Taste improvement of the HVP contribution to the muon g-2 à la BMWc

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Budapest-Marseille-Wuppertal collaboration [BMWc]

Borsanyi, Fodor, Guenther, Hoelbling, Katz, LL, Lippert, Szabo, Parato, Stokes, Toth, Torok, Varnhorst

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Taste improvement: how to reduce cutoff effects

SχPT: Bernard, Golterman, Sharpe, Van de Water, Aubin, Blum, Bailey, ...
Application to HVP: HPQCD, Aubin *et al*, ...

- Leading cutoff effect in staggered: pion tower that shrinks as $a \to 0$
  $a = 0.119\,\mathrm{fm}: \quad 135 - 200 - 250 - 290 - 330\,\mathrm{MeV}$
  $a = 0.064\,\mathrm{fm}: \quad 135 - 145 - 155 - 165 - 175\,\mathrm{MeV}$
- For quanties that do not vanish in χ-limit (e.g ρ mass), this effect is far smaller (in relative terms)

  Discretization effects are mostly taste effects, and these are mainly mass-difference effects

  For large Euclidean times t and for $M_\pi \lesssim 300\,\mathrm{MeV}$ these effects can be described in SχPT or SMLLGS
- The question then becomes: starting at asymptotically large t how far down in t do SχPT/SMLLGS remain applicable
- Can answer this question directly by comparing SχPT/SMLLGS prediction for discretization errors to difference of lattice results from fine and coarse simulations

Taste improvement: how to reduce cutoff effects

Consider two independent improvements

1. NNLO SχPT (Bijnens, Colangelo, Ecker, Talavera, Lee, Sharpe, Van de Water, Bailey, Aubin *et al.*, . . .)

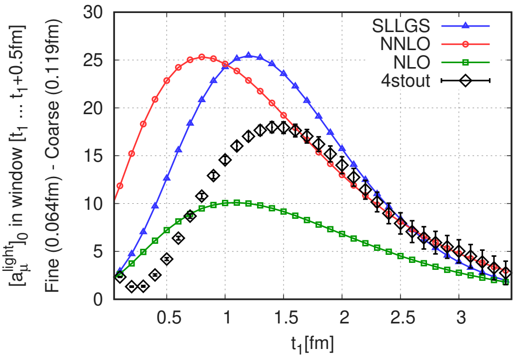
   $\longrightarrow$ depends only on 2 LECs (F & l_6) and masses of pion tastes

   $\longrightarrow$ compute NNLO taste corrections (BMWc '20)

2. SMLLGS (Meyer '11, Francis *et al.* '13, Lüscher, Lellouch, . . .)

   $\longrightarrow$ apply taste averaging to MLLGS (BMWc '20)

Taste improvement: direct comparison to lattice results



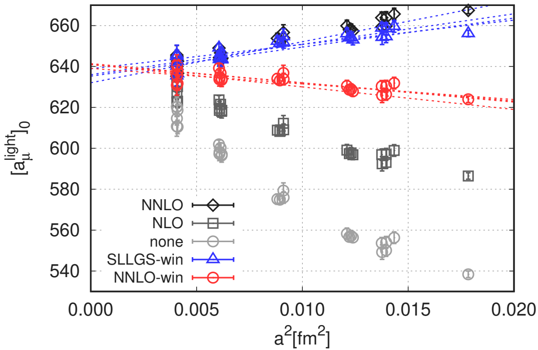
- All analytical descriptions work at large $t_1 \sim 3.5\,\mathrm{fm}$
- As t_1 is reduced:
  - NLO SχPT fails rapidly: it does not have any information about the ρ
  - SMLLGS includes ρ but has only 8 states: works down to $t_1 \sim 2\,\mathrm{fm}$
  - NNLO SχPT knows about ρ through l_6 and has 2-pion tower: works down to $t_1 \sim 1.5\,\mathrm{fm}$
  - All analytical descriptions capture at least some of the taste breaking

Taste improvement: conclusions

- Not dealing with the total value of a_μ but just with a_μ in the long-distance region, $t \gtrsim 1.5\,\mathrm{fm}$
- Not even dealing with total value of a_μ at long distances

  $\longrightarrow$ merely with its lattice-spacing dependence $\simeq$ differences in a_μ due differences in pion mass
- Expect this procedure to describe most of the physics behind taste violations: confirmed by our lattice results
- Remaining small effects are taken care of by a linear plus quadratic extrapolation in a^2

Taste improvement: illustrative continuum exptrap.



