Disconnected HVP from BMWc

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Noise reduction

1. Use same random vectors for l and s [Gülpers *et al.* 2014]

$$\tilde{j}^{(l)} - \tilde{j}^{(s)} = \left\langle \frac{1}{N} \sum_{r=1}^{N} \mathrm{Im} \left\langle \xi^{(r)} \middle| U_\mu^{+-} \left(M_l^{-1} - M_s^{-1} \right) \xi^{(r)} \right\rangle_t \right\rangle$$

$$M_l^{-1} - M_s^{-1} = \frac{m_s - m_l}{M_l \, M_s}$$

$\longrightarrow$ UV-part of noise is suppressed

2. Compute lowest N_{eig} eigenpairs of M: λ_i, v_i

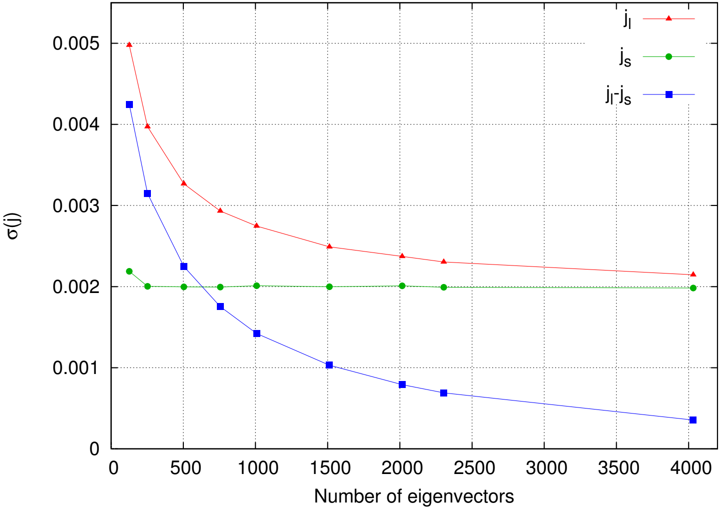
$$M^{-1} = \sum_{i=1}^{N_{\mathrm{eig}}} \frac{1}{\lambda_i} \left| v_i \right\rangle \left\langle v_i \right| + M^{-1} P, \qquad P = \left(1 - \sum_{i=1}^{N_{\mathrm{eig}}} \left| v_i \right\rangle \left\langle v_i \right| \right)$$

$$\tilde{j}_\mu(t) = \sum_{i=1}^{N_{\mathrm{eig}}} \mathrm{Im} \left[\frac{1}{\lambda_i} \left\langle v_i \middle| U_\mu^{+-} v_i \right\rangle_t \right] + \left\langle \frac{1}{N} \sum_{r=1}^{N} \mathrm{Im} \left\langle P \xi^{(r)} \middle| U_\mu^{+-} M^{-1} P \xi^{(r)} \right\rangle_t \right\rangle$$

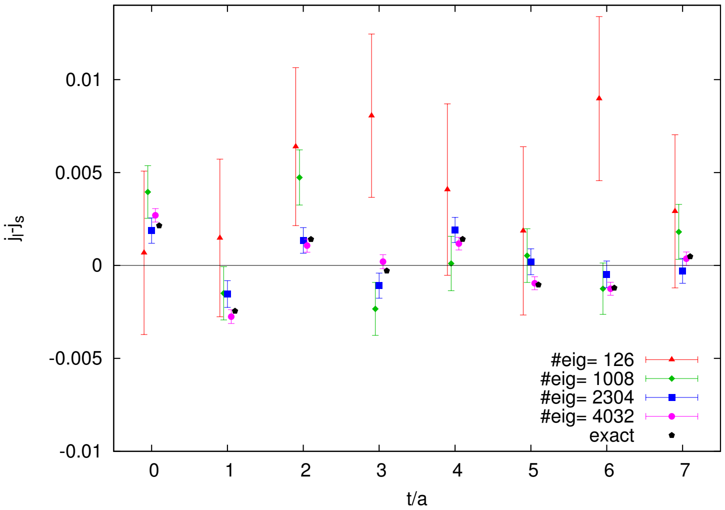
$\longrightarrow$ IR-part of noise is suppressed

