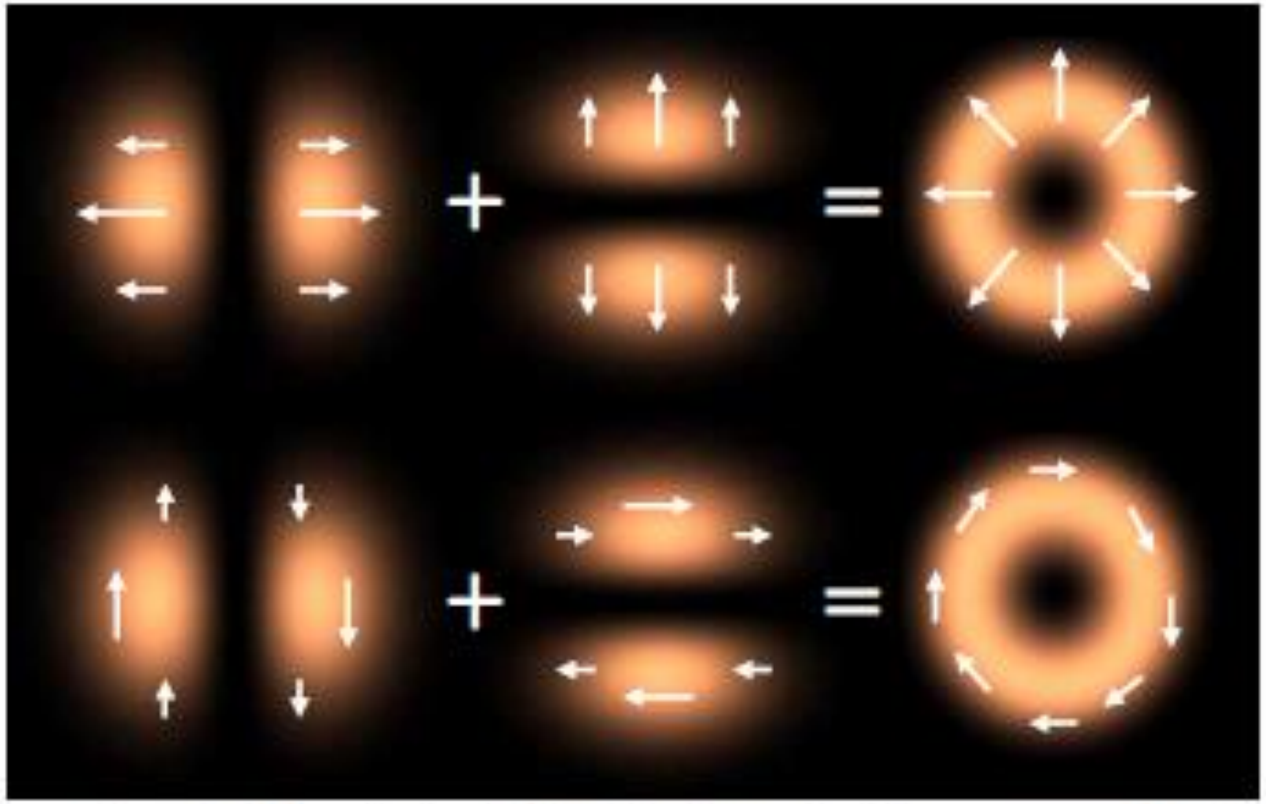


Star   Lemon

Hermit Gaussian Modes

Using linear polarisation cylindrical vector beams can be expressed in terms of Hermite-Gaussian modes.

$$E = HG_{mn}\hat{\boldsymbol{e}}_{\boldsymbol{x}} + HG_{mn}\hat{\boldsymbol{e}}_{\boldsymbol{y}}$$



[1] Qiwen Zhan, "*Cylindrical vector beams: from mathematical concepts to applications*," Advances in Optics and Photonics. 1.1, 161-1-57 (2009)

Generation methods

Techniques for generating Poincaré beams include:

- interferometric techniques
- q-plates
- liquid crystal spatial light modulators.

These use external conversion optics which superimpose orthogonally polarized transverse modes.

This limits the wavelength and power of the Poincaré beams generated through these methods.

No optics method currently exists at X-rays.

We want to generate bight generating bright, tunable, coherent Poincaré beams with wavelengths as short as X-rays.

Solution: Use a FEL.

Free Electron Lasers, FELs

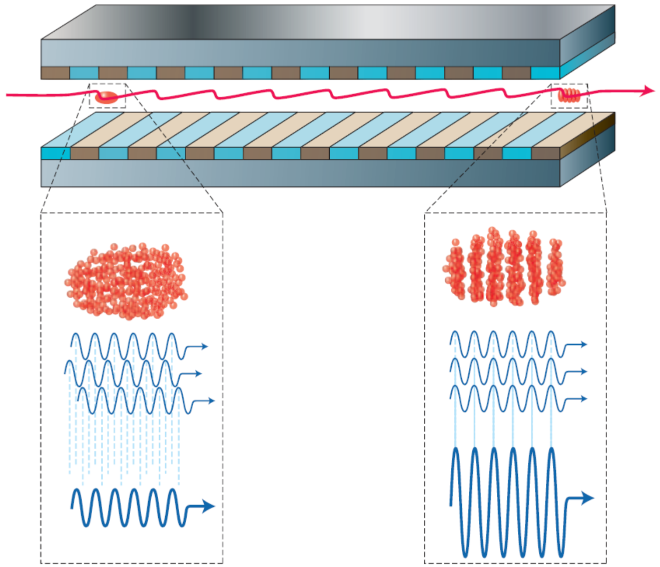


Figure : Electrons exchange energy with the radiation field causing them to bunch together and emit coherently[1].

