Coulomb-nuclear interference in elastic scattering: eikonal calculation to all orders of α

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J. Kašpar
Institute of Physics of the ASCR and CERN

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- elastic scattering of (anti)protons
  - mediated by: electromagnetic (*Coulomb*) and strong (*nuclear*) force
  - at low four momentum transfer squared, $|t|$: the two forces of similar strength $\rightarrow$ observable interference effects = *Coulomb-nuclear interference (CNI)*
    - (possibly) more than a sum of two (complex) amplitudes – consider diagrams including both Coulomb and nuclear exchanges

- motivation for revisiting: recent activity
  - TOTEM extraction of ρ [1] at 13 TeV: a first indication for Odderon existence
  - CNI studies by Petrov
    - new mathematical approach applied to CNI [2][3] $\rightarrow$ new CNI formula with one term less wrt. Cahn [4] and Kundrát-Lokajíček (KL) [5] (used e.g. by TOTEM)
    - indicated possible flaws in the preceding works [6][7]
  - proposal by Godizov [8]: amplitude-level effects negligible – low overlap of Coulomb and nuclear eikonals
    - similar conclusion made by Donnachie and Landshoff [9]
  - Khoze et al. [10]: amplitude-level effects re-confirmed, studied the effect of inelastic intermediate states

- goal of the presented study: check the above statements and controversies
  - also: check the effect of various approximations made in the above analytic calculation

- numerical calculation
  - advantages
    - a complementary approach to previous analytic derivations – good for checks
    - easy to include all powers of α, the fine-structure constants – up to now, for practical reasons, $\mathcal{O}(\alpha)$ formulae used
  - disadvantages
    - can only be done for few hadronic amplitudes $\rightarrow$ focus on physically relevant ones
    - needs a special handling of the infrared regulator (see later) $\rightarrow$ I believe under control

- eikonal framework
  - based on additivity of eikonals

$$\delta^{\mathrm{C+N}} = \delta^{\mathrm{C}} + \delta^{\mathrm{N}} \tag{1}$$

  - Coulomb eikonal from the Born amplitude ($\mathcal{F}$ = form factor, λ fictious photon mass = infrared regulator)

$$\delta^{\mathrm{C}}(b) = \frac{1}{s}\int_0^\infty \mathrm{d}q\, q\, J_0(bq)\, F^{\mathrm{C}}_{\mathrm{Born}}(-q^2)\ , \quad F^{\mathrm{C}}_{\mathrm{Born}}(t) = \pm\frac{\alpha s}{t-\lambda^2}\mathcal{F}^2(t) \tag{2}$$

  - nuclear amplitude from the (inverse) Fourier-Bessel transform

$$\delta^{\mathrm{N}}(b) = \frac{1}{2\mathrm{i}}\ln\left(2\mathrm{i}\, A^{\mathrm{N}}(b) + 1\right)\ , \quad A^{\mathrm{N}}(b) = \frac{1}{s}\int_0^\infty \mathrm{d}q\, q\, J_0(bq)\, F^{\mathrm{N}}(-q^2) \tag{3}$$

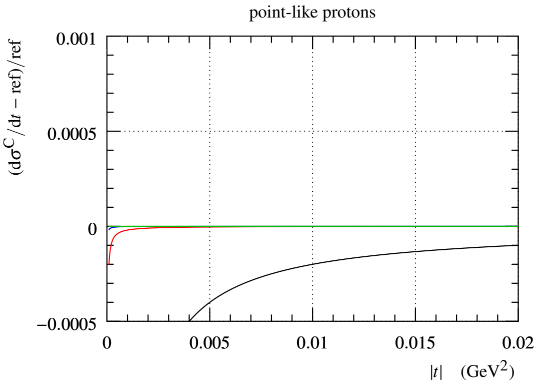
  - complete amplitude by the Fourier-Bessel transform

