Isospin Breaking Corrections to the HVP

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THE UNIVERSITY *of* EDINBURGH

Isospin Breaking Corrections

- lattice calculations usually done in the isospin symmetric limit
- two sources of isospin breaking effects
  - different masses for up- and down quark (of $\mathcal{O}((m_d - m_u)/\Lambda_{\mathrm{QCD}})$)
  - Quarks have electrical charge (of $\mathcal{O}(\alpha)$)
- lattice calculation aiming at $\mathbf{1\%}$ precision requires to include isospin breaking
- separation of strong IB and QED effects requires renormalization scheme
- definition of "physical point" in a "QCD only world" also scheme dependent

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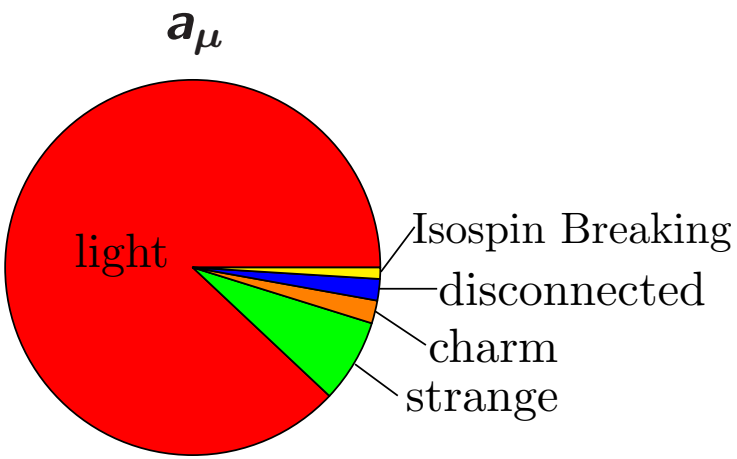
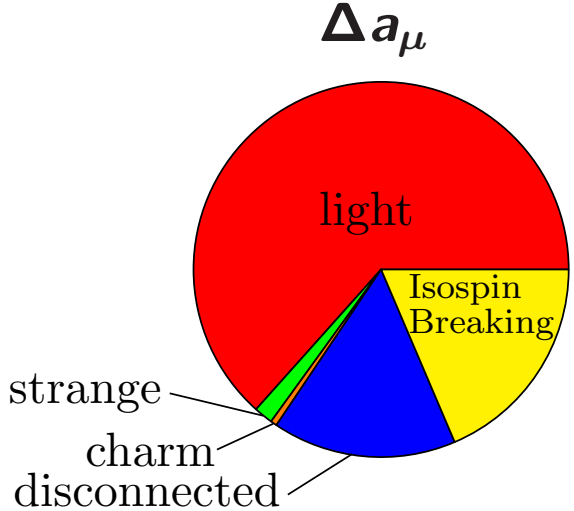
- IB contribution included in final lattice result from the WP [arXiv:2006.04822]





QED

- Euclidean path integral including QED

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[\Psi, \overline{\Psi}] \mathcal{D}[U] \mathcal{D}[A] \; O \; e^{-S_F[\Psi, \overline{\Psi}, U, A]} \, e^{-S_G[U]} \, e^{-S_\gamma[A]}$$

- photons in a box: finite volume corrections
  $\rightarrow 1/(m_\pi L)^3$ for QED corrections to HVP in QED$_L$
  [J. Bijnens *et al*, Phys. Rev. D 100, 014508 (2019)], [D.Giusti *et al*, JHEP 1710 (2017) 157]
- stochastic QED using $U(1)$ gauge configurations
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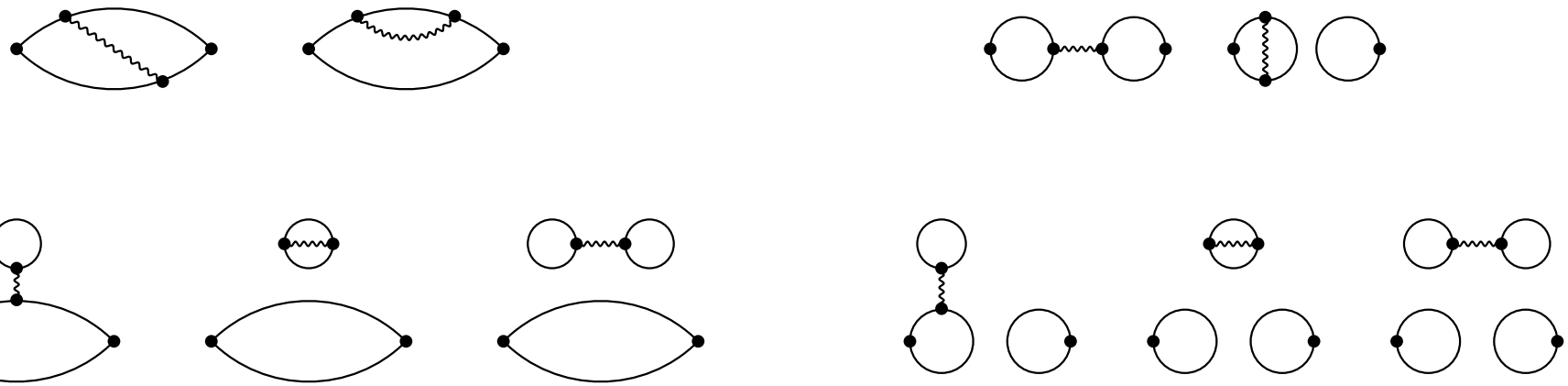

