Lepton $(g_\ell - 2)$ in Lattice QCD+QED from the ETM Collaboration

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The Hadronic Vacuum Polarization from Lattice QCD at high precision

16th – 20th November

OUTLINE

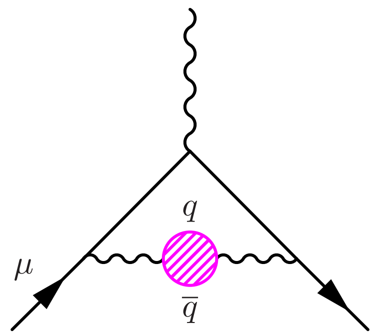
- Results for the light, strange and charm quark contributions to a_ℓ^{HVP}
- Ratios of the HVP contributions to the lepton $(g_\ell - 2)$

D. Giusti and S. Simula

ArXiv:1707.03019; ArXiv:1808.00887; ArXiv:1901.10462; ArXiv:1910.03874; ArXiv:2003.12086

Extended Twisted Mass Collaboration

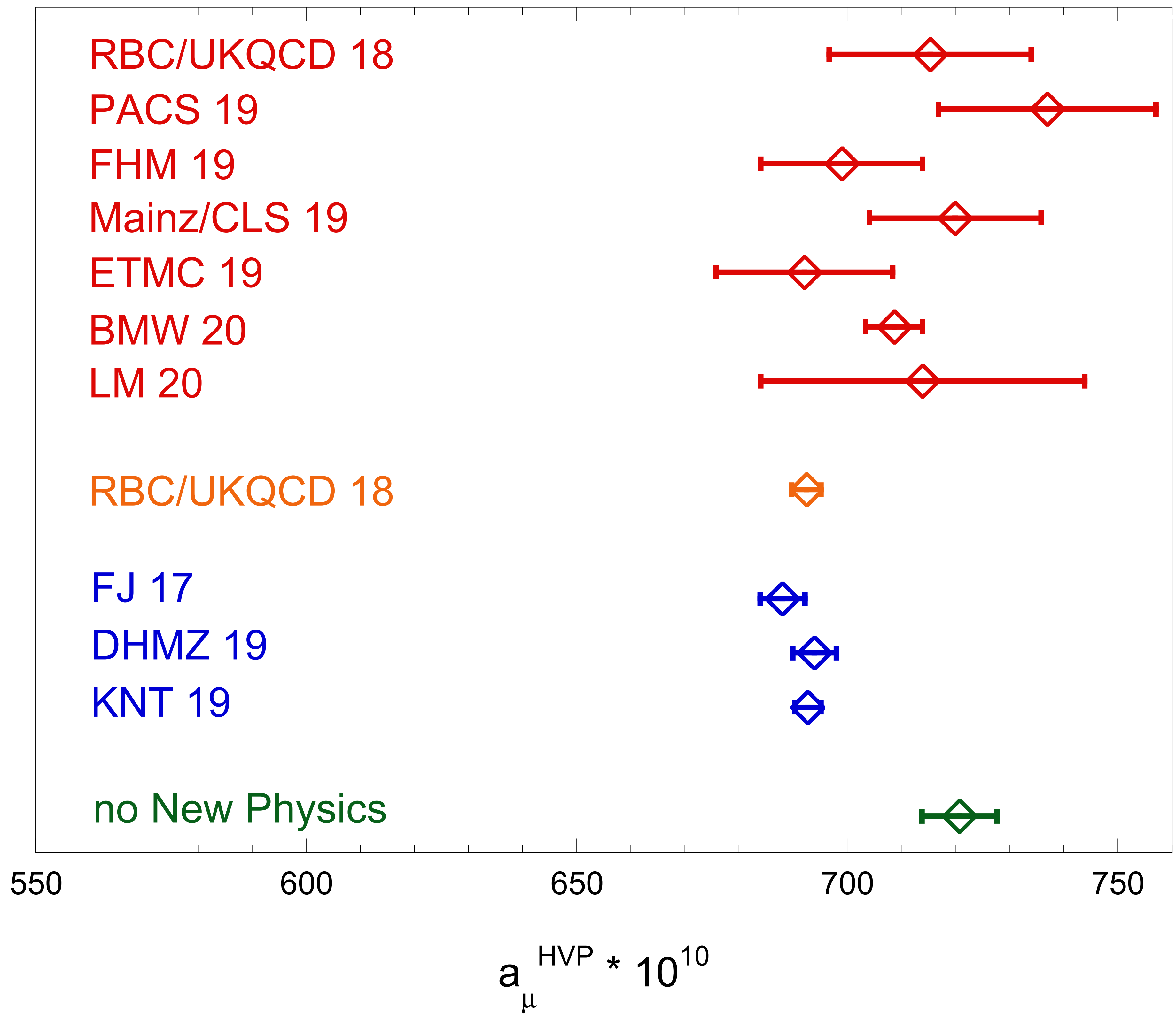
Hadronic Vacuum Polarization



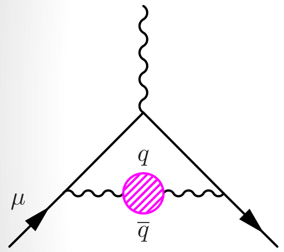
lattice data
100%

lattice + e^+e^-
~ 30% + 70%

e^+e^- data
100%



Lepton anomalous magnetic moments from ETMC

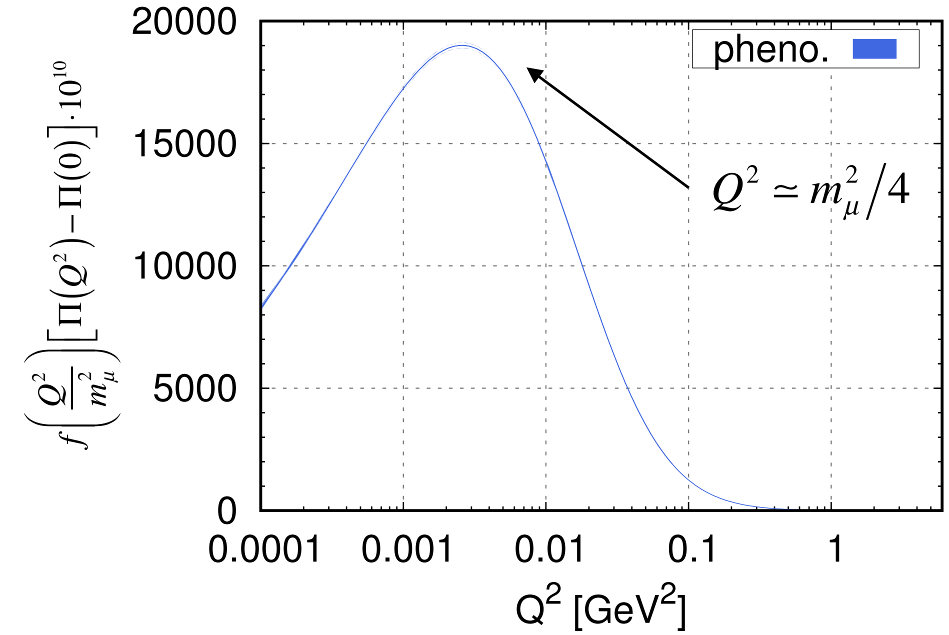


HVP from LQCD

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x} \langle J_\mu(x) J_\nu(0)\rangle = \left[\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu\right] \Pi(Q^2)$$

$$a_\ell^{\rm HVP} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \, \frac{1}{m_\ell^2} f\left(\frac{Q^2}{m_\ell^2}\right) \left[\Pi(Q^2) - \Pi(0)\right]$$

B. E. Lautrup *et al.*, 1972; T. Blum, 2002



F. Jegerlehner, "alphaQEDc17"

Time-Momentum Representation

$$a_\ell^{\rm HVP} = 4\alpha_{em}^2 \int_0^\infty dt \; K_\ell(t)\, V(t)$$

D. Bernecker and H. B. Meyer, 2011

$$V(t) \equiv \frac{1}{3} \sum_{i=1,2,3} \int d\vec{x} \, \langle J_i(\vec{x},t) J_i(0)\rangle$$

$$\boxed{a_\ell^{\rm HVP} = \sum_{f=u,d,s,c} 4\alpha_{em}^2 \left\{ \sum_{t=0}^{T_{data}} K_\ell(t) V^f(t) + \sum_{t=T_{data}+a}^{\infty} K_\ell(t) \frac{G_V^f}{2M_V^f} e^{-M_V^f t} \right\}}$$

$t \leq T_{data} < T/2$ (avoid bw signals)

$t > T_{data} > t_{min}$ (ground-state dom.)

quark-connected terms only

lattice data

local vector currents

analytic representation

Details of the lattice simulation

We have used the gauge field configurations generated by ETMC, European Twisted Mass Collaboration, in the pure isosymmetric QCD theory with Nf=2+1+1 dynamical quarks

ensemble	β	V/a^4	$a\mu_{ud}$	$a\mu_\sigma$	$a\mu_\delta$	N_{cf}	$a\mu_s$	M_π (MeV)	M_K (MeV)
$A40.40$	1.90	$40^3 \cdot 80$	0.0040	0.15	0.19	100	0.02363	317(12)	576(22)
$A30.32$		$32^3 \cdot 64$	0.0030			150		275(10)	568(22)
$A40.32$			0.0040			100		316(12)	578(22)
$A50.32$			0.0050			150		350(13)	586(22)
$A40.24$		$24^3 \cdot 48$	0.0040			150		322(13)	582(23)
$A60.24$			0.0060			150		386(15)	599(23)
$A80.24$			0.0080			150		442(17)	618(14)
$A100.24$			0.0100			150		495(19)	639(24)
$A40.20$		$20^3 \cdot 48$	0.0040			150		330(13)	586(23)
$B25.32$	1.95	$32^3 \cdot 64$	0.0025	0.135	0.170	150	0.02094	259 (9)	546(19)
$B35.32$			0.0035			150		302(10)	555(19)
$B55.32$			0.0055			150		375(13)	578(20)
$B75.32$			0.0075			80		436(15)	599(21)
$B85.24$		$24^3 \cdot 48$	0.0085			150		468(16)	613(21)
$D15.48$	2.10	$48^3 \cdot 96$	0.0015	0.1200	0.1385	100	0.01612	223 (6)	529(14)
$D20.48$			0.0020			100		256 (7)	535(14)
$D30.48$			0.0030			100		312 (8)	550(14)

- Gluon action: Iwasaki
- Quark action: twisted mass at maximal twist (automatically O(a) improved)

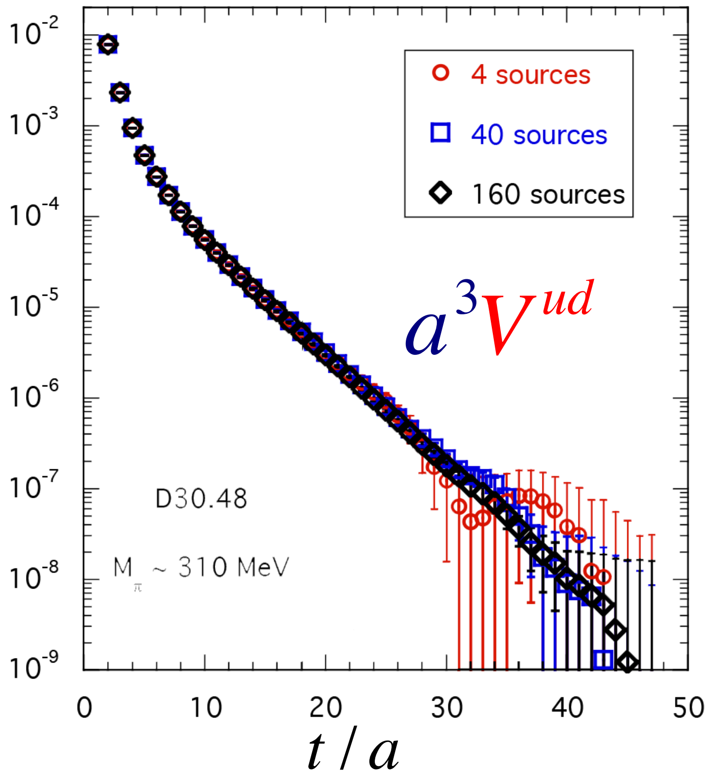
OS for *s* and *c* valence quarks

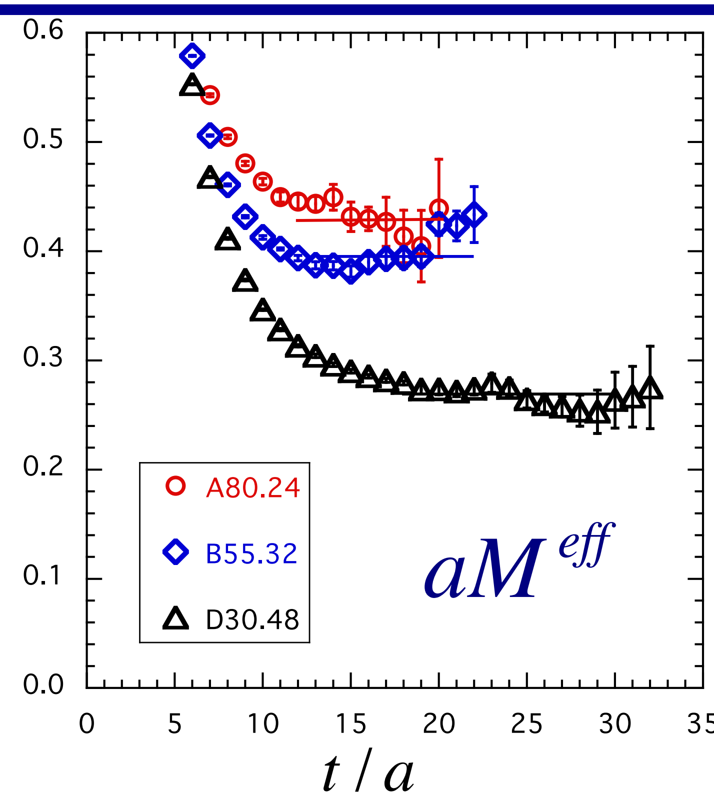
Pion masses in the range 220 - 490 MeV

4 volumes @ $M_\pi \simeq 320$ MeV and $a \simeq 0.09$ fm

$M_\pi L \simeq 3.0 \div 5.8$

Light quark contribution



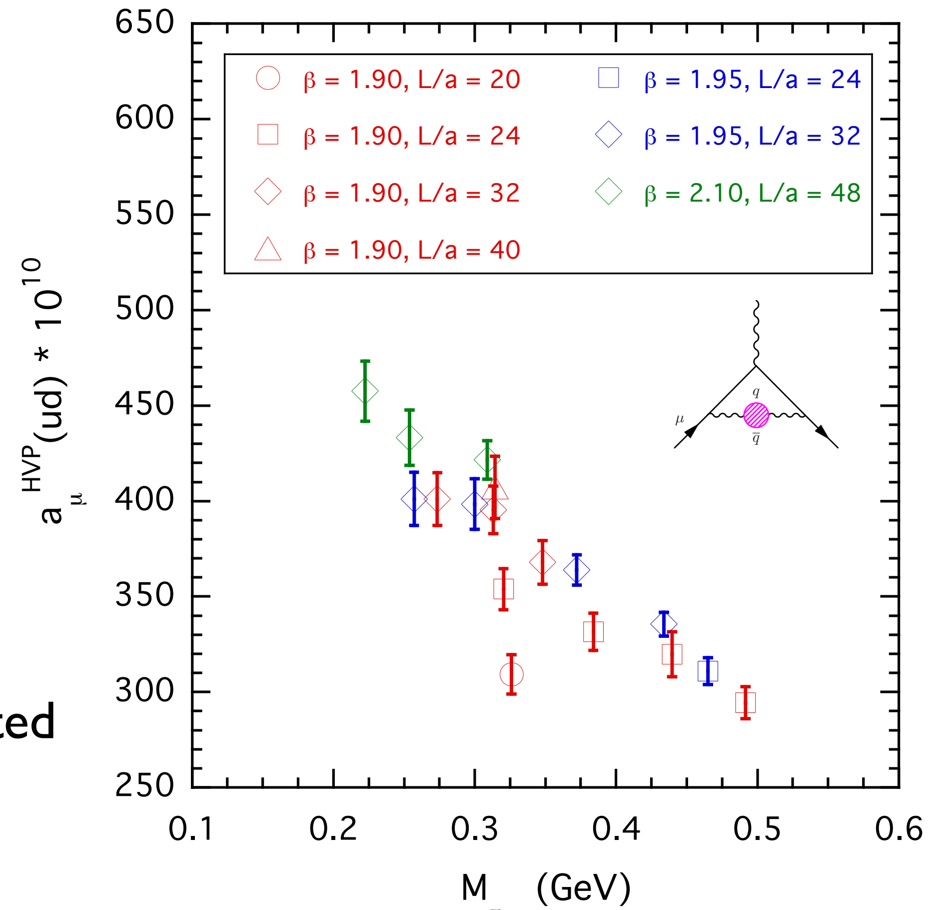


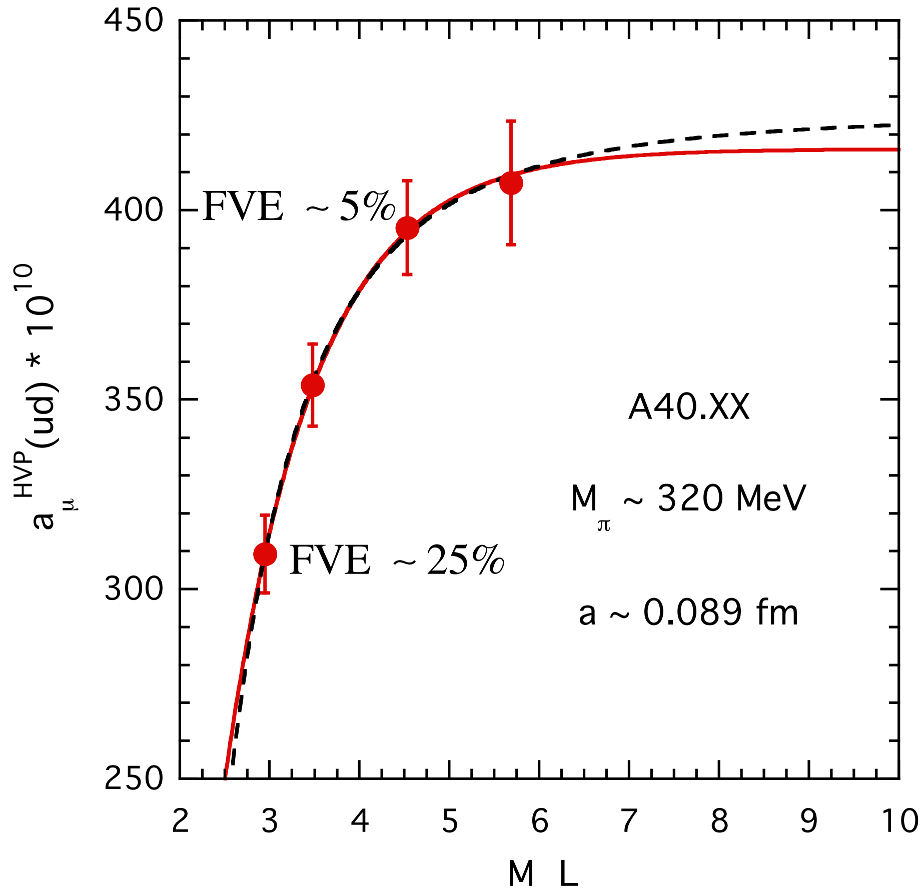
StN: $\propto e^{-(M_\rho - M_\pi)t}$

G. Parisi, 1984;
G. P. Lepage, 1989

160 stoch. sources / gauge conf.

DG *et al.*, 2018
PRD98(2018)114504





quark-connected terms only

Correlator representation

$$\boxed{V^{ud}(t) = V_{dual}(t) + V_{\pi\pi}(t)}$$

<u>low and intermediate time distances</u>

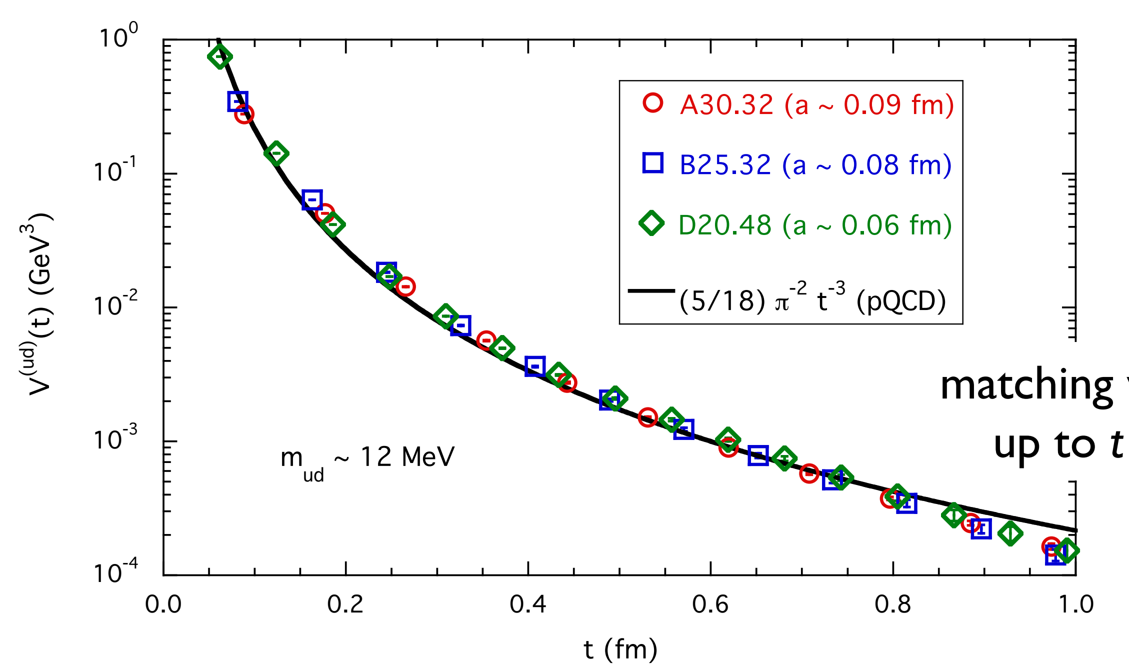
$$V_{dual}(t) \equiv \frac{1}{24\pi^2}\int_{s_{dual}}^{\infty} ds\sqrt{s}\; e^{-\sqrt{s}t} R^{pQCD}(s)$$

$$s_{dual} = \left(M_\rho + E_{dual}\right)^2 \qquad R_{dual} = 1 + O\left(\frac{m_{ud}^4}{s_{dual}^2}\right) + O(\alpha_s) + O(a^2)$$

$$V_{dual}(t) = \frac{5}{18\pi^2}\frac{R_{dual}}{t^3} e^{-(M_\rho+E_{dual})t}\left[1 + (M_\rho + E_{dual})t + \frac{1}{2}(M_\rho + E_{dual})^2 t^2\right]$$