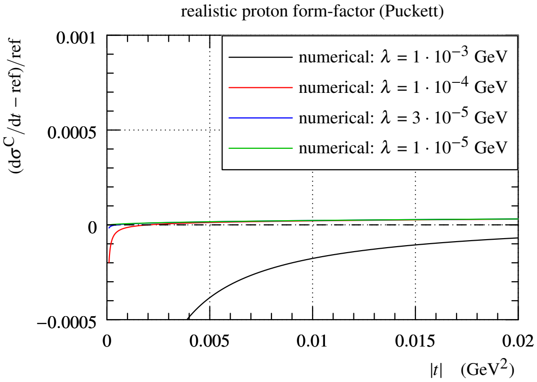
$$F^{\mathrm{C+N}}(t) = \frac{s}{2\mathrm{i}}\int_0^\infty \mathrm{d}b\, b\, J_0(b\sqrt{-t})\left(\mathrm{e}^{2\mathrm{i}\delta^{\mathrm{C+N}}(b)} - 1\right) \tag{4}$$

- the following CNI calculations/formulae will be compared
  - *"numerical"*: numerical evaluation of Eq. (4) on slide 3 (all orders of α)
  - *"Cahn"*: Eq. (30) in Ref. [4] ($\mathcal{O}(\alpha)$ approximation)
  - *"KL"*: Eq. (26) in Ref. [5] ($\mathcal{O}(\alpha)$ approximation)
  - *"Petrov"*: Eq. (17) in Ref. [2] (taking into account the erratum [3]) – formula in $\mathcal{O}(\alpha)$ approximation, one term less wrt. Cahn/KL
    - Eq. (12) in Ref. [2] (to all orders of α) not considered here – difficult for computer-program implementation
  - *"SWY"* (simplified formula by West and Yennie): Eq. (26) in Ref. [11] ($\mathcal{O}(\alpha)$ approximation, assumes slow variation of nuclear phase and exponential decrease of nuclear modulus)
  - *"trivial"*: plain sum of the Coulomb and nuclear amplitude, as suggested e.g. in Ref. [8]

- form-factor choices
  - for *point-like* protons ($\mathcal{F} \equiv 1$) – unrealistic, but useful for benchmarking the "numerical" calculation
  - *realistic form factor* – in particular by Puckett et al. [12]

- nuclear amplitude choices – two extreme alternatives as published by TOTEM at 8 TeV [13] – modulus constrained strongly by data $\rightarrow$ make choices for phase:
  - *"central"*: slowly varying phase, elastic scattering predominantly at low impact parameter
  - *"peripheral"*: rapidly varying phase, impact-parameter profile peaks at a larger b

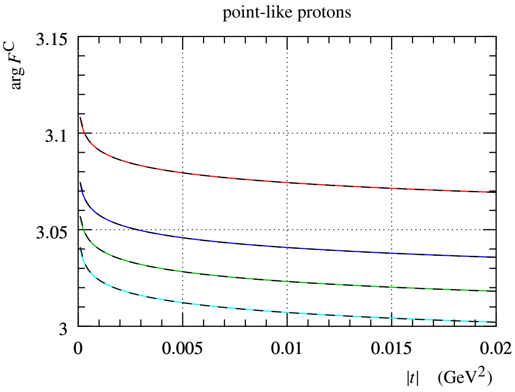


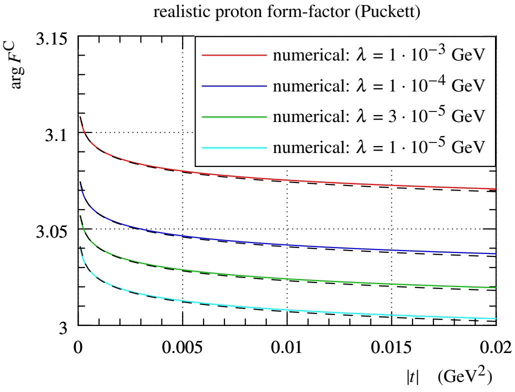


- "complete" = eikonal resummation to all orders of α, i.e. Eq. (4) with δ^{C} instead of δ^{C+N}

