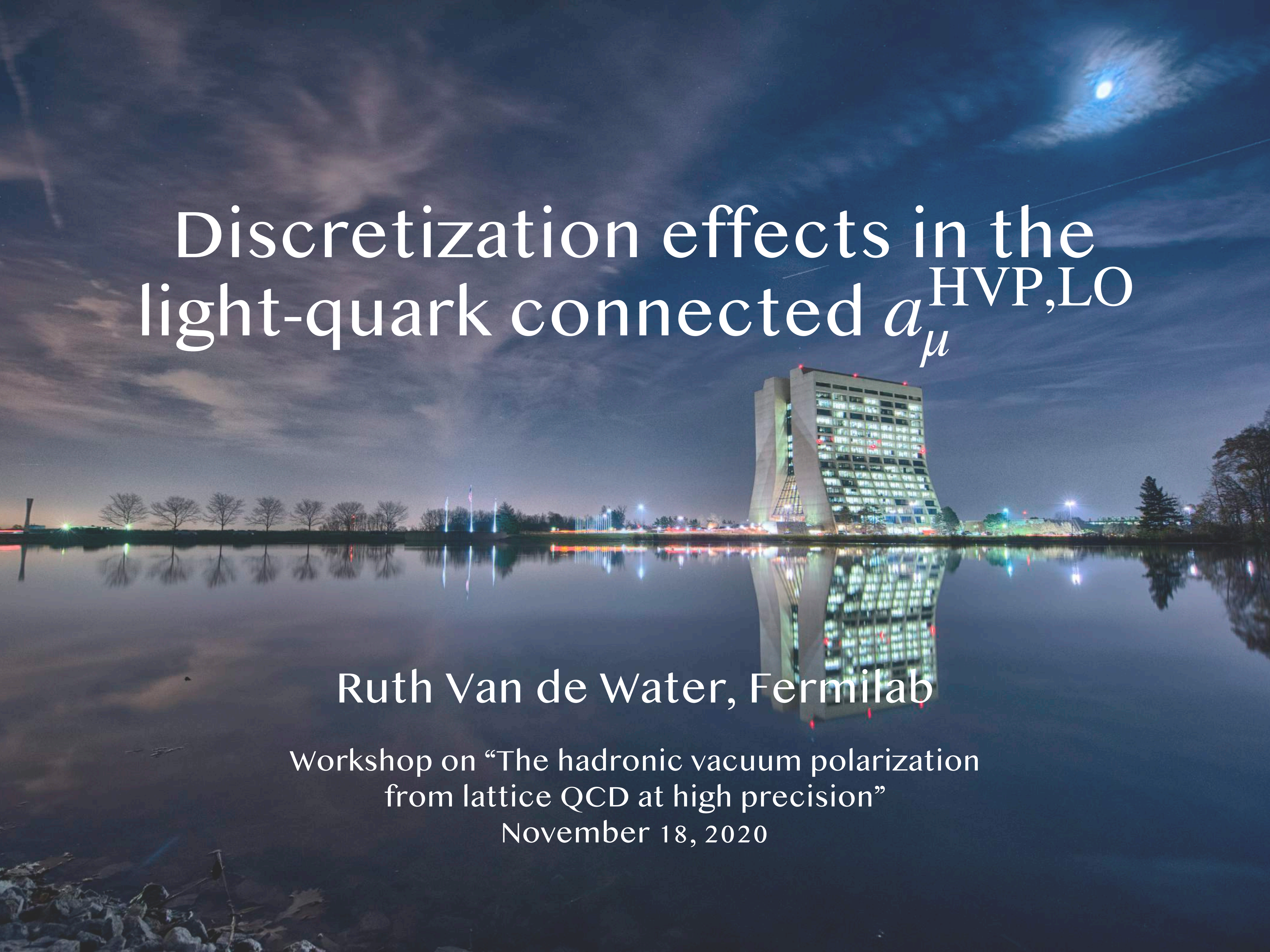

Discretization effects in the light-quark connected $a_\mu^{\mathrm{HVP,LO}}$

Ruth Van de Water, Fermilab

Workshop on "The hadronic vacuum polarization from lattice QCD at high precision"
November 18, 2020

Framing the discussion

- **All lattice simulations suffer from generic discretization effects**; such effects are O(a) for unimproved Wilson fermions and O(a^2) for domain-wall, improved Wilson, staggered, and twisted-mass fermions
- Staggered-fermion simulations also suffer from taste-breaking discretization errors that alter the pion spectrum; taste breaking primarily impacts $a_\mu^{ll,\mathrm{conn.}}$ through the value of m_π in the leading pion bubble diagram
- Because different lattice quark actions have different discretization effects, results for $a_\mu^{ll,\mathrm{conn.}}$ will differ at fixed, nonzero lattice spacing; must all agree in continuum limit
- If discretization effects are large and / or continuum extrapolation is far, need solid understanding theoretical lattice-spacing dependence to guide continuum extrapolation (*can often use Symanzik effective theory*)
- If discretization effects are small and continuum extrapolation is short, *any fit function will do*

Thoughts on comparing results

- Comparison of discretization effects in $a_\mu^{ll,\text{conn.}}$ from different simulations complicated by the fact that, for recent staggered simulations (Aubin *et al.*, BMW, Fermilab/HPQCD/MILC), taste-breaking effects are much larger than generic discretization effects at the largest lattice spacings
- To make an apples-to-apples comparison, can first remove taste-breaking effects with staggered χPT (*more on this later*)
- Alternatively, to compare generic discretization effects from different lattice fermion actions, useful to identify quantities that are less sensitive to $\pi\pi$ contributions, and hence less sensitive to taste breaking
- Because lightest states in correlator are most important at large Euclidean times, quantities derived from the correlator at shorter Euclidean times are less sensitive to $\pi\pi$ states; examples include the **leading Taylor coefficient Π_1** and the **intermediate window a_μ^{win}(0.4 — 1 fm)****

* Intermediate window *wholly derived from data* ⇒ *free from assumptions about large-t behavior of correlator*