quark-hadron duality *à la SVZ*

SVZ, 1979



matching with pQCD up to $t \sim 0.8$ fm

<u>long time distances</u>

$$V_{\pi\pi}(t) = \sum_n \nu_n |A_n|^2 e^{-\omega_n t}$$

M. Lüscher 1991

$$\omega_n = 2\sqrt{M_\pi^2 + k_n^2}$$

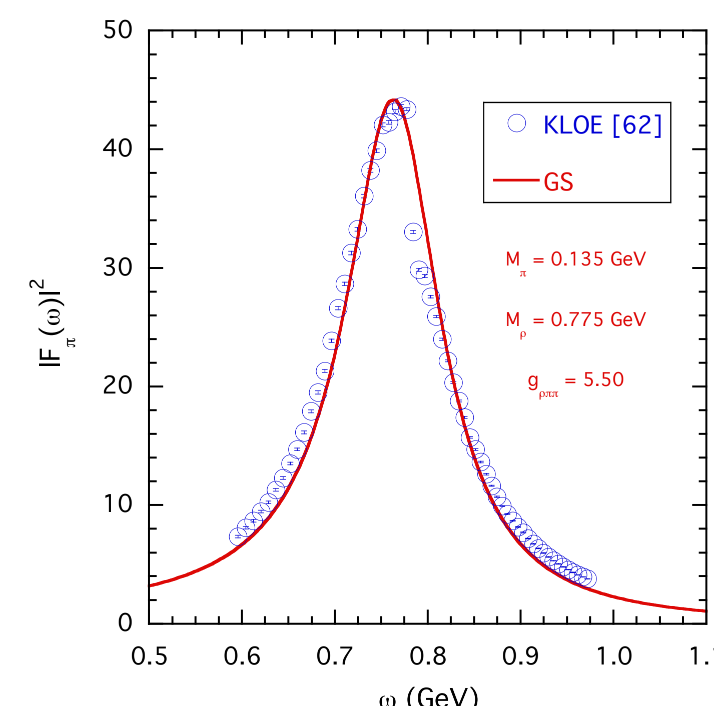
Lüscher condition

L. Lellouch and M. Lüscher, 2001
H.B. Meyer, 2011

$$|A_n|^2 \rightarrow |F_\pi(\omega_n)|^2$$

Gounaris-Sakurai parameterization

M_ρ, $g_{\rho\pi\pi}$ GS, 1968

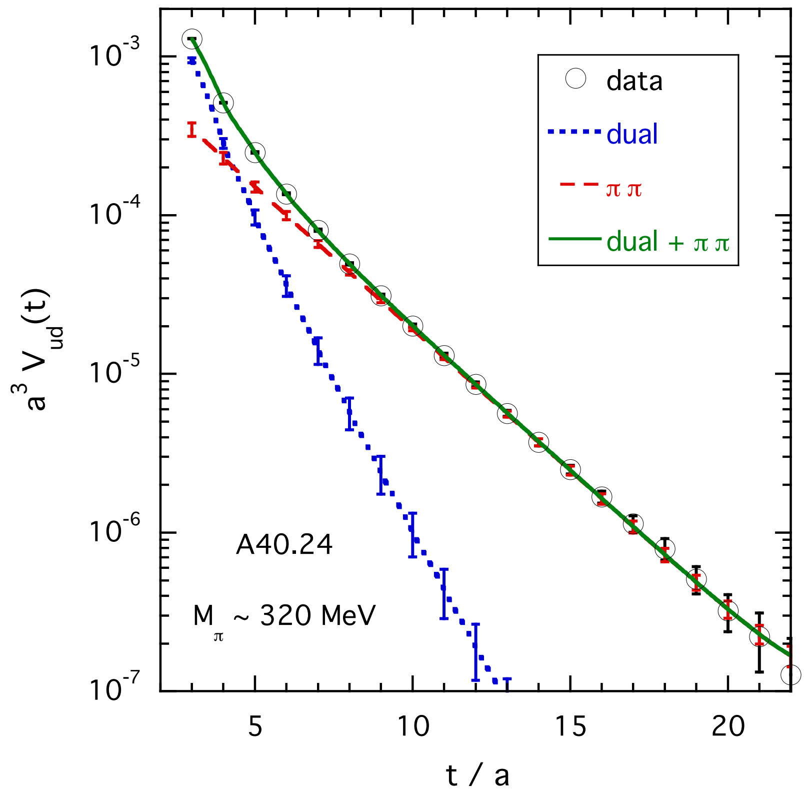


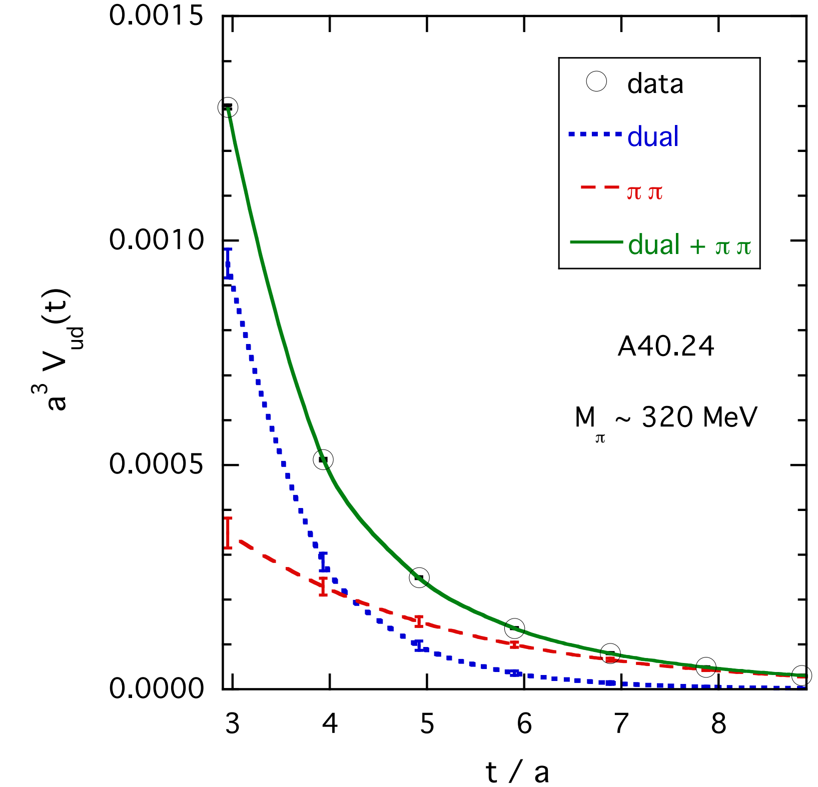
Subtraction of FVEs

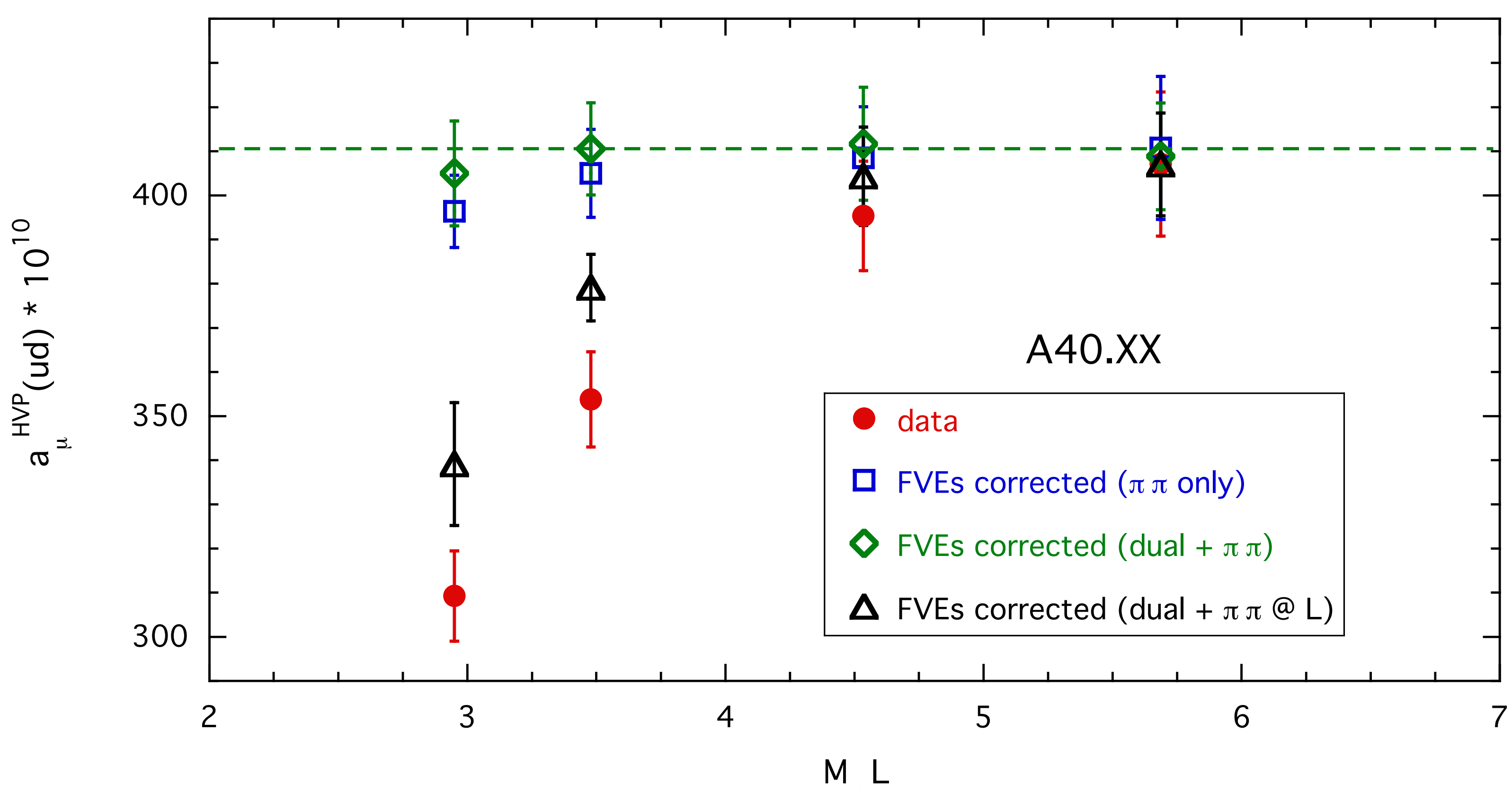
Accurate reproduction
for all the ETMC ensembles
$t \geq 0.2$ fm

$$R_{dual}, \ \frac{E_{dual}}{M_\pi}, \ g_{\rho\pi\pi}, \ \frac{M_\rho}{M_\pi}$$

π-π : 4 energy levels







$$a_\mu^{\mathrm{HVP}}(\infty) = a_\mu^{\mathrm{HVP}}(L) + \Delta_{\mathrm{FVE}} a_\mu^{\mathrm{HVP}}$$

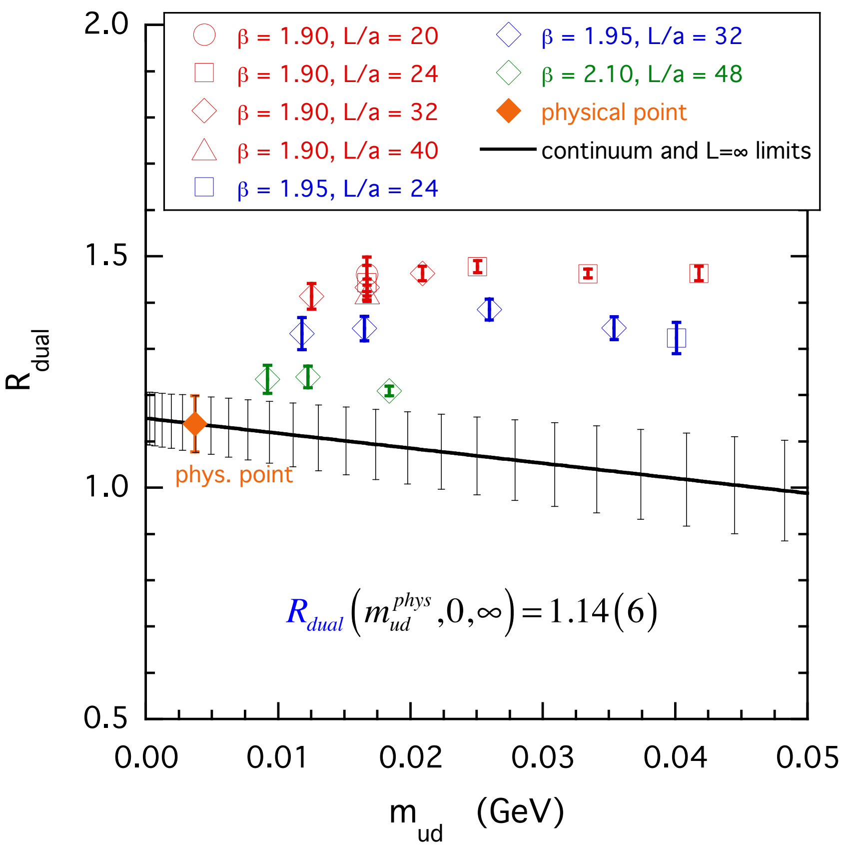
infinite-volume limit

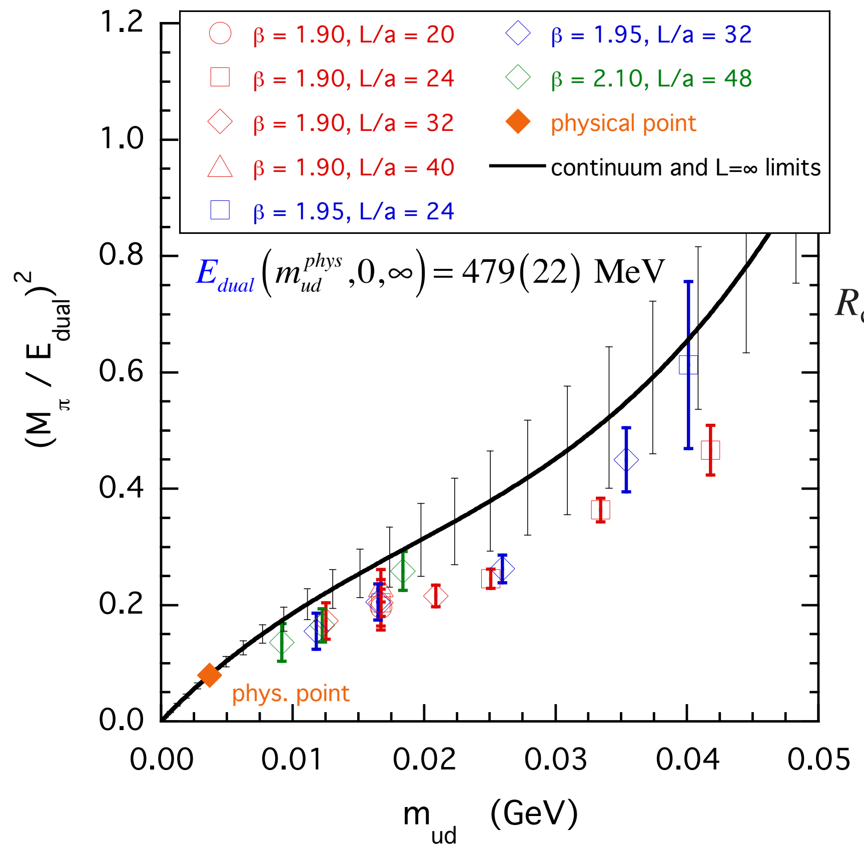
$$V_{dual}^\infty(t): \ R_{dual}^\infty \ M_\rho^\infty \ E_{dual}^\infty$$

$$V_{\pi\pi}^\infty(t) = \frac{1}{48\pi^2} \int_{2M_\pi^\infty}^\infty d\omega \ \omega^2 \left[1 - \frac{(2M_\pi^\infty)^2}{\omega^2}\right]^{3/2} \left|F_\pi^\infty(\omega)\right|^2 e^{-\omega t}$$

H. B. Meyer, 2011

Parameters

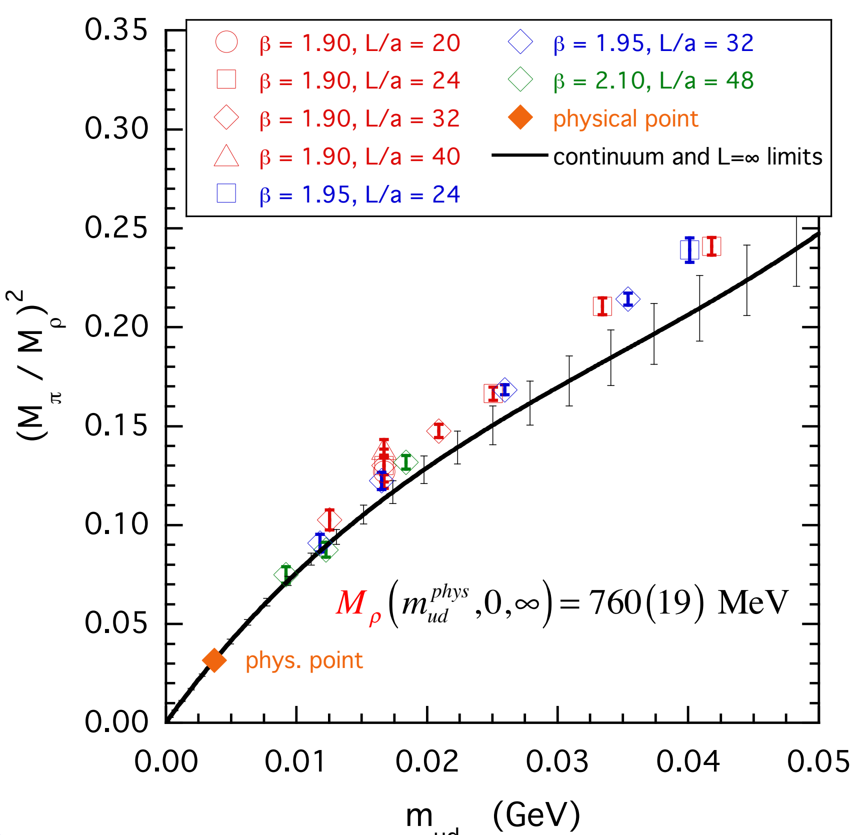


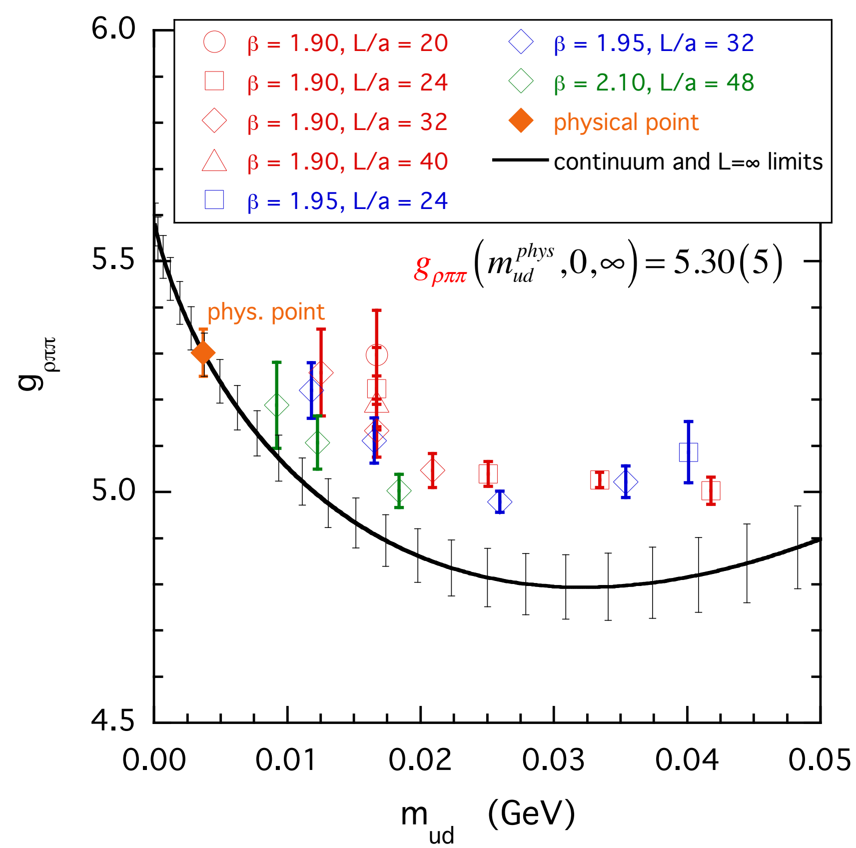


good control of FVEs

$$R_{\rm dual}(m_{ud}, a^2, L) = R_0[1 + R_1 m_{ud} + R_a a^2 + R_{am} a^2 m_{ud}] \times \left[1 + R_{\rm FVE}\,\xi \frac{e^{-ML}}{(ML)^{3/2}}\right]$$

$$\frac{M_\pi^2}{E_{\rm dual}^2}(m_{ud}, a^2, L) = E_0 m_{ud}[1 + E_1 m_{ud} + \xi \log(\xi) + E_2 m_{ud}^2 + E_a a^2]$$



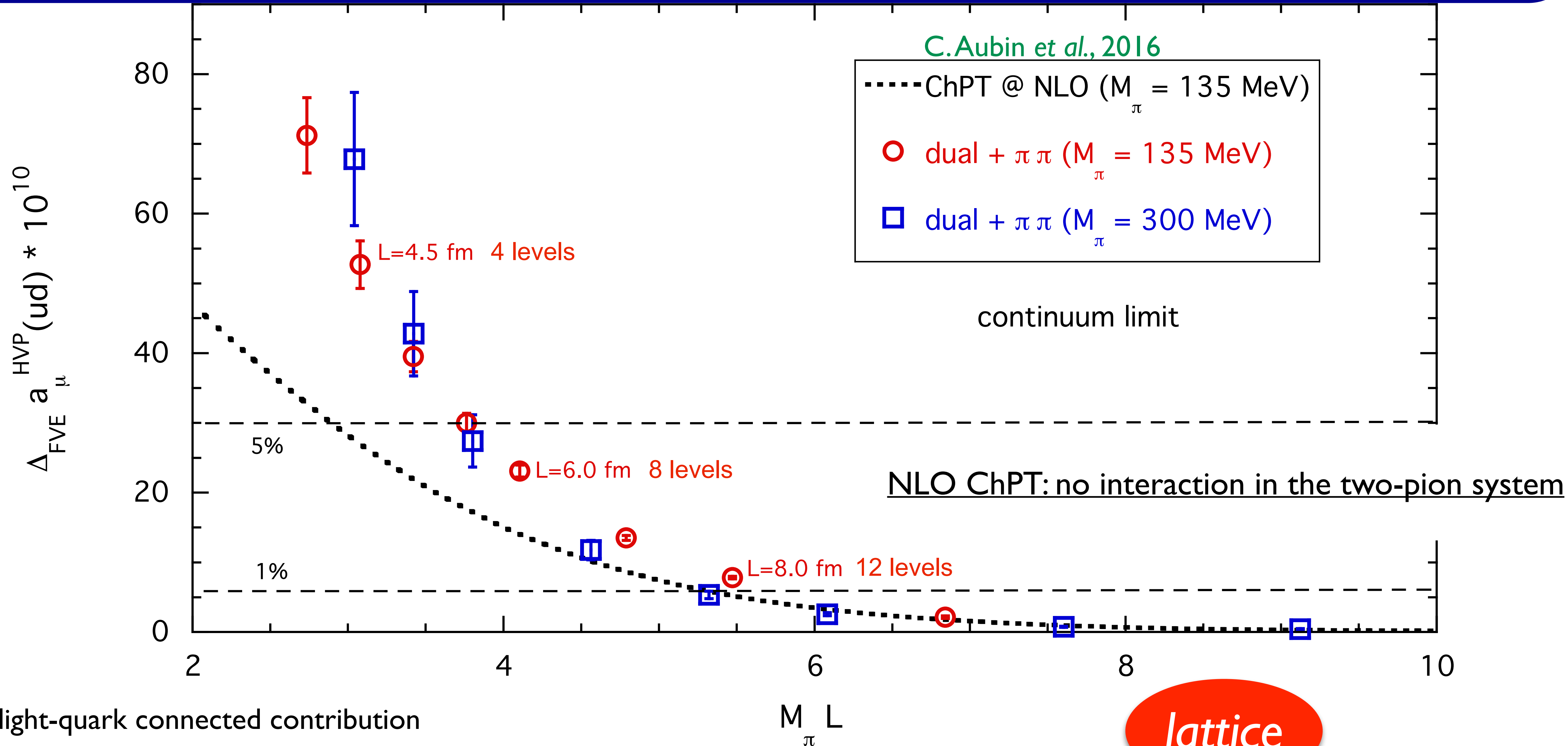


$$\frac{M_\pi^2}{M_\rho^2}(m_{ud}, a^2, L) = V_0 m_{ud}[1 + V_1 m_{ud} + \xi \log(\xi) + V_2 m_{ud}^2 + V_a a^2]$$

$$g_{\rho\pi\pi}(m_{ud}, a^2, L) = g_0[1 + g_1 m_{ud} + 2\xi \log(\xi) + g_a a^2] \times \left[1 + g_{\rm FVE}\,\xi \frac{e^{-ML}}{(ML)^{3/2}}\right]$$

$$M_\pi^2(m_{ud}, a^2, L) = 2B_0 m_{ud}[1 + P_1 m_{ud} + \xi \log(\xi) + P_2 m_{ud}^2 + P_a a^2] \cdot \left[1 + P_{\rm FVE}\,\xi \frac{e^{-ML}}{(ML)^{3/2}}\right]$$

