Cross Check Quantities for the Workshop "The HVP from Lattice QCD at high precision"

1 Overview

The goal is to provide a list of quantities to be used for cross checks between various lattice calculations during the workshop "The hadronic vacuum polarisation from Lattice QCD at high precision"[1] from the 16th-20th November 2020. We ask every collaboration to provide as many of the quantities listed below as possible (in order of priority):

1. Intermediate window contribution (see equation (4)) for $t_0 = 0.4$ fm, $t_1 = 1.0$ fm, $\Delta = 0.15$ fm
2. further Windows:
   - short distance window (see equation (3))
   - long distance window (see equation (5))
3. Taylor coefficients Π_1 and Π_2 from the first two moments of the vector correlator

For more details/definitions see below.

All quantities should be given for the **light-quark connected HVP** contribution to a_μ with

- continuum extrapolation $a = 0$
- including finite volume corrections
- in the isospin symmetric limit using $M_\pi = 135$ MeV

Please contact the members of the Organising Committee if you have any questions/comments.

2 Window Method

May

$$C(t) = -\frac{1}{3}\sum_{j=0}^{2}\int \mathrm{d}\vec{x}\ \langle J_j(\vec{x},t)\, J_j(0)\rangle \qquad \text{with} \qquad J_\mu(x) = \frac{2}{3}\,\overline{u}(x)\gamma_\mu u(x) - \frac{1}{3}\,\overline{d}(x)\gamma_\mu d(x) \quad (1)$$

be the light-quark vector-vector two-point function and $w(t)$ the appropriate integration kernel such that

$$a_\mu^{\mathrm{HVP,LO}}(ud) = \int_0^\infty \mathrm{d}t\ C(t)\, w(t)\,. \quad (2)$$

We define the window contributions as in [1]

short distance
$$a_\mu^{\mathrm{SD}} = \int_0^\infty \mathrm{d}t\ C(t)\, w(t)\ [1 - \Theta(t,t_0,\Delta)] \quad (3)$$

intermediate distance
$$a_\mu^{\mathrm{W}} = \int_0^\infty \mathrm{d}t\ C(t)\, w(t)\ [\Theta(t,t_0,\Delta) - \Theta(t,t_1,\Delta)] \quad (4)$$

long distance
$$a_\mu^{\mathrm{LD}} = \int_0^\infty \mathrm{d}t\ C(t)\, w(t)\, \Theta(t,t_1,\Delta) \quad (5)$$

[1] https://indico.cern.ch/event/956699/

with $t_0 = 0.4$ fm, $t_1 = 1.0$ fm, $\Delta = 0.15$ fm and

$$\Theta(t, t', \Delta) \equiv \frac{1}{2}\left(1 + \tanh\left[(t - t')/\Delta\right]\right) . \tag{6}$$

3 Moments

We define the time moments of the vector-vector two-point function as first introduced in [2]

$$G_{2n} = \int_0^\infty \mathrm{d}t\, t^{2n} C(t) \tag{7}$$

and the Taylor coefficients as

$$\Pi_n = \frac{(-1)^{n+1}}{(2n+2)!}\, G_{2n+2}\,. \tag{8}$$

References

[1] T. Blum, P.A. Boyle, V. Gülpers, T. Izubuchi, L. Jin, C. Jung, A. Jüttner, C. Lehner, A. Portelli, and J.T. Tsang. Calculation of the hadronic vacuum polarization contribution to the muon anomalous magnetic moment. *Phys. Rev. Lett.*, 121(2):022003, 2018.

[2] Bipasha Chakraborty, C.T.H. Davies, G.C. Donald, R.J. Dowdall, J. Koponen, G.P. Lepage, and T. Teubner. Strange and charm quark contributions to the anomalous magnetic moment of the muon. *Phys. Rev. D*, 89(11):114501, 2014.