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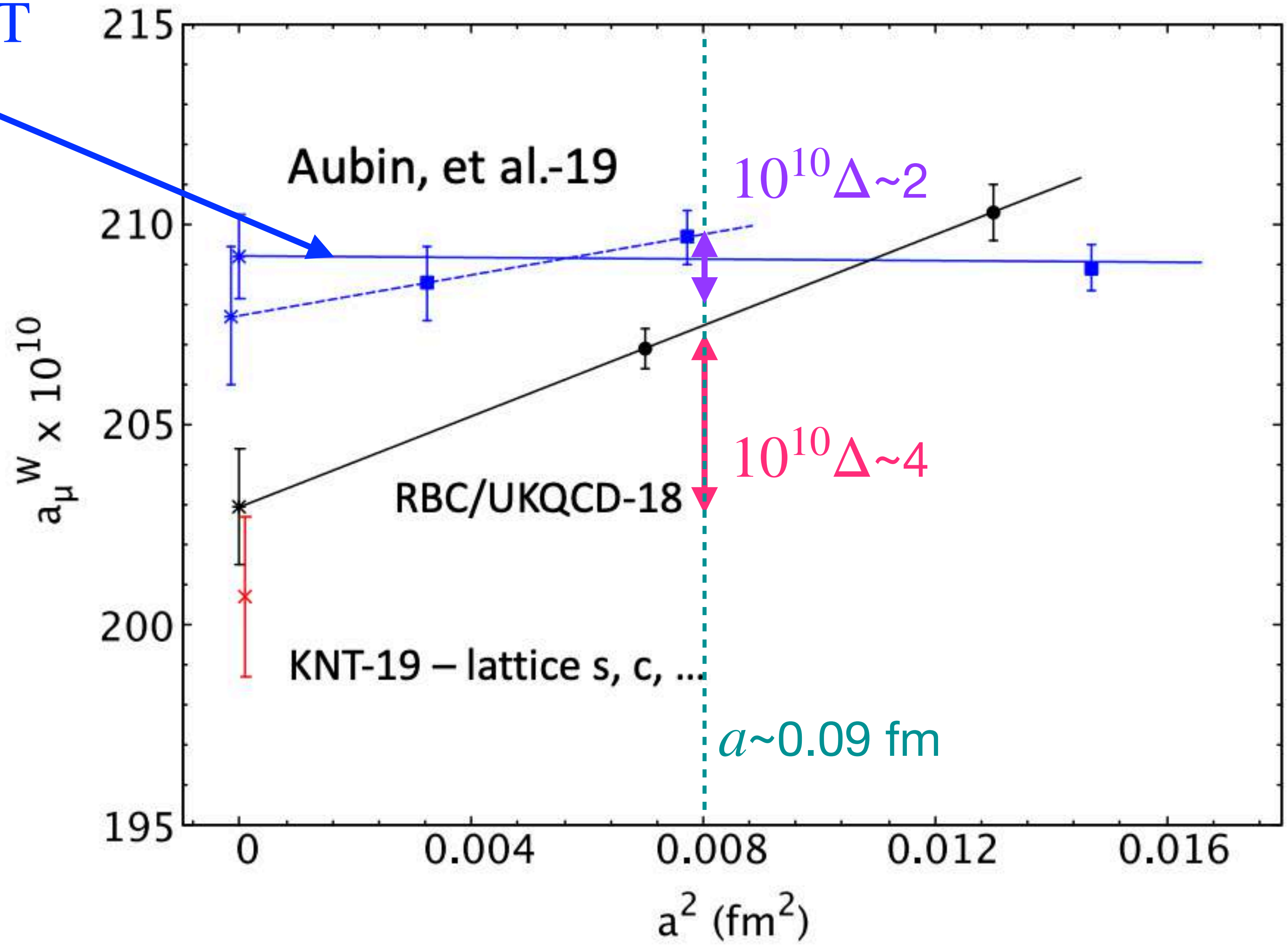
Discretization effects in $a_\mu^{\rm win}(0.4\text{—}1\ \mathrm{fm})$

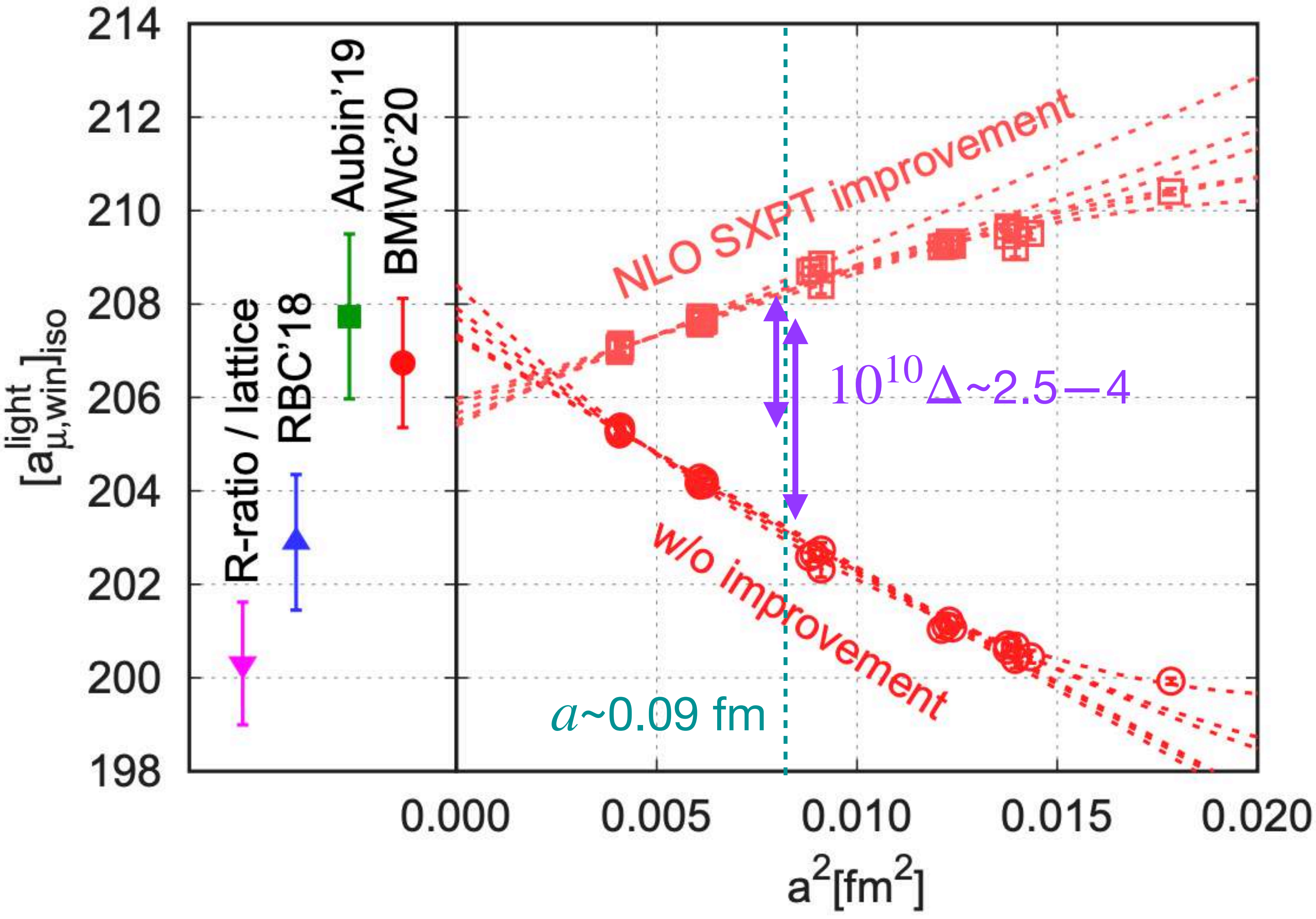
Data corrected for taste breaking with NLO SχPT

