Anomalies of 1-form symmetries and consistent gauge groups

Based on:
2008.09117 with F. Apruzzi & M. Dierigl
2008.10605 with M. Cvetič, M. Dierigl and H. Y. Zhang

Ling Lin, Sep 21, 2020

What is the Standard Model's gauge group?

i.e. $SU(3) \times SU(2) \times U(1)$ vs. $[SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_n,\ n = 2, 3, 6$

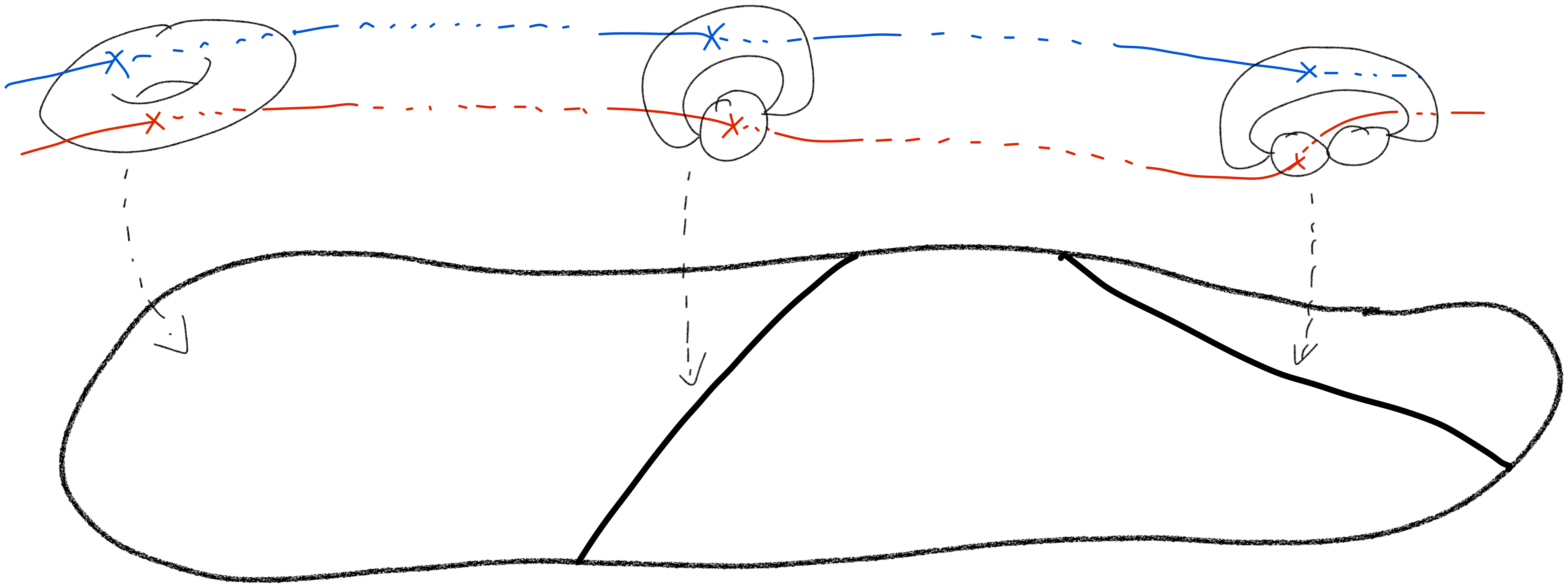
- Simpler example: $SU(2) \quad \text{vs.} \quad SU(2)/\mathbb{Z}_2 \cong SO(3)$
- Restricts *allowed* matter representations.
- Not detectable by local operators.
- Affect periodicity of theta angle.
- In quantum gravity: different electric charges for monopoles.

Given gauge algebra, is any global form allowed?

- No known effective field theory constraints.
- Swampland program: more subtle restrictions when gravity present.
  E.g., not all gauge algebras are allowed (in higher dimensions).
- In string compactifications: limitations from geometry,
  with intricate, "arithmetic" structures.