FVEs correction @ $a^2 \to 0$



light-quark connected contribution

$M_\pi^{phys} L$	L (fm)	$\Delta^{\rm lat}_{\rm FVE}(L)/\Delta^{\rm ChPT,NLO}_{\rm FVE}(L)$
2.7	4.0	2.17 (17)
3.1	4.5	1.95 (13)
3.4	5.0	1.79 (10)
3.8	5.5	1.68 (8)
4.1	6.0	1.60 (6)
4.8	7.0	1.48 (4)
5.5	8.0	1.37 (5)

lattice

DG et al., 2019

$$\frac{\Delta^{\rm lat}_{\rm FVE}}{\Delta^{\rm ChPT,NLO}_{\rm FVE}}\left(L = 5 \div 6 \ {\rm fm}\right) \simeq 1.7(1)$$

$(5.4 \ {\rm fm} \to 10.8 \ {\rm fm})$

$$\frac{\Delta^{\rm lat}_{\rm FVE}}{\Delta^{\rm ChPT,NLO}_{\rm FVE}} \simeq 1.74(71)$$

PACS19

C.Aubin et al., 2019

NNLO ChPT

$$\frac{\Delta^{\rm ChPT,NNLO}_{\rm FVE}}{\Delta^{\rm ChPT,NLO}_{\rm FVE}}\left(L = 5 \div 6 \ {\rm fm}\right) \simeq 1.4(2)$$

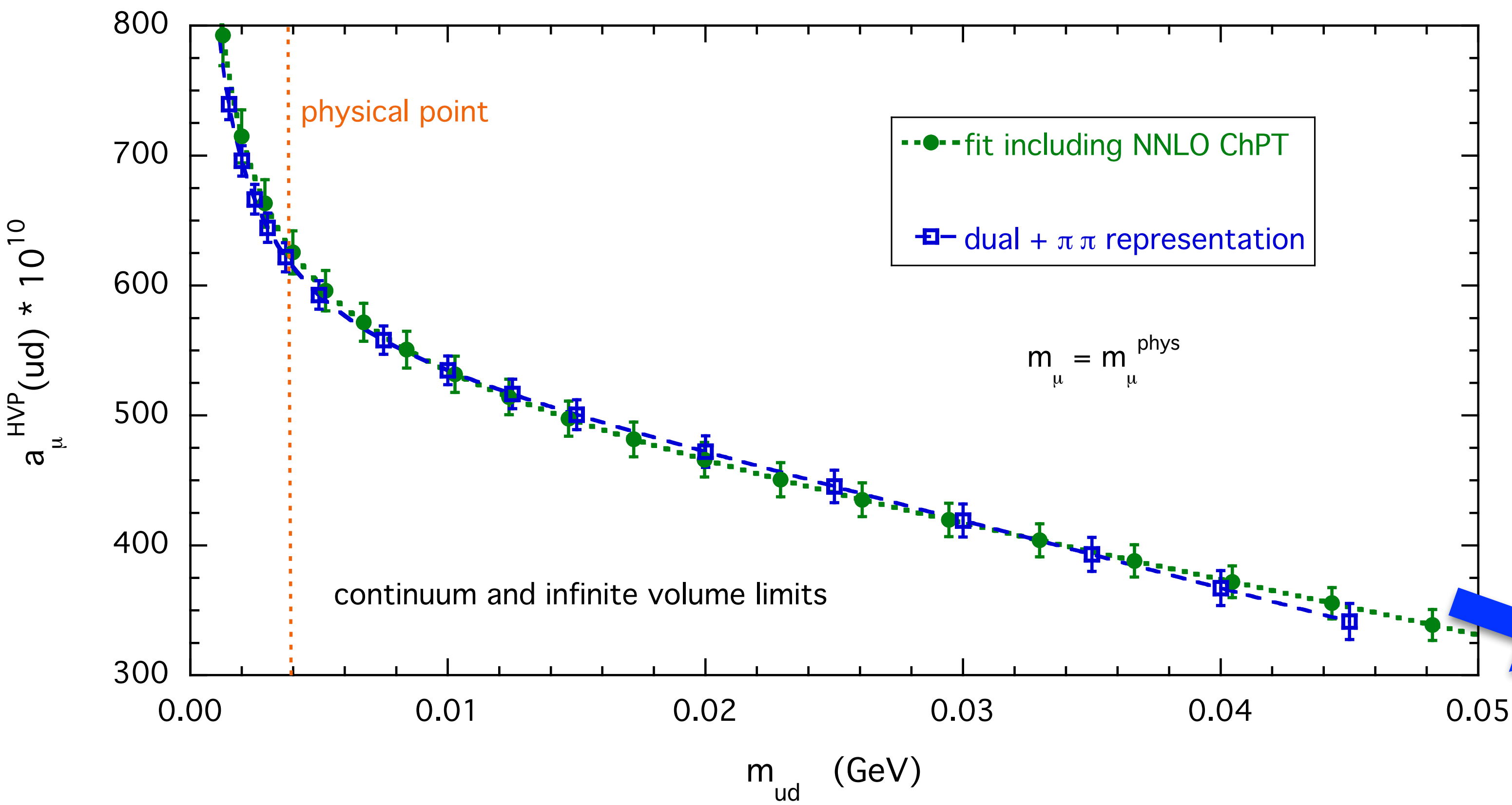
Extrapolation to M_π^{phys}

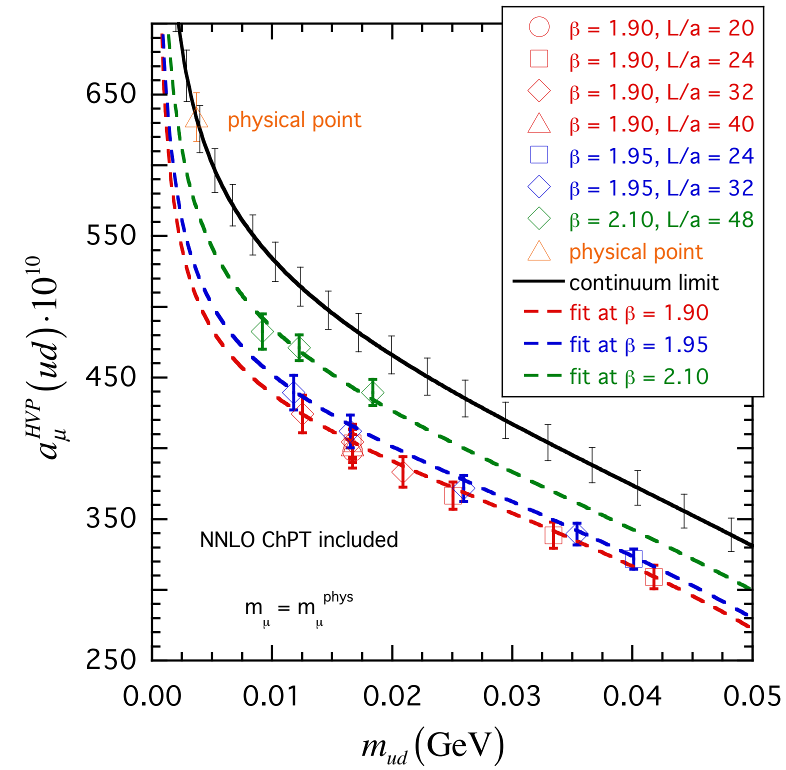
$a_\mu^{\rm HVP}(ud)$ diverges in the chiral limit ➡ $a_\mu^{\rm HVP}(ud) = \left\{ \left[a_\mu^{\rm HVP} \right]^{NLO} + \left[a_\mu^{\rm HVP} \right]^{NNLO}_{L_9, C_{93}} + A_0 + A_1 m_{ud} \right\} \left(1 + D_0 a^2 + D_1 a^2 m_{ud} \right)$

$\ln(m_{ud})$ LECs-independent

E. Golowich and J. Kambor, 1995; G. Amoros *et al.*, 2000
J. Bijnens and J. Refelors, 2016; M. Golterman *et al.*, 2017

Using the (dual + π-π) analytic representation





The blue points do not contain chiral logs explicitly

Using the analytic representation $a_\mu^{\rm HVP}(ud)$ does not depend on the absolute scale setting

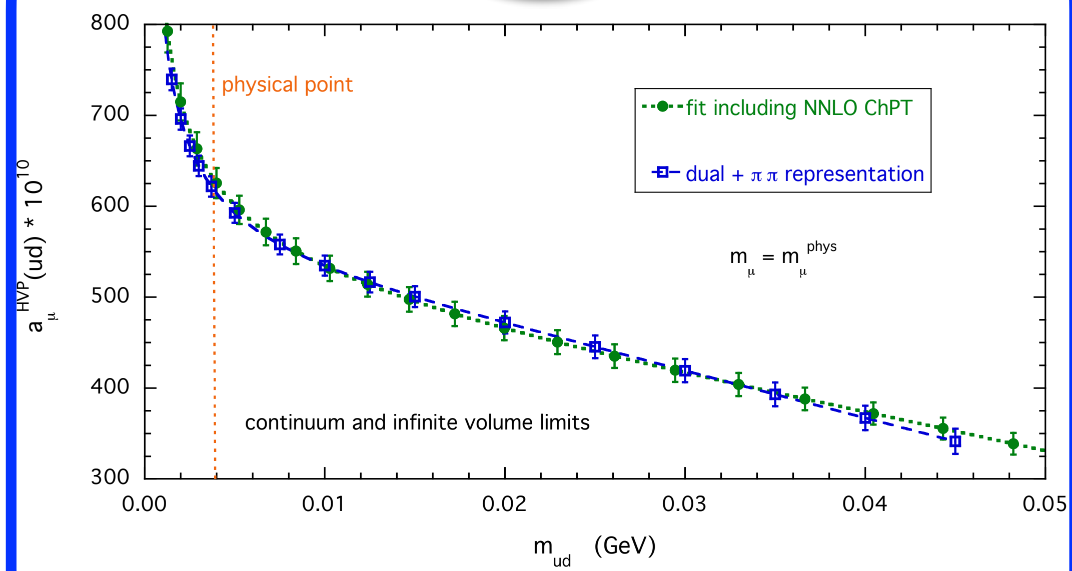
$$V_{dual+\pi\pi}(t) = M_\pi^3 \widetilde{V}\left(\tau_\pi; R_{dual}, \frac{E_{dual}}{M_\pi}, \frac{M_\rho}{M_\pi}, g_{\rho\pi\pi} \right)$$

$\tau_\pi = M_\pi t$ dimensionless quantities

$$\boxed{a_\ell^{\rm HVP}(ud) = 4\alpha_{em}^2 \int_0^\infty d\tau_\pi \widetilde{K}_\ell(\tau_\pi) \widetilde{V}(\tau_\pi)}$$

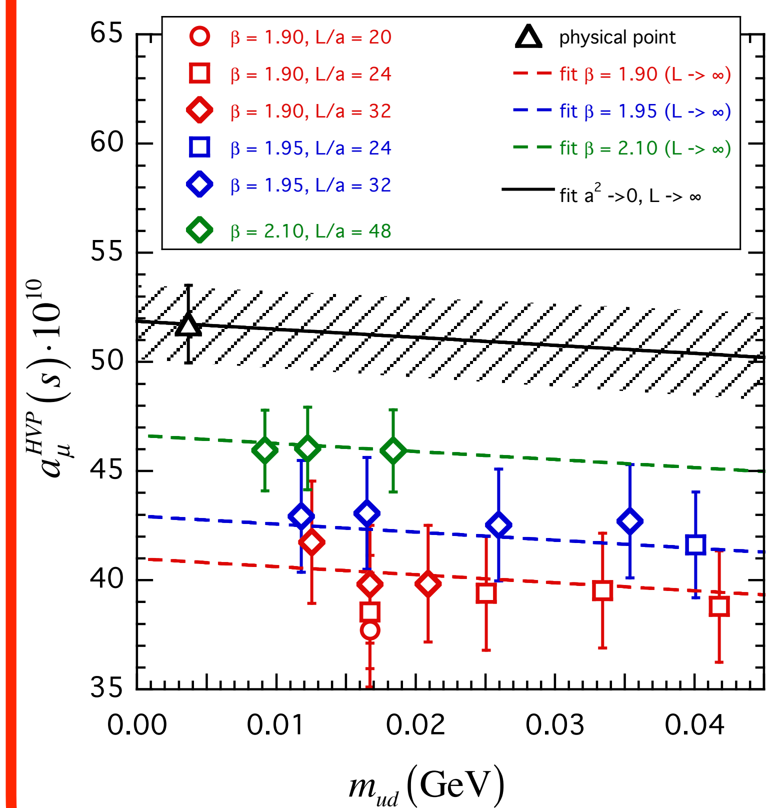
udsc-quark contributions

ud



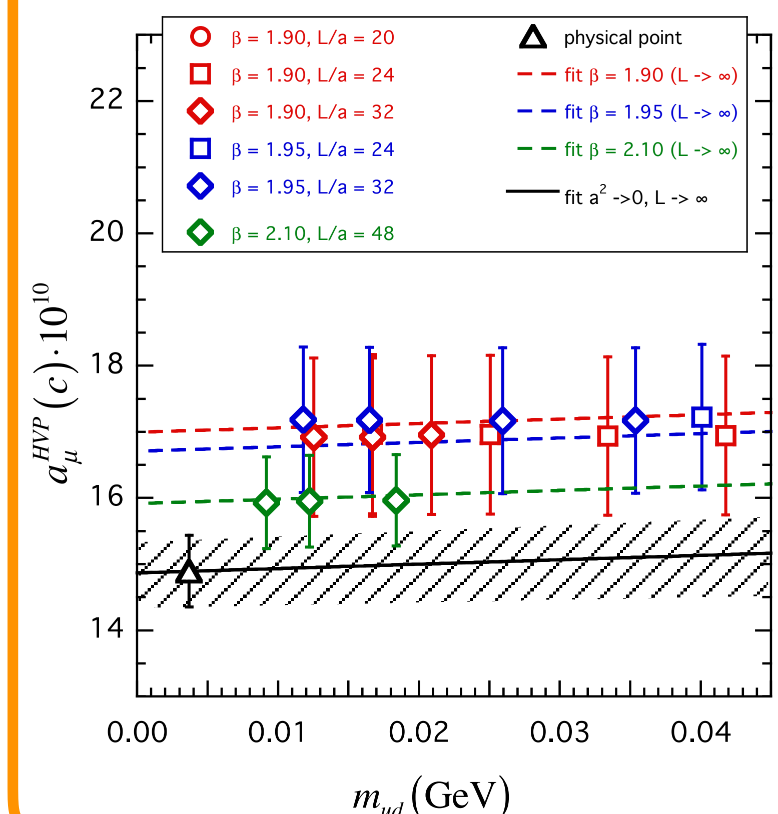
$$a_\mu^{\rm HVP}(ud) = 629.1(11.5)(7.5)[13.7] \cdot 10^{-10}$$

s



$$a_\mu^{\rm HVP}(s) = 53.1(2.5)\cdot 10^{-10}$$

c



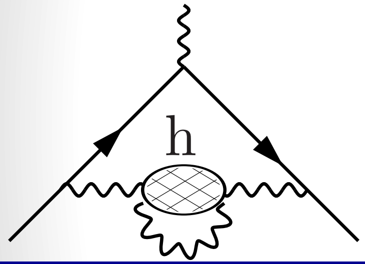
$$a_\mu^{\rm HVP}(c) = 14.75(56)\cdot 10^{-10}$$

DG *et al.*, 2018 — PRD98(2018)114504

DG and S. Simula, 2019 — ArXiv:1910.03874

DG *et al.*, 2017 — JHEP1710(2017)157

quark-connected terms only



LIB corrections

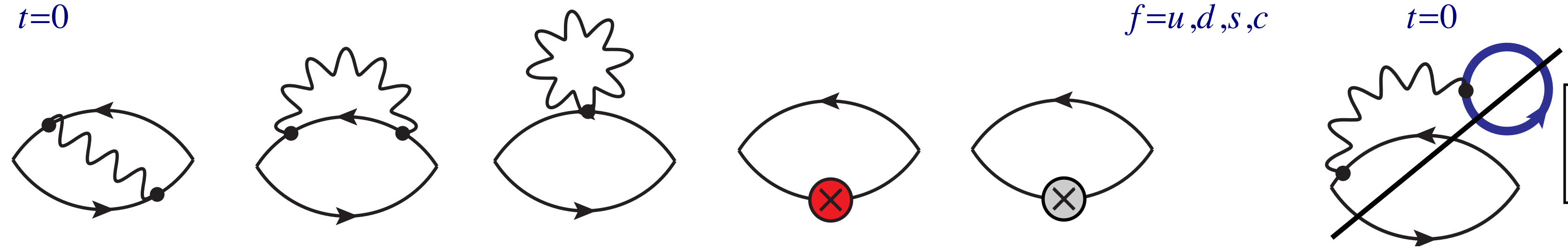
quark-connected terms only

$$\boxed{\delta a_\ell^{\mathrm{HVP}} = \delta a_\ell^{\mathrm{HVP}}(QCD) + \delta a_\ell^{\mathrm{HVP}}(QED)}$$

photon zero-mode: QED_L
M. Hayakawa and S. Uno, 2008

$$\delta a_\ell^{\mathrm{HVP}}(QCD) = \sum_{f=u,d,s,c} 4\alpha_{em}^2 \sum_{t=0}^{\infty} K_\ell(t)\,\delta V_f^{QCD}(t) \qquad \delta a_\ell^{\mathrm{HVP}}(QED) = \sum_{f=u,d,s,c} 4\alpha_{em}^2 \sum_{t=0}^{\infty} K_\ell(t)\,\delta V_f^{QED}(t)$$

RM123 method
G. M. de Divitiis *et al.*, 2012; 2013

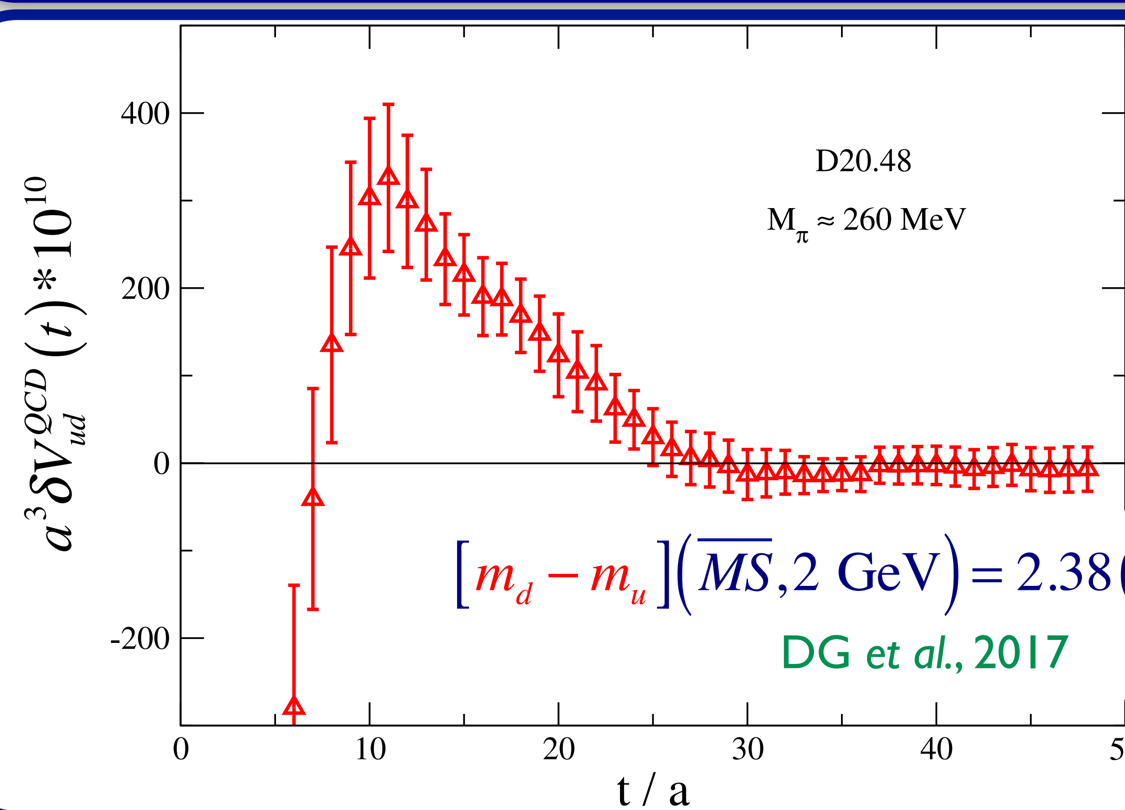


qQED approximation

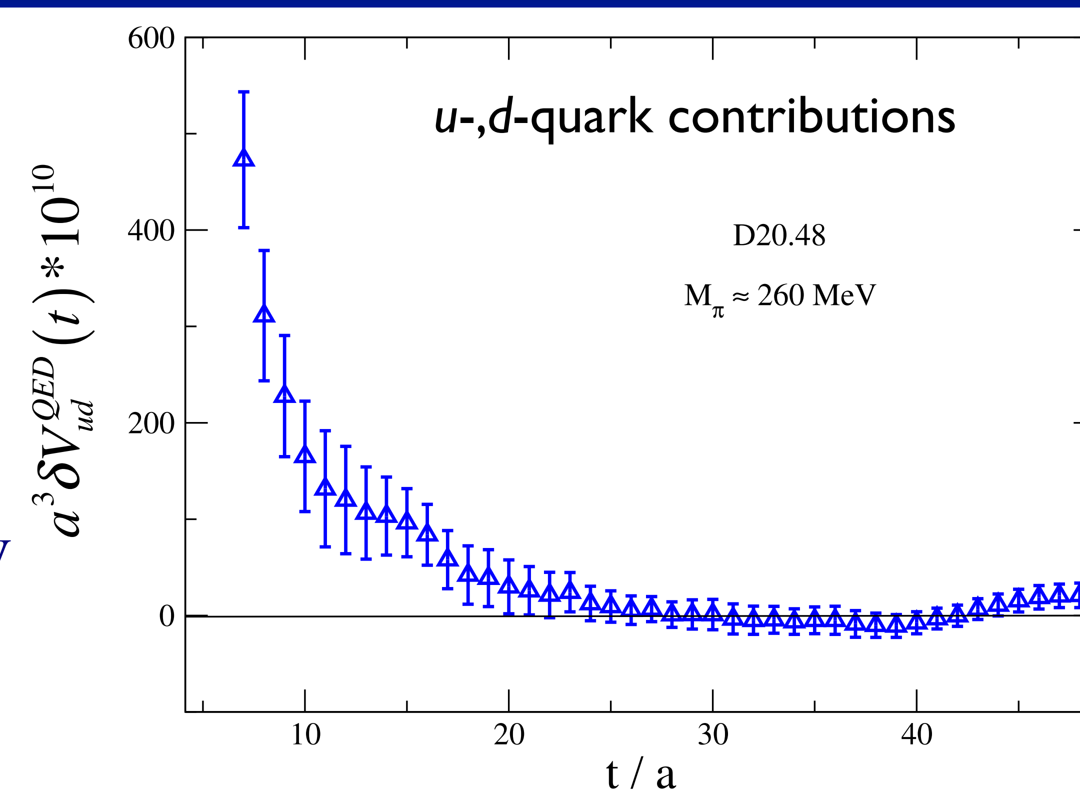
isoQCD/IB separation: consistent prescriptions adopted

QCD/QED separation is scheme and scale dependent $\longrightarrow$ $m_f(\overline{MS}, 2\ \mathrm{GeV}) = m_f^0(\overline{MS}, 2\ \mathrm{GeV})$
J. Gasser *et al.*, 2003

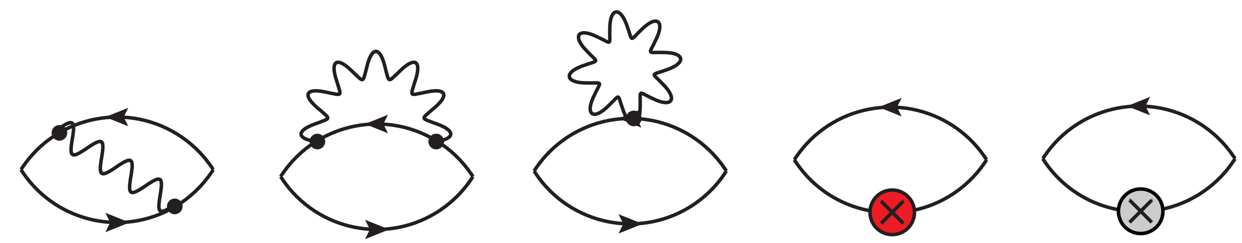
What is QCD in the full QCD+QED theory? see M. Di Carlo *et al.*, 2019

