The hadronic contribution to $\Delta\alpha_{\mathrm{QED}}$

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[1] Speaker

Background

We parametrize the running of the QED coupling as

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha(Q^2)}$$

The hadronic contribution to $\Delta\alpha$ is

$$\Delta\alpha_{\text{had}}(Q^2) = 4\pi\alpha\bar{\Pi}(Q^2),$$

In the time-momentum representation (TMR) [Francis et al. 2013a; Bernecker and Meyer 2011],

$$\bar{\Pi}(Q^2) = \int_0^\infty dx_0\, G(x_0) \left[x_0^2 - \frac{4}{Q^2} sin^2\left(\frac{Qx_0}{2}\right)\right],$$

with the electromagnetic correlator (b-quark contribution neglected)

$$G(x) = G^{33}(x) + \frac{1}{3}G^{88}(x) + \frac{4}{9}C^{c,c}(x),$$

$$G^{33}(x) = \frac{1}{2}C^{\ell,\ell}(x),$$

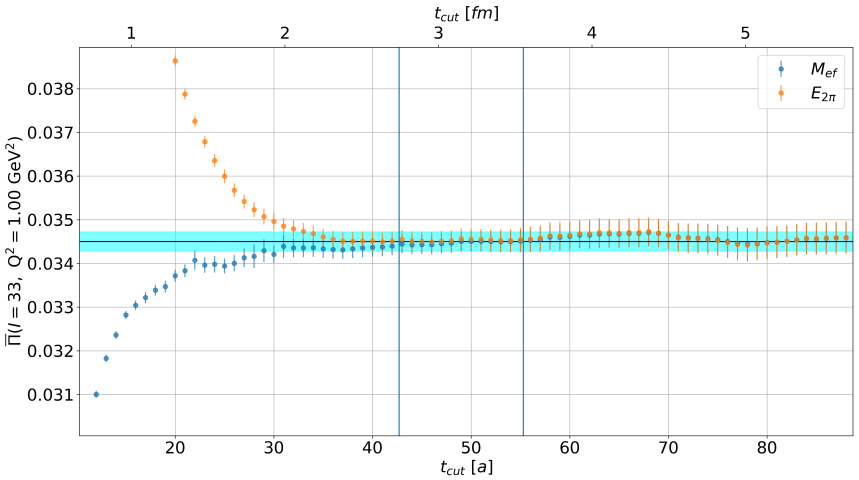
$$G^{88}(x) = \frac{1}{6}\left[C^{\ell,\ell}(x) + 2C^{s,s}(x) + 2D^{\ell-s,\ell-s}(x)\right]$$

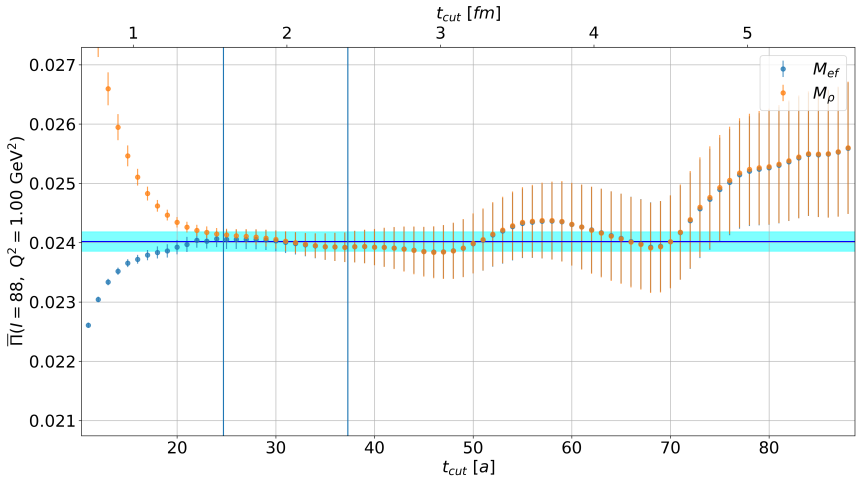
In our study we use the same $N_f = 2 + 1$ set of CLS ensembles and similar techniques as in the $g - 2$ analysis. [A. Gerardin talk on Monday]

Error budget on ensemble E250 at 1 GeV^2

Ensemble at the physical pion mass ($M_\pi \sim 130$ MeV) with $L = 6.2$ fm, $M_\pi L = 4.1$ and $96^3 \times 192$ lattice sites:

1. The initial statistical error of the renormalized, $O(a)$-improved [Gerardin, Harris, and Meyer 2019] correlators on ensemble E250 is:
   - Isovector: 1%
   - Isoscalar: 5%
   - Charm: 0.05%
2. We apply the bounding method [Gérardin et al. 2019; Blum et al. 2018] to isovector and isoscalar only. Errors drop slightly below 1%.