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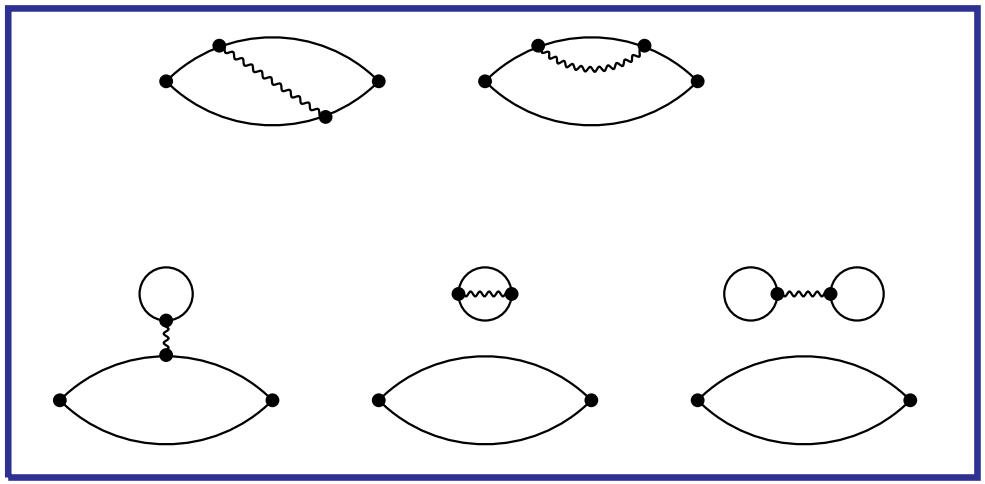
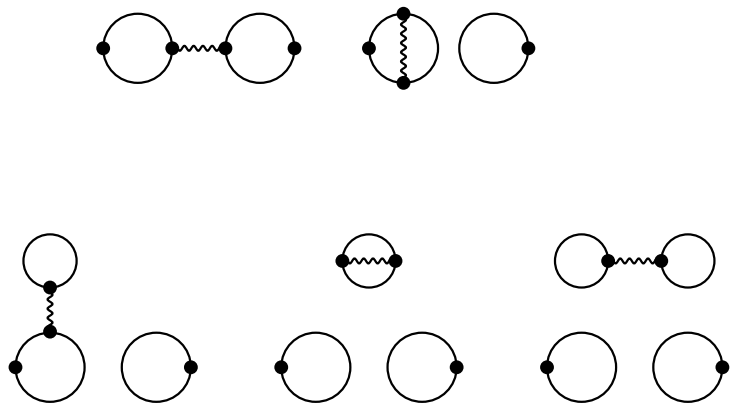