3. AMA/Truncated solver method: stop at lower precision, compute correction on few vectors

Noise of j_μ on one configuration, $\beta = 3.7000$



Noise of j_μ on one configuration, $\beta = 3.7000$



Hopping parameter expansion

[Thron *et al.* 1998; Bali *et al.* 2010]

- $M = m + D, \qquad M^\dagger = m - D, \qquad M^\dagger M = m^2 - D^2$

$$M^{-1} = M^\dagger \left(M^\dagger M\right)^{-1} = \frac{m - D}{m^2 - D^2}$$

- Then the current:

$$\tilde{j}_\mu(t) = \sum_{\underline{x}} \mathrm{Im}\,\mathrm{tr}_c \left(\left(U_\mu \frac{m - D}{m^2 - D^2} \right)_{\underline{x},t\,;\,\underline{x},t} \right) =$$

$$= \sum_{\underline{x}} \mathrm{Im}\,\mathrm{tr}_c \left(\left(U_\mu (-D) \frac{1}{m^2 - D^2} \right)_{\underline{x},t\,;\,\underline{x},t} \right)$$

- Rewrite the inverse using n^{th} degree polynomial $a_0 + a_1 x + \cdots + a_n x^n$

$$\frac{1}{m^2 - D^2} = \sum_{k=0}^{n} a_k \left(D^2\right)^k + \frac{1}{m^2 - D^2} \sum_{k=0}^{n+1} \underbrace{\left(a_{k-1} - m^2\, a_k\right)}_{b_k} \cdot \left(D^2\right)^k$$

with $a_{-1} = 1$ and $a_{n+1} = 0$.

Hopping parameter expansion

- Current consists of two parts:

$$\tilde{j}_\mu(t) = \sum_{k=0}^{n} a_k \overbrace{\sum_{\underline{x}} \mathrm{Im}\,\mathrm{tr}_c \left(U_\mu(-D)\,(D^2)^k \right)_{\underline{x},t\,;\,\underline{x},t}}^{K_\mu^{(k)}(t)} + \\ + \sum_{k=0}^{n+1} b_k \underbrace{\sum_{\underline{x}} \mathrm{Im}\,\mathrm{tr}_c \left(U_\mu(-D)\,\frac{(D^2)^k}{m^2 - D^2} \right)_{\underline{x},t\,;\,\underline{x},t}}_{R_\mu^{(k)}(t)}$$

- Recipe:
  - Calculate $K_\mu^{(k)}(t)$ exactly.
  - Calculate $R_\mu^{(k)}(t)$ using random vectors:

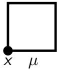
$$R_\mu^{(k)}(t) = \frac{1}{N} \sum_{r=1}^{N} \mathrm{Im} \left\langle \xi^{(r)} \middle| U_\mu\,(-D)\,\frac{(D^2)^k}{m^2 - D^2}\,\xi^{(r)} \right\rangle_t$$

    Use same random vector set for all $k = 0, 1, \ldots, n+1$.
  - Choose coefficients a_k such that the noise of $\sum_k b_k R_\mu^{(k)}(t)$ is minimal.

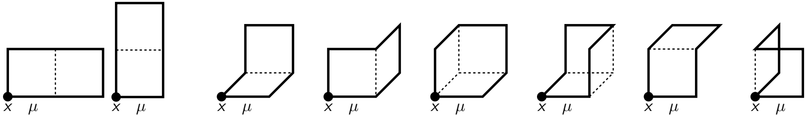
Hopping parameter expansion

$$K_\mu^{(k)}(t) = \sum_{\underline{x}} \mathrm{Im}\,\mathrm{tr}_c \left(U_\mu(-D)\,(D^2)^k \right)_{\underline{x},t\,;\,\underline{x},t}$$

- Computing $K_\mu^{(k)}(t)$ $\longrightarrow$ calculate loops
  - $k=0$ $K_\mu^{(0)}(t)=0$
  - $k=1$ 1 loop of length 4 $\longrightarrow$



  - $k=2$ 8 additional loops of length 6



  - $k=3$ 167 additional loops of length 8
  - $k=4$ 4402 additional loops of length 10
  - ...

Hopping parameter expansion