The hadronic contribution to $\Delta\alpha_{\mathrm{QED}}$

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Background

We parametrize the running of the QED coupling as

$$(Q^2) = \frac{}{1 \quad \Delta \quad (Q^2)}$$

The hadronic contribution to Δ is

$$\Delta \quad_{\mathrm{had}}(Q^2) = 4 \quad \bar{\Pi}(Q^2);$$

In the time-momentum representation (TMR) [Francis et al. 2013a; Bernecker and Meyer 2011],

$$\bar{\Pi}(Q^2) = \int_0^{1} dx_0 G(x_0) \left[x_0^2 \quad \frac{4}{Q^2} sin^2\left(\frac{Qx_0}{2}\right)\right];$$

with the electromagnetic correlator (b-quark contribution neglected)

$$G(x) = G^{33}(x) + \frac{1}{3} G^{88}(x) + \frac{4}{9} C^{c;c}(x);$$

$$G^{33}(x) = \frac{1}{2} C^{`;`}(x);$$

$$G^{88}(x) = \frac{1}{6}\left[C^{`;`}(x) + 2C^{s;s}(x) + 2D^{` \quad s;` \quad s}(x)\right]$$

In our study we use the same $N_f = 2+1$ set of CLS ensembles and similar techniques as in the $g \quad 2$ analysis. [A. Gerardin talk on Monday]

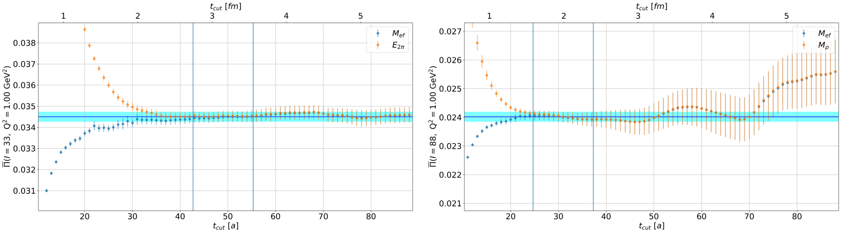
Error budget on ensemble E250 at 1 GeV^2

Ensemble at the physical pion mass (M   130 MeV) with L = 6:2 fm, M L = 4:1 and 96^3 192 lattice sites:

The initial statistical error of the renormalized, $O(a)$-improved [Gerardin, Harris, and Meyer 2019] correlators on ensemble E250 is:

- Isovector: 1%
- Isoscalar: 5%
- Charm: 0:05%

We apply the bounding method [Gérardin et al. 2019; Blum et al. 2018] to isovector and isoscalar only. Errors drop slightly below 1%.



Finite volume correction, Hansen-Patella method [Hansen and Patella 2020, 2019] and MLLGS-method [Francis et al. 2013b; Meyer 2011; Lellouch and Luscher 2001]

- Isovector: $+36 \times 10$ ^{5} (1%).

The size of the discretization effects is   5%

Global error budget

After chiral and continuum extrapolation, at the isospin symmetric physical point,

$Q^2[\mathrm{GeV}^2]$	$\bar{\Pi}^{33}$ 10^5	$\bar{\Pi}^{88}$ 10^5	$\bar{\Pi}^{c}$ 10^5	Δ 10^6	Δ error 10^6
1.00	3264 (18)(30)(6)	739 (5)(9)(0)	176.6 (0.8)(3.6)(0.2)	3832 (19)(39)(6)(28)	52 (1.4)
6.00	5815 (42)(30)(29)	1549 (14)(10)(8)	820.6 (2.7)(13.1)(0.9)	7506 (50)(49)(34)(34)	85 (1.1)

Statistical error

- a^3 lattice artifacts included for $Q^2 > 1$ GeV^2.
- It grows with the energy.

Scale setting error

- Estimated using standard propagation of errors.

Extrapolation error

- Estimated excluding ensembles with M > 400 MeV.
- Subleading.

Missing IB corrections error [A. Risch talk on Wednesday]

- Based on computation done on ensembles N200 and H102.