Aubin *et al.*, RBC/UKQCD



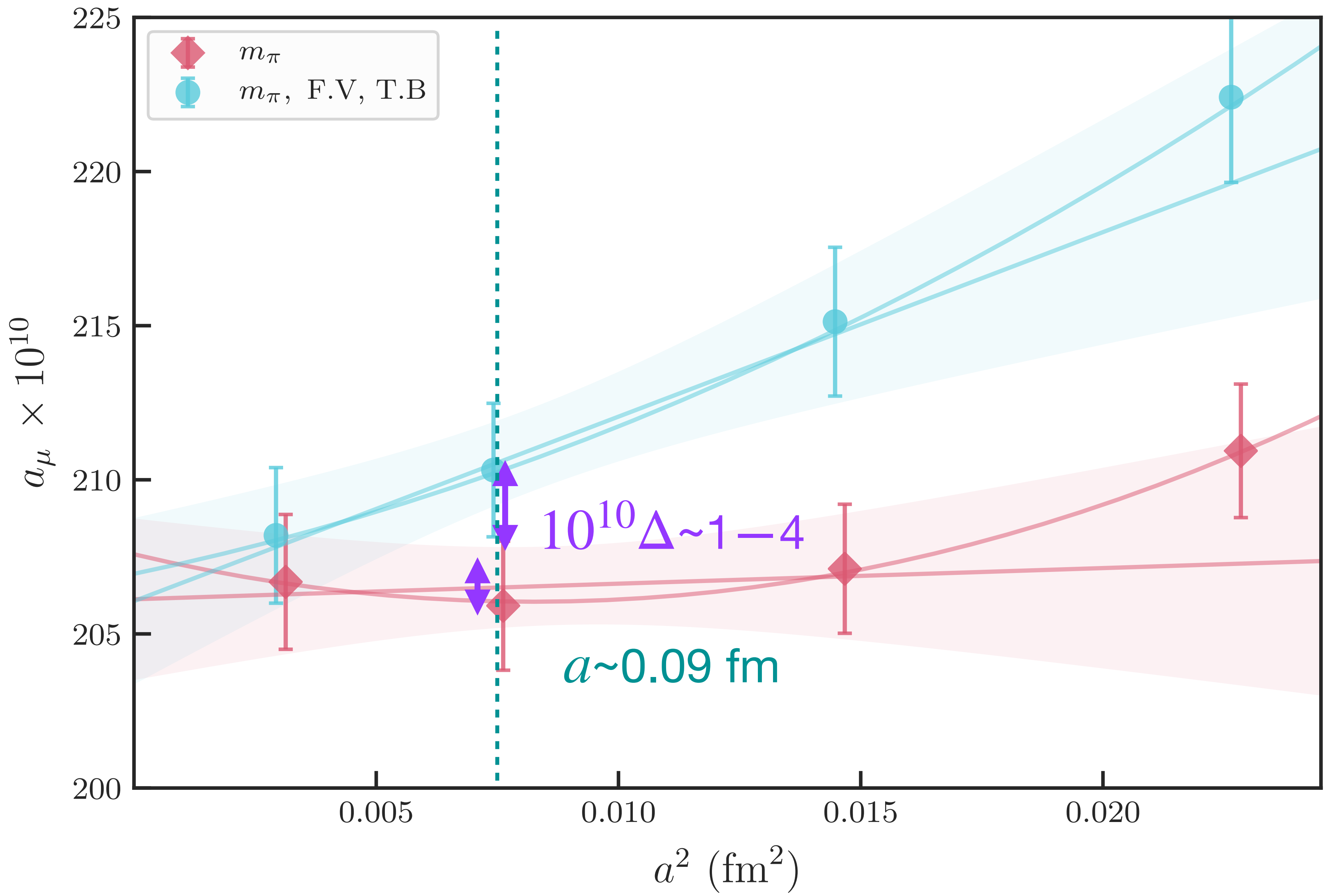
Discretization effects in $a_\mu^{\rm win}(0.4—1\ {\rm fm})$

BMW



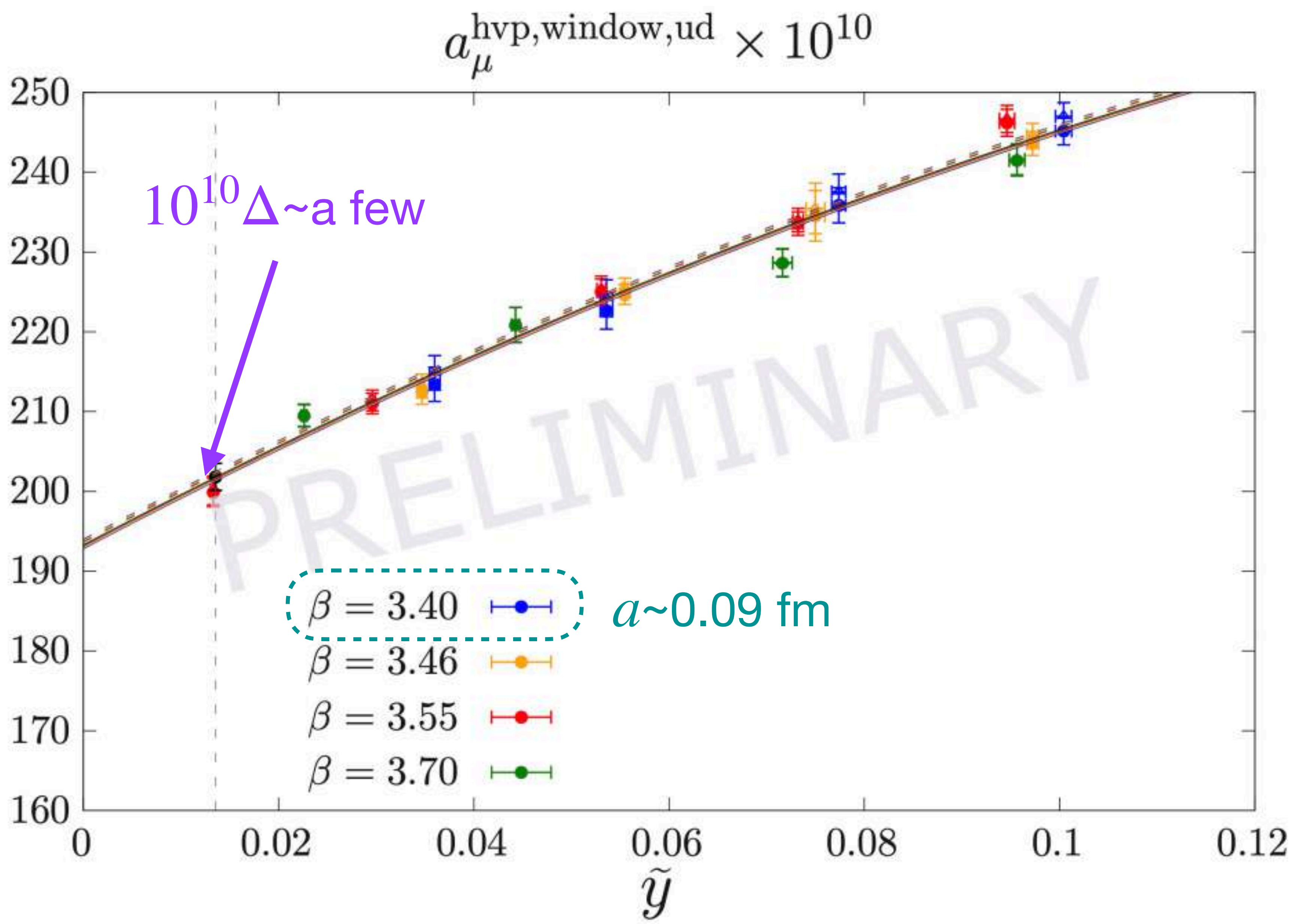
Discretization effects in $a_\mu^{\rm win}(0.4—1\ {\rm fm})$

