Comparison with e^+e^- data

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Muon $g-2$ Theory Initiative virtual workshop

“The hadronic vacuum polarization from lattice QCD at high precision”

HVP: time-like and space-like master formulae

- **Time-like formulation**: for $e^+e^- \to$ hadrons data Bouchiat, Michel 1961, Brodsky, de Rafael 1968

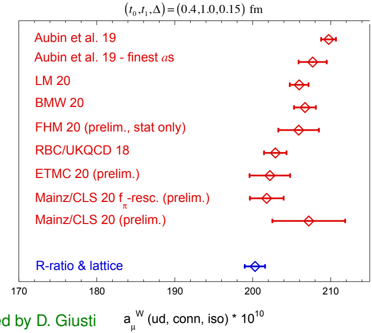
$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{\text{thr}}}^{\infty} ds \frac{\hat{K}(s)}{s^2} R_{\text{had}}(s) \qquad R_{\text{had}}(s) = \frac{3s}{4\pi\alpha^2}\sigma(e^+e^- \to \text{hadrons})$$

- **Space-like formulation**: for lattice QCD Blum 2003

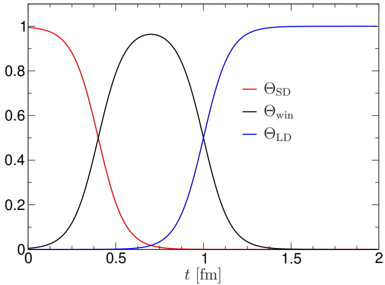
$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{\infty} ds\, f(s)\hat{\Pi}(-s) \qquad \hat{\Pi}(s) = 4\pi^2\Big[\Pi(s) - \Pi(0)\Big]$$

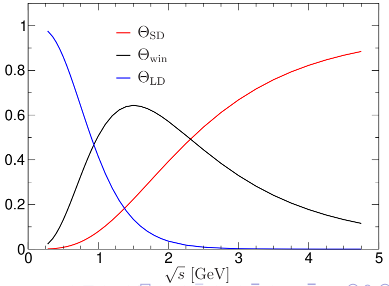
- Both are equivalent, but not obvious how to best compare beyond $a_\mu^{\text{HVP, LO}}$

Windows in Euclidean time



Compiled by D. Giusti





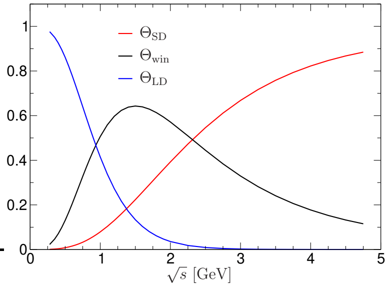
- Windows defined in **Euclidean time**

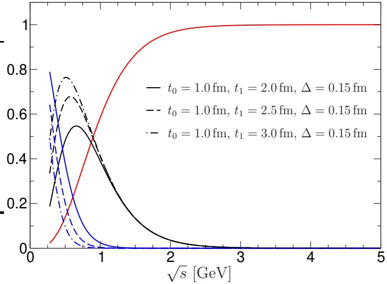
  $\hookrightarrow$ $t_0 = 0.4\,\mathrm{fm}$, $t_1 = 1.0\,\mathrm{fm}$, $\Delta = 0.15\,\mathrm{fm}$
- Less clear separation in $\sqrt{s}$

  $\hookrightarrow$ long tail of window part
- Windows for connected *ud* only or for the full thing?

- Standard window not necessarily best suited to help with KLOE/BaBar tension

	percentage captured of $\pi\pi$ channel $\leq$ 1 GeV		
window	SD	intermediate	LD
$[0.4, 1.0]$ fm	3	28	69
$[1.0, 2.0]$ fm	31	51	18
$[1.0, 2.5]$ fm	31	61	9
$[1.0, 3.0]$ fm	31	65	4





Tension between BMWc and e^+e^- data

	e^+e^- from WP	lattice average from WP	BMWc v2
$a_\mu^{\text{HVP, LO}} \times 10^{11}$	6 931(40)	7 116(184)	7 087(53)
difference to e^+e^-		$+1.0\sigma$	$+2.3\sigma$

