Constraints on the two-pion contribution to HVP

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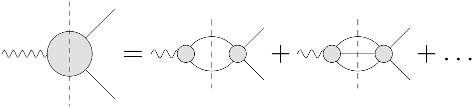
Topical Muon $g-2$ Theory Initiative workshop

“The hadronic vacuum polarization from lattice QCD at high precision”

Addressing the lattice vs pheno puzzle

- BMWc studies suggest HVP modifications in region $< 2\,$GeV
- $\pi\pi$ **channel** dominates $\Rightarrow$ **unitarity/analyticity** constraints

Dispersive representation of pion VFF



$$F_\pi^V(s) = \Omega_1^1(s) \times G_\omega(s) \times G_{\rm in}^N(s)$$

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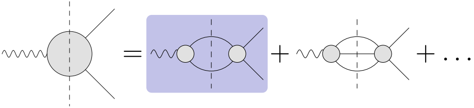

$$F_\pi^V(s) = \Omega_1^1(s) \times G_\omega(s) \times G_{\rm in}^N(s)$$

- Omnès function with $\pi\pi$ P-wave phase shift $\delta_1^1(s)$ as input:

$$\Omega_1^1(s) = \exp\left\{ \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\delta_1^1(s')}{s'(s'-s)} \right\}$$

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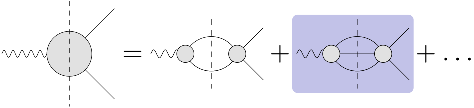

$$F_\pi^V(s) = \Omega_1^1(s) \times G_\omega(s) \times G_{\text{in}}^N(s)$$

- isospin-breaking 3π intermediate state: ρ–ω interference

$$G_\omega(s) \approx g_\omega(s) = 1 + \epsilon_\omega \frac{s}{(M_\omega - \frac{i}{2}\Gamma_\omega)^2 - s}$$

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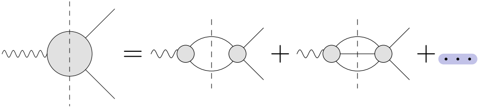


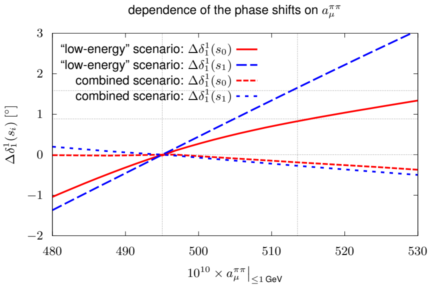
$$F_\pi^V(s) = \Omega_1^1(s) \times G_\omega(s) \times G_{\text{in}}^N(s)$$

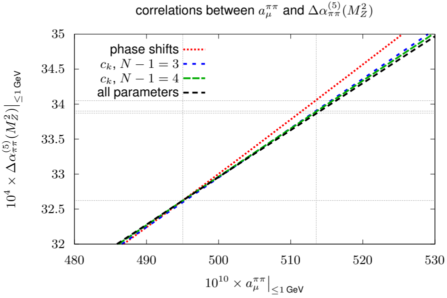
- heavier intermediate states: use conformal polynomial

$$G_{\text{in}}^N(s) = 1 + \sum_{k=1}^{N} c_k (z^k(s) - z^k(0))$$

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

- include “lattice” datum with tiny uncertainty in fit
- three different scenarios:
  - “low-energy” physics: **$\pi\pi$ phase shifts**
  - “high-energy” physics: **inelastic effects, c_k**
  - **all parameters free**
- effects on **phase shifts, hadronic running of $\alpha_{\text{QED}}^{\text{eff}}$**, pion charge radius, space-like VFF





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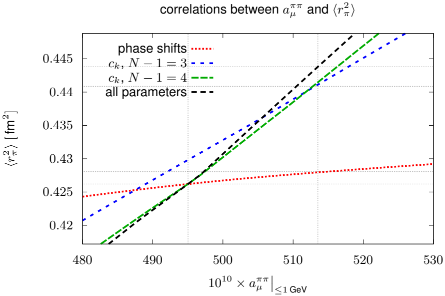

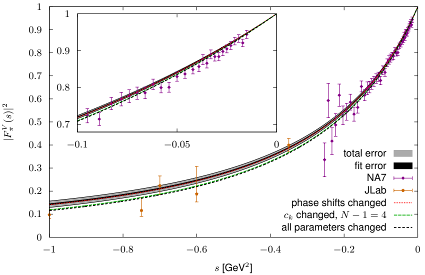

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

- "low-energy" scenario: local changes in cross section of **$\sim 8\%$ around ρ**
- "high-energy" scenario: impact on **pion charge radius** and space-like VFF $\Rightarrow$ chance for **independent lattice-QCD checks**
- requires **factor ~ 3 improvement** over χQCD result:
  $\langle r_\pi^2 \rangle = 0.433(9)(13)\,\text{fm}^2$
  $\rightarrow$ arXiv:2006.05431 [hep-ph]