HVP from Lattice QCD: Long distance statistical errors

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Virtual meeting,
November 19th, 2020

Large-time behavior in the time-momentum representation

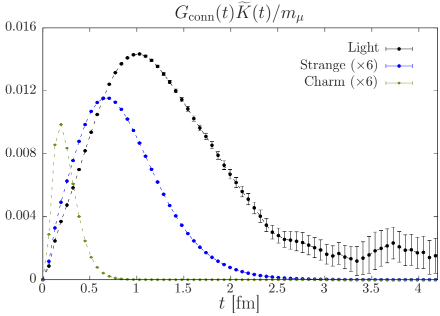
- Basic expression:

$$a_\mu^{HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}x_0 \, C(x_0) \, \tilde{f}(x_0)$$

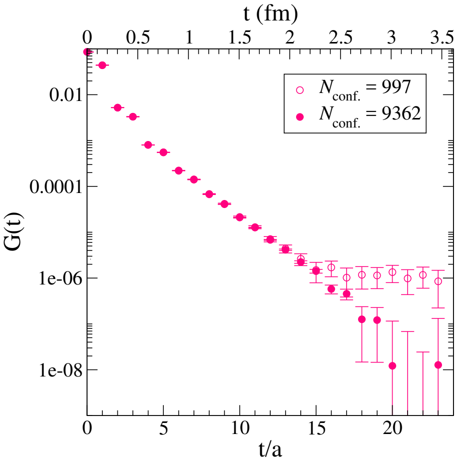
  with Kernel function $\tilde{f}(x_0)$.
- Challenge: Determine the large x_0 behavior of the integrand
  - Large-time behavior suffers from an exponential growth of the statistical uncertainty
  - Data in this region is affected by finite-volume and potentially by finite-time extent effects
- Consequences:
  - Direct correlator data is typically only used for $x_0 \leq x_0^{cut}$
  - Relying on data at largish time separations to model the behavior above the cut is dangerous.

Illustration of the challenge

Two recent examples from the literature:



A. Gerardin *et al.* PRD 100 014510 (2019)



Davies *et al.* PRD 101 034512 (2020)

Methods to address the uncertainty in the tail

There are a number of methods on the market

- Bounding method
- Improved bounding method(s)
- Multi-state fits starting from lower time-separations
- Dedicated spectroscopy studies
- Modeling/determination of the timelike pion form factors

Based on the spectral representation in a finite volume

$$C(x_0) = \sum_{n=1}^{\infty} A_n \mathrm{e}^{-E_n x_0}$$

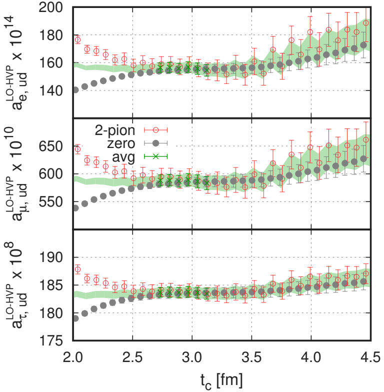
Bounding method

Suggested by BMW and RBC/UKQCD and used in other publications

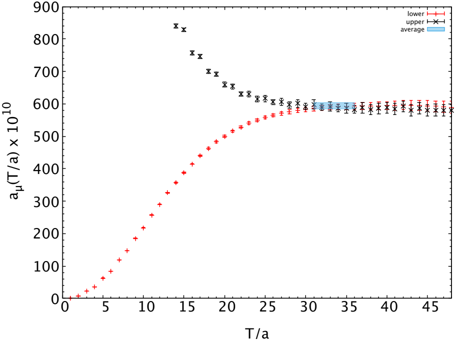
$$0 \le C_{ii}^{ud}(t) \le C_{ii}^{ud}(t_c)\frac{\varphi(t)}{\varphi(t_c)}$$

$$\text{with}\varphi(t) = \cosh\left[E_{2\pi}(T/2 - t)\right]$$

Borsanyi *et al.* PRL 121 022002 (2018)
Blum *et al.* PRL 121, 022003 (2018)
Borsanyi et al. arXiv:2002.12347



Borsanyi *et al.* PRL 121 022002



Aubin *et al.* PRD 101 014503 (2020)