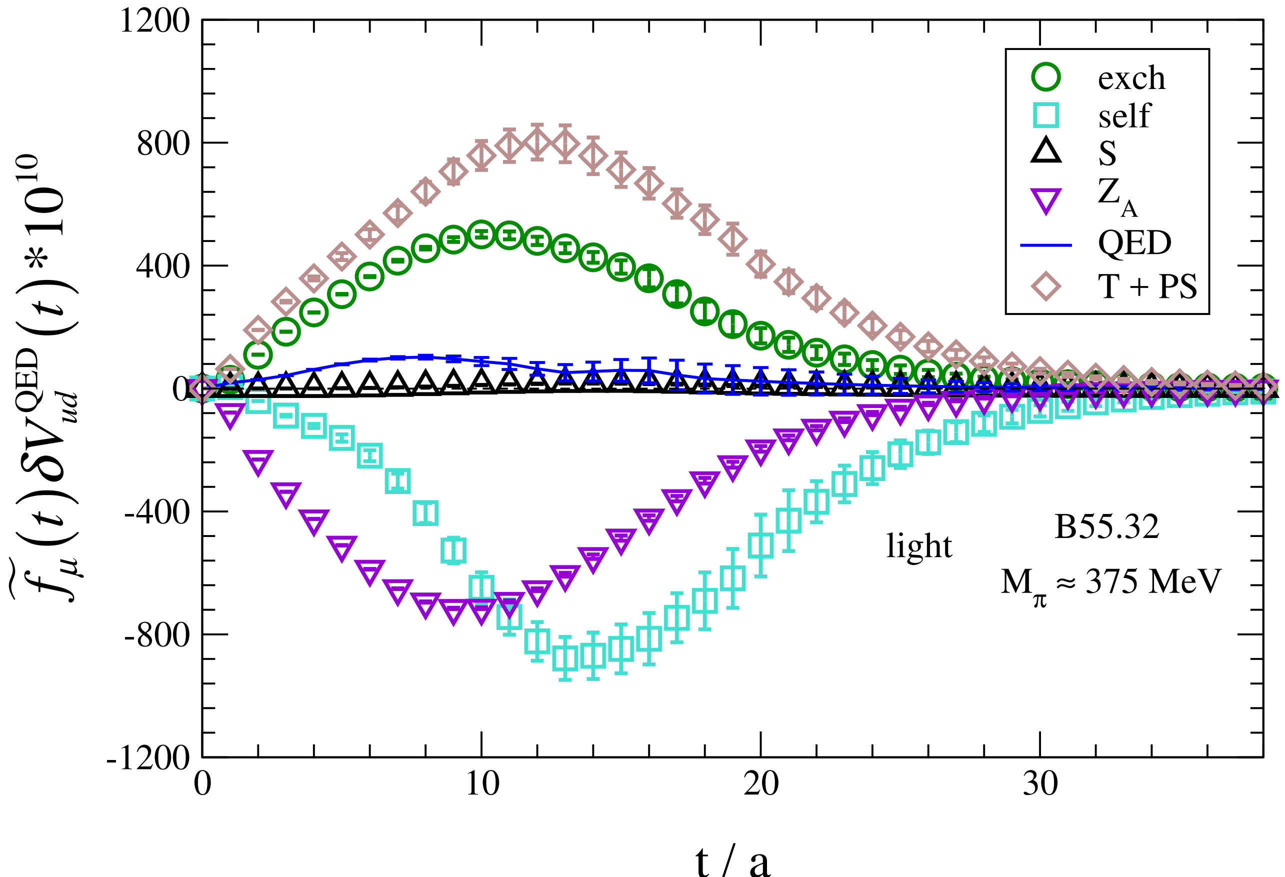


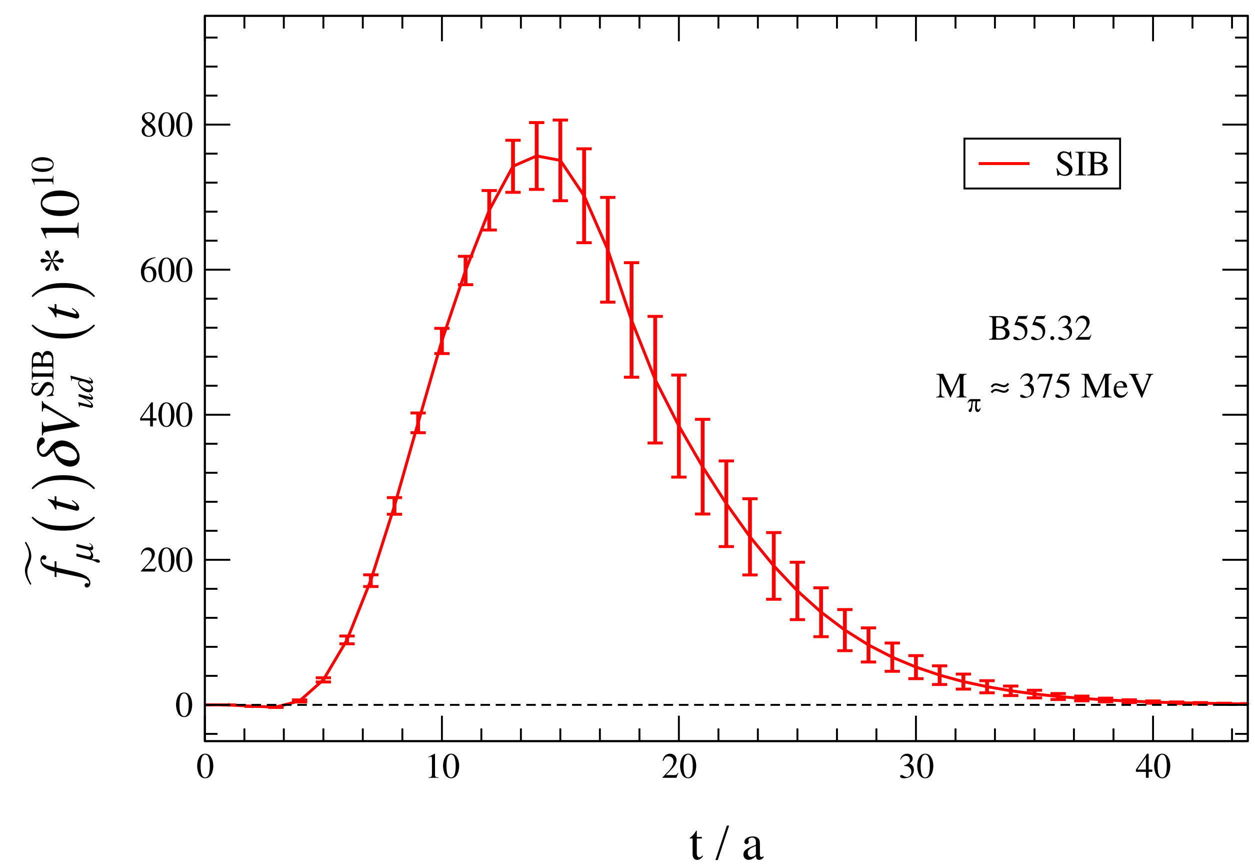
$$[m_d - m_u](\overline{MS}, 2\ \mathrm{GeV}) = 2.38(0.18)\ \mathrm{MeV}$$
DG *et al.*, 2017



LIB corr.:







$$Z_A = Z_A^{(0)}\left(1+\frac{\alpha_{em}}{4\pi}\delta Z_A^{QED} Z_A^{fact}\right)+O\left(\alpha_{em}^m \alpha_s^n\right)$$

$$\delta Z_A^{QED} = -15.7963\; q_f^2$$

perturbative estimate at LO

G. Martinelli and Y.-C. Zhang, 1982

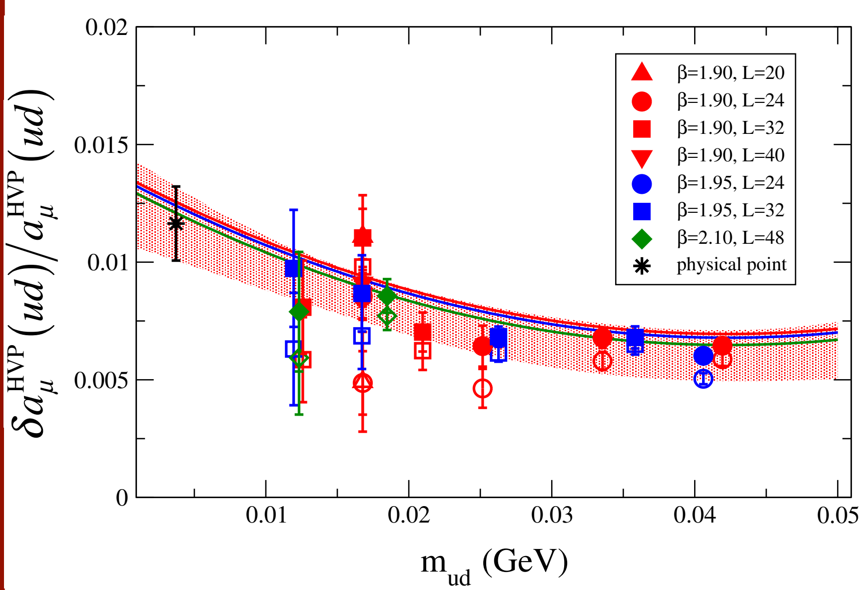
$$\boxed{\delta V_f^{Z_A}(t) = \frac{\alpha_{em}}{4\pi}\delta Z_A^{QED} Z_A^{fact}\;\; V^f(t)}$$

β	$Z_m^{\rm fact}$ (M1)	$Z_A^{\rm fact}$ (M1)	$Z_m^{\rm fact}$ (M2)	$Z_A^{\rm fact}$ (M2)
1.90	1.629 (41)	0.859 (15)	1.637 (14)	0.990 (9)
1.95	1.514 (33)	0.873 (13)	1.585 (12)	0.980 (8)
2.10	1.459 (17)	0.909 (6)	1.462 (6)	0.958 (3)

RI'-MOM @ $O\left(\alpha_{em}\alpha_s^n\right)$ DG *et al.*, 2019; M. Di Carlo *et al.*, 2019

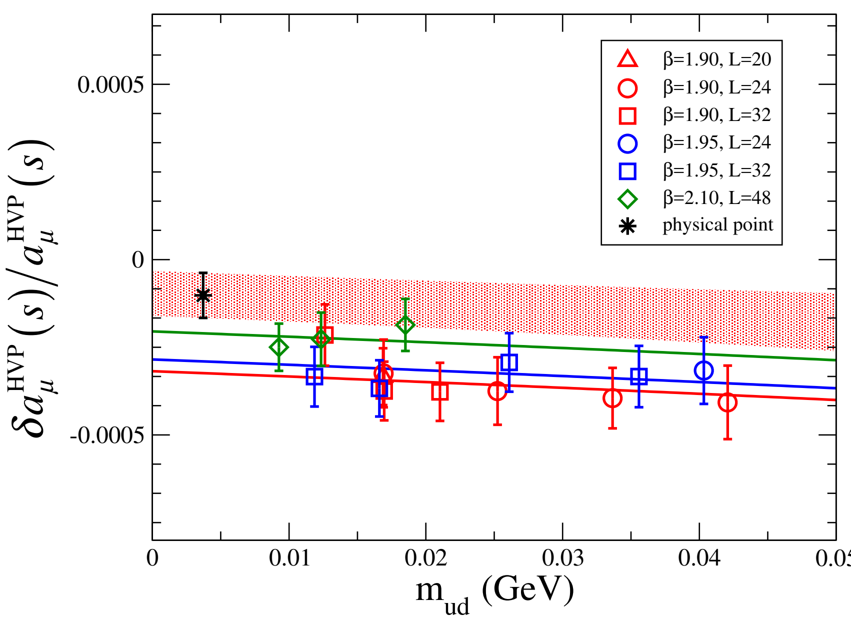
LIB corr.: *udsc*-quark contr.

ud



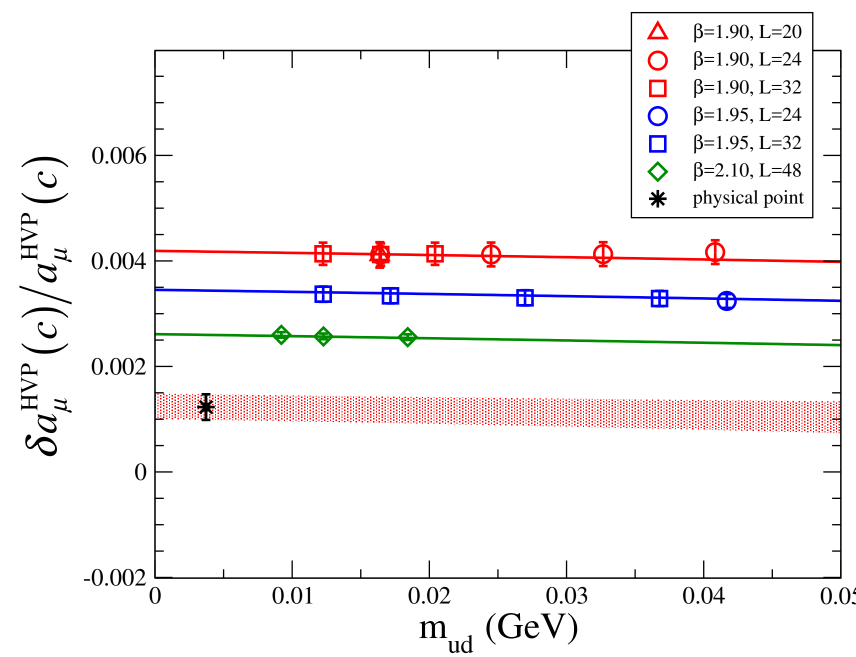
$$\delta a_\mu^{HVP}(ud) = 7.1(2.5)\cdot 10^{-10}$$

s



$$\delta a_\mu^{HVP}(s) = -0.0053(33)\cdot 10^{-10}$$

c



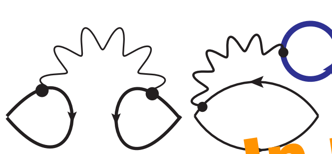
$$\delta a_\mu^{HVP}(c) = 0.0182(36)\cdot 10^{-10}$$

DG *et al.*, 2019

PRD99(2019)114502

$$\delta a_\mu^{HVP} = 7.1(2.6)(1.2)_{qQED+disc}\cdot 10^{-10}$$

$$= 7.1(2.9)\cdot 10^{-10}$$



In progress

a_e^{HVP} and a_τ^{HVP} : Lattice results

LO

ArXiv:1910.03874

IB

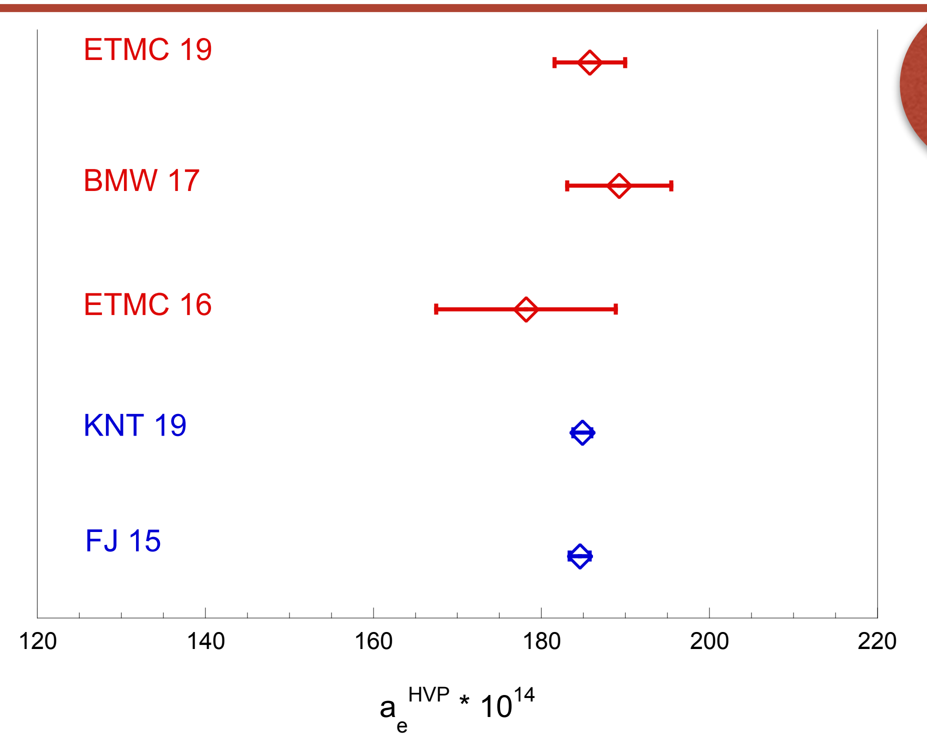
f	$a_e^{\text{HVP}}(f)\cdot 10^{14}$	$a_\tau^{\text{HVP}}(f)\cdot 10^{8}$
ud	170.7 (3.9)	273.3 (6.6)
s	13.5 (0.8)	36.2 (1.1)
c	3.5 (0.2)	25.8 (0.8)
disc BMW 17	−3.8 (0.4)	−2.4 (0.3)

f	$\delta a_e^{\text{HVP}}(f)\cdot 10^{14}$	$\delta a_\tau^{\text{HVP}}(f)\cdot 10^{8}$
ud	1.9 (0.8)	3.0 (1.1)
s	−0.002 (0.001)	0.001 (0.002)
c	0.004 (0.001)	0.032 (0.006)
total	1.9 (1.0)	3.0 (1.3)

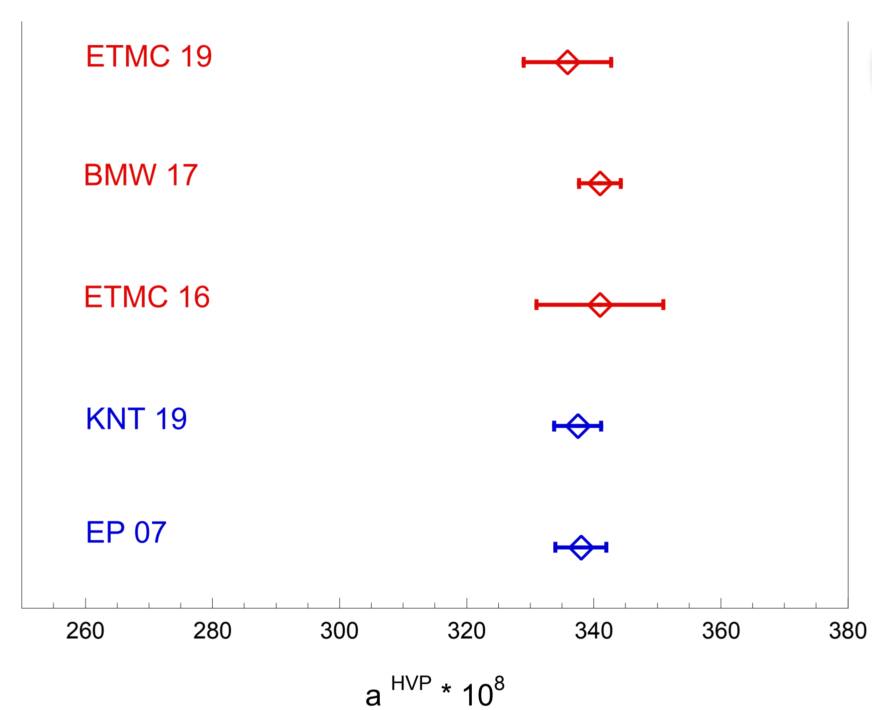
$$a_e^{\text{HVP}} = 185.8(4.2)\cdot 10^{-14}$$

$$a_\tau^{\text{HVP}} = 335.9(6.9)\cdot 10^{-8}$$

e

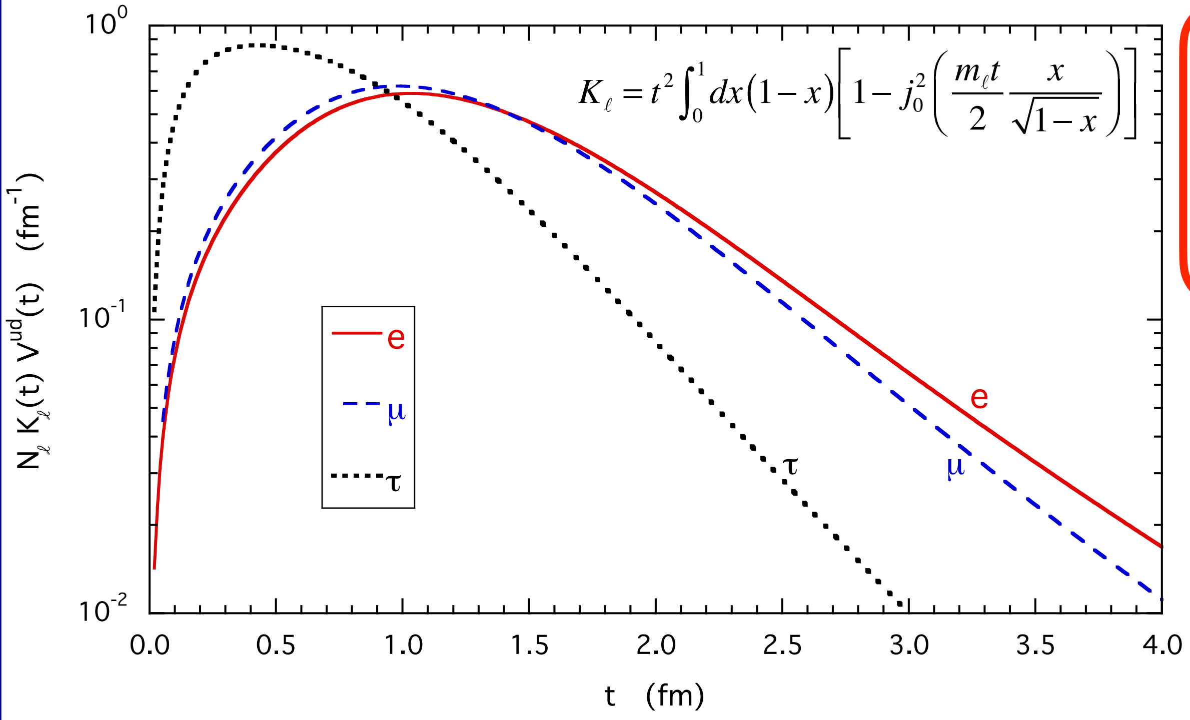


τ



Ratios of the HVP contributions to the lepton g-2

Ratio electron/muon



$$R_{e/\mu} \equiv \left(\frac{m_\mu}{m_e}\right)^2 \frac{a_e^{\rm HVP}}{a_\mu^{\rm HVP}}$$

- numerator and denominator share the same hadronic input
- hadronic uncertainties strongly correlated (~ 98%) and largely cancel out