Fermilab/HPQCD/MILC (***preliminary***)



Discretization effects in a_μ^{win}(0.4—1 fm)

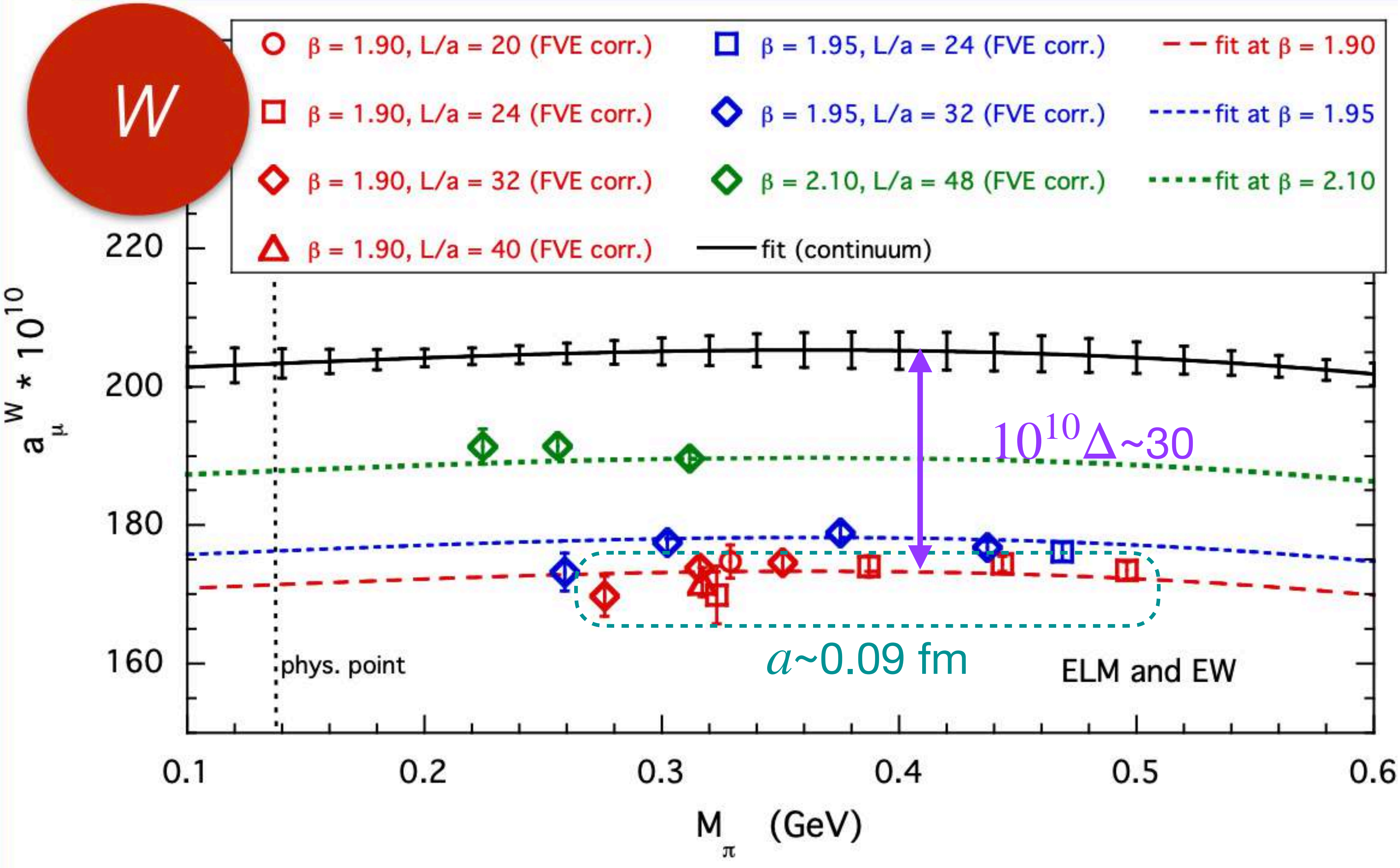
Mainz (***preliminary,*** *m_μ in kernel rescaled by f_π*)



☆ For total a_μ^{ll}, discretization effects are approximately 3x larger without f_π rescaling

