Two-pion contributions to the hadronic vacuum polarization with staggered quarks

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ON BEHALF OF THE FERMILAB LATTICE & MILC COLLABORATIONS

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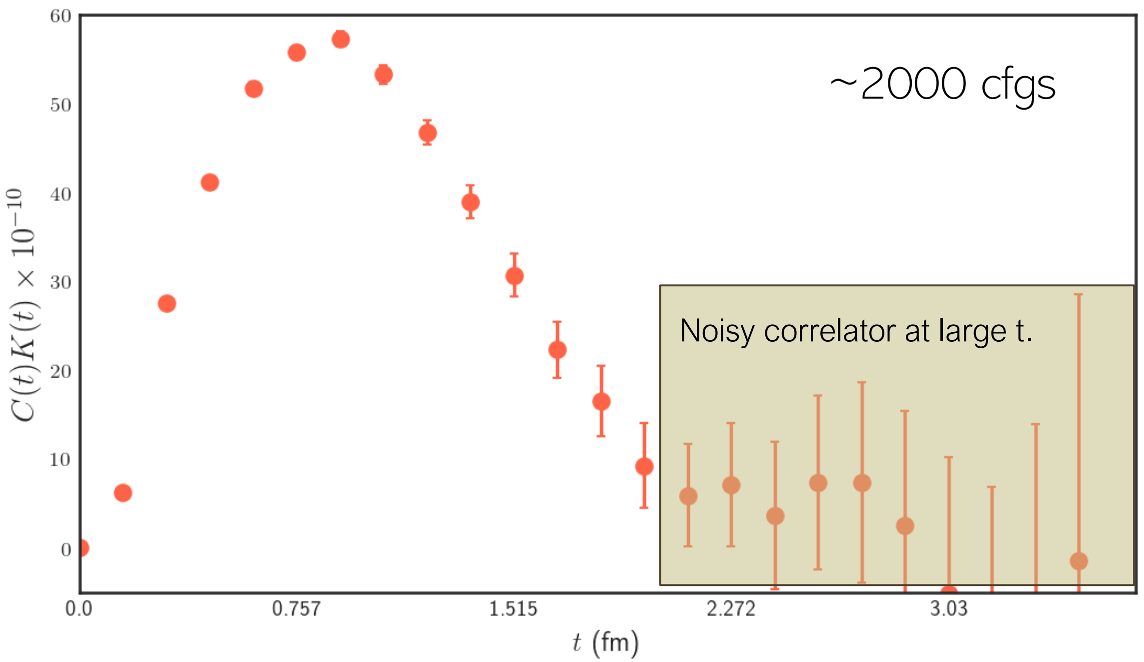
Noise reduction strategy

$$a_\mu^{\text{HVP, LO}} = 4\alpha^2 \int_0^\infty dt\, C(t) K(t)\ , \quad C(t) = \frac{1}{3}\sum_{i,\vec{x}} \langle J^i(\vec{x},t) J^i(\vec{0},0)\rangle$$

Bernecker & Meyer, 2011

Connected light-quark contribution from lattice data at 0.15fm



Correlator has noisy tail due to $J^i(\vec{x},t)$ operator overlapping strongly with rho but poorly with lowest energy two-pion states.

Strategy

- Explicitly compute two pion contributions from a GEVP of correlator matrix $C_{ij}(t)$ with $\mathcal{O}_{J^i}, \tilde{\mathcal{O}}_{J^i}, \mathcal{O}_{\pi\pi}$ operators.
- Replace C(t) after some t* with spectral reconstruction: $\tilde{C}(t) = \sum_n Z_n e^{-E_n t}$

$$C_{ij}(t) = \begin{pmatrix} \{J^i, \tilde{J}^i\} \to \{J^i, \tilde{J}^i\} & \{J^i, \tilde{J}^i\} \to \pi\pi \\ \pi\pi \to \{J^i, \tilde{J}^i\} & \pi\pi \to \pi\pi \end{pmatrix}$$

Staggered two-pion states

- 15 additional copies of each meson with taste quantum number. $\xi \in \{1, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \gamma_\nu\gamma_\mu\}$

- Taste singlet vector current $(\gamma_i \otimes 1) \rightarrow$ require two-pion operators which transform under this irrep.