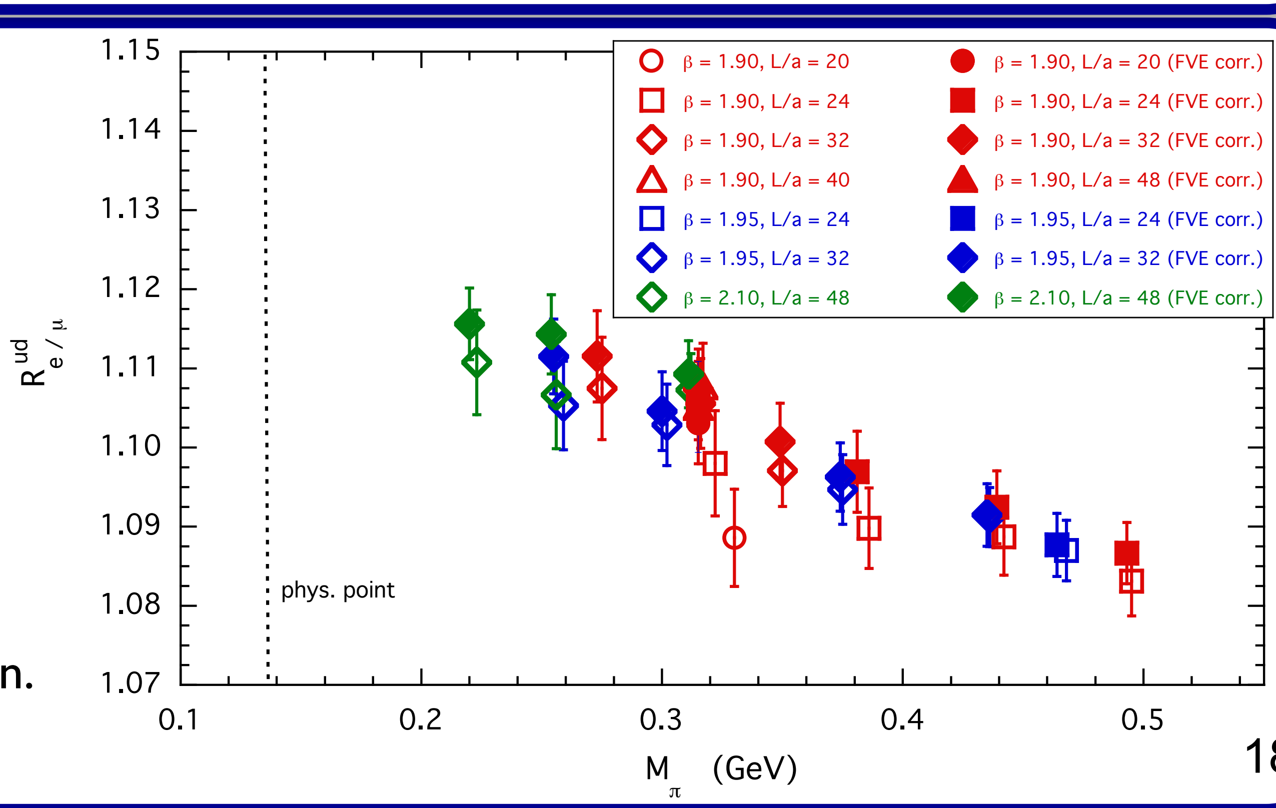
$$R_{e/\mu} \equiv R^{ud}_{e/\mu} \cdot \widetilde{R}_{e/\mu}$$

$$R^{ud}_{e/\mu} \equiv \left(\frac{m_\mu}{m_e}\right)^2 \frac{a_e^{\rm HVP}(ud)}{a_\mu^{\rm HVP}(ud)}$$

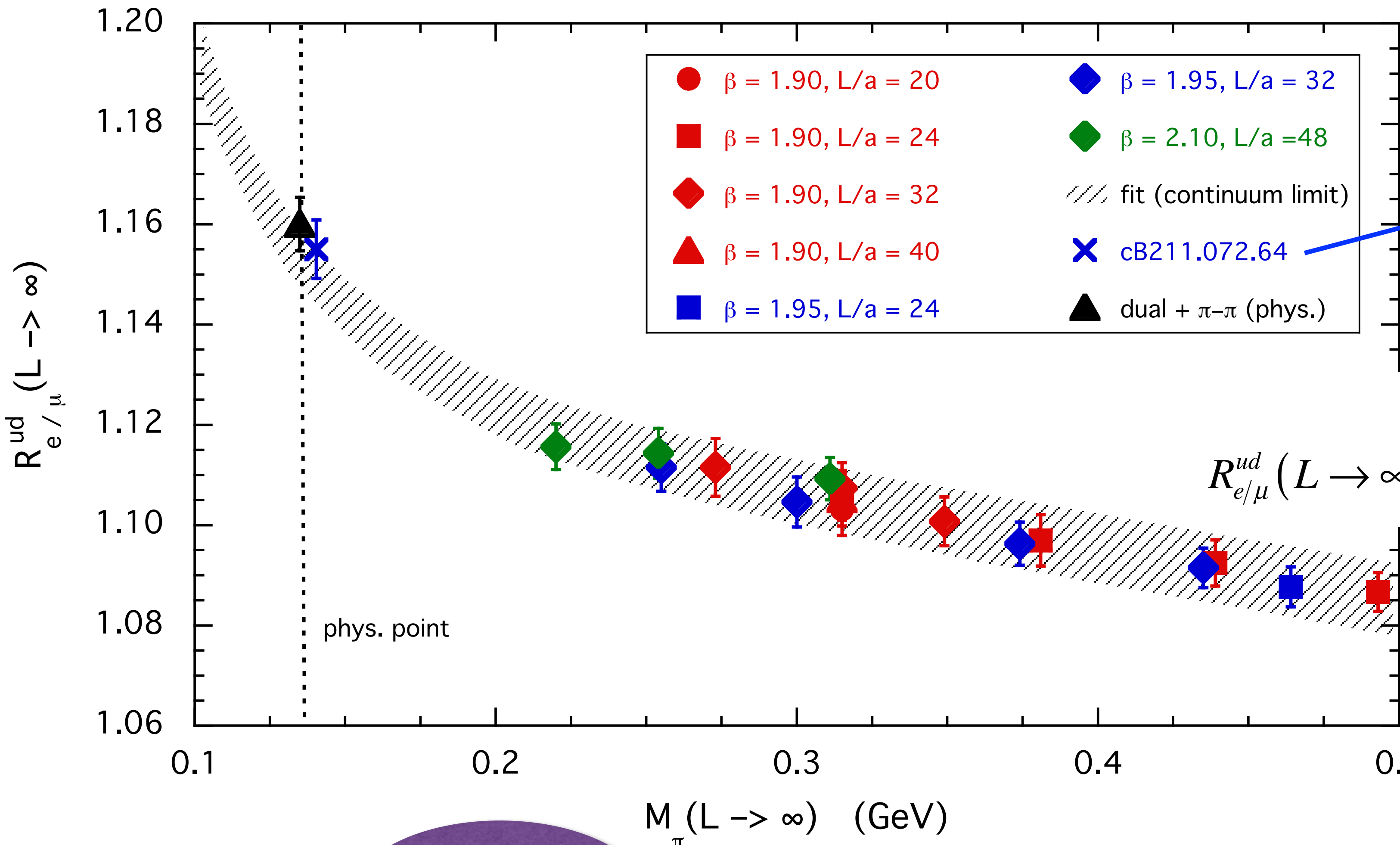
$$\widetilde{R}_{e/\mu} \equiv \frac{1+\sum_{j=s,c,IB,disc} \frac{a_e^{\rm HVP}(j)}{a_e^{\rm HVP}(ud)}}{1+\sum_{j=s,c,IB,disc} \frac{a_\mu^{\rm HVP}(j)}{a_\mu^{\rm HVP}(ud)}}$$

$R^{ud}_{e/\mu}$

- Precision of the data ≈ 4 times better than the individual HVP terms
- Discretization and scale setting errors play a minor role
- Non-trivial pion mass dependence
- Visible FVEs, removed using the analytic representation. The correction does not exceed ~ 1.3%



Ratio electron/muon



$N_f = 2+1+1$

$M_\pi = 139(1)$ MeV

$a = 0.0803(4)$ fm

$L \simeq 5.1$ fm

$$R^{ud}_{e/\mu}(L \to \infty) = \left[\frac{m_\mu^2}{m_e^2} \left(\frac{a_e^{\mathrm{HVP}}(ud)}{a_\mu^{\mathrm{HVP}}(ud)} \right)^{NNLO}_{L_9, C_{93}} + A_0 + A_1 M_\pi^2 \right] \left(1 + Da^2\right)$$

DG and S. Simula, 2020

PRD102(2020)054503

$R^{ud}_{e/\mu}$

$$R^{ud}_{e/\mu} = 1.1578(52)_{stat}(39)_{syst}[65]$$

$\tilde{R}_{e/\mu}$

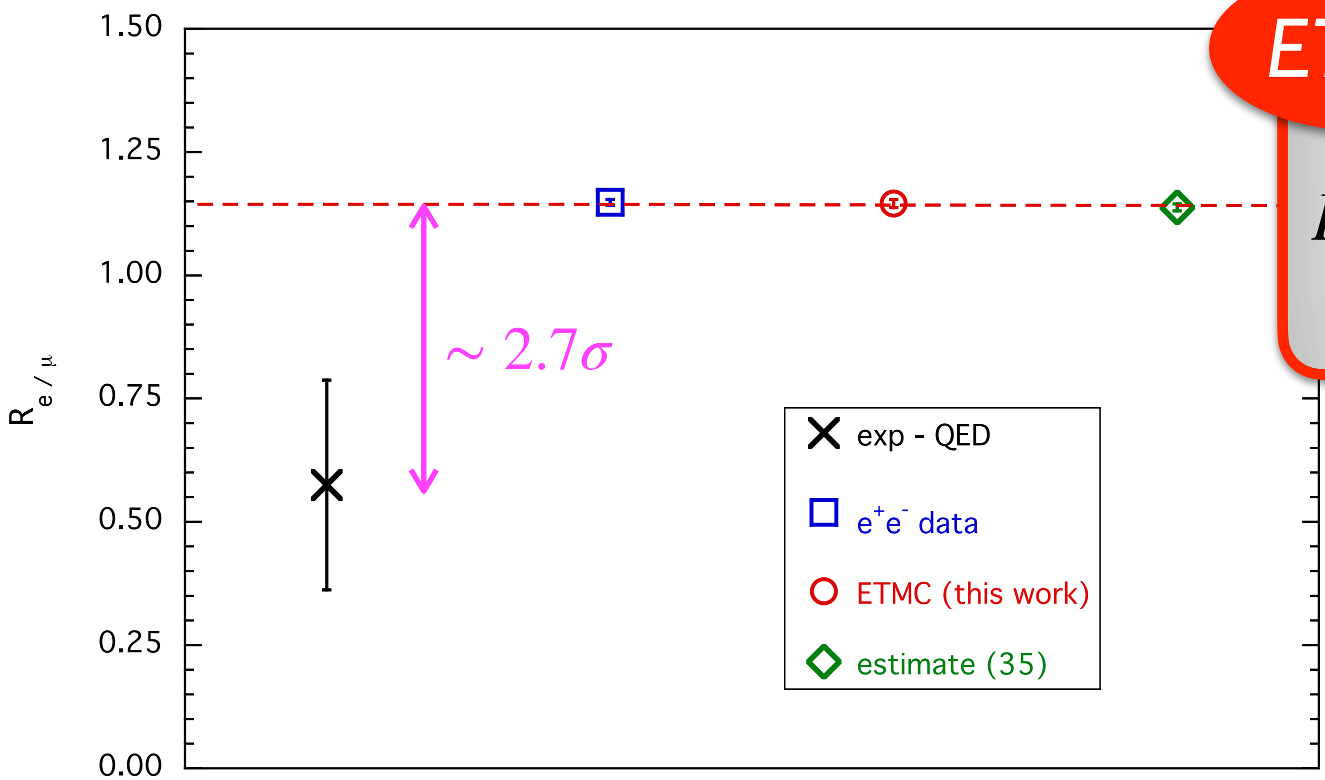
$$\tilde{R}_{e/\mu} \equiv \frac{1 + \sum_{j=s,c,IB,disc} \frac{a_e^{\mathrm{HVP}}(j)}{a_e^{\mathrm{HVP}}(ud)}}{1 + \sum_{j=s,c,IB,disc} \frac{a_\mu^{\mathrm{HVP}}(j)}{a_\mu^{\mathrm{HVP}}(ud)}}$$

$$\tilde{R}_{e/\mu} = 0.9895(32)_{stat}(31)_{syst}[45]$$

$a_\mu^{\mathrm{HVP}}(disc) = -12(4) \cdot 10^{-10}$

BMW 17 & RBC/UKQCD 18

Results



ETMC

$$R_{e/\mu} = \left(\frac{m_\mu}{m_e}\right)^2 \frac{a_e^{\mathrm{HVP}}}{a_\mu^{\mathrm{HVP}}} = 1.1456(63)_{stat}(54)_{syst}[83]$$

$$R_{e/\mu} = 1.1381(72)$$

based on BMW 17 (assuming 100% correlation)

$$R_{e/\mu}^{e^+e^-} = 1.148343(62)$$

KNT 19

- The ratio $R_{e/\mu}$ is less sensitive to possible tensions between lattice and dispersive results occurring for the individual HVP terms

$$R_{e/\mu}^{\mathrm{exp}-QED} = 0.575(213)_e(6)_\mu[213]$$

- Twofold improvement in a_e^{exp} and a_e^{QED} would lead to $\sim 5\sigma$ tension

$R_{e/\mu}$	$R_{e/\tau}$	$R_{\mu/\tau}$
1.1456 (83)	6.69 (20)	5.83 (17)

numerator and denominator almost uncorrelated

Benchmark quantities

Windows

$$a_\mu^{\text{HVP}} = a_\mu^{SD} + a_\mu^{W} + a_\mu^{LD}$$

light-quark connected contribution, isoQCD

$$a_\mu^{SD}(t_0;\Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\ K_\mu(t) V^{ud}(t)\left[1-\Theta(t,t_0;\Delta)\right]$$

$$a_\mu^{W}(t_0,t_1;\Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\ K_\mu(t) V^{ud}(t)\left[\Theta(t,t_0;\Delta)-\Theta(t,t_1;\Delta)\right]$$

$$a_\mu^{LD}(t_1;\Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\ K_\mu(t) V^{ud}(t)\Theta(t,t_1;\Delta)$$

$$\Theta(t,t';\Delta) = \frac{1}{1+e^{-2(t-t')/\Delta}}$$

$$t_0 = 0.4 \text{ fm} \quad t_1 = 1.0 \text{ fm}$$

$$\Delta = 0.15 \text{ fm}$$

Effective lepton mass and effective windows

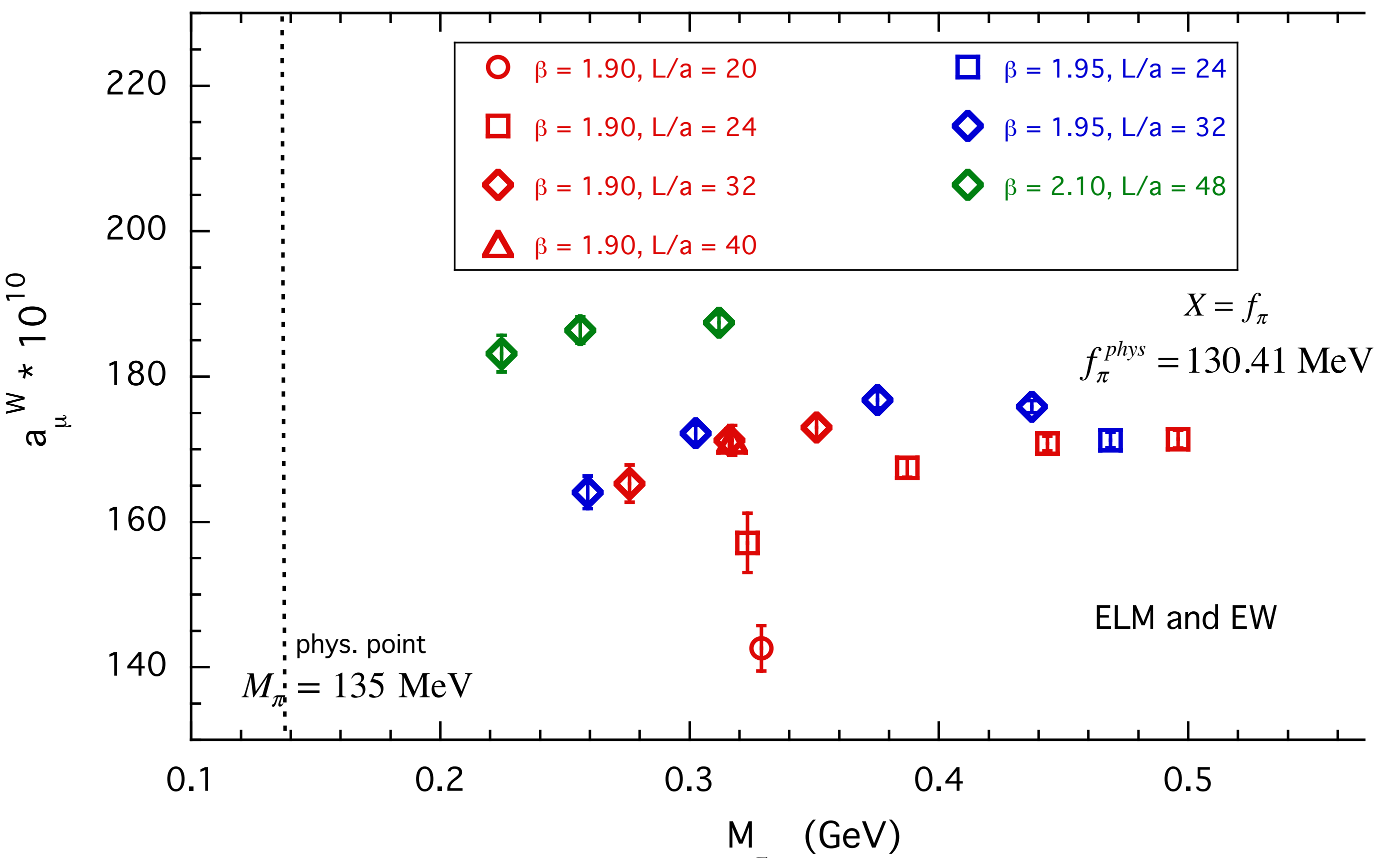
$$m_\mu^{eff} \equiv \left(m_\mu / X^{phys}\right) X \qquad t_1^{eff} \equiv t_1 X^{phys}/X$$

$$\Delta^{eff} \equiv \Delta X^{phys}/X \qquad t_0^{eff} \equiv t_0 X^{phys}/X$$

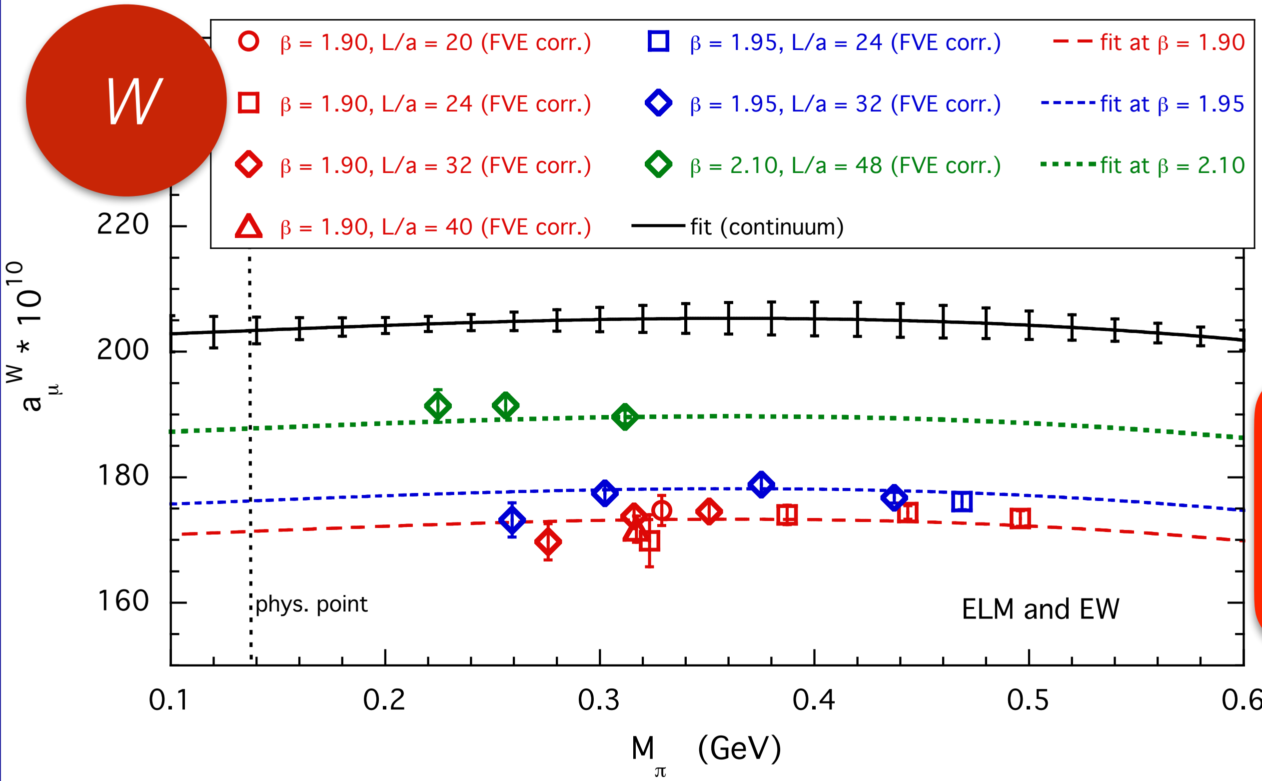
$$a_\mu^W\left(t_0^{eff},t_1^{eff};\Delta^{eff}\right) = 4\alpha_{em}^2 \frac{1}{m_\mu^2}\left(\frac{X^{phys}}{aX}\right)^2 \sum_{n=1}^{N_T} \widetilde{K}_\mu\left(m_\mu \frac{aX}{X^{phys}} n\right) a^3 V^{ud}(an)$$

$$\cdot\left[\Theta\left(aXn, t_0 X^{phys}; \Delta X^{phys}\right) - \Theta\left(aXn, t_1 X^{phys}; \Delta X^{phys}\right)\right]$$

- **Advantage**: uncertainty of the scale setting does not play any role
- For $X = f_\pi$ the pion mass dependence is mild
- Visible FVEs and large discretization effects



Preliminary results



$$a_\mu^W = A_0\left[1 + A_{1\ell} M_\pi^2 \log\left(M_\pi^2\right) + A_1 M_\pi^2 + A_2 M_\pi^4 + D a^2\right] \cdot \left[1 + F_0 M_\pi^2 e^{-M_\pi L} / \left(M_\pi L\right)^p\right]$$

optimal choice $p \simeq 2$

$$a_\mu^W = 202.2(2.0)_{stat}(0.4)_{chir}(1.5)_{disc}(0.7)_{FVE}[2.6] \cdot 10^{-10}$$

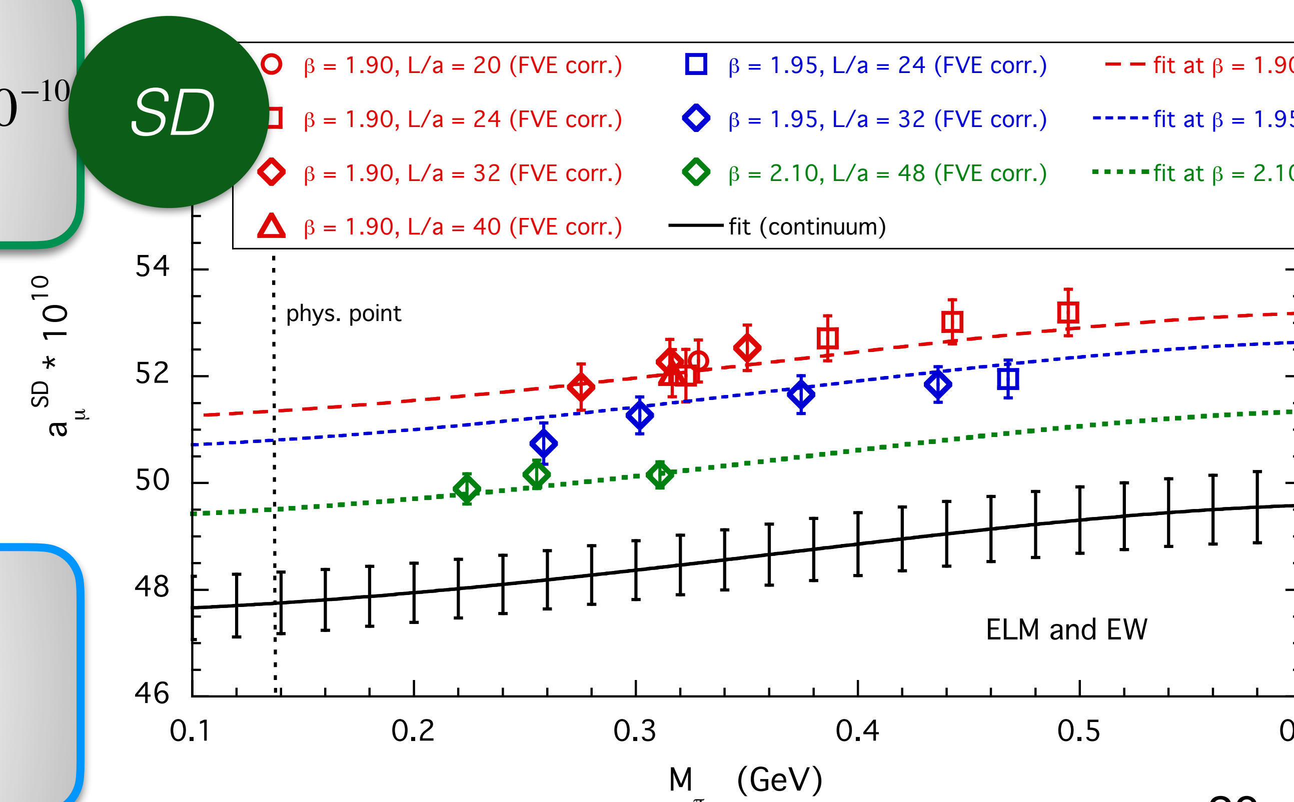
$$a_\mu^W = 198.0(3.4)_{stat}(4.7)_{syst}[5.8] \cdot 10^{-10}$$

analytic representation

$$a_\mu^{SD} = 48.21(0.56)_{stat}(0.10)_{chir}(0.50)_{disc}(0.25)_{FVE}[0.80] \cdot 10^{-10}$$

$$a_\mu^{SD} = 48.6(1.8)_{stat}(1.0)_{syst}[2.0] \cdot 10^{-10}$$

analytic representation



$$a_\mu^{LD} = 382.5(10.5)_{stat}(5.2)_{syst}[11.7] \cdot 10^{-10}$$

analytic representation

Moments of the HVP function

light-quark connected contribution

$$\Pi_{n+1}^{ud} \equiv (-)^n \frac{(n+1)!}{(2n+4)!} \frac{18}{5} \int_0^\infty dt\ t^{2n+4} V^{ud}(t)$$

