Leading isospin breaking effects for the HVP contribution to a_μ on CLS $N_\mathrm{f} = 2 + 1$ $\mathrm{QCD_{iso}}$ ensembles

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Mainz setup based on CLS $N_\mathrm{f} = 2+1$ $\mathrm{QCD_{iso}}$ ensembles [Bruno et al. 2015]:

- $\mathrm{QED_L}$ prescription for IR regularisation [Hayakawa & Uno 2008]
- Open [Lüscher & Schaefer 2011] and (anti-)periodic temporal boundary conditions
- Relate QCD+QED and $\mathrm{QCD_{iso}}$ via reweighting and a perturbative expansion [de Divitiis et al. 2013] in

$$\Delta\varepsilon = (\Delta m_\mathrm{u}, \Delta m_\mathrm{d}, \Delta m_\mathrm{s}, \Delta\beta, e^2)$$

- Leading order calculation ⇒ no UV Landau pole ⇒ continuum limit exists

Intermediate scheme for matching of $\mathrm{QCD_{iso}}$ and QCD+QED:

- Fix parameters $\Delta\varepsilon$ on each ensemble:

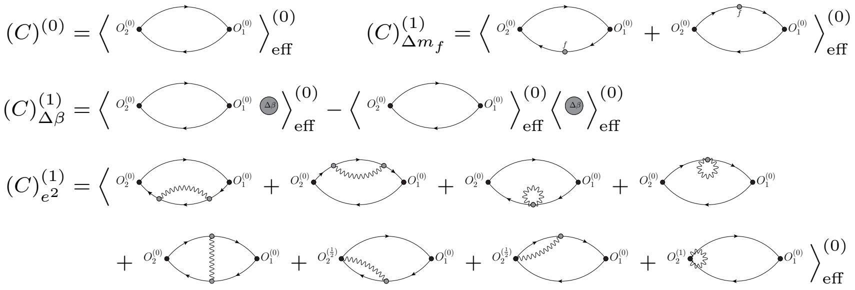
$$m_{\pi^+} + m_{\pi^0} = (m_{\pi^+} + m_{\pi^0})^{(0)} \quad m_{K^+} - m_{K^0} = 3.934(20)\,\mathrm{MeV}$$
$$m_{K^+} + m_{K^0} = (m_{K^+} + m_{K^0})^{(0)} \qquad \Delta\beta = 0 \qquad e^2 = 4\pi\alpha$$

- Scale setting based on $\frac{2}{3}(f_K + \frac{1}{2}f_\pi)$ [Bruno et al. 2017] ⇒ neglect IB corrections

Expansion of operators and correlation functions:

$$\mathcal{O} = \mathcal{O}^{(0)} + e\mathcal{O}^{(\frac{1}{2})} + \frac{1}{2}e^2\mathcal{O}^{(1)} + O(e^{\frac{3}{2}})$$
$$C = C^{(0)} + \sum_l \Delta\varepsilon_l C_l^{(1)} + O(\Delta\varepsilon^2)$$

Quark-connected contributions of $\langle \mathcal{P}\mathcal{P} \rangle$, $\langle \mathcal{V}_{\mathrm{c}}\mathcal{V}_{\mathrm{l}} \rangle$, $\langle \mathcal{V}_{\mathrm{l}}\mathcal{V}_{\mathrm{l}} \rangle$:



Vector current renormalisation $\mathcal{V}_{\mathrm{R}} = Z_{\mathcal{V}_{\mathrm{R}}\mathcal{V}}\mathcal{V}$, assuming $Z_{\mathcal{V}_{\mathrm{c,R}}\mathcal{V}_{\mathrm{c}}} = \mathbb{1}$:

$$\langle \mathcal{V}_{\mathrm{c}}\mathcal{V}_{\mathrm{l}} \rangle - Z_{\mathcal{V}_{\mathrm{l,R}}\mathcal{V}_{\mathrm{l}}} \langle \mathcal{V}_{\mathrm{l}}\mathcal{V}_{\mathrm{l}} \rangle = O(a) \qquad \mathcal{V}_{\mathrm{R}}^{\gamma} = \sum_{i=0,3,8} \left(Z_{\mathcal{V}_{\mathrm{R}}^{3}\mathcal{V}^{i}} + \frac{1}{\sqrt{3}} Z_{\mathcal{V}_{\mathrm{R}}^{8}\mathcal{V}^{i}} \right) \mathcal{V}^{i}$$