- treatment of infrared regulator λ
  - cannot be strictly set to zero
  - consider series of λ values and check if the behaviour as expected
  - reminder – denominator in F^{C}_{Born}, Eq. (2): $t - \lambda^2$
    - if $\lambda^2 \ll |t|$: impact of λ negligible
    - as λ is reduced: its impact expected to smaller and smaller $|t|$ values $\rightarrow$ effectively observed in the plots
    - conclusion: λ can be chosen small enough to have negligible impact in the $|t|$ range of interest

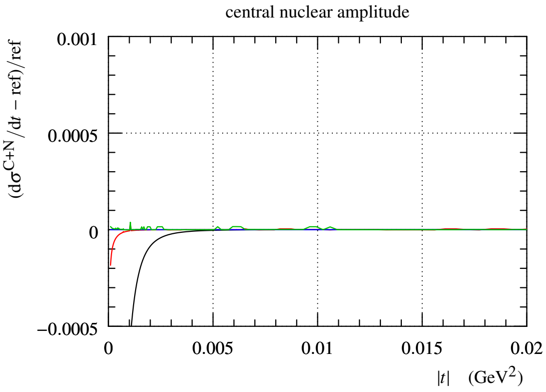
- plots: relative $d\sigma^{C}/dt$ deviation from reference
  - reference = Born Coulomb amplitude (Eq. (2) with $\lambda = 0$)
  - solid lines: numerical calculation, colours = different values of λ
  - top: for point-like protons
    - analytic Calculation by Cahn: in this case, the effect of "completing" (eikonal resummation) should only impact amplitude phase $\rightarrow$ as expected, no deviation found
  - bottom: with realistic form factor
    - small deviation found: $\mathcal{O}(10^{-4})$
    - this effect neglected in formulae by Cahn and KL, cf. Eq. (27) in [4]

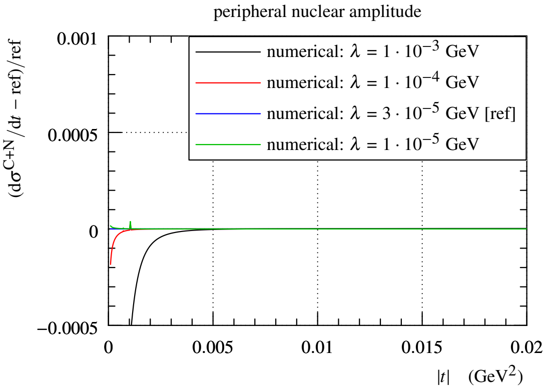




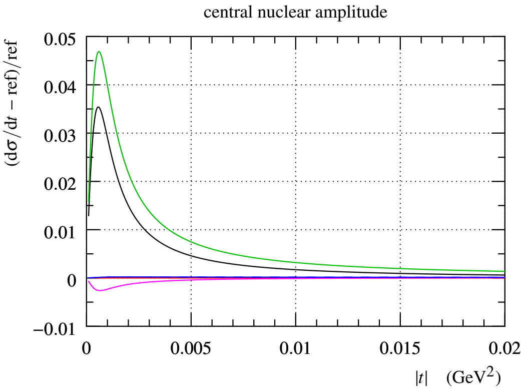
- "complete" = eikonal resummation to all orders of α, i.e. Eq. (4) with δ^{C} instead of $\delta^{\mathrm{C+N}}$