Tuneable wavelength

High-Brightness

Short wavelength : *(down to a few Armstrong)*

Short pulse durations : *(few femto-seconds.)*

Resonant wavelength:

$$\lambda_n = \frac{\lambda_u}{2n\gamma^2}(1 + K^2 + \theta^2\gamma^2)$$

λ_u - undulator wavelength
n - harmonic number
θ – angle from propagation axis
γ - electron energy
K- rms undulator parameter

[1] Brian WJ McNeil and Neil R Thompson. X-ray free-electron lasers. *Nature photonics*, 4(12):814, 2010.

Transverse modes from harmonic emission

Emission from one electron in an undulator.

On axis radiation, ($\theta = 0$):

Helical undulator – No harmonic emission

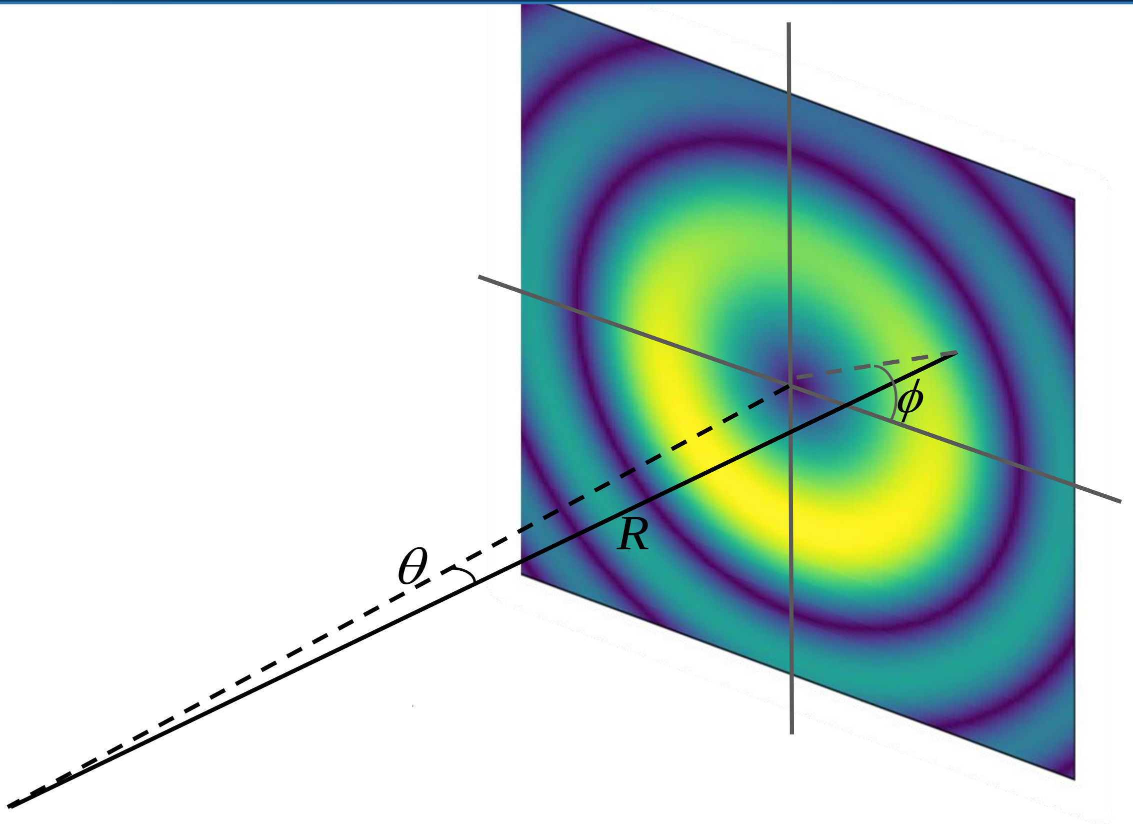
Planar undulator - No **even** harmonic emission

Off axis,

Helical undulator - The phase of the field varies with azimuthal position for $h > 1$.

Helical phase structure carries OAM with $|l| = h - 1$.

Planar undulator – At $\phi = 0$ and $\phi = \pi$ the radiation is 180° out of phase corresponding to the HG modes.



For a Gaussian electrons beam: The angular emission is determined by the beams finite electron beam directions.

Bunching in the e-beam determines the spectrum radiated.

Shigemi Sasaki and Ian McNulty. Proposal for generating brilliant x-ray beams carrying orbital angular momentum. Phys. Rev. Lett., 100(12):124801, 2008.

Higher order modes carry OAM

Helical undulator – LG modes.
Planar undulator – HG modes.

Can access the higher order modes through an afterburner set up:

Electrons bunched at the second harmonic of the undulator, $\lambda_b = \lambda_r/\mathrm{h},$ radiate with higher order transverse modes.