connected HVP



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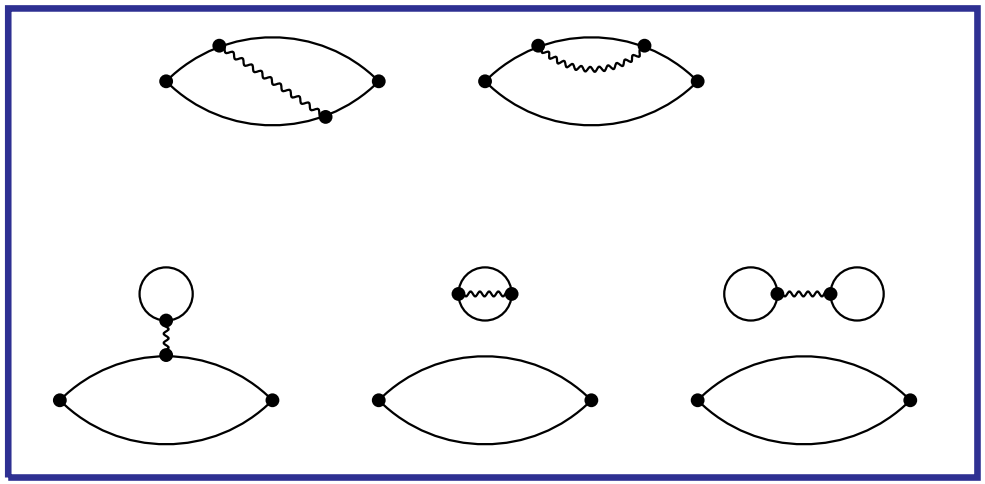

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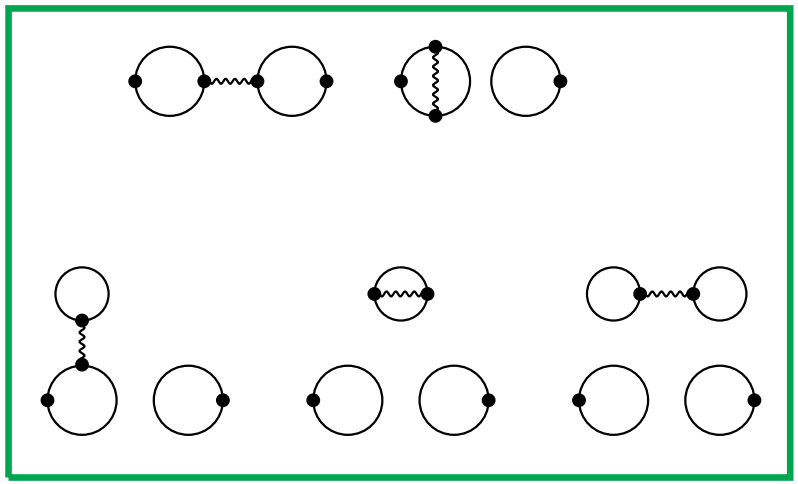

disconnected HVP

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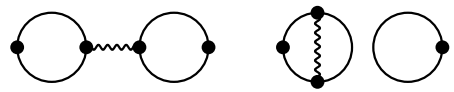

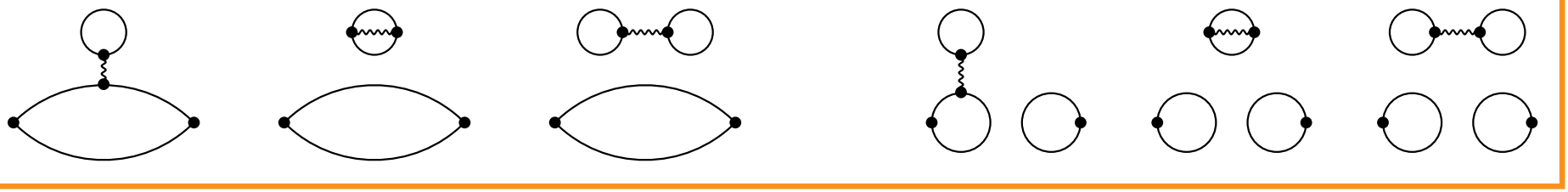

sea-quark effects

Strong isospin Corrections / Corrections due to quark-mass shifts

- Different up- and down-quark masses $\delta m = m_u - m_d$
- simulation using different up, down quark masses
- sea quark effects: $\longrightarrow$ configurations with different up, down masses
- perturbative expansion in the quark masses [G.M. de Divitiis *et al*, JHEP 1204 (2012) 124]

$$\langle O \rangle = \langle O \rangle_{m_i = \hat{m}_i} + \sum_i \Delta m_i \left. \frac{\partial}{\partial m_i} \langle O \rangle \right|_{m_i = \hat{m}} + \mathcal{O}\left(\Delta m^2\right)$$

with $\Delta m_i = (m_i - \hat{m}_i)$ and $\hat{m}_i$ quark mass in isospin symmetric "world"

