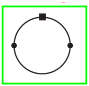
Connected and disconnected strong isospin-breaking and finite-volume corrections

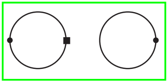
contents of talk also in C. Lehner, A. Meyer Phys.Rev.D 102 (2020) 5, 054509

Two points for this talk:

1) Attempt to explain the apparent cancelation of connected and disconnected SIB, see BMW-20:



connected
6.59(63)(53)



disconnected
-4.63(54)(69)

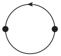
$\times 10^{-10}$

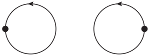
2) For connected diagram itself, found significant FV correction in LM-20:

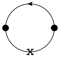
infinite volume:	$9.0(0.8)(1.2)\times 10^{-10}$
$L = 5.5$fm:	$5.25(76)(29)\times 10^{-10}$

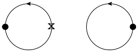
Volume adjustment for comparison of connected diagram in isolation required

Define for up and down contributions

$\frac{5}{9}c(m_v) =$ 

$-\frac{1}{9}d(m_v) =$ 

$-\frac{2}{3}\Delta m_u M(m_v) =$ 

$\frac{2}{3}\Delta m_u O(m_v) =$ 

with valence mass m_v and a symmetric expansion point $\Delta m_u = -\Delta m_d$.

The valence-mass derivative then relates $c'(m_v) = -2M(m_v)$ and $d'(m_v) = -2O(m_v)$.

In LM-20 we derive in NLO PQChPT

$$C^{\mathrm{NLO,PQ\chi PT,conn.}}(t) = \frac{10}{9}\frac{1}{3}\frac{1}{L^3}\sum_{\vec{p}} \frac{\vec{p}^2}{(E_p^{vl})^2} e^{-2E_p^{vl} t}\,,$$

$$C^{\mathrm{NLO,PQ\chi PT,disc.}}(t) = -\frac{1}{9}\frac{1}{3}\frac{1}{L^3}\sum_{\vec{p}} \frac{\vec{p}^2}{(E_p^{vv})^2} e^{-2E_p^{vv} t} \qquad (1)$$

with

$$E_p^{vl} = \sqrt{(m_\pi^{vl})^2 + \vec{p}^2}\,, \qquad E_p^{vv} = \sqrt{(m_\pi^{vv})^2 + \vec{p}^2}\,,$$

$$(m_\pi^{vl})^2 = B(m_l + m_v)\,, \qquad (m_\pi^{vv})^2 = 2Bm_v\,.$$

In these expressions L^3 is the spatial volume. Note the valence and sea masses m_v and m_l.

We can then isolate c and d defined before in NLO PQChPT by

$$\frac{\partial}{\partial m_v} C^{\mathrm{NLO,PQ\chi PT,conn.}} = \frac{5}{9}\frac{\partial}{\partial m_v} c = -\frac{10}{9} M \,,$$
$$\frac{\partial}{\partial m_v} C^{\mathrm{NLO,PQ\chi PT,disc.}} = -\frac{1}{9}\frac{\partial}{\partial m_v} d = \frac{2}{9} O \,.$$

From Eq. (1) it then follows that

$$\frac{\partial}{\partial m_v} C^{\mathrm{NLO,PQ\chi PT,conn.}}\Big|_{m_v=m_l} = -5\frac{\partial}{\partial m_v} C^{\mathrm{NLO,PQ\chi PT,disc.}}\Big|_{m_v=m_l}$$

and therefore that within NLO PQChPT

$$M = O \,.$$

Since the connected plus disconnected SIB enters as $M - O$ (see page 1), in NLO PQChPT we expect complete cancellation between connected and disconnected SIB but individually they are non-zero.

NLO PQChPT then also predicts for the individual contributions

$$a_\mu^{\mathrm{SIB,conn.,NLOPQ\chi PT}} = -a_\mu^{\mathrm{SIB,disc.,NLOPQ\chi PT}} \approx 6.9 \times 10^{-10}$$

which compares reasonably well with the finite-volume lattice results

$$a_\mu^{\mathrm{SIB,conn.,BMW-20}} = 6.59(63)(53) \times 10^{-10}\,,$$
$$a_\mu^{\mathrm{SIB,disc.,BMW-20}} = -4.63(54)(69) \times 10^{-10}\,.$$

In LM-20 we then use FV NLO PQChPT to compute the finite-volume corrections for the connected SIB contribution individually, which are significant:

LM-20, infinite volume:	$9.0(0.8)(1.2)\ \times 10^{-10}$
LM-20, $L = 5.5$fm:	$5.25(76)(29)\ \times 10^{-10}$.

For comparisons of connected/disconnected SIB contributions in isolation, FV corrections non-negligible!

Thanks!