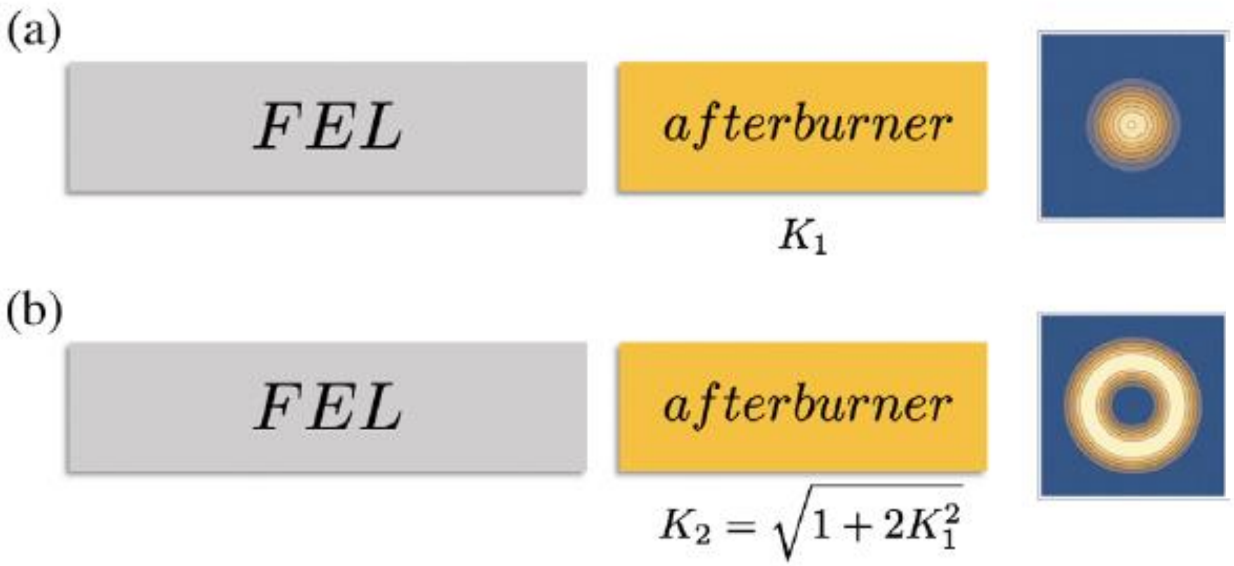
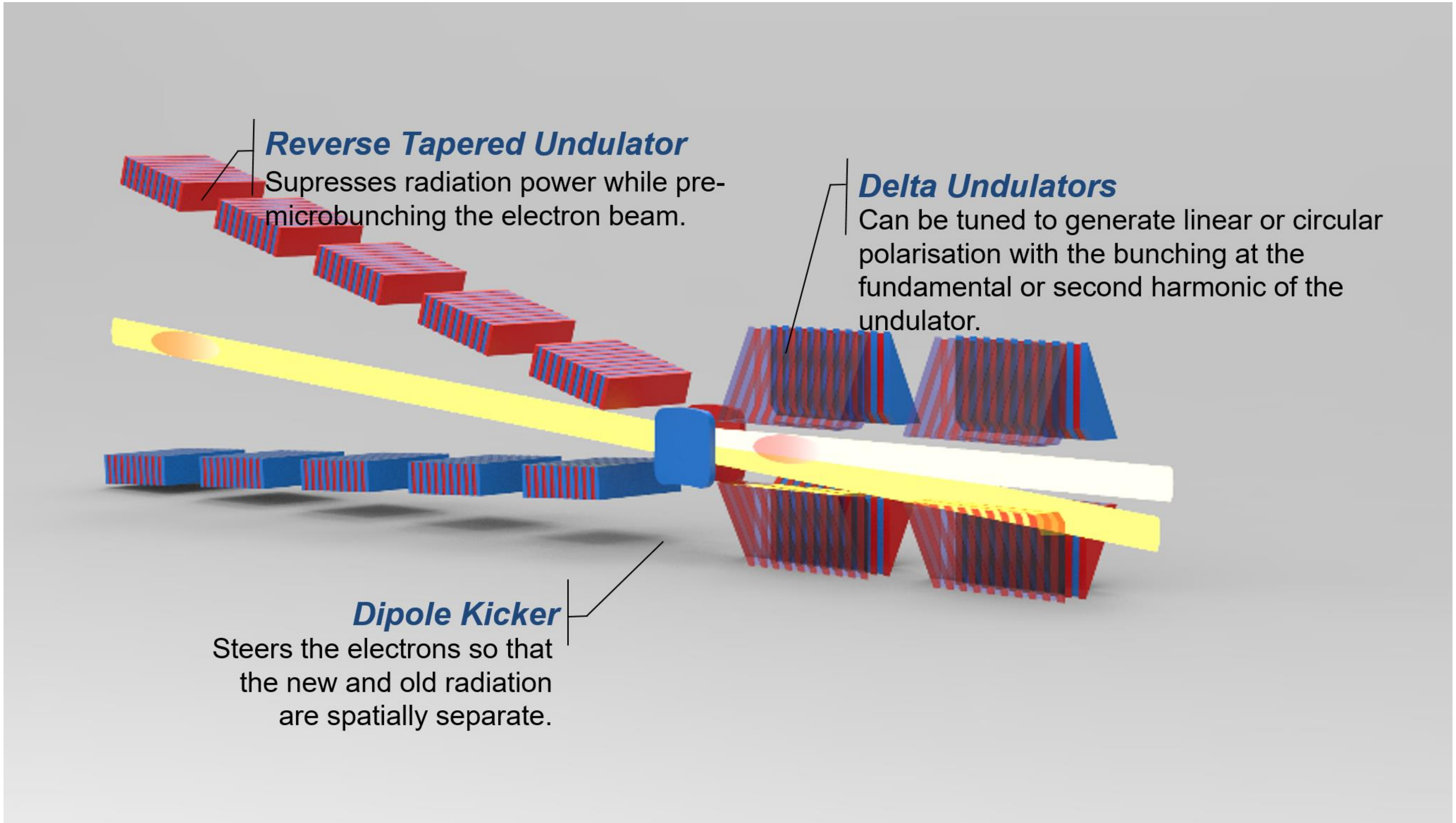


TABLE I. Possible transverse mode and polarisation combinations.

Undulator	Polarisation	Transverse mode
Left-Hand Helical	Right-hand circular	LG_0^{-1}
Right-Hand Helical	Left-hand circular	LG_0^{+1}
x-Poled Plannar	y linear	HM_{01}
y-Poled Plannar	x linear	HM_{10}

Hemsing, E. (2020). Coherent photons with angular momentum in a helical afterburner. *Physical Review Accelerators and Beams*, *23*(2), 020703

Schematic of set up



Schematic of set up

The FEL is modelled using the FEL simulation code Puffin[1].

Electrons are bunched in a tapered FEL amplifier based on LCLSII parameters.