Total (incl. s,c-quak;disc.;IB)

$$\boxed{\Pi_1^{tot} = 0.1002(23)\ \mathrm{GeV}^{-2}}$$

$\Pi_1^{tot} = 0.1000(30)\ \mathrm{GeV}^{-2}$ Sz. Borsanyi *et al.*, 2016

$\Pi_1^{tot} = 0.1000(23)\ \mathrm{GeV}^{-2}$ FHM19

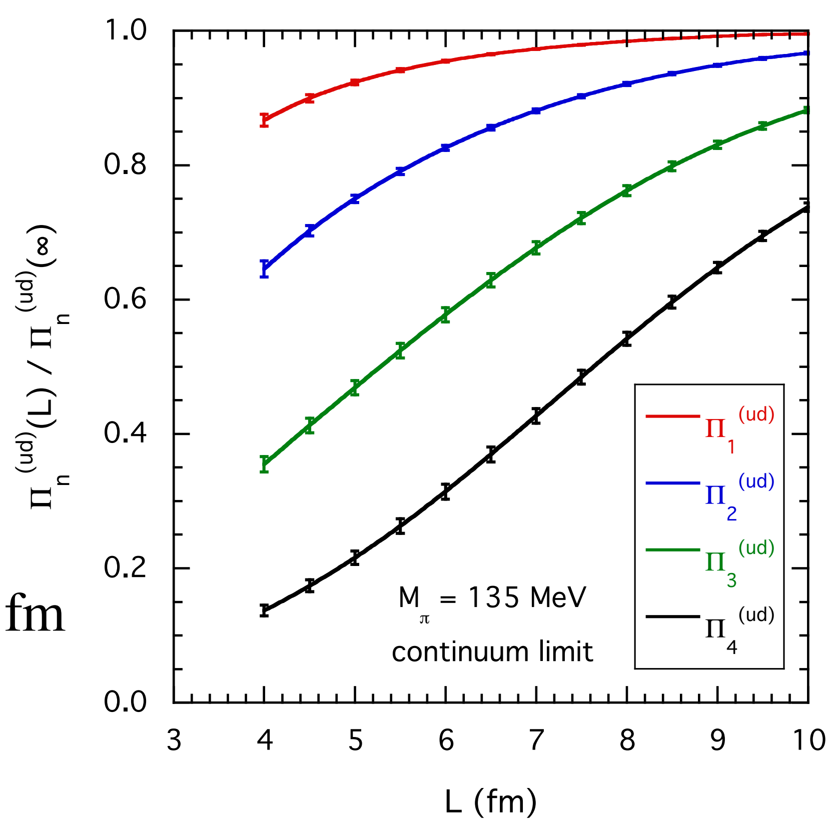
(dual + π-π) representation

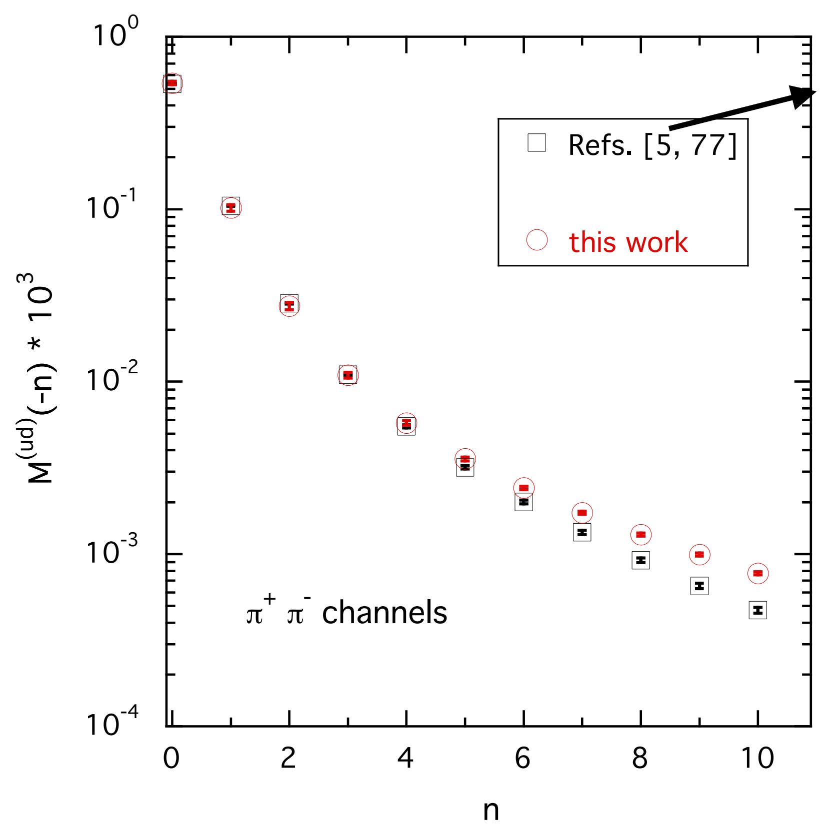
$\Pi_1^{ud} = 0.1642(33)\ \mathrm{GeV}^{-2}$ $\Pi_2^{ud} = -0.383(16)\ \mathrm{GeV}^{-4}$

$\Pi_3^{ud} = 1.394(65)\ \mathrm{GeV}^{-6}$ $\Pi_4^{ud} = -7.60(28)\ \mathrm{GeV}^{-8}$

for Π_2^{ud} $@L \sim 10$ fm

FVEs: $\simeq 3-4\%$





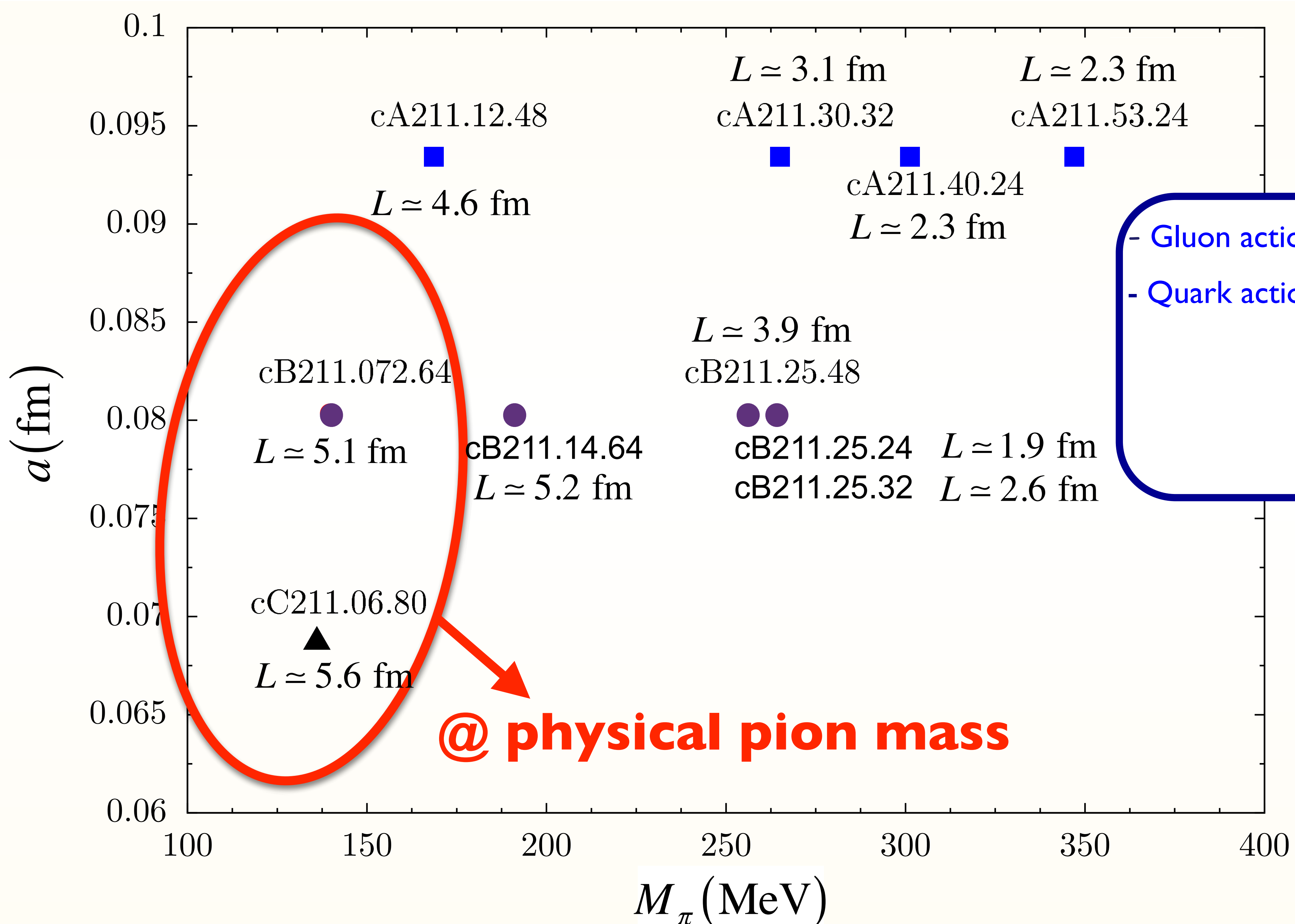
$\sigma(e^+e^- \to \pi^+\pi^-)$

A. Keshavarzi *et al.*, 2018

large time-distance behavior of $V^{ud}(t)$ reliably evaluated by using the repr.

New ETMC setup

We are generating new gauge field configurations with Nf=2+1+1 dynamical quarks, including ensembles at the physical pion mass



- Gluon action: Iwasaki
- Quark action: twisted mass at maximal twist (automatically O(a) improved)

+ clover term

OS for *s* and *c* valence quarks

Conclusions

- The HVP contribution is currently one of the most important sources of the theoretical uncertainty to the muon (g-2) → LQCD

- We have performed a first-principles lattice QCD+QED calculation of $a_\ell^{\rm HVP}$. Our results agree with recent determinations based on dispersive analyses.

$$a_e^{\rm HVP} = 185.8(4.2)\cdot 10^{-14}$$

$$a_\mu^{\rm HVP} = 692.1(16.3)\cdot 10^{-10}$$

DG and S. Simula, 2019

$$a_\tau^{\rm HVP} = 335.9(6.9)\cdot 10^{-8}$$

In progress...

- evaluation of the quark-disconnected terms and relaxation of the *qQED* approximation

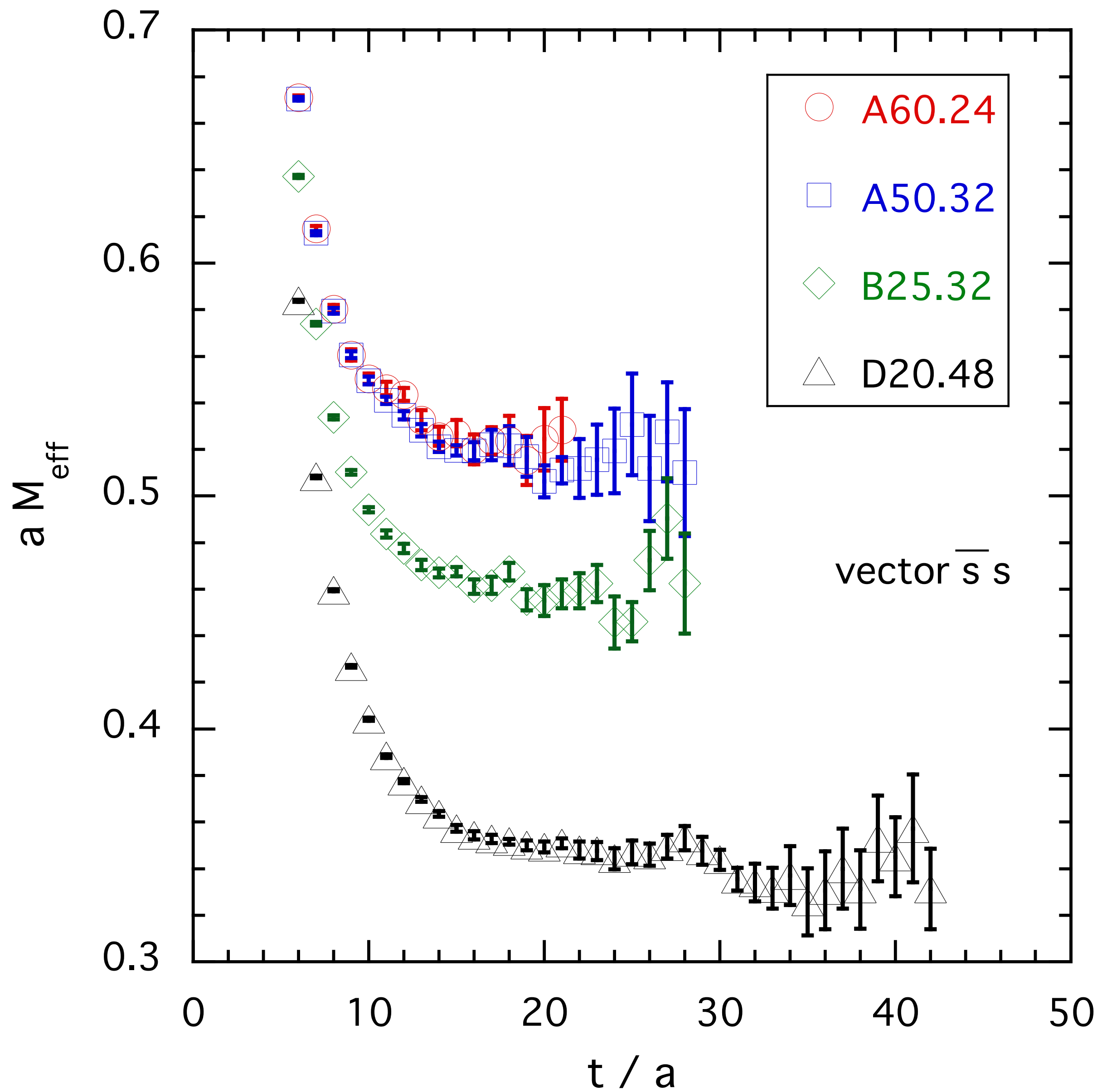
- development of an analytic representation of the correlator for *s*- and *c*-quark contributions