Global gauge group structure in F-theory

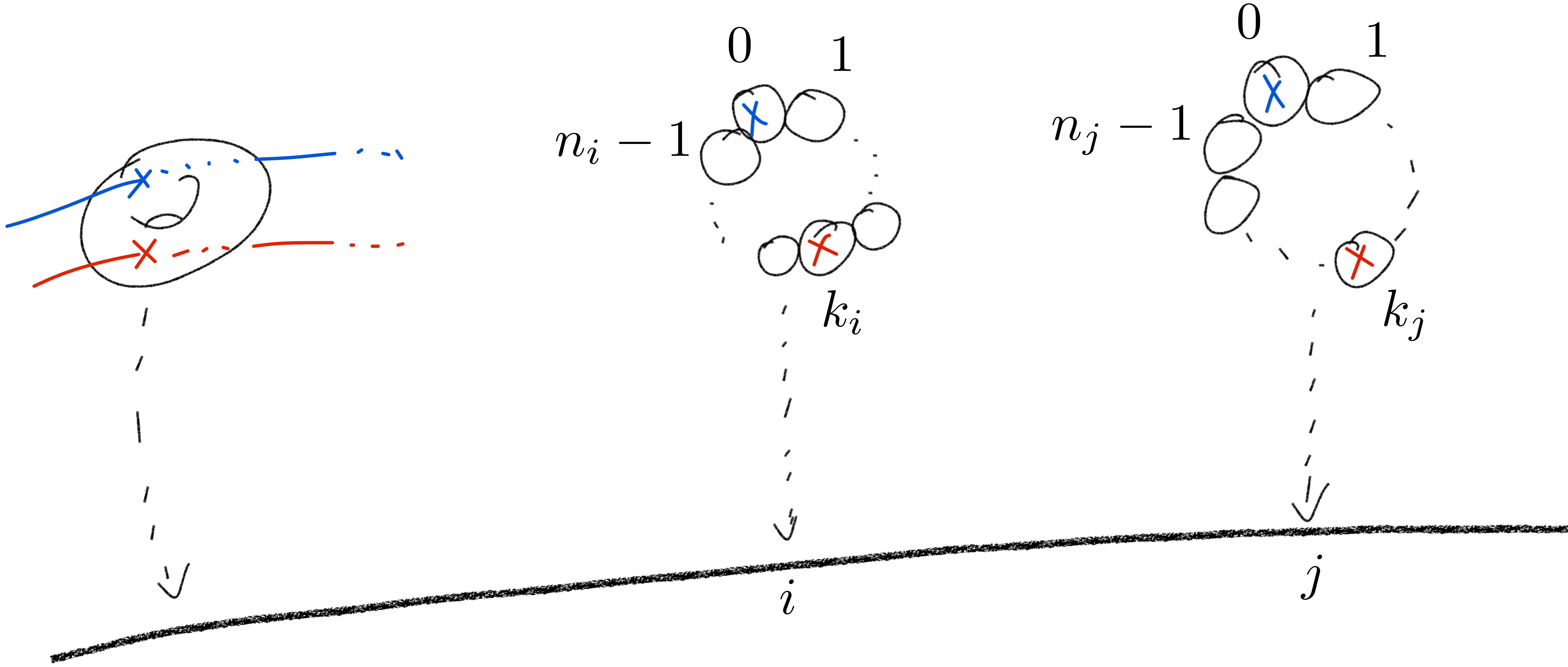
Aspinwall/Morrison ’98: G/Z determined by *Mordell—Weil* group.



MW-group = “addition” of sections ⟷ elliptic curve cryptography

Mordell—Weil group on K3 surfaces ($\rightsquigarrow$ F-theory to **8d**) fully classified.

Assume $G = \prod_i SU(n_i)$; which G/Z with $Z \subset Z(G)$ realizable?



$(k_1, k_2, ...) \in \prod_i \mathbb{Z}_{n_i} = Z(G)$

generates $\mathbb{Z}_\ell$ factor of Z.

([Mayrhofer/Morrison/Till/Weigand ’14], [Cvetič/LL ’17])

Geometric fact ([Miranda/Persson ’89]): $\sum_i \frac{n_i - 1}{2n_i} k_i^2 \in \mathbb{Z}$ ⟶ restricts possible G/Z,

also satisfied in heterotic on T^2 and CHL on S^1; e.g., no $SU(n)/\mathbb{Z}_n$!

More examples: $Z = \mathbb{Z}_7 : G = SU(7)^3$ and $Z = \mathbb{Z}_8 : G = SU(8)^2 \times SU(4) \times SU(2)$ and no solution for $\mathbb{Z}_l$ with $l > 8$ (and gauge rank < 19 [see. Montero’s talk next week]).

Can be explained field theoretically using *higher-form symmetries!*

- Point-like (0-dim) operators charged under 0-form symmetry.
- p-dim operators charged under p-form symmetry. ([Gaiotto/Kapustin/Seiberg/Willet ’14])
- Must be abelian for p>0; can couple to a (p+1)-form gauge field.