The electrons are extracted from the amplifier with a bunching parameter $|b| = 0.45$

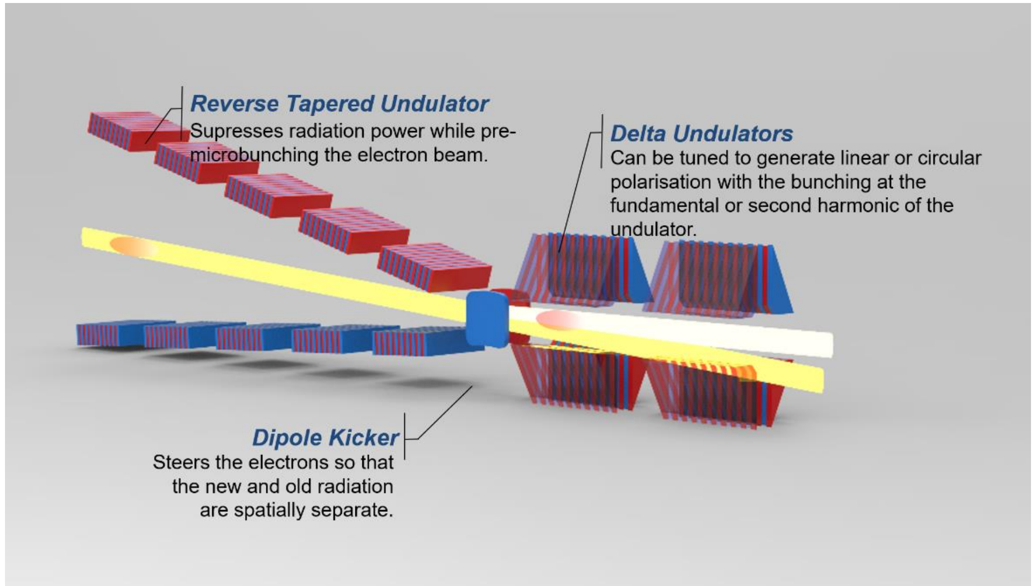


TABLE I. Simulation Parameters

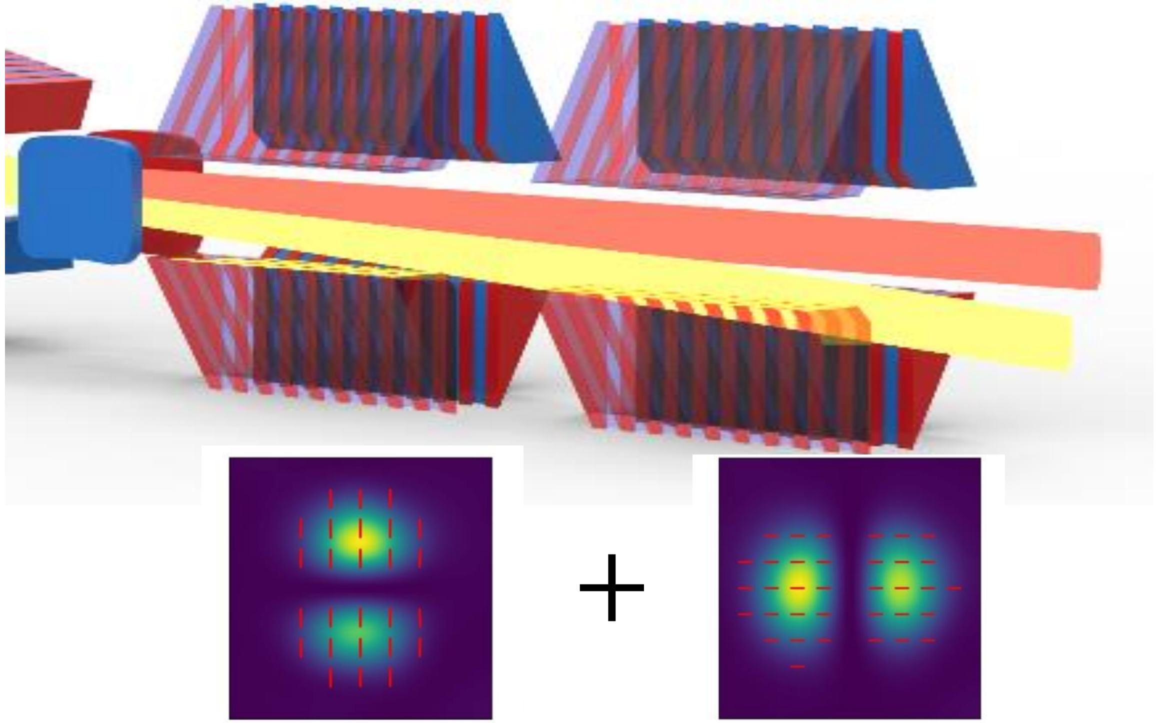
Parameter	Value	
Bunching Stage		
Electron beam energy [GeV]	4	
Peak current, I_0 [kA]	1	
rms energy spread σ_γ/γ	0.0125%	
Normalized emittance [mm-mrad]	0.45	
rms beam size σ_x [μm]	26	
Resonant wavelength λ_r [nm]	1.25	
Undulator period λ_u[cm]	3.9	
Afterburner	Delta 1	Delta 2
Number of periods N_u	20	20
Cylindrical vector λ_r [nm]	2.5	2.5
Poincaré vector λ_r [nm]	2.5	1.25

[1] Campbell, L.T. and McNeil, B.W.J., *Phys. Plasmas*. **19**, 093119 (2012)

