Correcting finite-size effects in the HVP contribution to the muon $g-2$ using large-volume lattice QCD simulations

Laurent Lellouch

CPT Marseille
CNRS & Aix-Marseille U.

Budapest-Marseille-Wuppertal collaboration [BMWc]

Borsanyi, Fodor, Guenther, Hoelbling, Katz, LL, Lippert, Szabo, Parato, Stokes, Toth, Torok, Varnhorst

arXiv:2002.12347v2

Finite-volume corrections: strategy

Early NLO estimate of these e^{-LM_π} effects (Aubin et al '16): $\sim 2\%$ on a_μ ($\equiv 10^{10} \times a_\mu^{\text{LO-HVP}}$) in our $L = 6$ simulations

$\rightarrow$ Large compared to target error and corrections to estimate could be $O(50\%)$

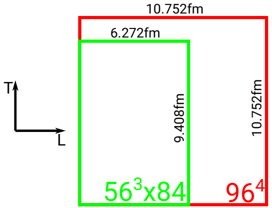
$\rightarrow$ Must control to better than 10%

FV correction in two steps

$L_{\text{ref}} = 6.272\,\text{fm} \quad T_{\text{ref}} = \frac{3}{2} L_{\text{ref}}$

$L_{\text{big}} = T_{\text{big}} = 10.752\,\text{fm}$

$$a_\mu(\infty,\infty) - a_\mu(L_{\text{ref}}, T_{\text{ref}}) =$$
$$[a_\mu(L_{\text{big}}, T_{\text{big}}) - a_\mu(L_{\text{ref}}, T_{\text{ref}})]_{\texttt{4HEX}}$$
$$+[a_\mu(\infty,\infty) - a_\mu(L_{\text{big}}, T_{\text{big}})]_{\text{NNLO}}$$



$\rightarrow$ Perform dedicated lattice study (see also PACS '19)

FV corrections: dedicated lattice study

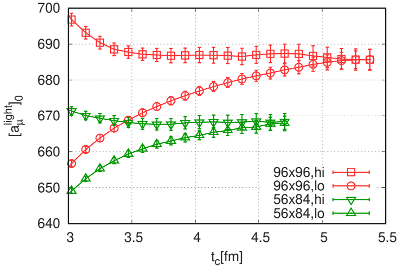
1) Compute $a_\mu(L_{\rm big}, T_{\rm big}) - a_\mu(L_{\rm ref}, T_{\rm ref})$ w/ staggered simulations

- Taste violations: average M_π in pion loops is larger than physical
  - $\Rightarrow$ FV effects of Goldstone pion are averaged w/ smaller ones of more massive partners
  - $\Rightarrow$ difficult to compute FV effects of Goldstone w/ staggered fermions
- Need very large L: can only be done w/ coarse lattices
  - $\Rightarrow$ FV corrections of Goldstone contribution even more difficult to isolate

$\Rightarrow$ designed action w/ small taste splittings

- DBW2 gauge action
- 4 steps of HEX smearing
- $\beta = 0.73,\ a = 0.112\,\text{fm}$
- $M_\pi = 104\,\text{MeV}$ and $M_\pi = 121\,\text{MeV}$
- Interpolate to $M_\pi = 110\,\text{MeV}$

$$\longrightarrow M^{-2}_{\pi,\rm HMS} \equiv \frac{1}{16}\sum_\alpha M^{-2}_{\pi,\alpha} = M^{-2}_{\pi^0,\text{phys}}$$

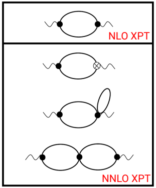


$$a_\mu(L_{\rm big}, T_{\rm big}) - a_\mu(L_{\rm ref}, T_{\rm ref}) = 18.1(2.0)_{\rm stat}(1.4)_{\rm cont}$$

FV: non-lattice approaches

FV effects on a_μ are dominated by long-distance $\pi\pi$ effects
$\Rightarrow$ accessible to χPT and hadronic models

- NLO and NNLO χPT (Gasser *et al* '85, Bijnens *et al* '99, Aubin *et al* '19)
- MLLGS: Meyer/Mainz application of Lüscher + Lellouch formalisms using Gounaris-Sakurai to model I=1 HVP (Meyer '11, Francis '13)
- HP: QFT relation to Compton scattering (Hansen *etl* '19-'20)



	lattice	NLO	NNLO	LLGS	HP
$a_\mu(L_{big}, T_{big}) - a_\mu(L_{ref}, T_{ref})$	18.1(2.0)(1.4)	11.6	15.7	17.8	16.3+0.4

2) Model validation $\Rightarrow$ compute $a_\mu(\infty,\infty) - a_\mu(L_{big}, T_{big})$ using NNLO χPT

	NLO	NNLO	HP
$a_\mu(\infty,\infty) - a_\mu(L_{big}, T_{big})$	0.3	0.6	1.4-0.8

3) Combine

$$a_\mu(\infty,\infty) - a_\mu(L_{ref}, T_{ref}) = 18.7(2.0)_{stat}(1.4)_{cont}(0.3)_{big}(0.6)_{I=0}(0.1)_{qed}[2.5]$$