3. Finite volume correction, Hansen-Patella method [Hansen and Patella 2020, 2019] and MLLGS-method [Francis et al. 2013b; Meyer 2011; Lellouch and Luscher 2001]
   - Isovector: $+36 \times 10^{-5}$ (1%).
4. The size of the discretization effects is $\sim 5\%$

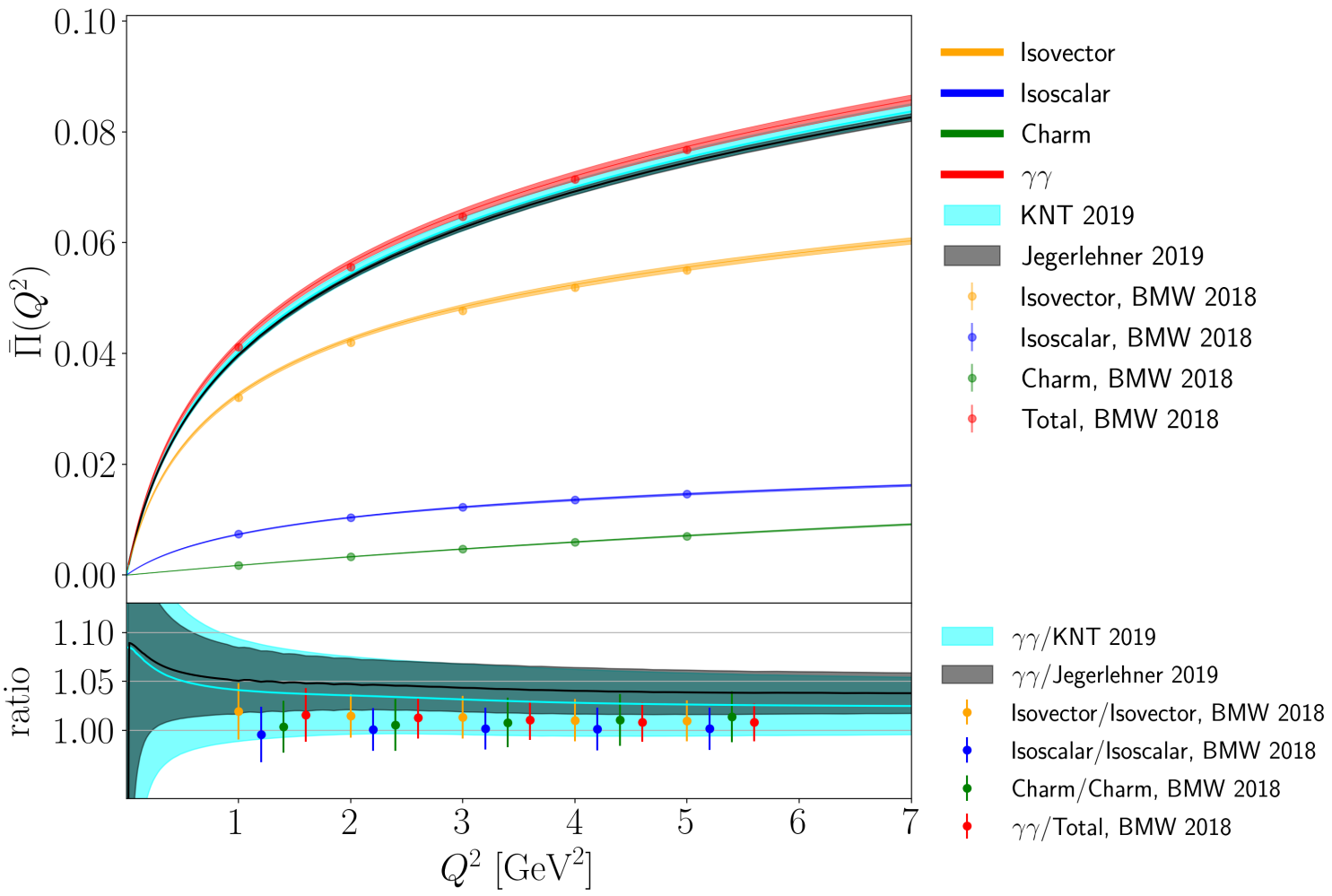
Global error budget

After chiral and continuum extrapolation, at the isospin symmetric physical point,

Q^2[GeV^2]	$\bar{\Pi}^{33} \times 10^5$	$\bar{\Pi}^{88} \times 10^5$	$\bar{\Pi}^{c} \times 10^5$	$\Delta\alpha^{\gamma\gamma} \times 10^6$	$\Delta\alpha^{\gamma\gamma}_{\text{error}} \times 10^6$
1.00	3264 (18)(30)(6)	739 (5)(9)(0)	176.6 (0.8)(3.6)(0.2)	3832 (19)(39)(6)(28)	52 (1.4)
6.00	5815 (42)(30)(29)	1549 (14)(10)(8)	820.6 (2.7)(13.1)(0.9)	7506 (50)(49)(34)(34)	85 (1.1)

1. Statistical error
   - a^3 lattice artifacts included for $Q^2 > 1$ GeV^2.
   - It grows with the energy.
2. Scale setting error
   - Estimated using standard propagation of errors.
3. Extrapolation error
   - Estimated excluding ensembles with $M_\pi > 400$ MeV.
   - Subleading.
4. Missing IB corrections error [A. Risch talk on Wednesday]
   - Based on computation done on ensembles N200 and H102.

Results and comparison



[Keshavarzi, Nomura, and Teubner 2020; Jegerlehner 2019; Borsanyi et al. 2018]