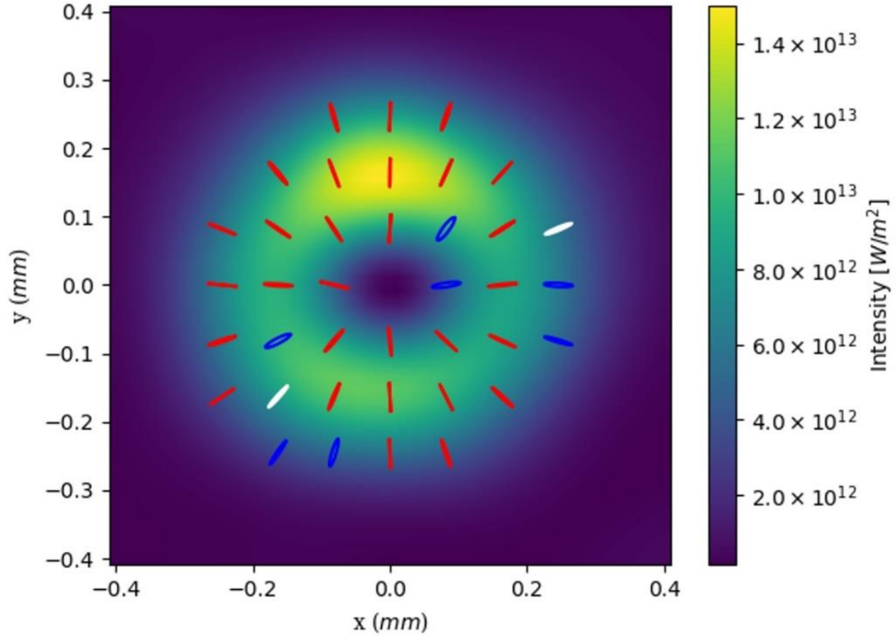
Combination of second harmonic radiation from two planar undulators produces a Radial Vector beam.

X-poled planar undulator
$\lambda_b = \lambda_r/2$

Y-poled planar undulator
$\lambda_b = \lambda_r/2$



$HG_{01}\hat{\boldsymbol{e}}_{\boldsymbol{y}}$ + $HG_{10}\hat{\boldsymbol{e}}_{\boldsymbol{x}}$ =



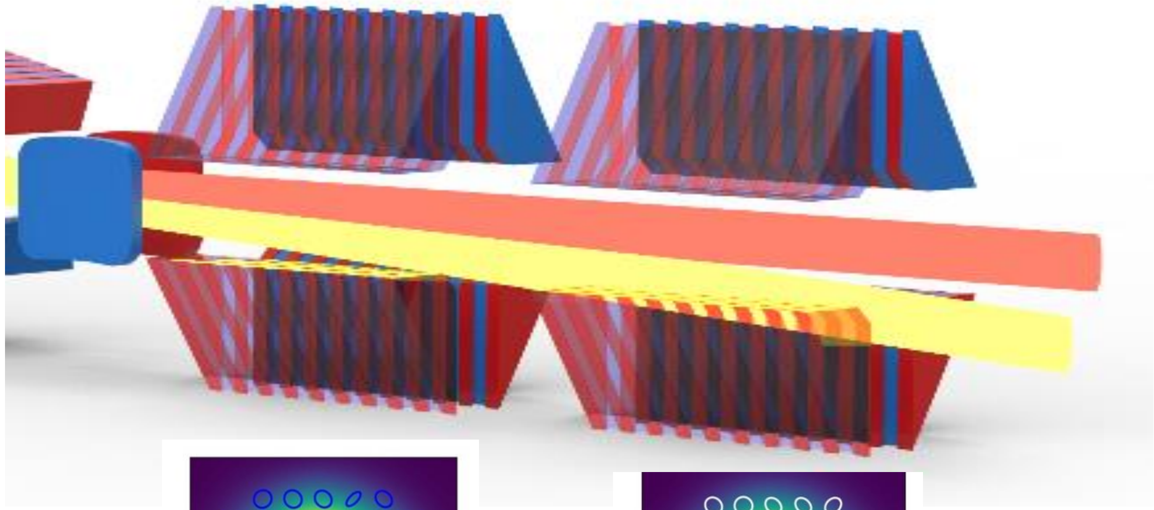
$$E = HG_{10}\hat{\boldsymbol{e}}_{\boldsymbol{x}} + HG_{10}\hat{\boldsymbol{e}}_{\boldsymbol{y}}$$

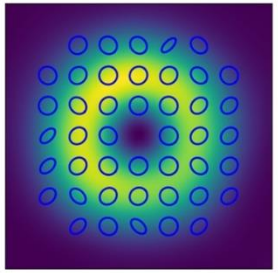
Results

Combination of second harmonic radiation from two helical undulators

Left-Hand Helical undulator
$\lambda_b = \lambda_r/2$

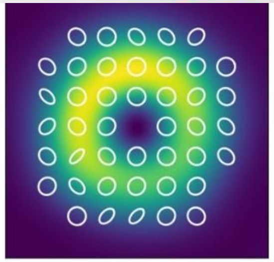
Right-Hand undulator
$\lambda_b = \lambda_r/2$





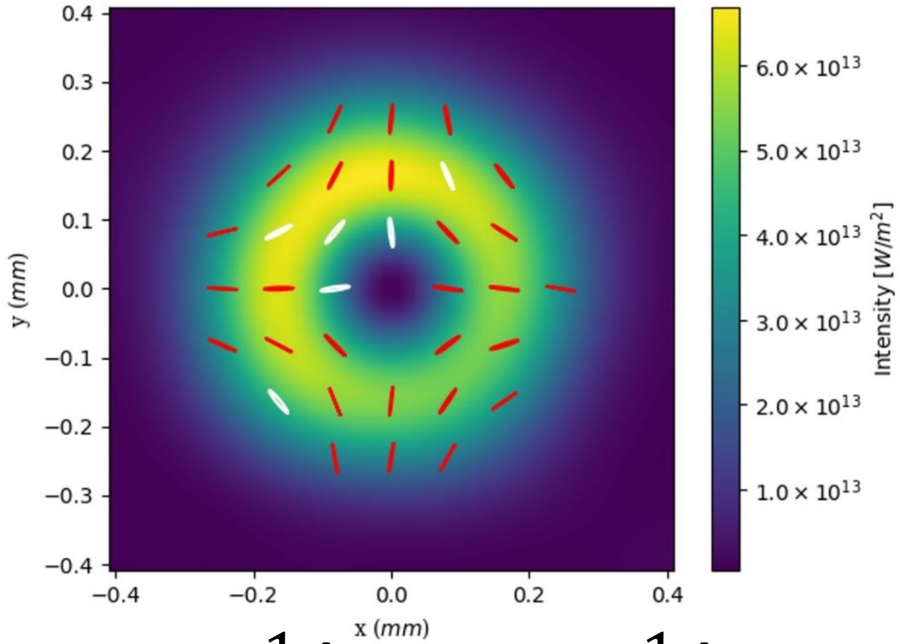
$LG_0^{-1}\hat{\boldsymbol{e}}_{\boldsymbol{R}}$