Staggered Group Structure

$$T_N^3 \rtimes \{\Xi_\mu, C_0\} \rtimes \{\tilde{R}_{ij}, I_S\} = Z_N^3 \rtimes \Gamma_{4,1} \rtimes W_3$$

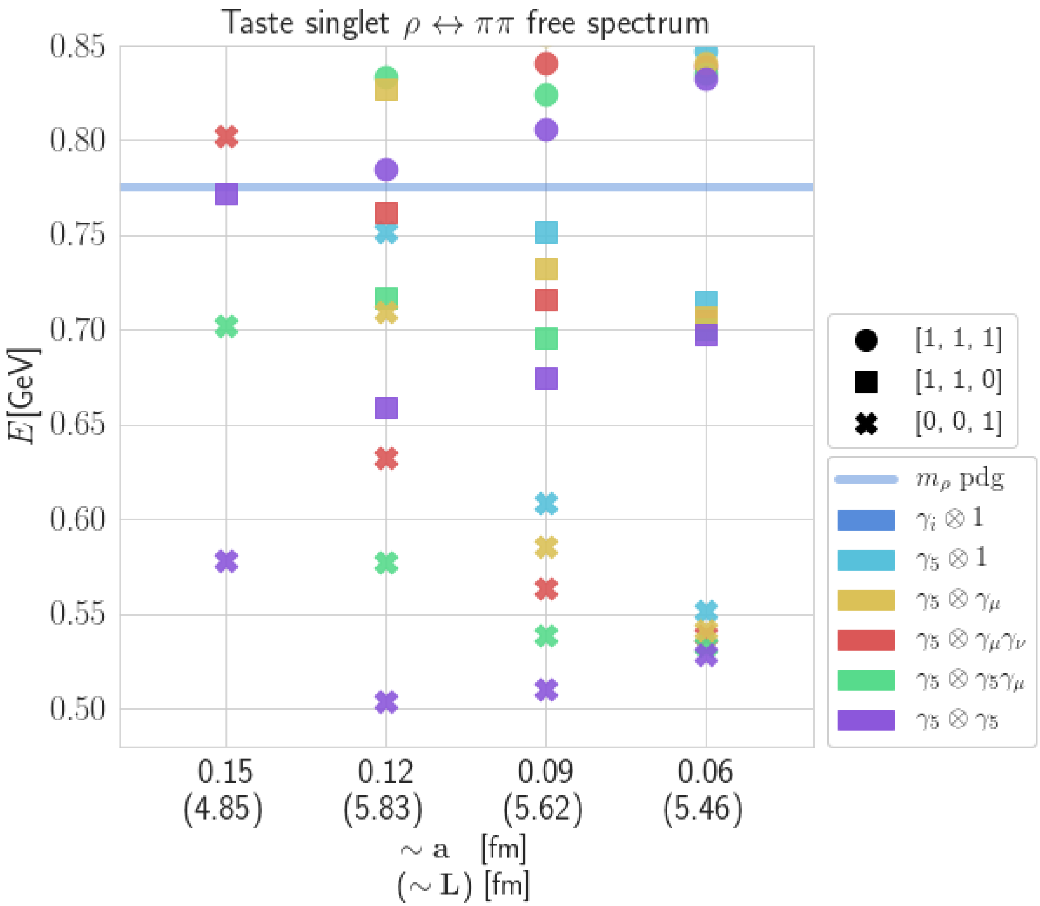
$$\mathcal{O}_{\pi\pi}(\vec{0})_{\gamma_i \otimes 1} = \sum_{\substack{\xi_1, \xi_2 \\ \vec{p} \in \{p\}^\star}} CG_{\text{stag, iso.}}(\vec{0}, \gamma_i \otimes 1, \{\vec{p}, \gamma_5 \otimes \xi_i\})\, \pi(\vec{p})_{\xi_1} \pi(-\vec{p})_{\xi_2}$$

Only taste diagonal two-pion states couple to taste singlet vector current $\pi_{\gamma_5}\pi_{\gamma_5}, \pi_{\gamma_\mu}\pi_{\gamma_\mu}$ etc..