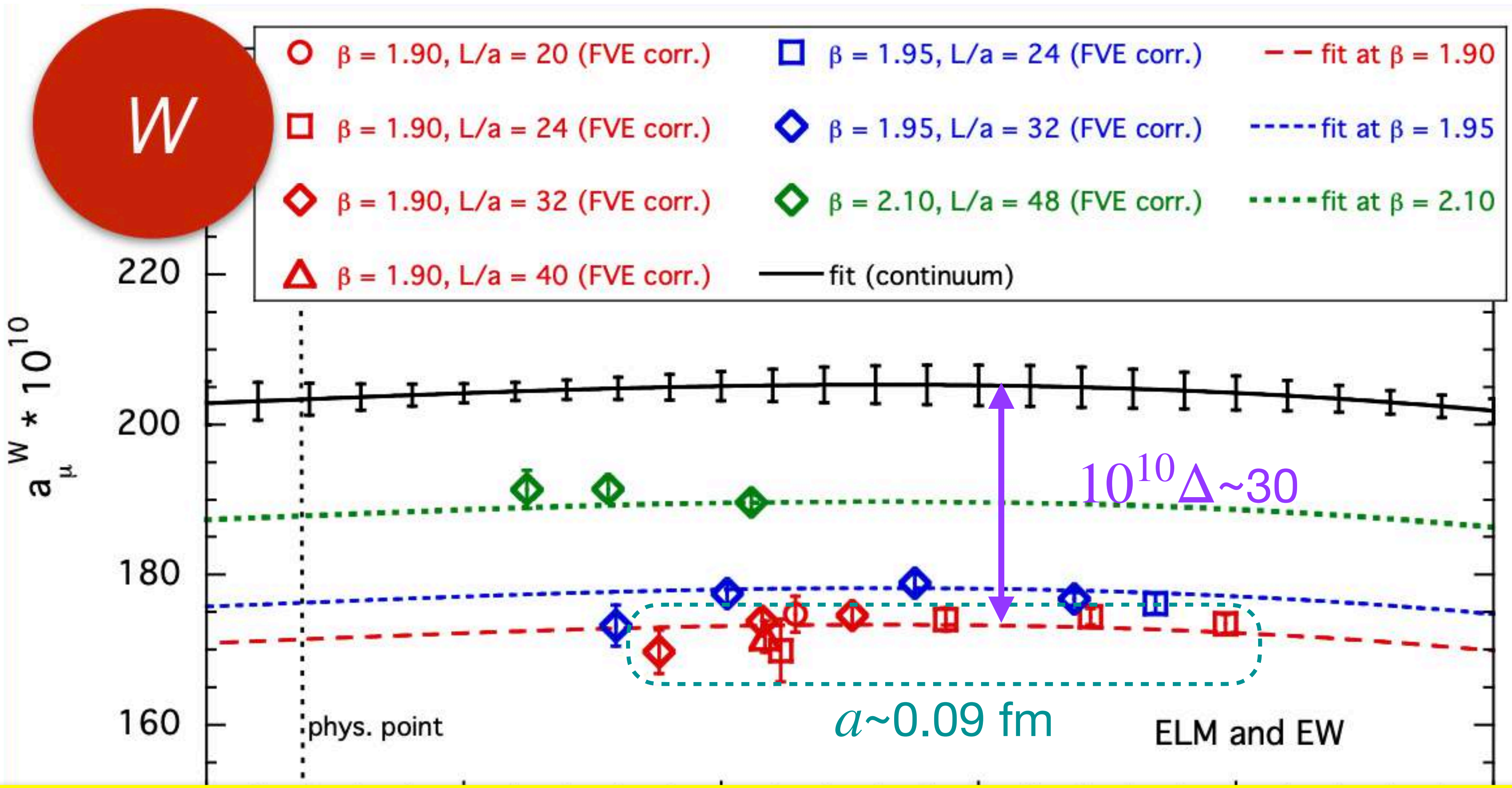
Discretization effects in $a_\mu^{\rm win}$(0.4—1 fm)

ETM (***preliminary***)



Discretization effects in $a_\mu^{\rm win}(0.4—1\ {\rm fm})$

ETM (***preliminary***)

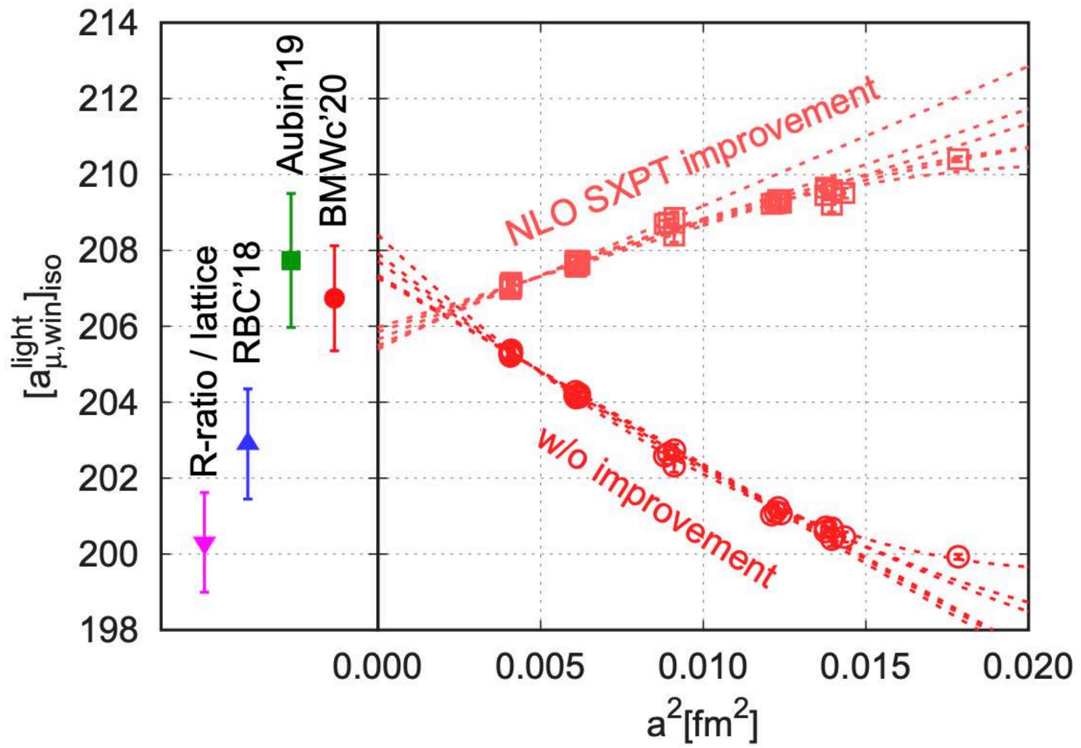


Discretization effects with O(a)-improved Wilson fermions several times larger than with (highly-improved / smeared) staggered fermions