Fit constant $Z_{\mathcal{V}_{\mathrm{l,R}}\mathcal{V}_{\mathrm{l}}}$ to $Z_{\mathrm{eff},\mathcal{V}_{\mathrm{l,R}}\mathcal{V}_{\mathrm{l}}}$ at large time distances:

$$Z_{\mathrm{eff},\mathcal{V}_{\mathrm{l,R}}\mathcal{V}_{\mathrm{l}}} = \langle \mathcal{V}_{\mathrm{c}}\mathcal{V}_{\mathrm{l}} \rangle (\langle \mathcal{V}_{\mathrm{l}}\mathcal{V}_{\mathrm{l}} \rangle)^{-1}$$

$a_{\mu,\mathrm{HVP}}$ via TMR, single state reconstruction and rescaling $(am_\mu)^{\mathrm{lat}} = \frac{m_\mu^{\mathrm{phys}}}{f_\pi^{\mathrm{phys}}}(af_\pi)^{\mathrm{lat}}$:

	N200	H102
a [fm]	$0.06426[76]_{\mathrm{tot}}$ fm	$0.08636[106]_{\mathrm{tot}}$ fm
m_{π^+} [MeV]	$285.6(5)_{\mathrm{st}}(3.4)_{\mathrm{a}}[3.4]_{\mathrm{tot}}$	$355.2(9)_{\mathrm{st}}(4.4)_{\mathrm{a}}[4.5]_{\mathrm{tot}}$
m_{π^0} [MeV]	$283.4(5)_{\mathrm{st}}(3.4)_{\mathrm{a}}[3.4]_{\mathrm{tot}}$	$353.2(9)_{\mathrm{st}}(4.3)_{\mathrm{a}}[4.4]_{\mathrm{tot}}$
m_{K^+} [MeV]	$460.1(4)_{\mathrm{st}}(5.4)_{\mathrm{a}}[5.5]_{\mathrm{tot}}$	$436.7(7)_{\mathrm{st}}(5.4)_{\mathrm{a}}[5.4]_{\mathrm{tot}}$
m_{K^0} [MeV]	$464.0(4)_{\mathrm{st}}(5.5)_{\mathrm{a}}[5.5]_{\mathrm{tot}}$	$440.6(7)_{\mathrm{st}}(5.4)_{\mathrm{a}}[5.4]_{\mathrm{tot}}$
$(a_{\mu,\mathrm{HVP}})^{(0)}$ $[10^{10}]$	$525[12]_{\mathrm{tot}}$	$490[7]_{\mathrm{tot}}$
$(a_{\mu,\mathrm{HVP}})^{(1)}$ $[10^{10}]$	$5[6]_{\mathrm{tot}}$	$2.5[4]_{\mathrm{tot}}$
$a_{\mu,\mathrm{HVP}}$ $[10^{10}]$	$529[13]_{\mathrm{tot}}$	$493[7]_{\mathrm{tot}}$

- $(a_{\mu,\mathrm{HVP}})^{(0)}$ differs from result in Mainz-2019 [Gérardin et al. 2019]: different renormalisation condition for $Z_{\mathcal{V}_{\mathrm{l,R}}\mathcal{V}_{\mathrm{l}}}$ and no $O(a)$-improvement
- $(a_{\mu,\mathrm{HVP}})^{(1)}$ is $O(1\%)$ of $(a_{\mu,\mathrm{HVP}})^{(0)}$ compatible with [Blum et al. 2018] [Giusti et al. 2019], $(a_{\mu,\mathrm{HVP}})^{(1)}$ dominated by error of $(a_{\mu,\mathrm{HVP}})^{(0)}$

In preparation (with A. Hanlon, P. Madanagopalan and A. Segner):

- Extend list of ensembles
- Scale setting based on baryon masses $\Rightarrow$ include IB corrections
- Define $Z_{\mathcal{V}_{\mathrm{R}}\mathcal{V}}$ via QCD+QED vector Ward identity $\Rightarrow$ Setup compatible with Mainz-2019 [Gérardin et al. 2019] with $O(a^2, a\Delta\varepsilon)$ continuum converges