- use of the new ETMC lattice setup @ the physical pion point

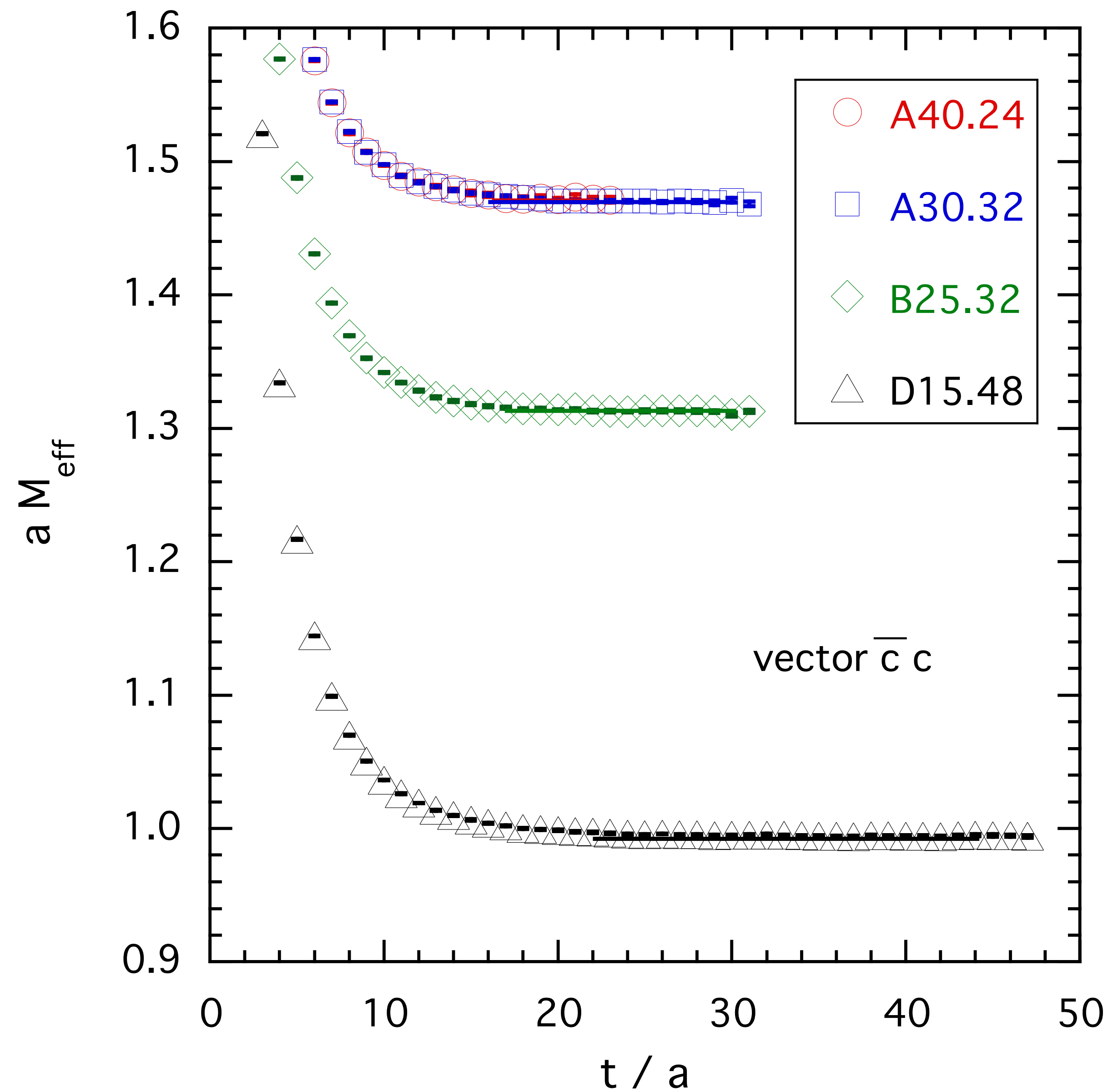
Backup slides

Ground-state identification

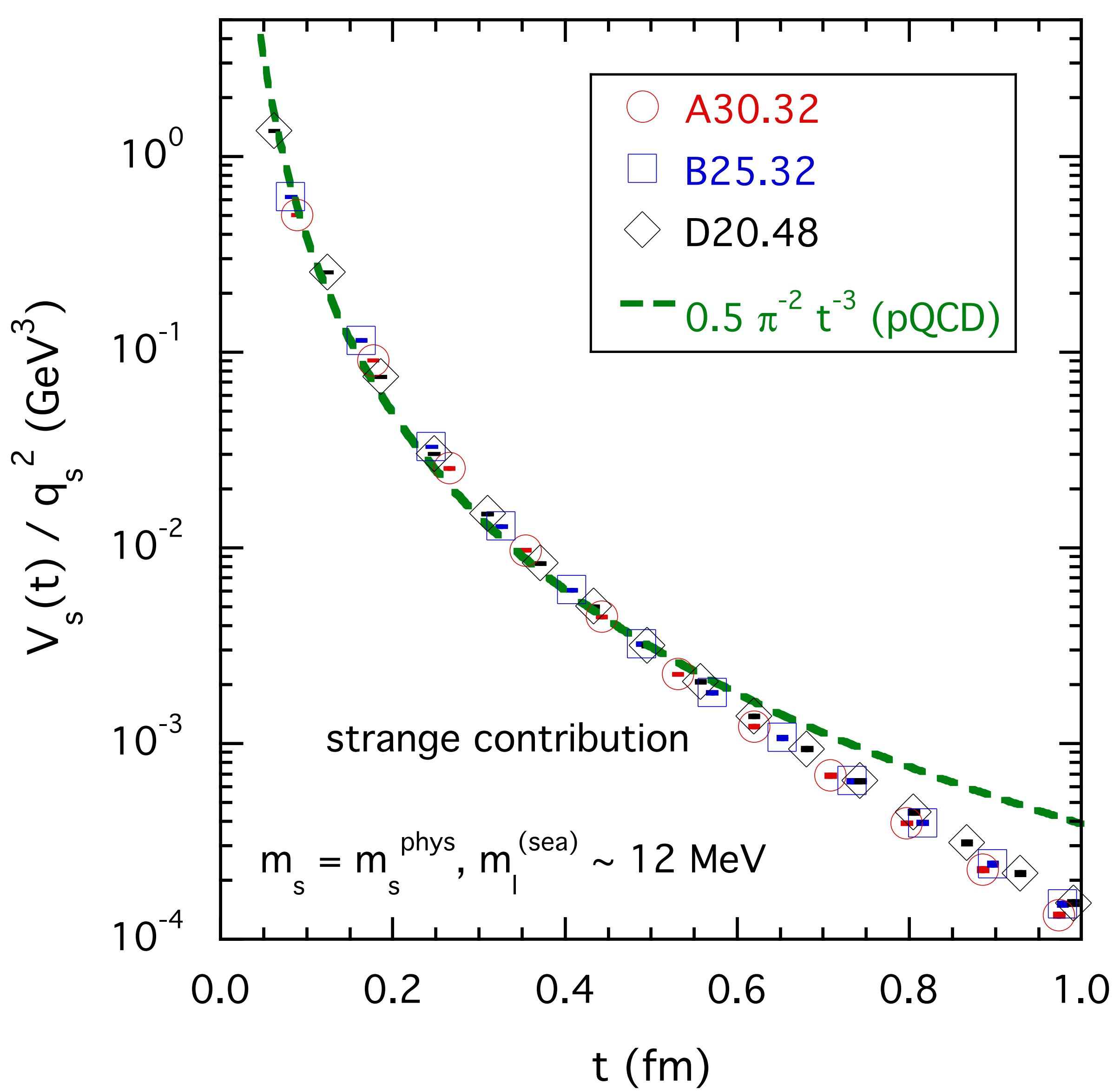
s-quark contribution

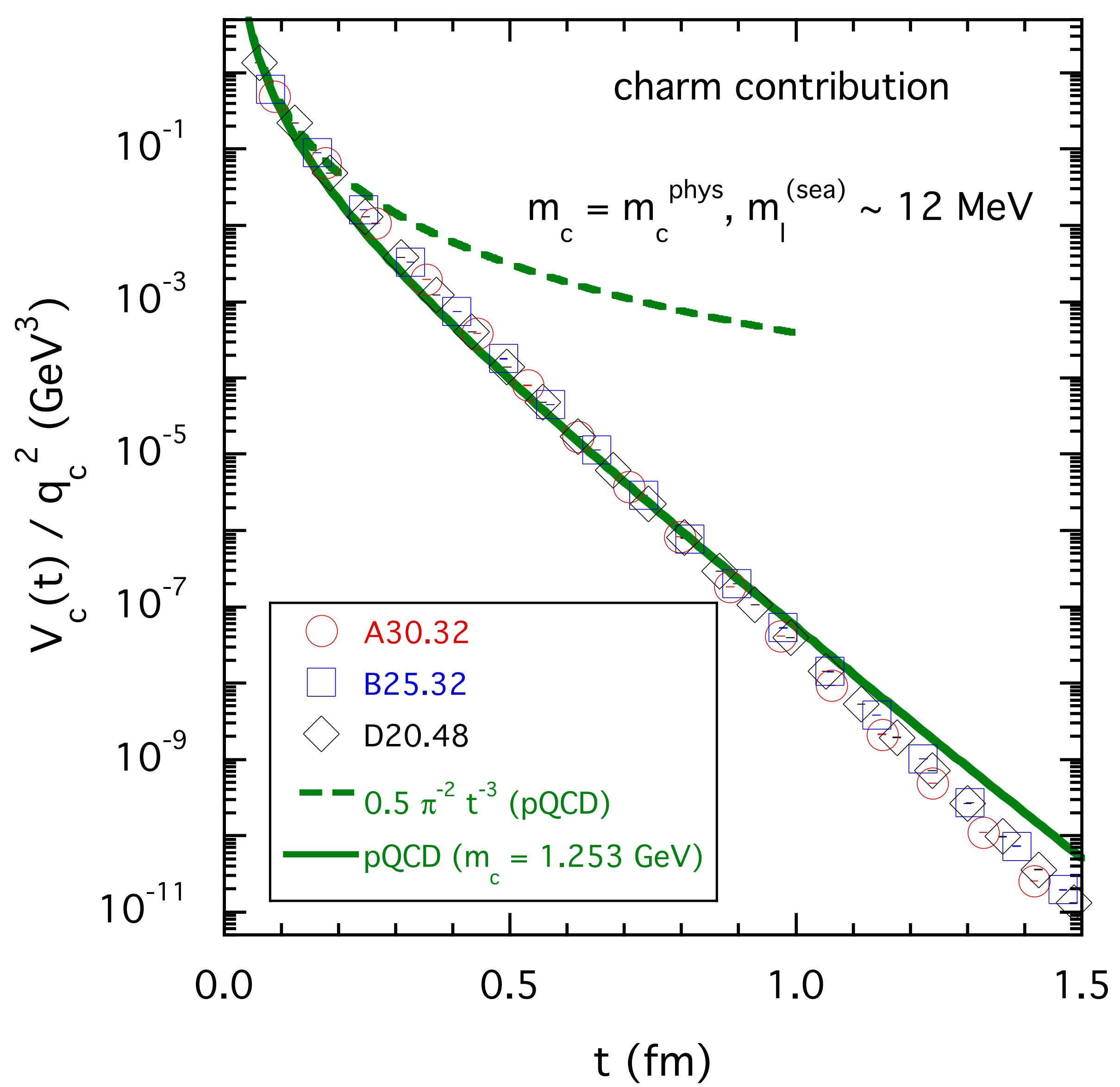


c-quark contribution

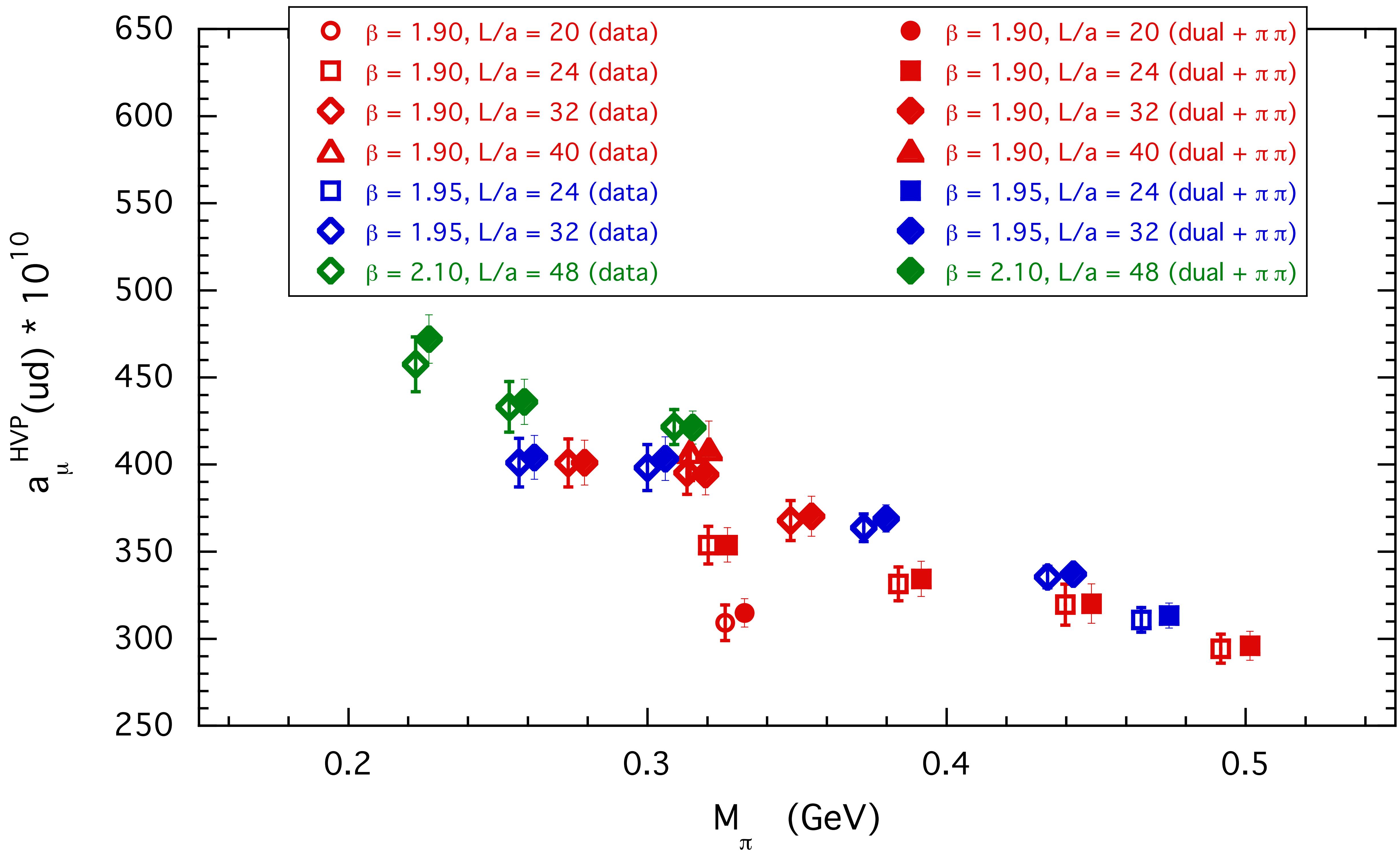


Quark-hadron duality

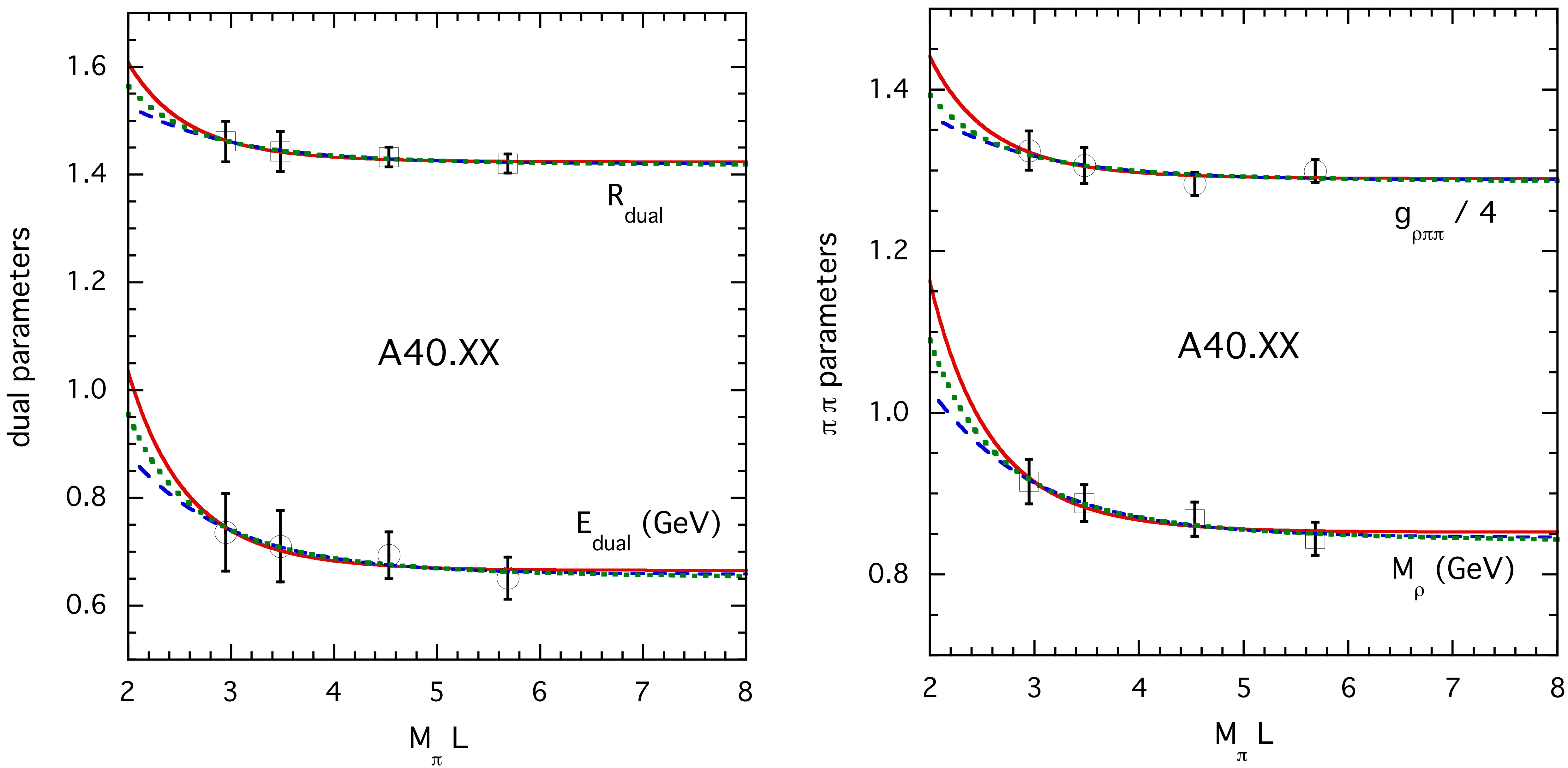




$a_\mu^{\mathrm{HVP}}(ud)$ lattice data



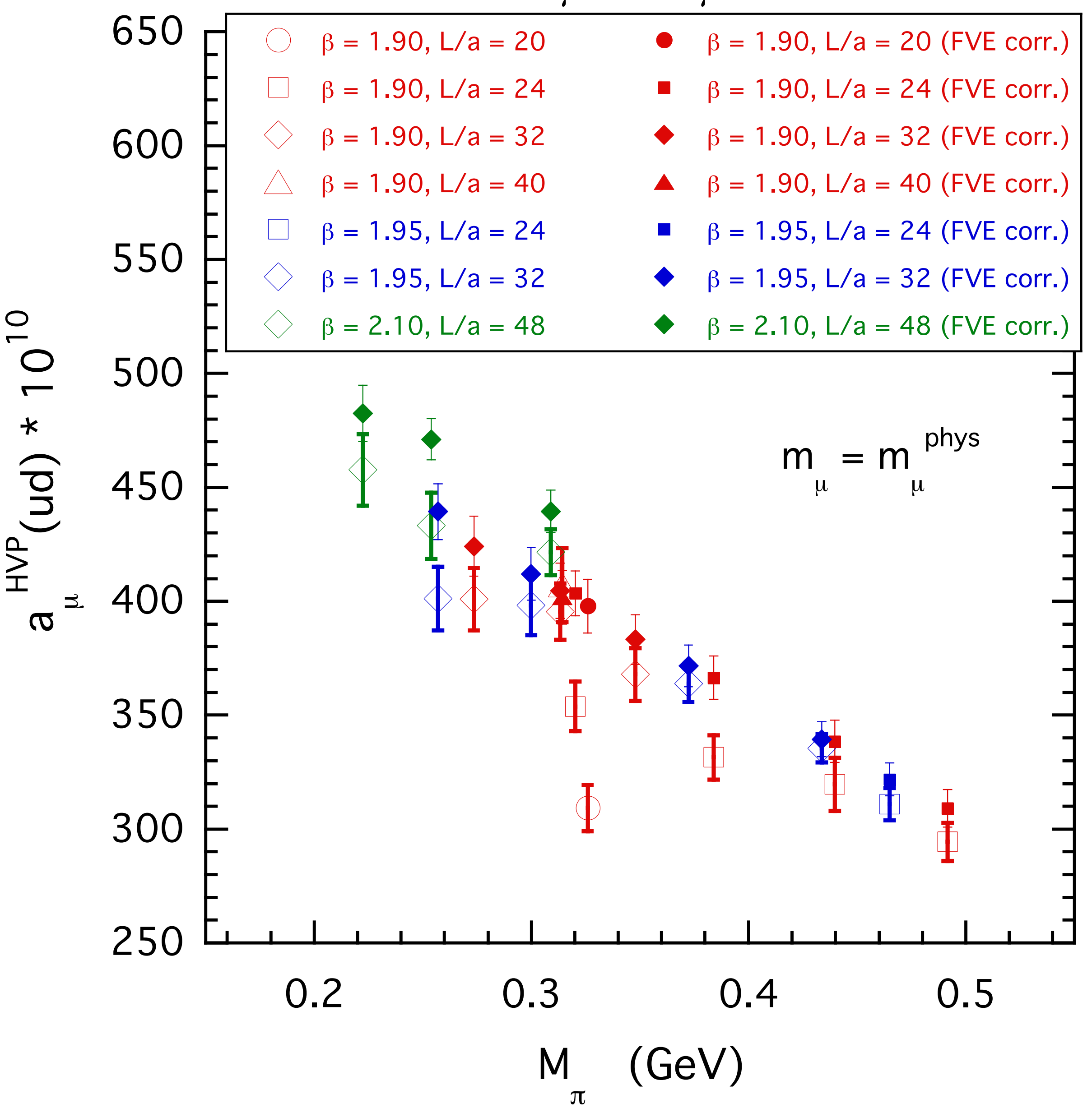
Subtraction of FVEs



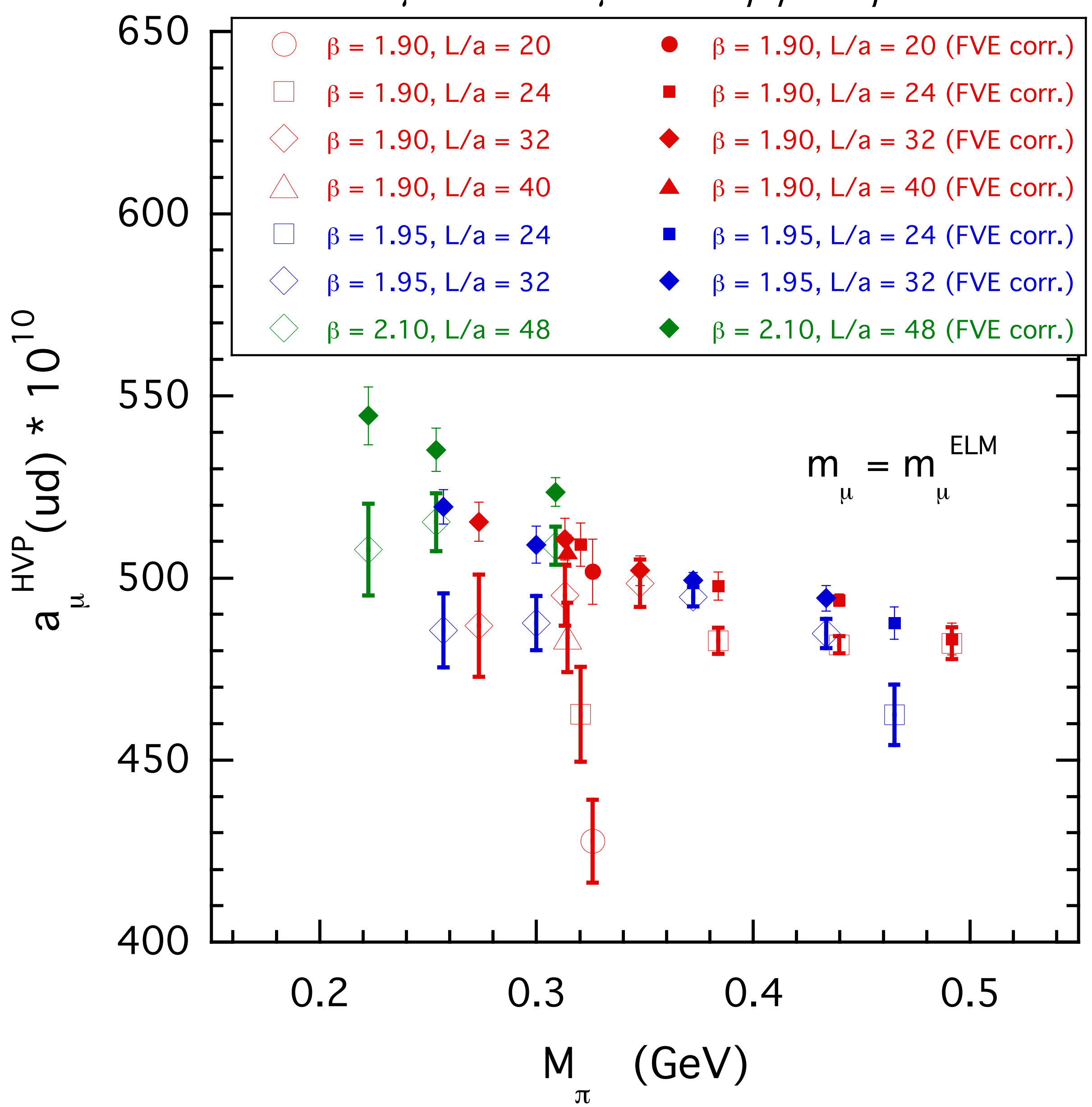
Subtraction of FVEs: $a_\mu^{\mathrm{HVP}}(ud)$

$$a_\mu^{\mathrm{HVP}}(\infty) = a_\mu^{\mathrm{HVP}}(L) + \left[a_\mu^{\mathrm{HVP}}(\infty) - a_\mu^{\mathrm{HVP}}(L)\right]_{dual+\pi\pi}$$

$m_\mu = m_\mu^{phys}$



$m_\mu^{ELM} = m_\mu^{phys} M_\rho / M_\rho^{phys}$



Isospin-breaking corrections to a_ℓ^{HVP}

A strategy for Lattice QCD:

The isospin-breaking part of the Lagrangian is treated as a perturbation

Expand in: $m_d - m_u$ + α_{em}

JHEP

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Isospin breaking effects due to the up-down mass difference in lattice QCD

RM123 collaboration

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PHYSICAL REVIEW D **87,** 114505 (2013)

Leading isospin breaking effects on the lattice

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(RM123 Collaboration)

ArXiv:1303.4896

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(Received 3 April 2013; published 7 June 2013)

RM123 Collaboration

The $(m_d - m_u)$ expansion

- Identify the isospin-breaking term in the QCD action

$$S_m = \sum_x \left[m_u \bar{u}u + m_d \bar{d}d \right] = \sum_x \left[\frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d) - \frac{1}{2}(m_d - m_u)(\bar{u}u - \bar{d}d) \right] =$$

$$= \sum_x \left[m_{ud}(\bar{u}u + \bar{d}d) - \Delta m(\bar{u}u - \bar{d}d) \right] = S_0 - \Delta m \hat{S} \longleftarrow \boxed{\hat{S} = \Sigma_x(\bar{u}u - \bar{d}d)}$$

- Expand the functional integral in powers of Δm

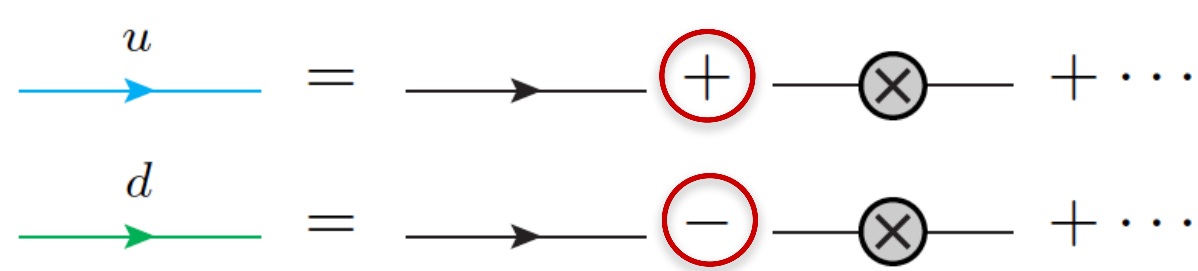
Advantage: factorised out

$$\langle O \rangle = \frac{\int D\phi\, O e^{-S_0 + \Delta m \hat{S}}}{\int D\phi\, e^{-S_0 + \Delta m \hat{S}}} \overset{1st}{\simeq} \frac{\int D\phi\, O e^{-S_0}(1 + \Delta m \hat{S})}{\int D\phi\, e^{-S_0}(1 + \Delta m \hat{S})} \simeq \frac{\langle O \rangle_0 + \Delta m \langle O \hat{S} \rangle_0}{1 + \cancel{\Delta m \langle \hat{S} \rangle_0}} = \langle O \rangle_0 + \Delta m \langle O \hat{S} \rangle_0$$

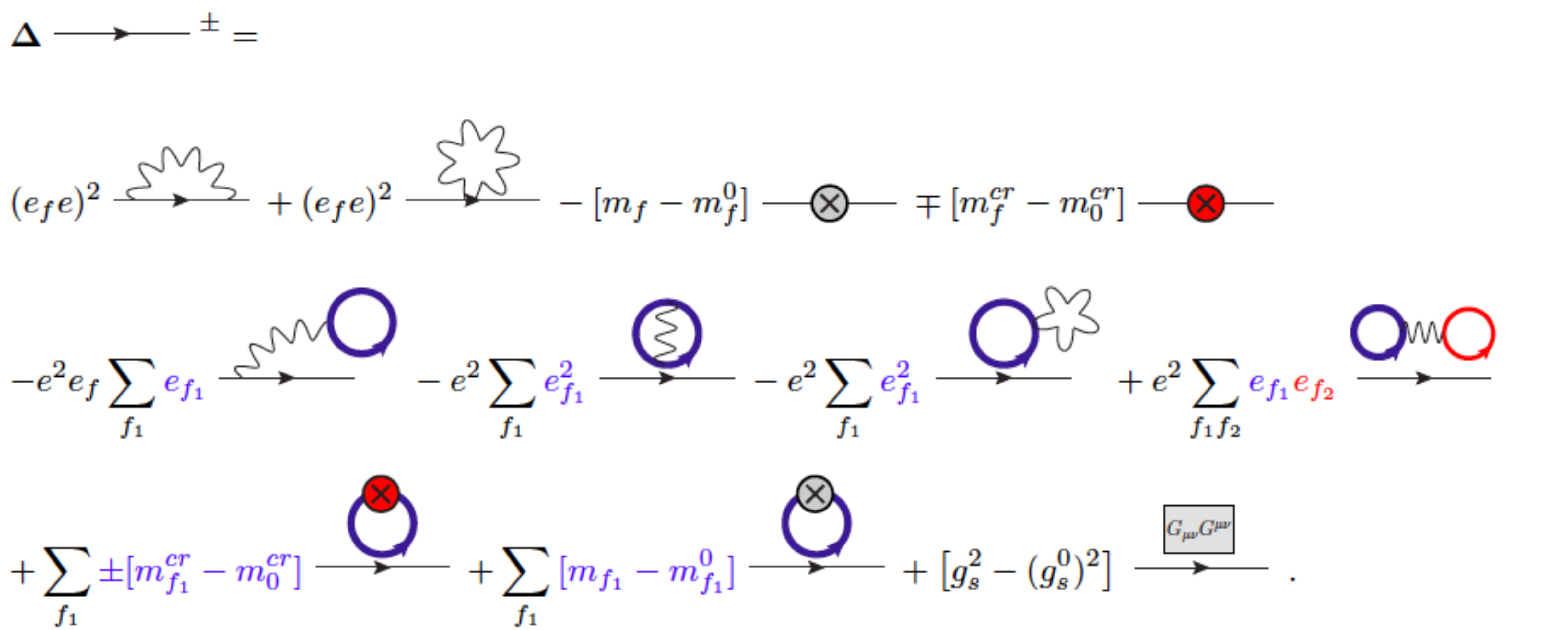
for isospin symmetry

- At leading order in Δm the corrections only appear in the valence-quark propagators:

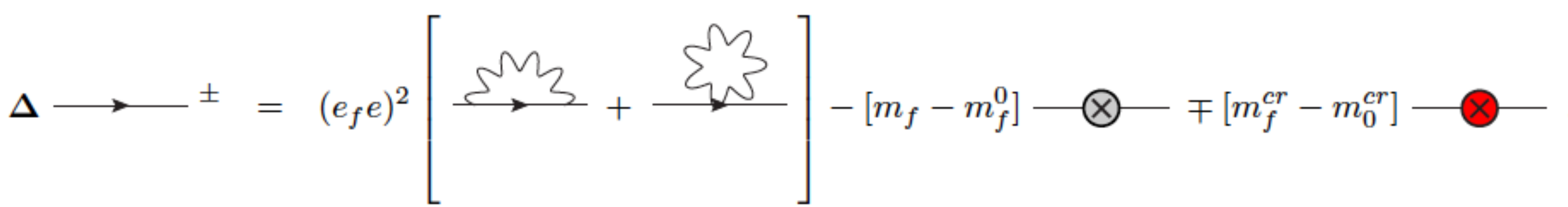
(disconnected contractions of $\bar{u}u$ and $\bar{d}d$ vanish due to isospin symmetry)



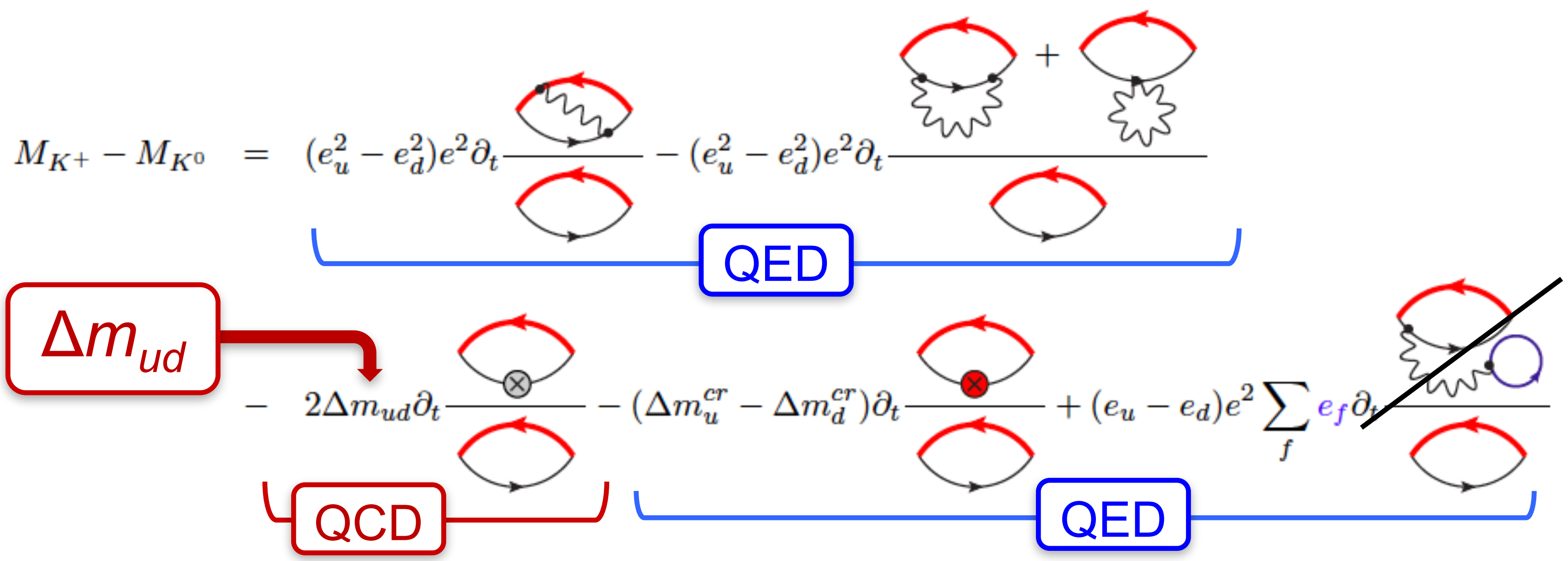
The **QED** expansion for the quark propagator

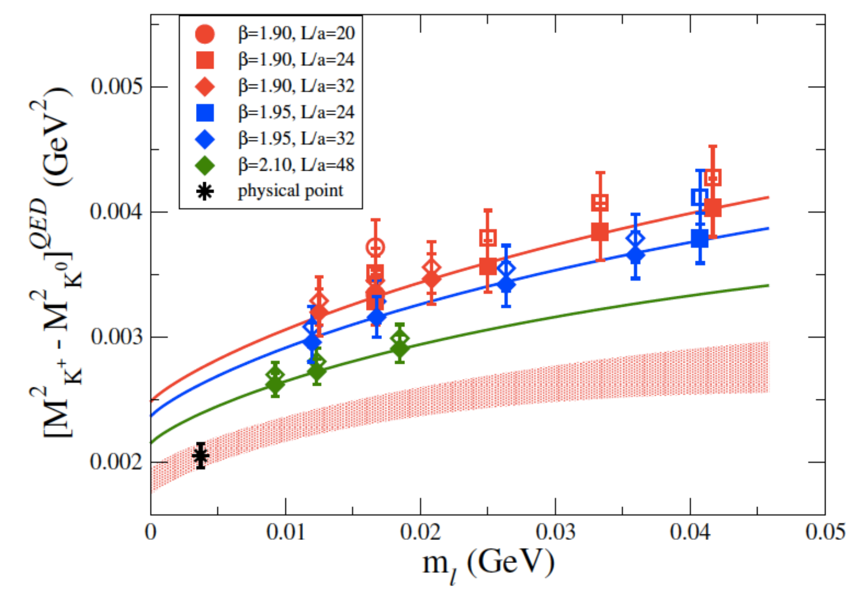


In the electro-quenched approximation:



The down- and up-quark mass difference

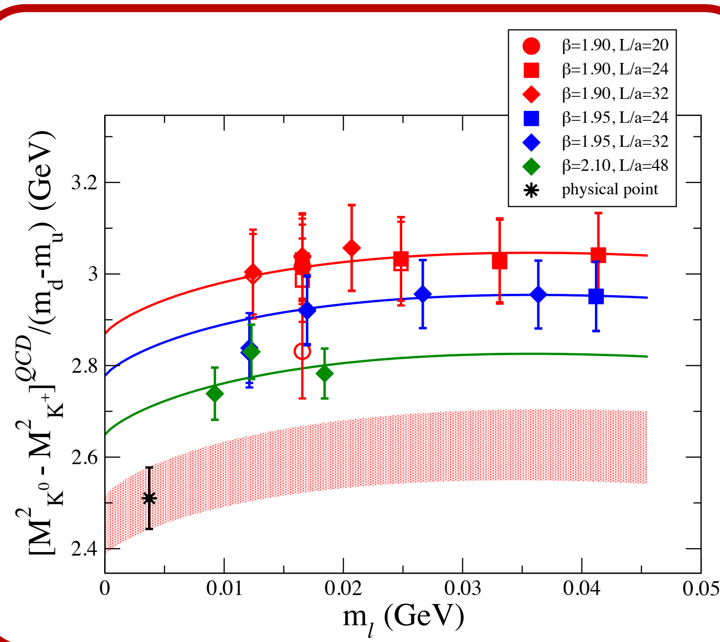


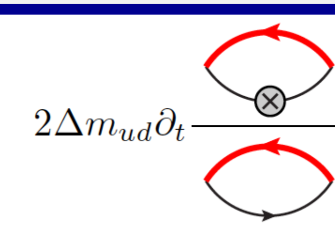


$$\left[M_{K^+} - M_{K^0}\right]^{QED} = 2.07(15)\ \text{MeV}$$

and from the experimental value

$$\left[M_{K^+} - M_{K^0}\right]^{QCD} = -6.00(15)\ \text{MeV}$$





$$\frac{\left[M^2_{K^0} - M^2_{K^+}\right]^{QCD}}{m_d - m_u} = 2.51(18)\ \text{GeV}$$

All masses in MSbar at 2 GeV

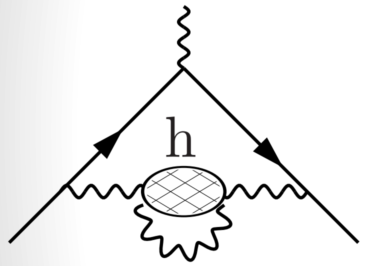
electro-quenched approximation

$$m_d - m_u = 2.38(18)\ \text{MeV}$$

$$m_u = 2.50(17)\ \text{MeV}$$
$$m_d = 4.88(20)\ \text{MeV}$$

RM123 Collaboration, PRD 95 (2017) 114504

LIB corrections



$$\delta a_\mu^{\mathrm{HVP}} = \sum_{f=u,d,s,c} 4\alpha_{em}^2 \left\{ \sum_{t=0}^{T_{data}} \widetilde{f}_\ell(t)\,\delta V^f(t) + \sum_{t=T_{data}+a}^{\infty} \widetilde{f}_\ell(t)\,\delta\left[\frac{G_V^f}{2M_V^f} e^{-M_V^f t}\right] \right\}$$

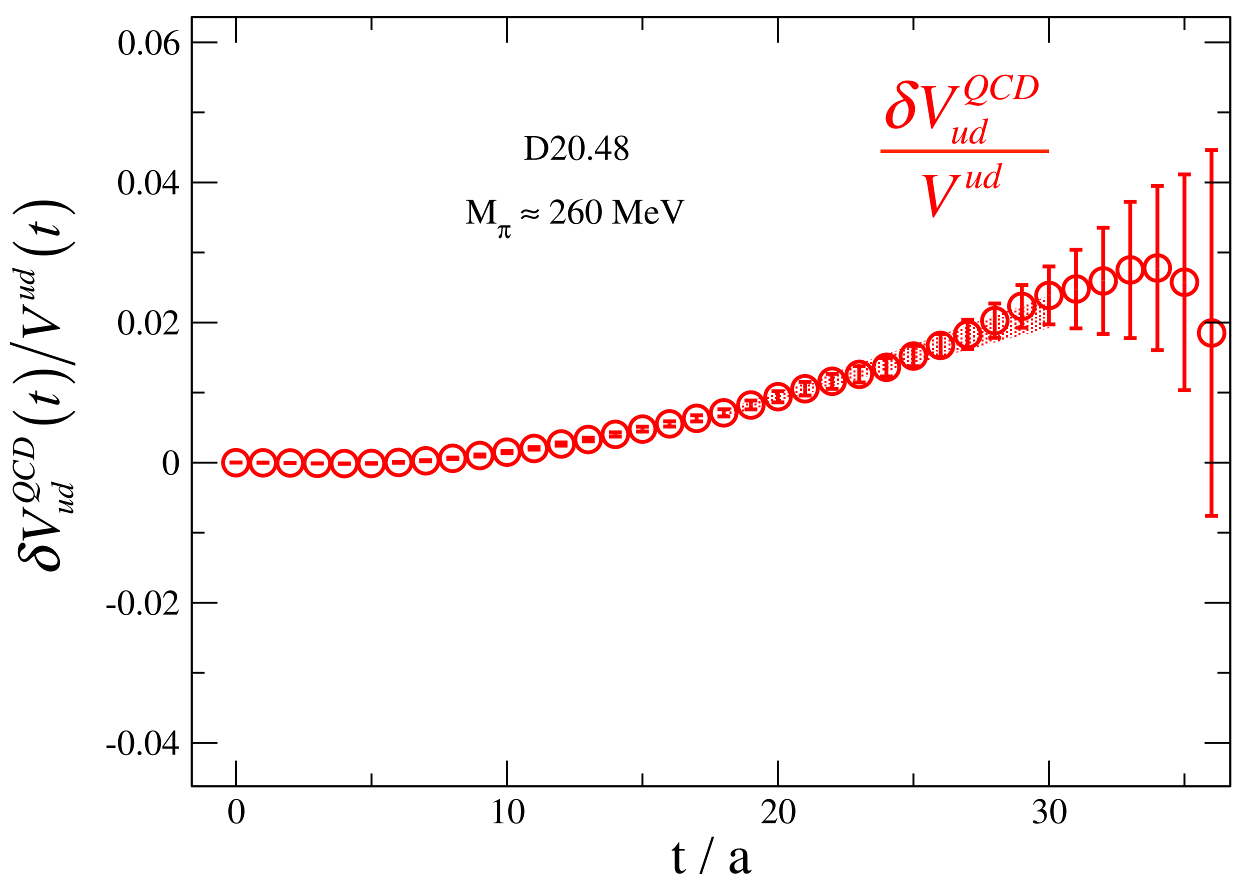
lattice data

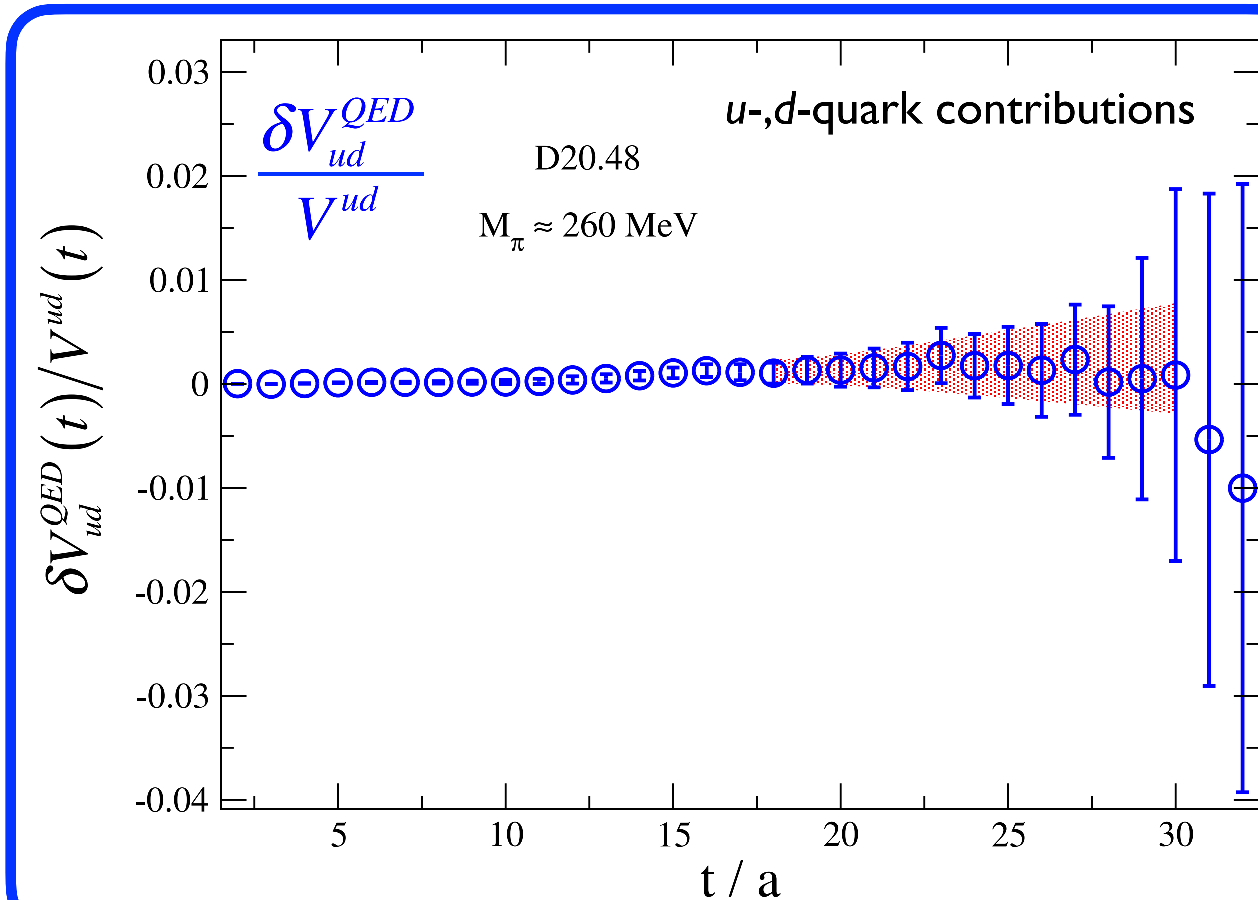
analytic repr.

<u>RM123 method</u>

G. M. de Divitiis *et al.*, 2012; 2013

$$\frac{\delta V(t)}{V(t)} \xrightarrow[t \gg a]{} \frac{\delta G_V}{G_V} - \frac{\delta M_V}{M_V}\left(1 + M_V t\right)$$



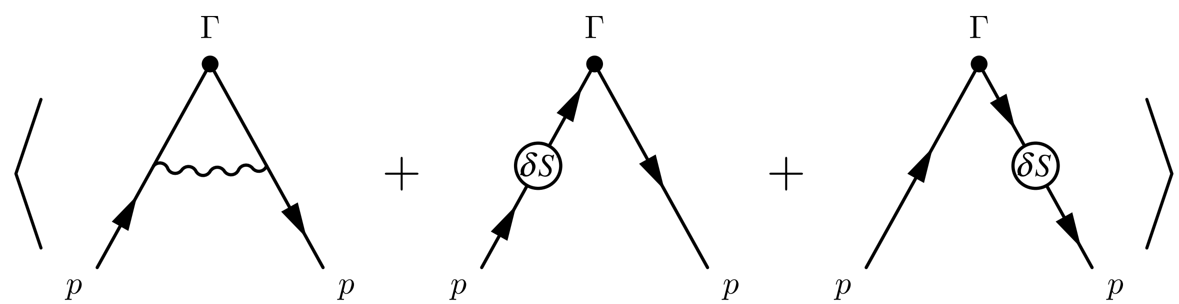


RI'-MOM in QCD+QED

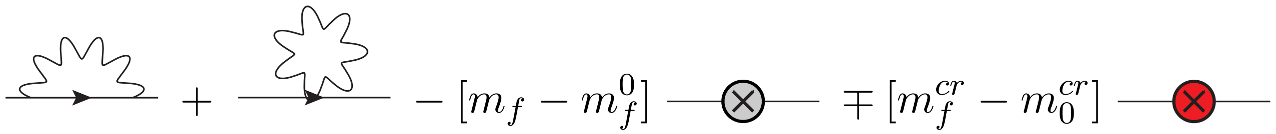
$$Z_O = Z_O^{(0)}\left(1+\frac{\alpha_{em}}{4\pi}\delta Z_O\right) \qquad Z_{q_1}^{-1/2} Z_{q_2}^{-1/2} Z_O \mathrm{Tr}\left[\Lambda_O(p) P_O\right]\Big|_{p^2=\mu^2} = 1$$

$$\delta Z_O = -\frac{\mathrm{Tr}\left[\delta\Lambda_O P_O\right]}{\mathrm{Tr}\left[\Lambda_O^{(0)} P_O\right]} + \frac{1}{2}\left(\frac{\delta Z_{q_1}}{Z_{q_1}^{(0)}} + \frac{\delta Z_{q_2}}{Z_{q_2}^{(0)}}\right)$$

$$\delta\Lambda_O(p) = \left\langle S_{q_2}^{(0)}(p)\right\rangle^{-1} \delta G_O(p)\gamma_5 \left\langle S_{q_1}^{(0)\dagger}(p)\right\rangle^{-1}\gamma_5 \quad + \text{corrections to the inverse propagators}$$



$$Z_O^{fact} = \frac{\delta Z_O}{\delta Z_O^{QED}}$$



β	Z_m^{fact} (M1)	Z_A^{fact} (M1)	Z_m^{fact} (M2)	Z_A^{fact} (M2)
1.90	1.629 (41)	0.859 (15)	1.637 (14)	0.990 (9)
1.95	1.514 (33)	0.873 (13)	1.585 (12)	0.980 (8)
2.10	1.459 (17)	0.909 (6)	1.462 (6)	0.958 (3)

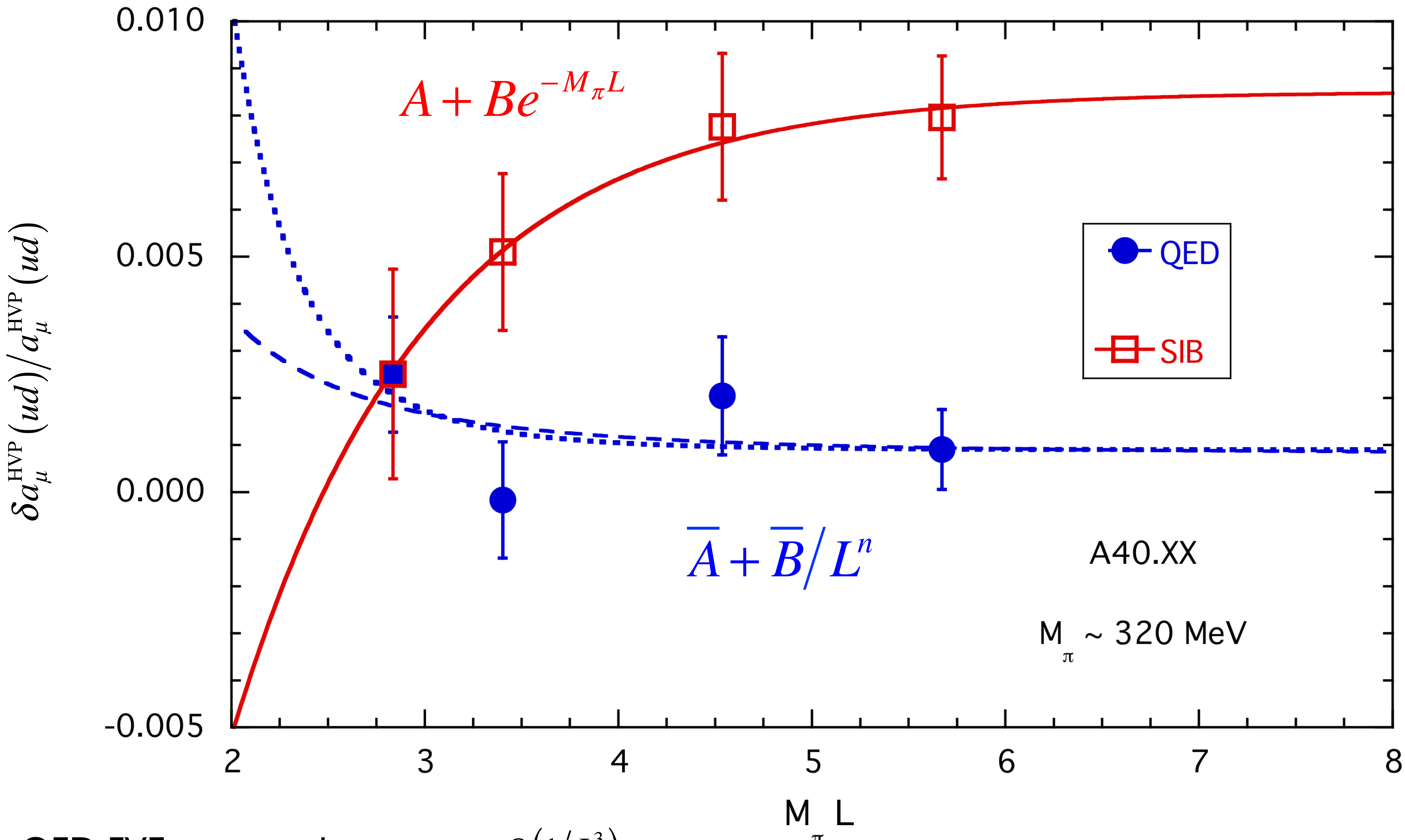
RI'-MOM

$\mu = 1/a$

M1: extrapolate to $a^2p^2 \to 0$

M2: interpolate at fixed a^2p^2

LIB corr.: *u*-,*d*-quark FVEs



QED FVEs expected to start at $O\left(1/L^3\right)$

DG *et al.*, 2017; DG *et al.*, 2019; J. Bijnens *et al.*, 2019