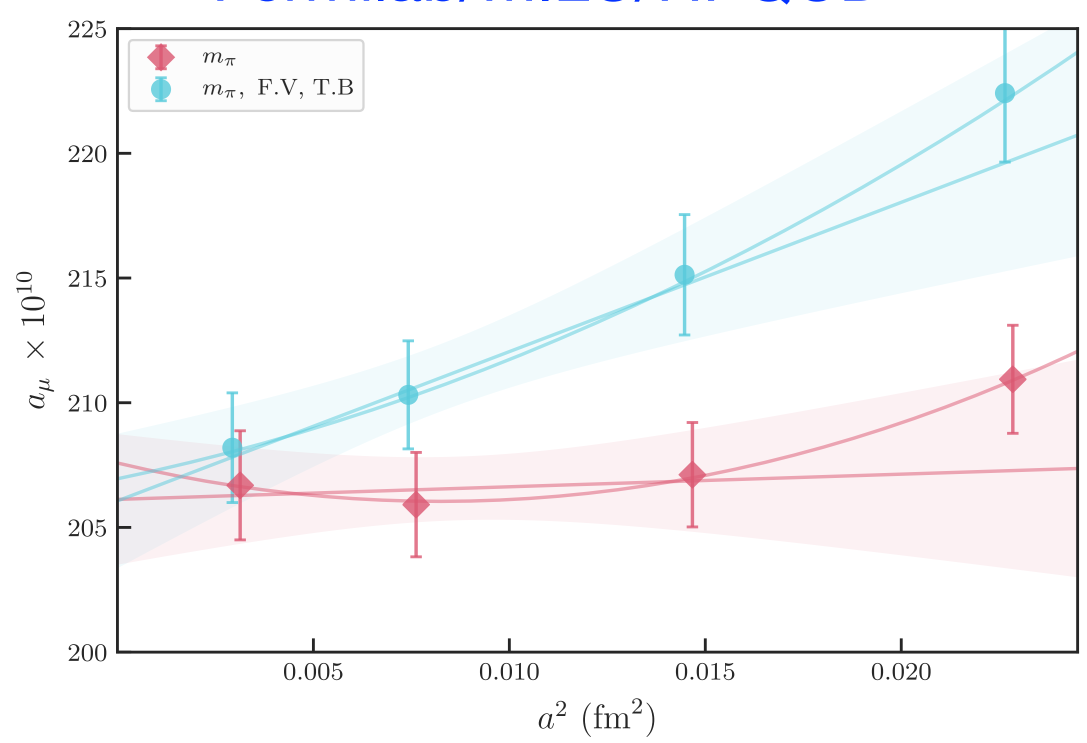
Taste-breaking in $a_\mu^{\rm win}(0.4$—$1\ {\rm fm})$

- Compare lattice-spacing dependence of Aubin *et al.*, BMW, and Fermilab/HPQCD/MILC, all of whom employ staggered fermions
- Because χPT is an effective theory of pions, do not expect it to describe vector-current correlator in Euclidean time range where ρ states are important

BMW



Fermilab/MILC/HPQCD



For BMW & FHM, slope of data is unimproved / worsened by inclusion of taste-breaking effects estimated in SχPT

Taste-breaking in $a_\mu^{\rm win}$(0.4—1 fm)

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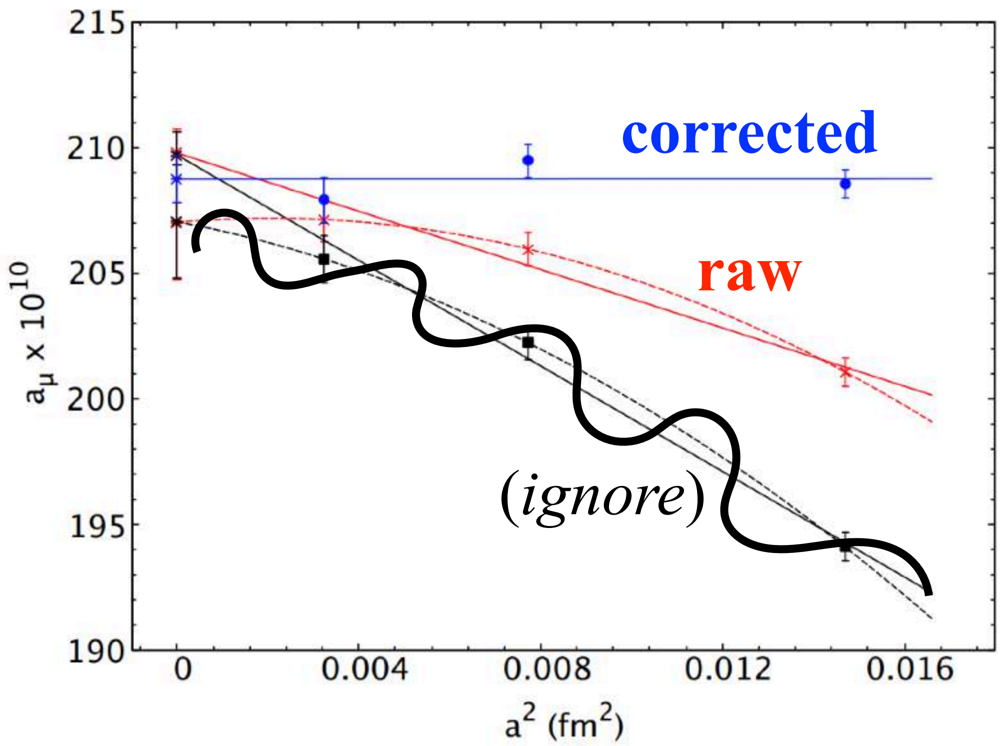
Aubin *et al.*



For Aubin et al., slope of data is reduced by including taste-breaking?

Taste-breaking in $a_\mu^{ll,\text{conn.}}$

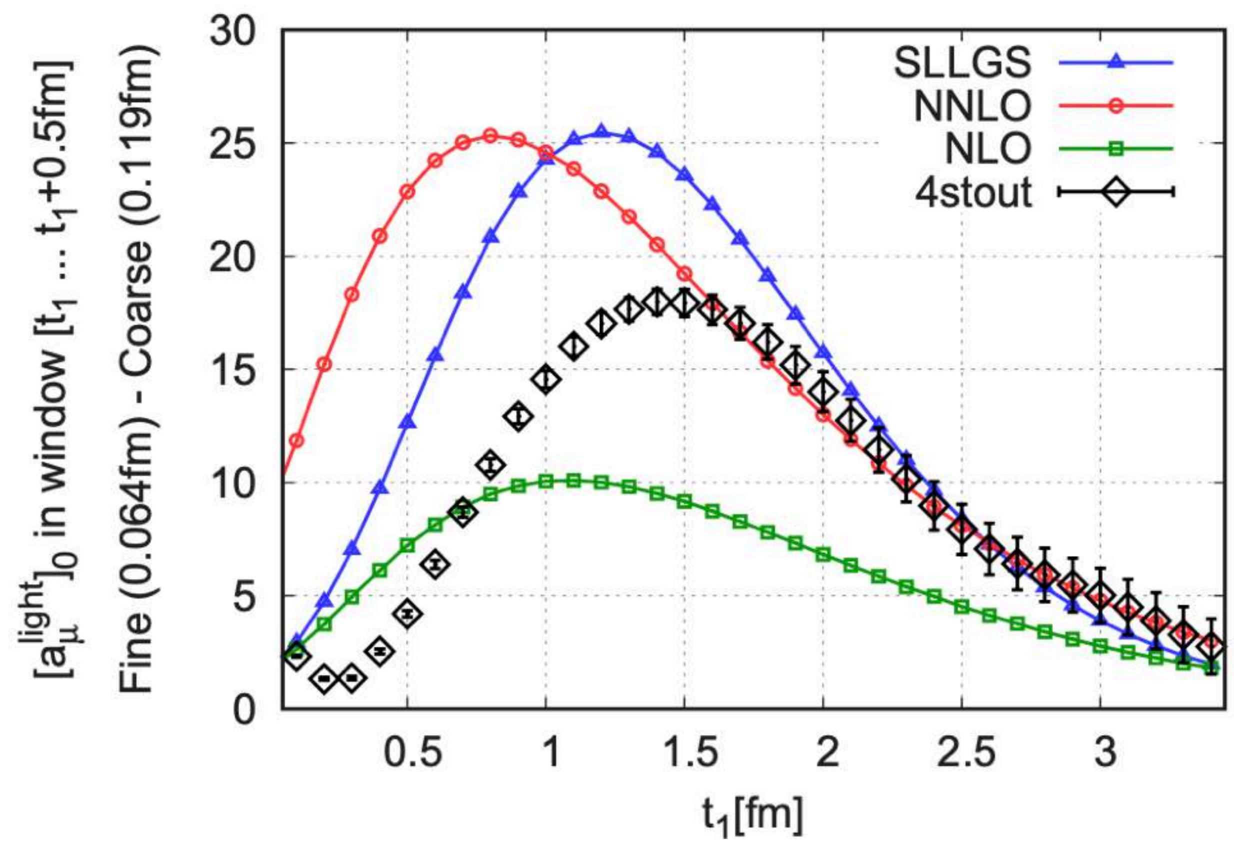
- Theoretical argument for using SχPT to estimate taste-breaking corrections is same as for using FVχPT to estimate finite-volume corrections:

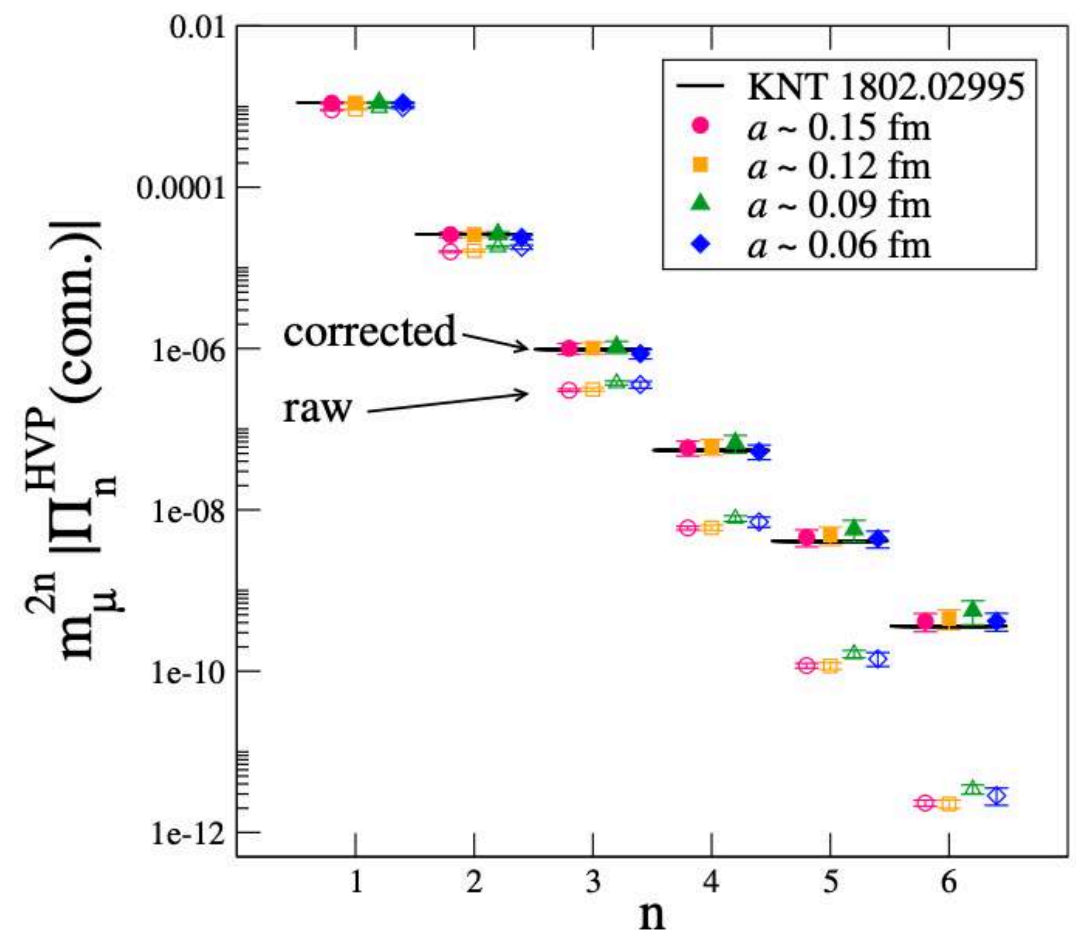
although pions do not comprise most of the value of a_μ^{ll}, they generate most of the finite-volume *and taste-breaking* effects

- *Ultimately, however, the proof is in the pudding...*

Taste-breaking in $a_\mu^{ll,\text{conn.}}$

- Expect (S)χPT to describe correlator at large Euclidean times — which is dominated by $\pi\pi$ states — reasonably well





- (*Left*) BMW observes that SχPT describes correlator differences well for t ≳ 2 fm
- (*Right*) FHM finds that Taylor coefficients corrected for finite volume and taste breaking with SχPT agree with determinations from R-ratio data