Improved bounding & dedicated spectroscopy studies

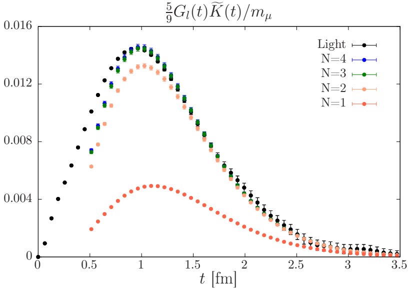
Version in the Mainz paper:

- Bounding applied to the subtracted correlator in the second equation

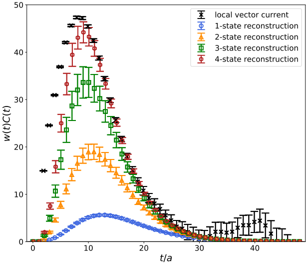
$$0 \le G(t_c)e^{-E_{\text{eff}}(t_c)(t-t_c)} \le G(t) \le G(t_c)e^{-E_N(t-t_c)}, \qquad t \ge t_c$$

$$\widetilde{G}(t) = G(t) - \sum_{n=0}^{N-1} \frac{Z_n^2}{2E_n} e^{-E_n t}$$

- obtain Z_n and E_n from a spectroscopy study in the rest frame



A. Gerardin *et al.* PRD 100 014510



Bruno *et al.* arXiv:1910.11745

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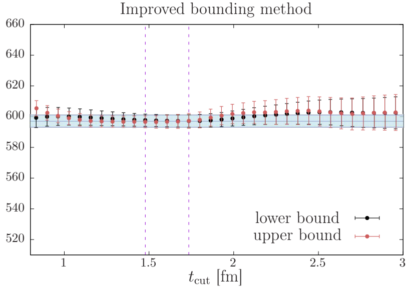

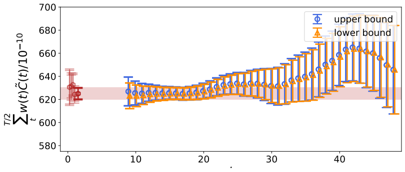

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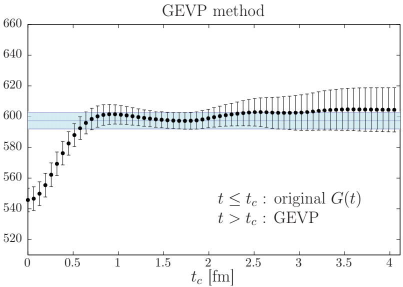

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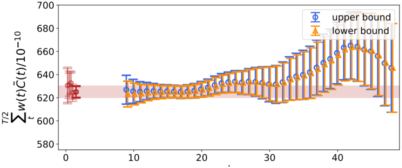

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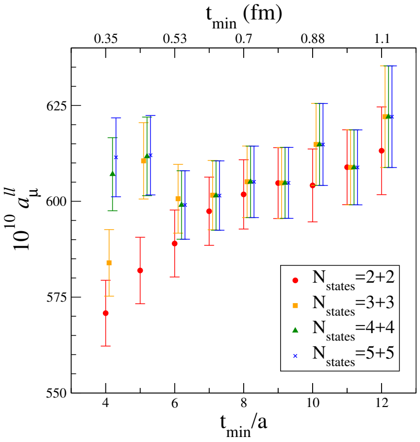
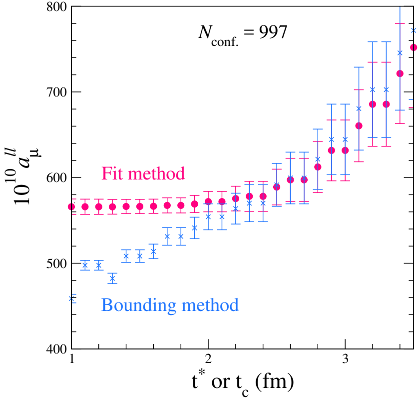
Multi-state fit from lower time-separations

The Fermilab Lattice, HPQCD and MILC collaborations use Bayesian multi-state fits of a 2×2 correlation matrix

Davies *et al.* PRD 101 034512 (2020)