$+$



$LG_0^{1}\hat{\boldsymbol{e}}_{\boldsymbol{L}}$

$=$

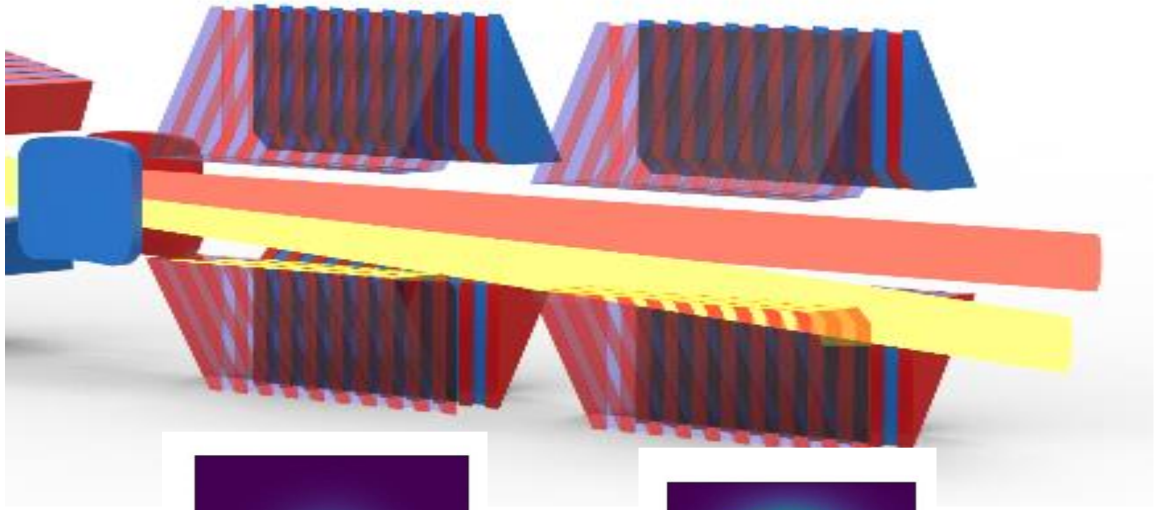


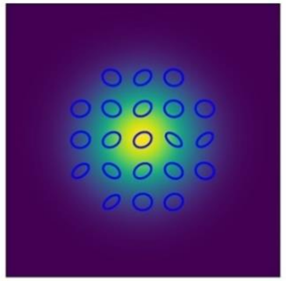
$$E = LG_0^{1}\hat{\boldsymbol{e}}_{\boldsymbol{L}} + LG_0^{-1}\hat{\boldsymbol{e}}_{\boldsymbol{R}}$$

Results

Left-Hand Helical undulator
$\lambda_b = \lambda_r$

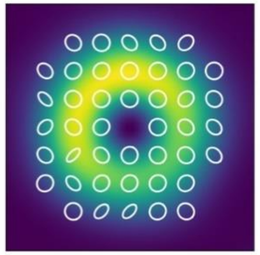
Right-Hand undulator
$\lambda_b = \lambda_r / 2$





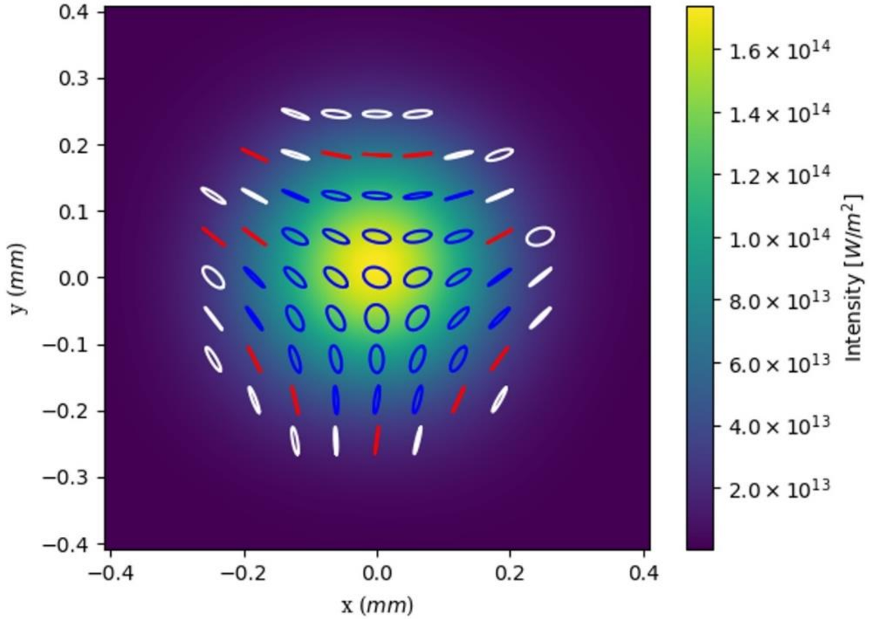
$LG_0^0 \hat{\boldsymbol{e}}_{\boldsymbol{R}}$

+



$LG_0^1 \hat{\boldsymbol{e}}_{\boldsymbol{L}}$

=



$$E = LG_0^1 \hat{\boldsymbol{e}}_{\boldsymbol{L}} + LG_0^0 \hat{\boldsymbol{e}}_{\boldsymbol{R}}$$