Taste-breaking in $a_\mu^{ll,\text{conn.}}$

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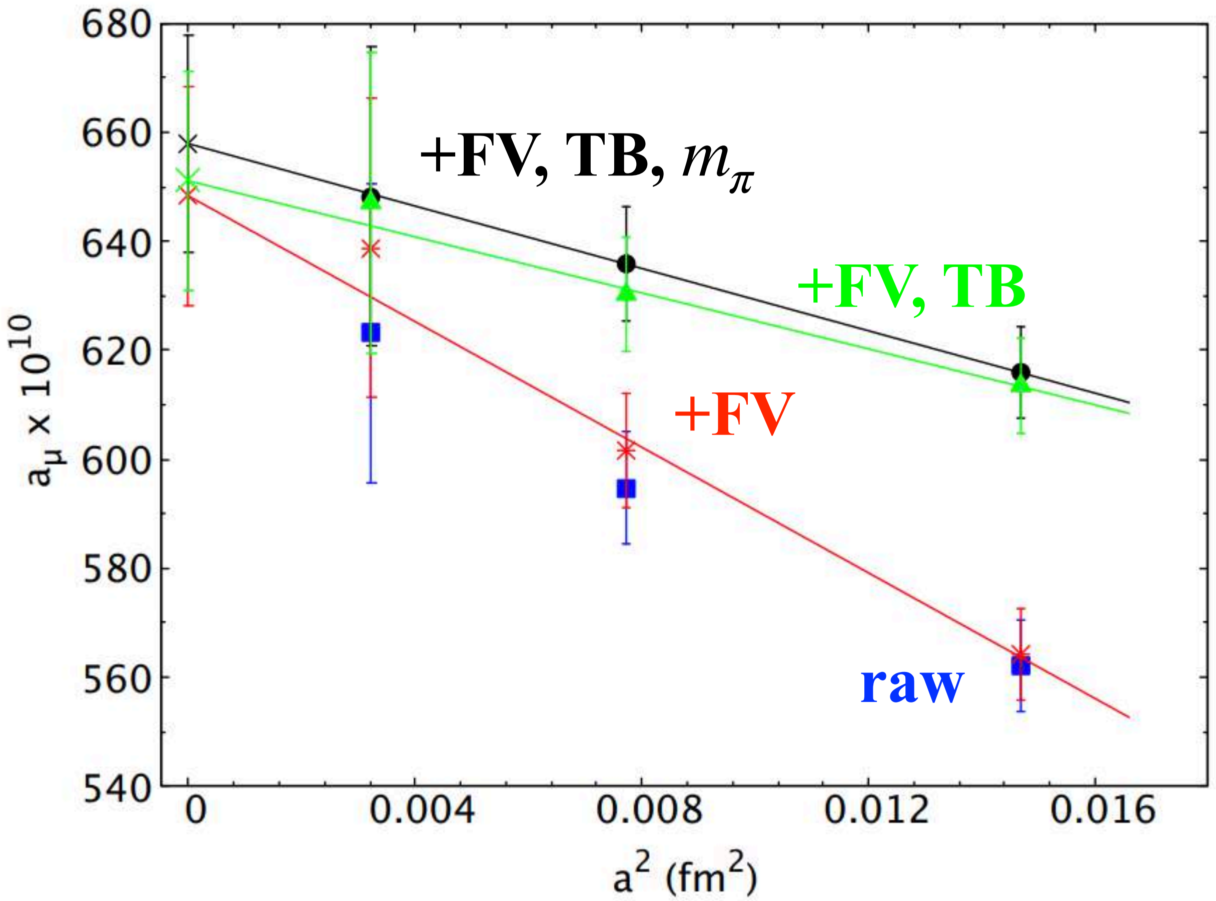
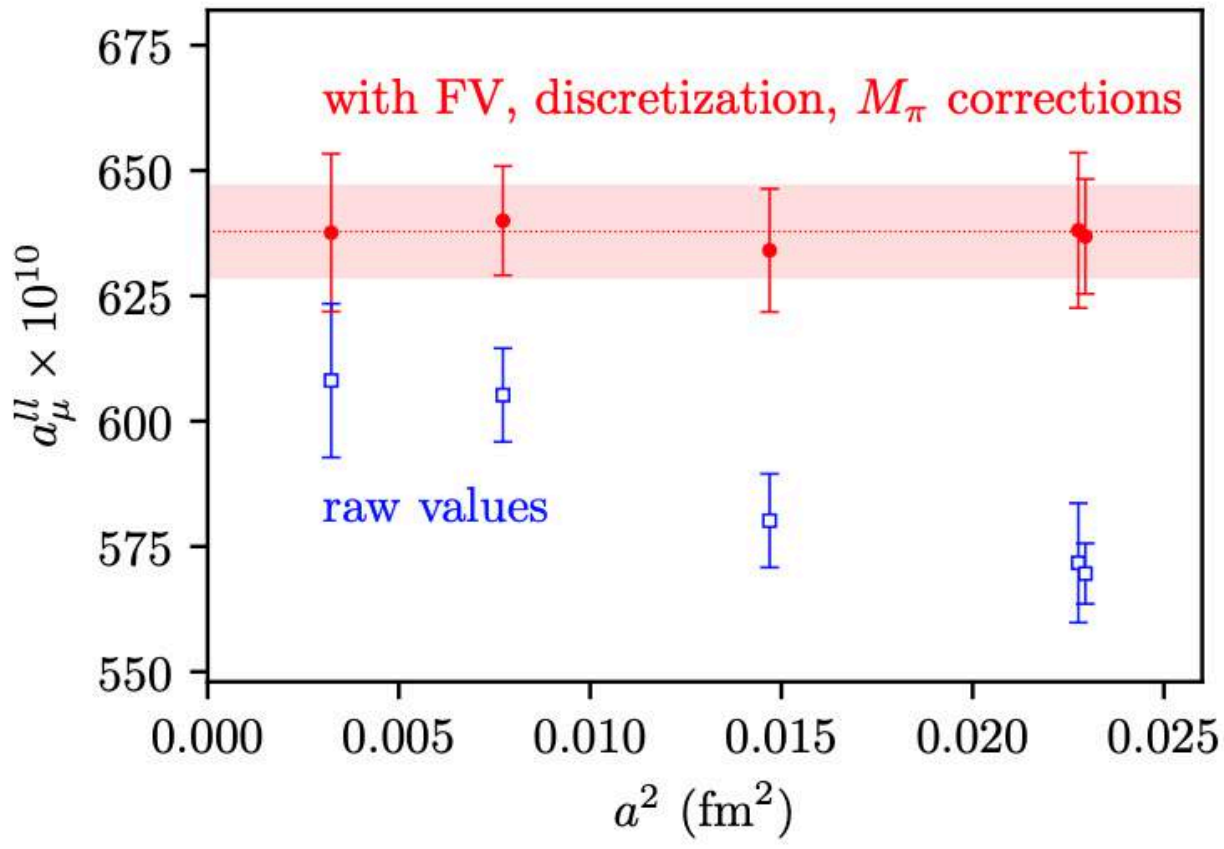


Fermilab/MILC/HPQCD



- Aubin et al., BMW, and FHM all observe that slope of $a_\mu^{ll,\text{conn.}}$ vs. a^2 is significantly reduced when taste-breaking corrections are included

Final thoughts

- Inclusion of taste-breaking discretization effects via SχPT **changes the approach to the continuum, but not the continuum-limit value**

- Because taste-breaking effects on a_μ^{ll} are primarily due to the altered pion spectrum, they happen to be calculable in effective field theory
  - ➡ *Allows lattice theorists to reduce distance from a=0 by applying corrections at finite lattice spacing before the continuum extrapolation*

- However, taste-breaking discretization effects need not be singled out: as any other discretization error, they are removed by taking the continuum limit

BMW