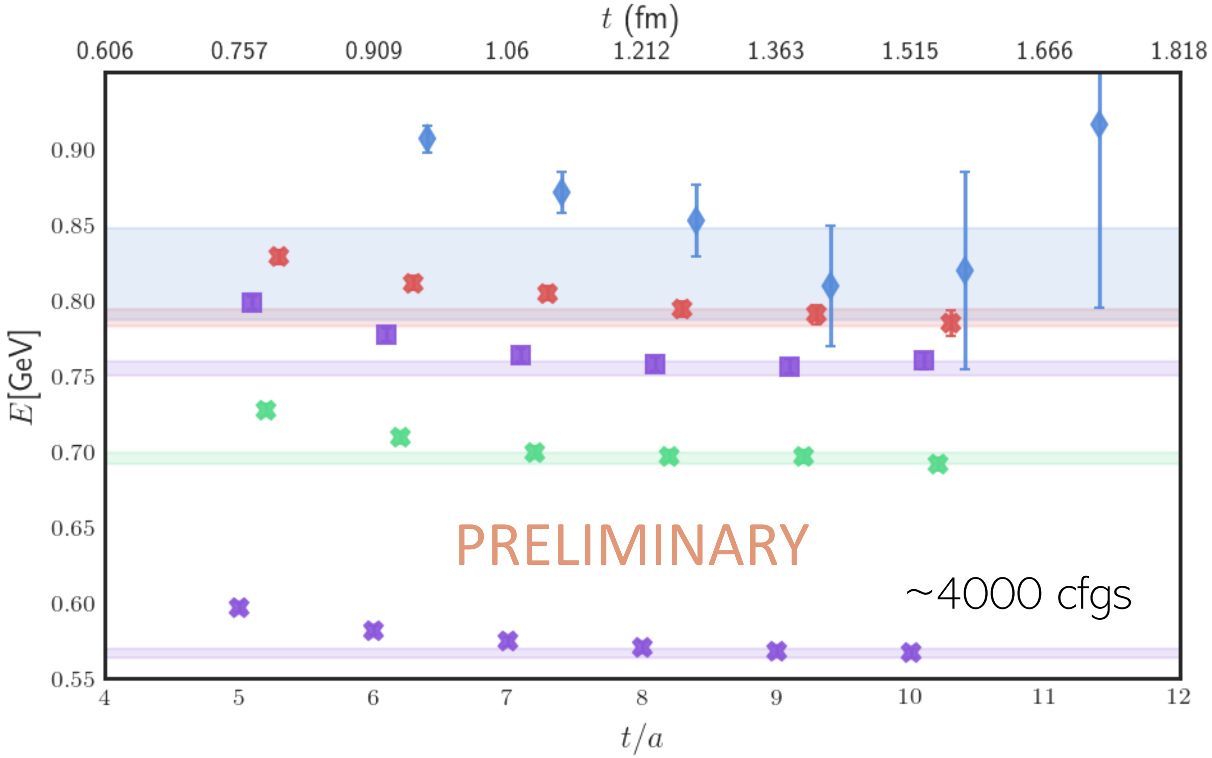
- Taste singlet pion combinations: π_{γ_5}, π_1 , situation is equivalent to Wilson fermions.

- Taste non-singlet pion combinations: there are additional complications due to coupling with taste, momentum and vector current direction (see appendix).

$\rho, \pi\pi$, f.v. spectrum on the phys. mass HISQ ensembles



Results from 0.15 physical mass HISQ ensemble.



$$E_{\pi\pi} = 2\sqrt{p^2 + m_\xi^2}$$

Effective mass w/ smearing for $\pi\pi \rightarrow \pi\pi$ diagonal four points.
Bands are results from multi-exponential fits.