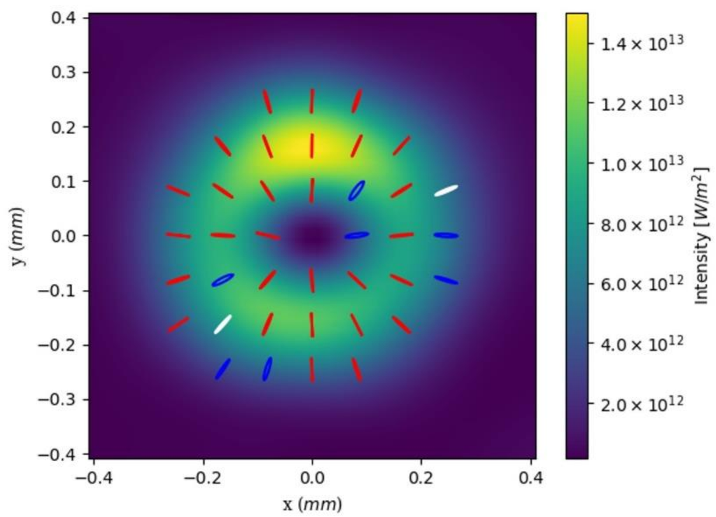
Results

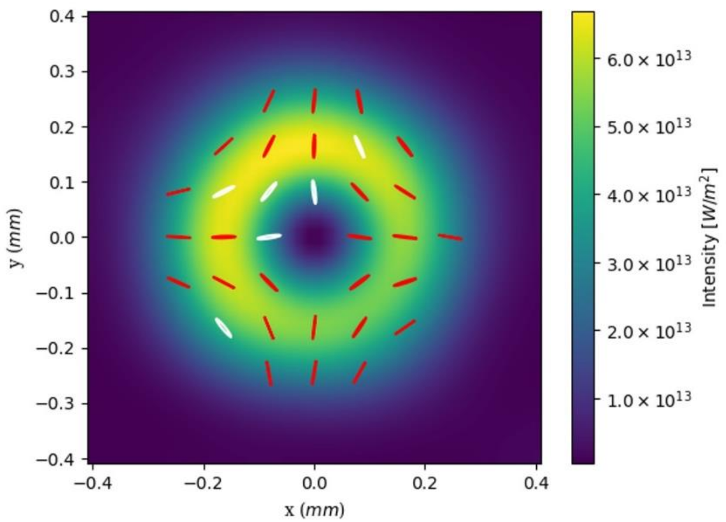
The finial power is,

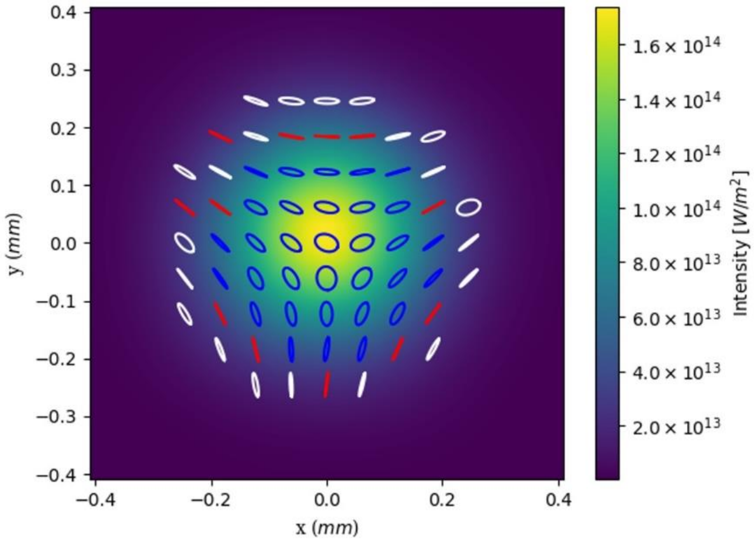
$$P = 4P_b b^2 \frac{I_0}{\gamma I_A}\left(\frac{K^2}{1+K^2}\right)\ln\left(\frac{1+4N^2}{4N^2}\right)$$

Where P_b is the peak electron beam power, I_A=17kA is the Alfven current, $N = k\sigma_x^2/L_u$ is the Fresnel nuber of the electron beam with $k = \frac{2\pi}{\lambda_b}$ and $L_u = N_u\lambda_u$.

The finial radiation power for the simulation parameters used is $P = 0.3$ MW







Future work

We have demonstrated three polarisation distributions by varying the polarisation and resonance of the afterburner undulators.

Other factors which will change the polarisation distribution includes,

- Phase between the two transverse modes
- Power ratio between the two transverse modes
- Detuning the resonance of one undulator to push radiation further off axis
- Radiating at even higher harmonics of a helical undulator.

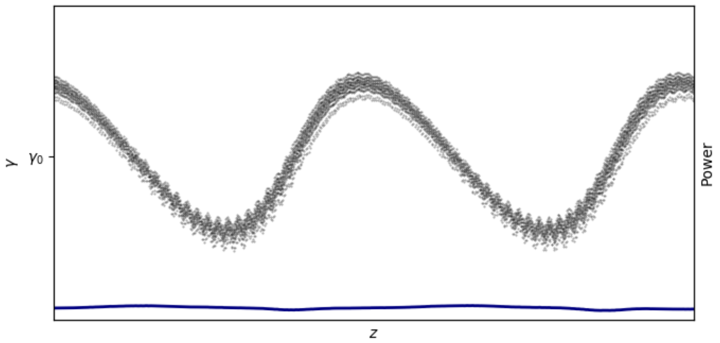
Pulses with alternating polarisation

A train of pulse can be generated through mode locking where the polarisation of each pulse alternates.