The timelike pion form factor from LQCD

- Form factor in timelike region can be calculated as

H. B. Meyer, PRL 107,072002 (2011)

$$|(F_\pi)^{\mathbf{d}}_{\Lambda}(E)|^2 = G^{\mathbf{d}}_{\Lambda}(\gamma)\left(q(\phi^{\mathbf{d}}_{\Lambda})'(q) + k\frac{\partial\delta(k)}{\partial k}\right)\frac{3\pi E^2}{k^5}|A_\Psi|^2$$

where the phase shift $\delta(k)$ is calculated with the Lüscher method and $|A_\Psi|^2 = |\langle\Omega|J(t)|n\rangle|^2$ can be determined from a ratio of optimized two point functions

Andersen *et al.* NPB 939 145 (2019)

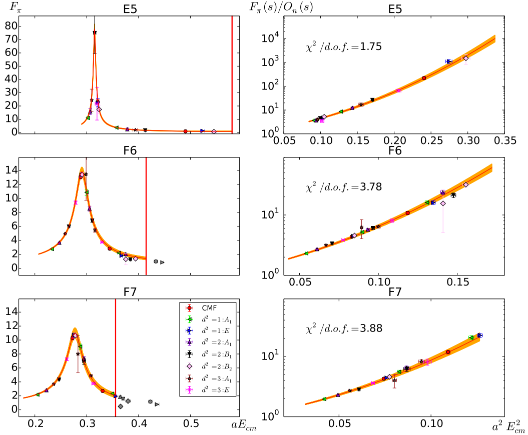
- The form factor has been calculated in recent years

Feng *et al.* PRD 91 054504 (2015)
Andersen *et al.* NPB 939 145 (2019)
F. Erben *et al.* PRD 101 054504 (2020)

Fits with an Omnés representation - three subtractions

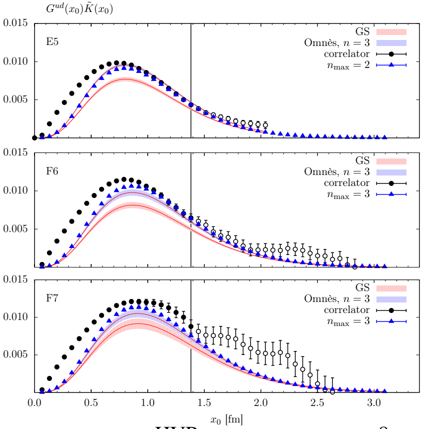
n-subtracted Omnès representation

$$F(t) = \exp\left(P_{n-1}(t)t + \frac{t^n}{\pi}\int_{4m_\pi^2}^{\infty} ds \frac{\delta_{11}(s)}{s^n(s-t-i\epsilon)}\right)$$



- Gounaris-Sakurai with the determined m_ρ and $g_{\rho\pi\pi}$ describes the data only roughly
- Omnés with three subtractions provides a good description
- Goodness of fit still not great

HVP – Infinite volume results from F_π



F. Erben, *et al.* PRD 101 054504 (2020)

- Below $4m_\pi$:

$$G^{ud}_{F_\pi}(x_0) = \frac{10}{9}\int_0^\infty d\omega\, \omega^2 \rho(\omega^2) e^{-\omega x_0}$$

$$\rho(s) = \frac{1}{48\pi^2}\left(1 - \frac{4m_\pi^2}{s}\right)^{\frac{3}{2}} |F_\pi(s)|^2$$

- In the paper we use multiple infinite-volume reconstructions of the correlator tail
- We compare finite and infinite-volume reconstructions

Values for a_μ^{HVP} in units of 10^{-8}:

	$m_\pi = 437$ MeV	$m_\pi = 311$ MeV	$m_\pi = 265$ MeV
FV correction, $n = 2$	0.043(12)	−0.032(31)	−0.001(50)
FV correction, $n = 3$	0.029(13)	−0.003(31)	0.028(54)
Mainz $g - 2$ paper		0.03	0.07

Discussing Gilberto's question from yesterday

Question: Should we do rescaling with m_ρ?

- Indeed the statistical uncertainty on the resonance mass is small (for example 0.3%-0.8% at quite moderate statistics)
  See `Andersen et al. NPB 939 145 (2019)` for low statistics CLS data
- The uncertainty is likely systematics-dominated and current spectroscopy studies do not (fully) quantify this uncertainty
- At physical pion mass, a rigorous study would only use data below the 4 pion threshold $\rightarrow$ potential loss of control over the resonance mass or model dependence
- Is the experimental value really as accurate as the PDG averages suggest?
- Possible alternative mentioned by Ben Hörz: What about the $K^*(892)$?