- e^+e^- error accounts for tensions among data sets Talk by M. Davier

  $\hookrightarrow$ lots of discussion at previous meetings

- Change in HVP affects other quantities

  $\hookrightarrow$ (most notably) hadronic running of α Passera, Marciano, Sirlin 2008

Hadronic running of α and global EW fit

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on BMWc
$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)\times 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

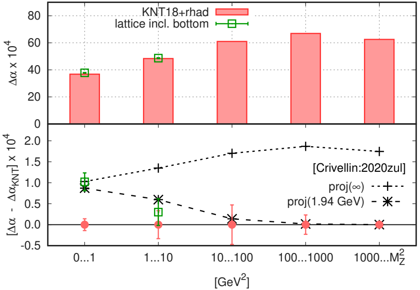
- **Time-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)=\frac{\alpha M_Z^2}{3\pi}P\int_{s_{\text{thr}}}^{\infty}ds\frac{R_{\text{had}}(s)}{s(M_Z^2-s)}$$

- **Space-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)=\frac{\alpha}{\pi}\hat{\Pi}(-M_Z^2)+\frac{\alpha}{\pi}\big(\hat{\Pi}(M_Z^2)-\hat{\Pi}(-M_Z^2)\big)$$

- Global EW fit
  - Difference between HEPFit and GFitter implementation mainly treatment of M_W
  - Pull goes into **opposite direction**



BMWc 2020

More in talks by M. Passera, B. Malaescu (phenomenology) and K. Miura, T. San José (lattice)

Changing HVP at low energies

- BMWc results for $\Delta\alpha_{\text{had}}$ suggest that the change needs to come from low energies

  $\hookrightarrow$ $\pi\pi$ channel

- $a_\mu^{\text{HVP, LO}}$ and $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$ expressed in terms of **pion vector form factor**

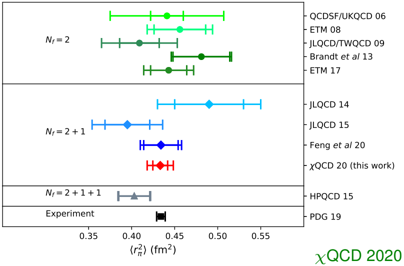
$$R_{\text{had}}(s) = \frac{1}{4}\bigg(1-\frac{4M_\pi^2}{s}\bigg)^{3/2}\big|F_\pi^V(s)\big|^2$$

- **Pion charge radius**

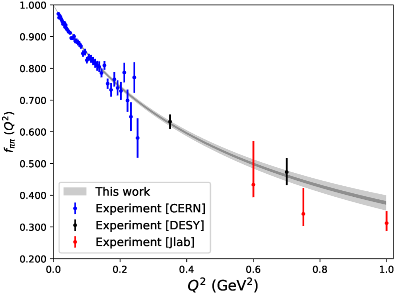
$$\langle r_\pi^2\rangle = \frac{6}{\pi}\int\limits_{4M_\pi^2}^{\infty} ds\frac{\text{Im}\,F_\pi^V(s)}{s^2}$$

  $\hookrightarrow$ can also be calculated on the lattice

More in talk by P. Stoffer



χQCD 2020



Other comparison quantities

- Other possible comparison quantities
  - Moments

$$\Pi_n = \frac{(-1)^n}{n!}\frac{\partial^n}{\partial s^n}\Pi(s)\bigg|_{s=0} = \frac{(-1)^n}{\pi}\int_{s_{\text{thr}}}^{\infty} ds \frac{\text{Im}\,\Pi(s)}{s^{n+1}}$$

  - Muon–electron ratio Talk by D. Giusti

$$R_{e/\mu} = \left(\frac{m_\mu}{m_e}\right)^2 \frac{a_e^{\text{HVP, LO}}}{a_\mu^{\text{HVP, LO}}}$$

- Both provide different energy weighting, but no additional (experimental/lattice) benchmarks
- 2.7σ tension in $R_{e/\mu}$ currently almost identical to the 2.5σ one in a_e
  $\hookrightarrow \Delta a_e^{\text{exp}} \sim 20\Delta a_e^{\text{HVP, LO}}$