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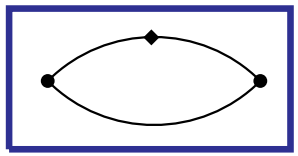

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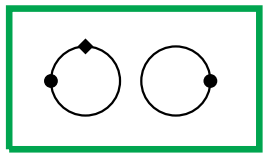

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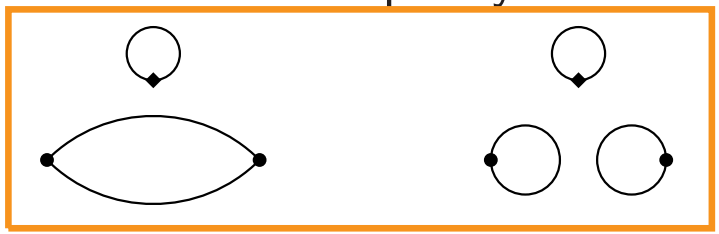

sea-quark effects
(zero at leading order IB)

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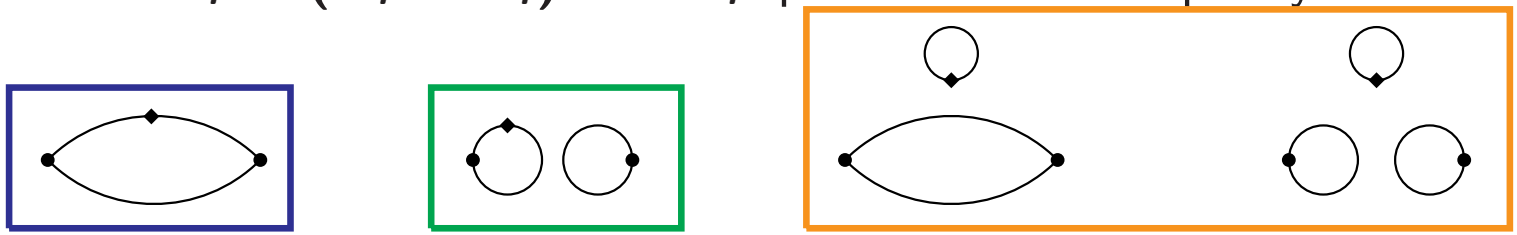

connected HVP | disconnected HVP | sea-quark effects (zero at leading order IB)

- same diagrams can be used to tune to physical point in QCD+QED, e.g.,

$$(m_u,\ m_d,\ m_s,\ \ldots) \longrightarrow (M_{\pi^+},\ M_{K^+},\ M_{K^0},\ \ldots)$$

(plus another mass to determine the scale)

Renormalization schemes

- separation of QED and strong IB breaking
- What is the "physical point" in a $\alpha = 0$, $m_u = m_d$ world?

$$X^{\phi} = \overline{X} + X^{SU(2)} + X^{\gamma} \qquad X^{\mathrm{QCD}} \equiv \overline{X} + X^{SU(2)}$$

observable at physical point

$m_u = m_d$, $\alpha = 0$ $SU(2)$-Breaking correction QED correction

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- define arbitrary unphysical values $\overline{\Pi}$ and Π^{QCD} for a set of quantities Π, e.g.,
  - GRS scheme: [J. Gasser *et al*, Eur. Phys. J. C32, 97]

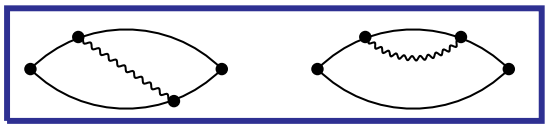
$$m_f^{\mathrm{QCD}}\left(\overline{MS}(2\ \mathrm{GeV})\right) = m_f^{\phi}\left(\overline{MS}(2\ \mathrm{GeV})\right)$$

  - Hadronic Schemes, e.g., [Sz. Borsanyi *et al*, Phys. Rev. Lett. 111 (25), 252001], [Sz. Borsanyi *et al*, 2002.12347]

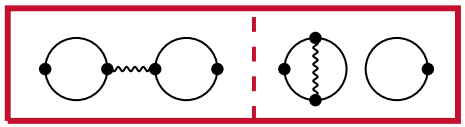
$$\Pi = \left(\left(\frac{1}{2}\left(M_{uu}^2 + M_{dd}^2\right)\right)^{1/2}, \left(M_{dd}^2 - M_{uu}^2\right)^{1/2}, M_{ss}\right) \cdot w_0 \qquad \Pi^{\phi} = \Pi^{\mathrm{QCD}}$$

