Logarithmic corrections to a^n behaviour

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Workshop: The hadronic vacuum polarization from lattice QCD at high precision

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based on

NH, P. Marquard and R. Sommer. Asymptotic behaviour of cutoff effects in Yang-Mills theory and in Wilson's lattice QCD. Eur. Phys. J. C, 80(3):200, 2020.

and new work.

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

DESY.

Symanzik effective theory.

$$S = S_{\mathrm{QCD}} + a^n \int \mathrm{d}^4x \sum_j c_j \mathcal{O}_j(x) + \mathrm{O}(a^{n+1})$$

with irrelevant operators $\mathcal{O}_j$, e.g.
$\mathcal{O}_1(x) = \frac{1}{g_0^2}\mathrm{tr}\{D_\mu F_{\nu\rho}(x) D_\mu F_{\nu\rho}(x)\}$.

Naive: $\Delta\mathcal{P}(a) = a^n \delta\mathcal{P} + \ldots$

EFT: $\Delta\mathcal{P}(a) = a^n [\alpha(1/a)]^{\hat{\gamma}_1} \delta\mathcal{P}_{1;\mathrm{RGI}} + \ldots$

- $\hat{\gamma}_1$ or full spectrum computable in continuum QCD.
- $\delta\mathcal{P}_{1;\mathrm{RGI}}$ is in general non-perturbative.

What needs to be done:

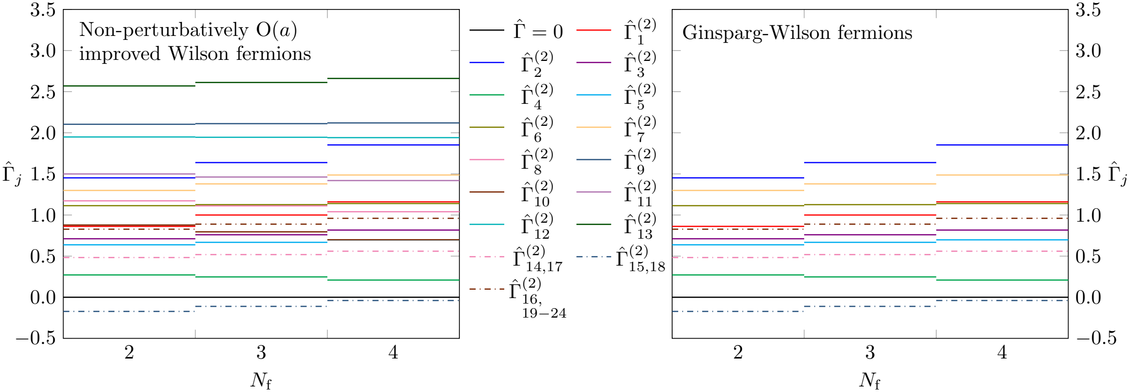
Matching coefficients $c_j(\alpha) = \bar{c}_j + \mathrm{O}(\alpha)$
TL coefficients $\bar{c}_j$ from classical a-expansion of S_{lattice}

$\hat{\gamma}_j$ from 1-loop renormalisation of operators in cont. QCD

} prediction of form of asymptotic a-dependence [free coefficients $\delta\mathcal{P}_{j;\mathrm{RGI}}$]

Leadings logarithms at $\mathrm{O}(a^2)$.

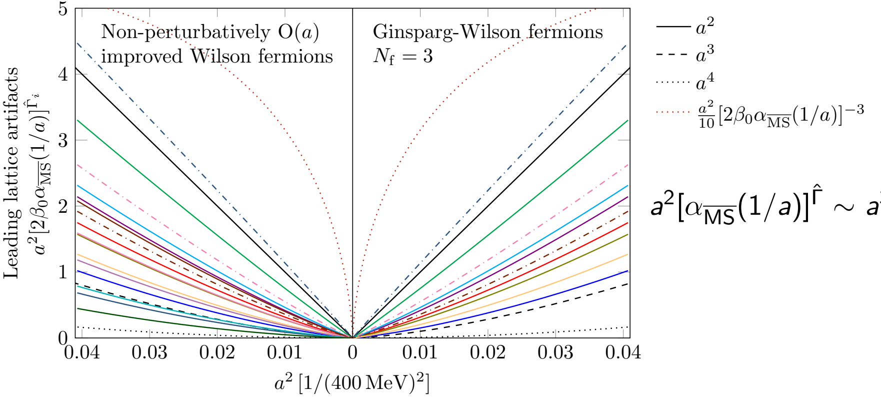
$$\Delta\mathcal{P}(a) = a^2 \sum_i [\alpha(1/a)]^{\hat{\Gamma}_i} \delta\mathcal{P}_{i;\mathrm{RGI}} + \ldots$$



solid: mass-independent **dash-dotted:** mass-dependent

Leadings logarithms at $O(a^2)$.

$$\Delta\mathcal{P}(a) = a^2 \sum_i [\alpha(1/a)]^{\hat{\Gamma}_i} \delta\mathcal{P}_{i;\mathrm{RGI}} + \ldots$$



$$a^2[\alpha_{\overline{MS}}(1/a)]^{\hat{\Gamma}} \sim a^3 \text{ if } \hat{\Gamma} \gtrsim 1.6$$

solid: mass-independent **dash-dotted:** mass-dependent

Conclusion.

- No bad behaviour in contrast to the O(3) model, i.e. $\hat{\Gamma} \gtrsim -0.1 \gg -3$.
- Leading anomalous dimensions of mass-independent contributions improve convergence as $a \searrow 0$.
- 4-fermion operators give a dense spectrum for $\hat{\Gamma}$, i.e. no clearly dominating contributions. This can lead to complicated lattice artifacts with cancellations and pile ups.
- Largest $\hat{\Gamma}_i$ are already close to a^3 (Ginsparg-Wilson) and a^4 (Wilson) behaviour.
- For staggered fermions $\hat{\Gamma}_i$ not yet known, but bounds from Ginsparg-Wilson fermions show that a significant spread will be there.
- Local fields introduce additional leading logarithms, which are still missing.
- Todo in practice:
  - Be aware that correct function is not $c_1 a^2 + c_2 a^4 + \ldots$
  - Try at least $c_1 a^2 [\alpha(1/a)]^{\hat{\Gamma}}$ for $\hat{\Gamma} = 1, 2, 3$ and include such changes in uncertainty.