- xxx-win → only apply improvements to long-distance window
- Improves approach to continuum limit → reduces uncertainties
- Does NOT modify this limit: all corrections vanish for $a \to 0$
- Systematics: cuts on a; SχPT vs SMLLGS; vary window boundaries

Taste-breaking effects from χPT explained

TOY MODEL SHOWN HERE ONLY FOR ILLUSTRATION: NOT USED IN BMWc ANALYSIS

$$C(t) = \frac{1}{3}\sum_{i=1}^{3} C_{ii}(t), \qquad C_{\mu\nu}(t)\int_{\vec{x}}\langle j_\mu(x) j_\nu(0)\rangle, \qquad j_\mu = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)$$

Toy model for $C(t)$: free 2π states $+$ a stable ρ (ρ dominance)

$$\begin{aligned} C(t, M_\pi^2) &\xrightarrow{|t|\gg 1/M_{\rho'}} C_{\text{LO-}\chi\text{PT}}(t, M_\pi^2) + C_\rho(t, M_\pi^2) \\ &= \underbrace{\frac{1}{3}\int_{\vec{p}} \frac{\vec{p}^2}{E_p^2}\exp\{-2E_p|t|\}}_{\chi\text{-logs}} + \frac{G_\rho^2 M_\rho^3}{2}\exp\{-M_\rho|t|\}, \qquad E_\pi = \sqrt{\vec{p}^2 + M_\pi^2} \end{aligned}$$

Consider $M_\pi^2 = M_{\pi,\phi}^2 + \Delta M_\pi^2$ w/ $\Delta M_\pi^2 \sim M_{\pi,\phi}^2$, so

$$M_\rho^2 = M_{\rho,\phi}^2\left[1 + C\frac{\Delta M_\pi^2}{M_{\rho,\phi}^2} + \cdots\right] \qquad \text{and} \qquad G_\rho^2 = G_{\rho,\phi}^2\left[1 + D\frac{\Delta M_\pi^2}{\Lambda^2} + \cdots\right]$$

Taste-breaking effects from χPT explained

Thus

$$
\begin{aligned}
\Delta_{M_\pi^2} C(t) &\equiv C(t, M^2_{\pi,\phi} + \Delta M_\pi^2) - C(t, M^2_{\pi\phi}) \\
&\overset{|t|\gg 1/M_{\rho'}}{\longrightarrow} \underbrace{C_{\text{NLO-}\chi\text{PT}}(t, M^2_{\pi,\phi} + \Delta M_\pi^2) - C_{\text{NLO-}\chi\text{PT}}(t, M^2_{\pi,\phi})}_{\equiv \Delta_{M_\pi^2} C(t)|_{\text{NLO-}\chi\text{PT}}\ :\ \text{leading effect; contains } \chi\text{-logs; computed in } \chi\text{PT}} \\
&\quad + \Delta M_\pi^2 \left(\frac{E}{\Lambda^2} + \frac{3}{2} \frac{D}{M^2_{\rho,\phi}} - \frac{D|t|}{M_{\rho,\phi}} \right) C_\rho(t, M^2_{\pi,\phi}) + O(\Delta M_\pi^4)
\end{aligned}
$$

Now, for staggered fermions, leading discretization effet is

$$\Delta M_\pi^2 \sim a^2 \Delta$$

w/ $\sqrt{a^2\Delta} \sim 60 \div 300\,\mathrm{MeV}$ for $a \sim 0.064 \div 0.119\,\mathrm{fm}$

Thus, SχPT corrected lattice result (i.e. with leading taste violations removed)

$$C_{\text{lat}}(t, a^2) - \Delta_{a^2\Delta} C(t)|_{\text{NLO-}\chi\text{PT}} \overset{|t|\gg 1/M_{\rho'}}{\longrightarrow} a^2\Delta \left(\frac{E}{\Lambda^2} + \frac{3}{2} \frac{D}{M^2_{\rho,\phi}} - \frac{D|t|}{M_{\rho,\phi}} \right) C_\rho(t, M^2_{\pi,\phi}) + O(a^4)$$