  - other hadronic schemes, e.g., [M. Di Carlo *et al*, Phys. Rev. D100 (3), 034514], [S. Basak *et al*, Phys. Rev. D 99, 034503 (2019)]
- g-2 Theory WP: isospin symmetric ($m_u = m_d$, $\alpha = 0$) defined by $\overline{M}_{\pi} = M_{\pi^0}^{\mathrm{exp}}$

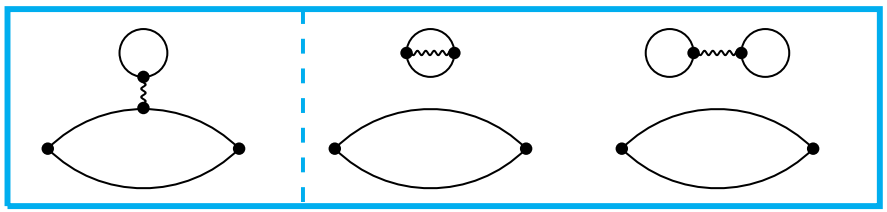
Overview of published results - contributions to $a_\mu \times 10^{10}$



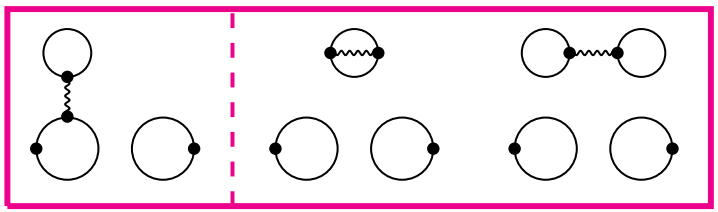
BMW $-1.27(40)(33)$
RBC/UKQCD $5.9(5.7)(1.7)$
ETM $1.1(1.0)$



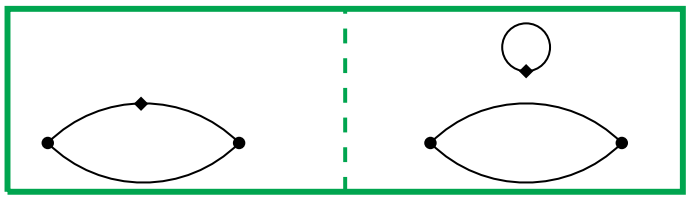
$-0.55(15)(11)$ BMW
$-6.9(2.1)(2.0)$ RBC/UKQCD



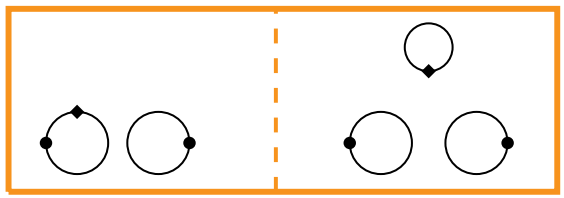
$-0.0095(86)(99)$ $0.42(20)(19)$ BMW



$0.011(24)(14)$ $-0.047(33)(23)$ BMW



$6.59(63)(53)$ BMW
$10.6(4.3)(6.8)$ RBC/UKQCD
$6.0(2.3)$ ETM
$7.7(3.7)$ $9.0(2.3)$ FHM
$9.0(0.8)(1.2)$ LM



$-4.63(54)(69)$ BMW

BMW [arXiv:2002.12347]
RBC/UKQCD [Phys.Rev.Lett. 121 (2018) 2, 022003]
ETM [Phys. Rev. D 99, 114502 (2019)]
FHM [Phys.Rev.Lett. 120 (2018) 15, 152001]
LM [Phys.Rev.D 101 (2020) 074515]

Summary and Outlook

- Summary
  - Inclusion of IB corrections important to reach sub-percent precision
  - "More, more precise ... IB calculations needed" [T. Blum Mon 15:10]

- Short Contributions
  - M. Krstić Marinković: HVP with C* boundary conditions
  - A. Risch: Leading isospin breaking effects in the anomalous magnetic moment of the muon (Status Mainz)

- possible discussion points
  - Disconnected HVP QED corrections (RBC/UKQCD vs BMW)
  - Consistency between results? Useful Crosschecks?
  - renormalization schemes
  - Scale setting with QED
  - Sea quark effects needed for per-mille precision on HVP?