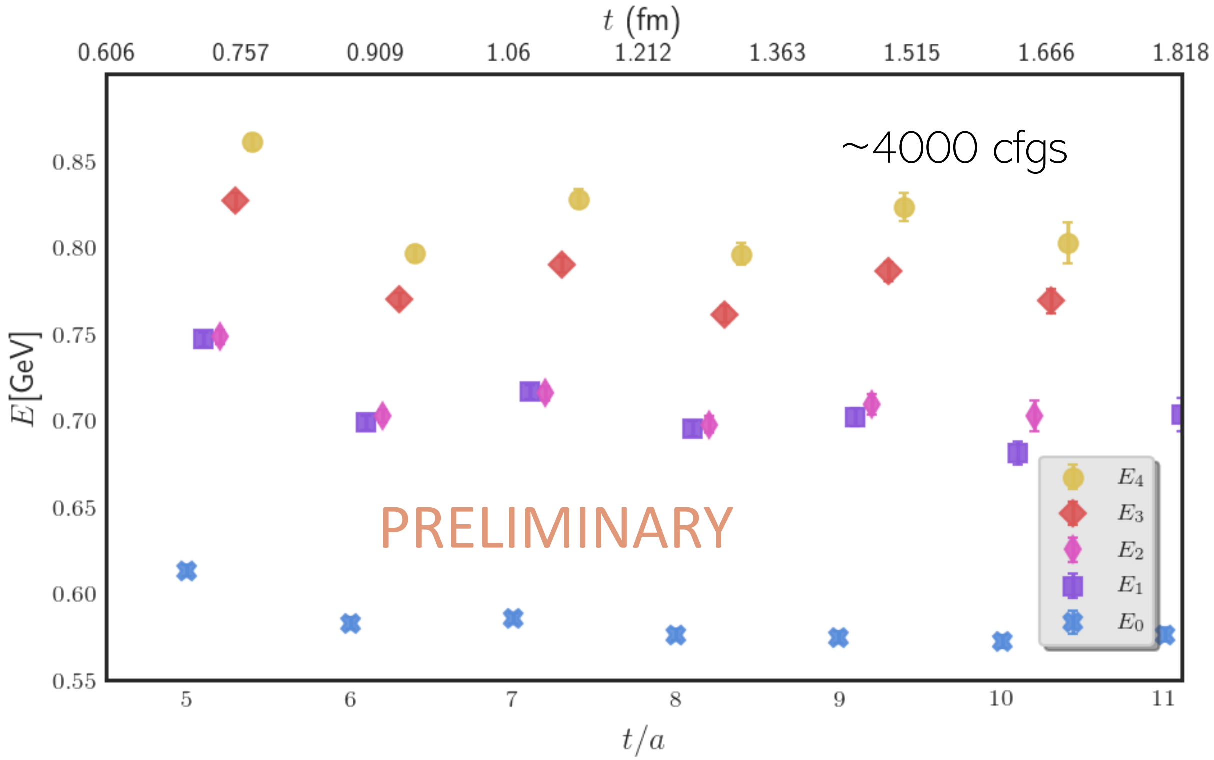
GEVP Analysis - First Look

$$C(t)v = \lambda C(t_0)v \quad \rightarrow \quad v_n^\dagger C(t) v_n \overset{t\to\infty}{\sim} Z_n^2 e^{-E_n t} + (-1)^t Z'^2_n e^{-E'_n t}$$

Oscillating states (fit or smear C(t) to remove)

Operator Basis

Vector current:	$\mathcal{O}_J, \tilde{\mathcal{O}}_J$	
Pseudo-scalar:	$\mathcal{O}_{\pi\pi}^{\otimes\gamma_5}$	$[0,0,1]\,,[1,1,0]$
Pseudo-vector:	$\mathcal{O}_{\pi\pi}^{\otimes\gamma_{5x/y}}, \mathcal{O}_{\pi\pi}^{\otimes\gamma_{5z}}$	$[0,0,1]$
(Temporal)Tensor:	$\mathcal{O}_{\pi\pi}^{\otimes\gamma_{x/y\,t}}, \mathcal{O}_{\pi\pi}^{\otimes\gamma_{z\,t}}$	$[0,0,1]$



- Analysis at 0.15 ongoing
- Generating data for 0.12 ensemble

Thank you for listening.

Staggered complications

Taste *orbit* splitting:

Taste non-singlet irreps split into multiple irreps at non-zero momentum. Originates from taste little group analysis.

$$\begin{aligned} \vec{p} = (0,0,0) \quad &\rightarrow \quad \vec{p} = (0,0,1) \\ \gamma_5 \otimes \gamma_i \quad &\rightarrow \quad \{\gamma_5 \otimes \gamma_z\} \oplus \{\gamma_5 \otimes \gamma_{x,y}\} \end{aligned}$$

$\Longrightarrow$ additional states to take into account.

- **Combinations of taste non-singlet pions:**

  Different couplings when taste (pseudo)vector, tensor in same(different) direction as vector current. Originates from Clebsch-Gordon calculation.

$$\pi\pi(\vec{0}) \equiv \pi^{\otimes\gamma_k}(\vec{p} = (1,1,0))\ \pi^{\otimes\gamma_k}(\vec{p} = (-1,-1,0))$$

different coupling / CG to singlet J^i when $k = i$ ($k \neq i$)