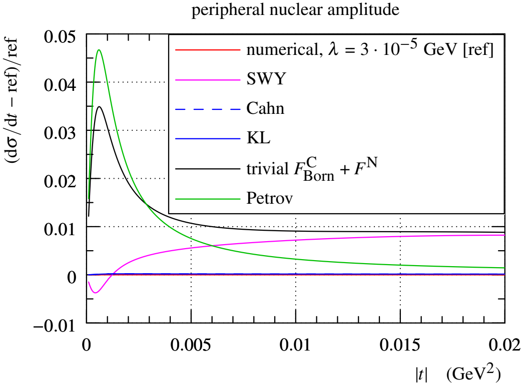
- plots: phase of the complete Coulomb amplitude
  - solid lines: numerical calculation
  - dashed lines: Cahn's analytic calculation (for point-like protons)
  - top: for point-like protons
    - as expected, numerical calculation in perfect agreement with the dashed lines
  - bottom: with realistic form factor
    - small deviation from the dashed line: $\mathcal{O}(10^{-3})$
    - this effect neglected in formulae by Cahn and KL, cf. Eq. (27) in [4]





- Coulomb+nuclear cross-section $d\sigma^{C+N}/dt$ calculated according to Eq. (4)
- plots: relative difference to reference
  - reference = result with $\lambda = 3 \cdot 10^{-5}$ GeV
  - top: central nuclear amplitude
  - bottom: peripheral nuclear amplitude
- interpretation: similar to slide 5: λ can be chosen small enough to have negligible impact in the $|t|$ range of interest
- green curve (smallest value of λ): some numerical glitches present





- Coulomb+nuclear cross-section $d\sigma^{C+N}/dt$ calculated according to different formulae
- plots: relative difference to reference
  - reference = numerical calculation with $\lambda = 3 \cdot 10^{-5}$ GeV
- formulae by Cahn and KL: very good agreement with the reference
  - relative deviations $\mathcal{O}(10^{-4})$
- “trivial sum”: deviations up to 3.5 %
- formula by Petrov: deviations up to 5 %
- SWY formula
  - central nuclear amplitude (top): good agreement, relative deviations $\mathcal{O}(10^{-3})$
  - peripheral nuclear amplitude (bottom), relative deviations up to 1 %
  - expected: SWY assumes slow phase variation
- NB: numerical calculation, Eq. (4), formulae by Cahn, KL and Petrov
  - identical start point (additivity of eikonals, expression for Born Coulomb amplitude)
  - thus expect identical results – deviations can be only due to approximations/mistakes in the works

- Cahn's approximation [4] of complete Coulomb amplitude verified to be inexact
  - correctly pointed out by Petrov [2]
  - with a realistic proton form factor deviations small: $\mathcal{O}(10^{-3})$ for phase, $\mathcal{O}(10^{-4})$ relatively for modulus – experimentally not observable
- presented: numerical calculation/evaluation of CNI effects
  - based directly on additivity of eikonals
  - to all orders of α, likely for the first time
  - multiple checks/validations presented
  - illustrated with two realistic nuclear-amplitude alternatives
- new CNI formula by Petrov ($\mathcal{O}(\alpha)$ approximation)
  - one term less wrt. formulae by Cahn and KL
  - $d\sigma/dt$ predictions differ up to by 5 % from the numerical-calculation reference
- plain sum of Coulomb and nuclear amplitudes (as proposed e.g. by Godizov)
  - $d\sigma/dt$ predictions differ up to by 3.5 % from the numerical-calculation reference
  - interpretation: proposal oversimplified
- SWY
  - reproduces the numerical reference well for central nuclear amplitude
  - for peripheral nuclear amplitude: deviations up to 1 %
  - expected: SWY assumes slow variation of phase (only compatible with central amplitude)

- CNI formulae by Cahn/KL
  - best reproduction of the numerical reference: relative deviations $\mathcal{O}(10^{-4})$
    - Cahn's inexact approximation of complete Coulomb amplitude and possibly early truncation of the series in powers of α (as suggested by Petrov) – no detrimental effect within experimental reach
  - from our selection: "best on the market" $\rightarrow$ recommendable for experimental data analyses
- numerical calculation, formulae by Cahn, KL and Petrov: exactly the same starting point
  - numerical calculation, formulae by Cahn and KL: mutually good agreement
  - formula by Petrov: significant difference wrt. the previous group
  - interpretation: indication of a problem in Petrov's formula
- outlook: study of another representation proposed by Petrov: Eq. (13) in [14]

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