Center Z(G) as 1-form symmetry

- Center Z ⟷ 1-form Z symmetry; charged operators: Wilson-loops.
- 1-form Z symmetry has Z-valued 2-form gauge field C_2 (more precisely: 2-cocycle).
- If Z = Z(G), non-trivial C_2 induces *fractional* instantons:

$$I(G) := \frac{1}{8\pi^2}\mathrm{Tr}(F \wedge F) = \alpha_G \, C_2 \cup C_2 \quad \mathrm{mod}\ \mathbb{Z}$$

- "Gauging" Z = summing over all C_2 configurations in path integral; results in gauge group G/Z.

Anomalies for center symmetries

- In general, Z has mixed ('t Hooft) anomalies with other symmetries.
- In 4d: mixed anomaly with periodicity of θ, related to Witten effect ([Witten '79]), confinement ([Aharony/Seiberg/Tachikawa '13]), time reversal symmetry ([Gaiotto/Kapustin/Komargodski/Seiberg '17]), etc.
- Mixed anomaly with another gauge symmetry breaks Z (similar to ABJ)
  ➡ cannot gauge, so G/Z inconsistent!

Anomalies for center symmetries in 8d

- In 8d $\mathcal{N}=1$ SYM (only vector multiplet): no restrictions on center 1-form.
- But with gravity multiplet: $S \supset \int \sum_i I(G_i) \wedge B_4$ ([Awada/Townsend '85)].
- Non-trivial $C_2^{(i)}$ background: $\Delta S = \int \sum_i \alpha_{G_i} (C_2^{(i)} \cup C_2^{(i)}) \wedge B_4 \mod \mathbb{Z}$
- B_4 enjoys U(1) gauge (3-form) symmetry!
- In $C_2^{(i)}$ background, partition function acquires non-trivial phase $(\alpha_{G_i} \notin \mathbb{Z})$

$$2\pi \int \sum_i \alpha_{G_i} (C_2^{(i)} \cup C_2^{(i)}) \cup b_4$$

under large U(1) transformation $B_4 \to B_4 + b_4$ ⟶ anomaly!

Constraining the gauge group in 8d

- Phase $2\pi \int \sum_i \alpha_{G_i} (C_2^{(i)} \cup C_2^{(i)}) \cup b_4$ in general non-trivial, so no $[\prod_i G_i]/[\prod_i Z(G_i)]$ gauge group allowed.

- Can find anomaly-free subgroup $Z \subset \prod_i Z(G_i)$, necessary for $[\prod_i G_i]/Z$.

- Take $G_i = SU(n_i)$, with $Z(G_i) = \mathbb{Z}_{n_i}$ and $\alpha_{G_i} = \frac{n_i - 1}{2n_i}$. Then, for $\mathbb{Z}_\ell \subset \prod_i \mathbb{Z}_{n_i}$ with generator $(k_1, ..., k_s)$, background field sets $C_2^{(i)} = k_i C_2$ ([Cordova/Freed/Lam/Seiberg '19]). Phase becomes:

$$2\pi \int \sum_i \alpha_{G_i} (C_2^{(i)} \cup C_2^{(i)}) \cup b_4 = 2\pi \left(\sum_i \frac{n_i - 1}{2n_i} k_i^2 \right) \int C_2 \cup C_2 \cup b_4$$

- Purely field theoretic constraint (with gravity), identical with geometry! Resulting groups also predicted by other Swampland arguments ([Montero/Vafa '20]).

Relationship to Quantum Gravity folklores

- No global symmetries & completeness hypothesis related to the center:
    - Also 1-form symmetries must be gauged or broken ([Ooguri/Harlow ’18]).
    - G and G/Z have different charge lattices: G has states charged under Z.
- If anomaly present: Z is broken, so gauge group is G.
  A consistent theory better supplies the necessary states!
- In 6d, come from excitations of BPS strings ([Apruzzi/Dierigl/LL ’20]),
  also present in non-gravitational theories / SCFTs.

Open questions

- Have not included U(1) factors.
- What about dual, magnetic (d-3)-form symmetry? Defect group structure?
  (5d story: [Morrison/Schäfer-Nameki/Willet ’20] and [Albertini/Del Zotto/García-Etxebarria/Hosseini ’20])
- Interplay with other gauge and continuous symmetries?

Thank you!