Mainz/CLS Running Coupling and Electroweak Precision Science

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Online Workshop: The HVP from LQCD at High Precision
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Mainz $\Delta^{\text{had}}\alpha(Q^2)$ Working Group

M. Cè, A. Géradin, H.B. Meyer, K. Miura, D. Mohler, K. Ottnad, T. San José, A. Risch, J. Wilhelm, and H. Wittig.

Reference: M. Cè et.al. PoS**Lattice2019** (2020), arXiv:1910.09525.
$\Delta^{\text{had}}\alpha(-Q^2)$ and $\Delta^{\text{had}}\sin^2\theta_W(-Q^2)$.

Running Electric Coupling

- $\alpha(s) = \frac{\alpha}{1-(\Delta^{had}\alpha(s)+\Delta^{lep}\alpha(s))}$, $\alpha = \frac{1}{137.03\cdots}$, $\Delta^{had}\alpha(-Q^2) = 4\pi\alpha\hat{\Pi}(Q^2)$.
- $\Delta^{had}\alpha^{(5)}(M_z^2)$ from Pheno. (Thanks for Bogdan):
  - $0.02761(11)$ [Keshavarzi et al. PRD2020].
  - $0.02760(10)$ [Davier et al. EPJC20].
  - $0.2722(41)$ EW-fit, $M_H = 94^{+20}_{-18} GeV$ [Keshavarzi et.al. PRD2020]
  - $0.02716(33)/0.02817(87)$ EW-fit w/wo M_H [Malaescu et al. 2008.08107].
  - $0.02716(39)$ EW-fit, $M_H = 90^{+21}_{-18} GeV$ [Gfitter. EPCJ18].
- [Crivellin et al. PRL2020]: If $a_\mu^{\text{LO-HVP}}$ gets closer to NoNewPhys $(720 \pm 7) \times 10^{-10}$, a tension would increase at electroweak global fit. c.f. [M.Passera et al. PRD08]
- [BMW-2020, 2002.12347]: The above tension is not necessarily suggested with naive looking at $\Delta^{had}\alpha(-10GeV^2) - \Delta^{had}\alpha(-1GeV^2)$.

Motivation

Running Electric Coupling

- $\alpha(s) = \frac{\alpha}{1-(\Delta^{had}\alpha(s)+\Delta^{lep}\alpha(s))}$, $\alpha = \frac{1}{137.03\cdots}$, $\Delta^{had}\alpha(-Q^2) = 4\pi\alpha\hat{\Pi}(Q^2)$.

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From LQCD to EW Precision Science

- Decompose [Jegerlehner hep-ph/0807.4206]:

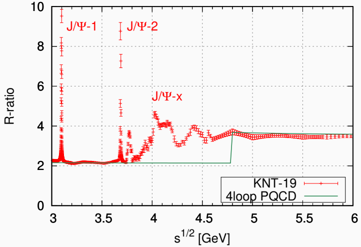
$$\Delta^{had}\alpha(M_z^2)=\Delta^{had}\alpha(-Q_0^2) \quad \longleftarrow \text{R-ratio/LQCD}$$
$$+[\Delta^{had}\alpha(-M_z^2) - \Delta^{had}\alpha(-Q_0^2)]_{PQCD'} \quad \longleftarrow \text{To Be Worked Out}$$
$$+[\Delta^{had}\alpha(M_z^2) - \Delta^{had}\alpha(-M_z^2)]_{PQCD}\,. \quad \longleftarrow 0.000038 \qquad (1)$$

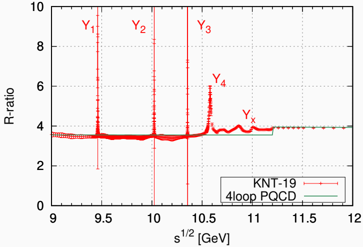
- $\Delta^{had}\alpha(-Q_0^2)$:
  Mainz/CLS (Teseo's Talk, ISB by A. Risch taken account)
  + b-quark effects [HPQCD-PRD15, 1408.5768].

- $[\Delta^{had}\alpha(-M_z^2) - \Delta^{had}\alpha(-Q_0^2)]_{PQCD'}$:
  4-loop 5-flavor Perturb. QCD (rhad-1.01) + J/Psi and Υ Resonances.

- $Q_0^2 \sim 5GeV^2$:
  Fix $\alpha_s(\mu)$ to $\overline{\text{MS}}$ at $\mu = Q_0$. Use physical pole quark masses.

J/Ψ and Υ

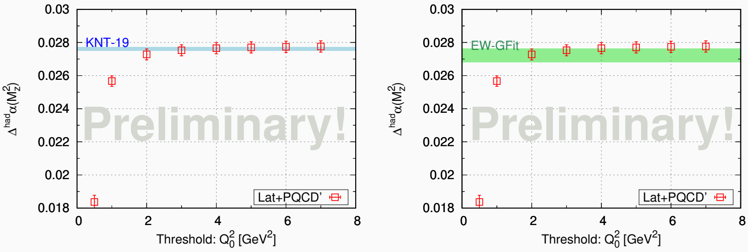




$$\left[\Delta^{\text{had}}\alpha(-M_Z^2) - \Delta^{\text{had}}\alpha(-Q_0^2)\right] \simeq \frac{(M_Z^2 - Q_0^2)\alpha_0}{3\pi} \int_{4M_\pi^2}^{\infty} ds \frac{R_{4loop}(s) + \delta R^{data}_{J/\Psi,\Upsilon}(s)}{(s+M_Z^2)(s+Q_0^2)} \ .$$

If Q_0^2 was sufficiently large, $1/(s + Q_0^2)$ in the integrand would suppress $\delta R^{data}_{J/\Psi,\Upsilon}(s)$ contribution. But this is not the case for our availability $Q_0^2 \sim 5GeV^2$.

Running Coupling at Z-Pole



- **Preliminary:** $\Delta^{\text{had}}\alpha^{(5)}(M_z^2)|_{Q_0^2=6GeV^2} = 0.02774(9)_{lat}(\cdots)_{PQCD'}$
- Less than 0.5% uncertainty from LQCD. Need estimate for uncertainty associated with Charm Sea-Quarks effects.
- Systematics from $PQCD'$ is about 1% and could be overestimated. Need more reasonablly precise estimate.