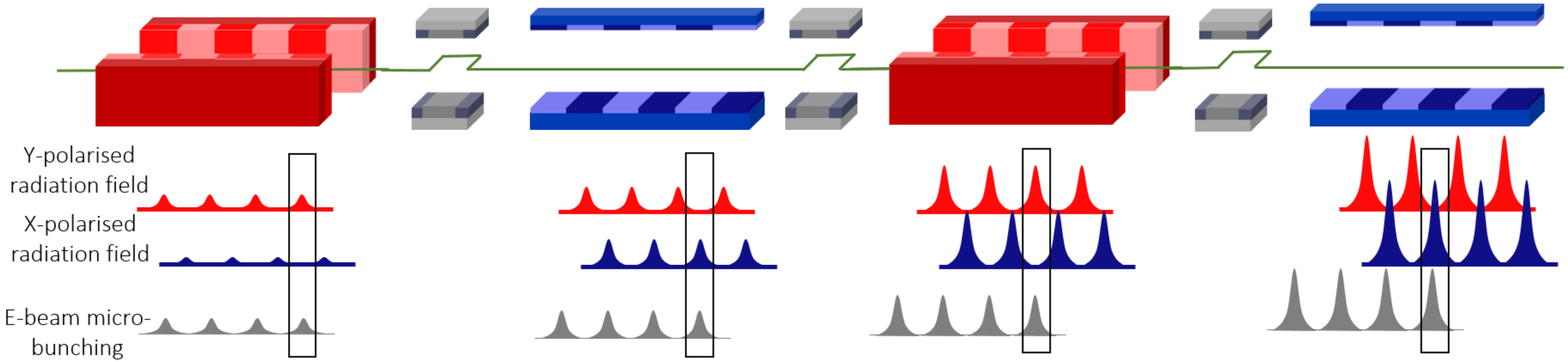
An energy modulated beam only experiences bunching at the energy minima – creating a micro bunching comb

In each undulator, those regions of the electron beam with modulated micro-bunching emit coherently.

Chicanes delay the electron beam between undulator modules so that those sections of high micro-bunching overlap with the appropriately polarised pulse.

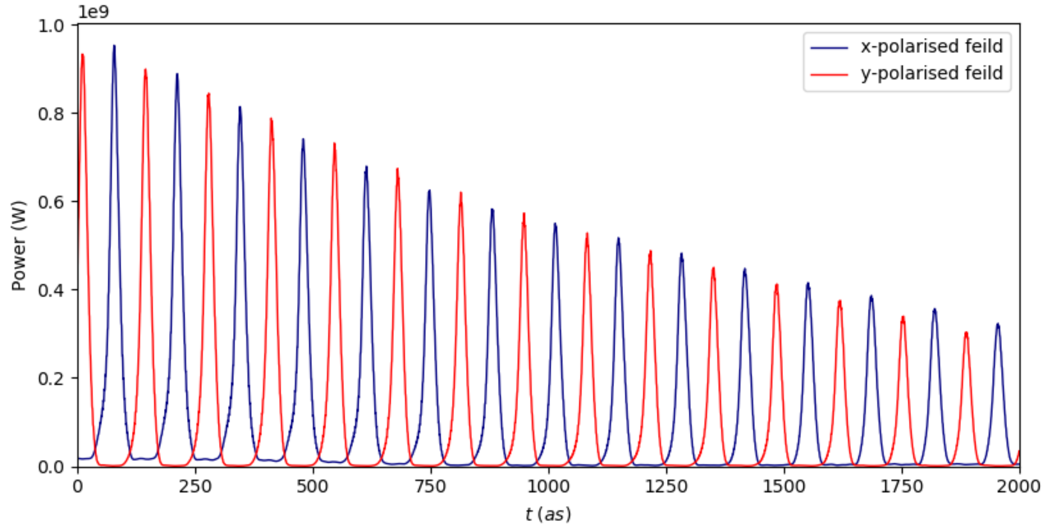


Energy modulation creates a microbunching comb



D Dunning et al, PRL **110,** 104801 (2013)

Pulses with alternating polarisation

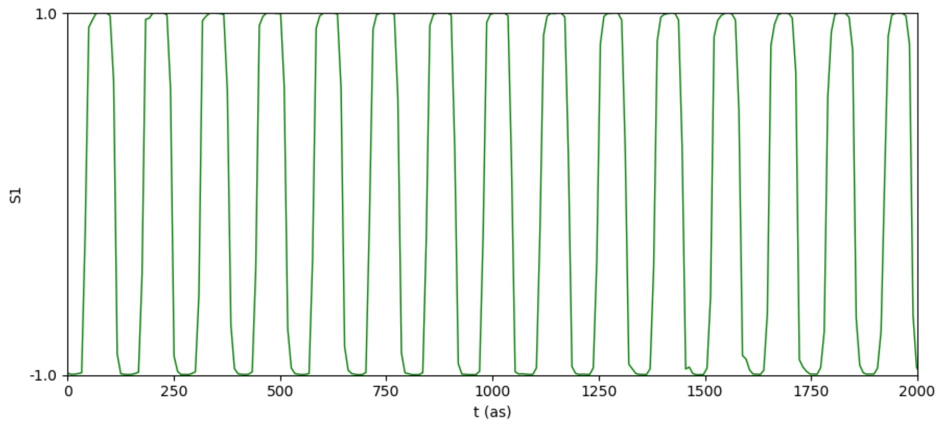


FWHM pulse duration $\tau_p = 19$ as

Separation between each pulse ≈ 67 as

Polarisation switching rate 15 PHz

$S_1 = |E_x|^2 - |E_y|^2$



Switching rate might be improved by moving to yet shorter wavelengths.

Further opportunities : alternating wavelengths
alternating Transverse modes

Thank you

Recent publication of this work is available,

Morgan, J., Hemsing, E., McNeil, B., & Yao, A. M. (2020). Free electron laser generation of X-ray Poincaré beams. *New Journal of Physics*.

Thank you to my collaborators: Erik Hemsing, Brian McNeil and Alison Yao