Lattice comparisons

Laurent Lellouch

CPT Marseille
CNRS & Aix-Marseille U.

Outline

$$a_\mu \equiv 10^{10} \times a_\mu^{\text{LO-HVP}} = \sum_{t/a \geq 0} K(tm_\mu) C(t)$$

$$C(t) = \frac{1}{3} \sum_{i=1}^{3} C_{ii}(t), \qquad C_{\mu\nu}(t) = \int_{\vec{x}} \langle j_\mu(x) j_\nu(0) \rangle$$

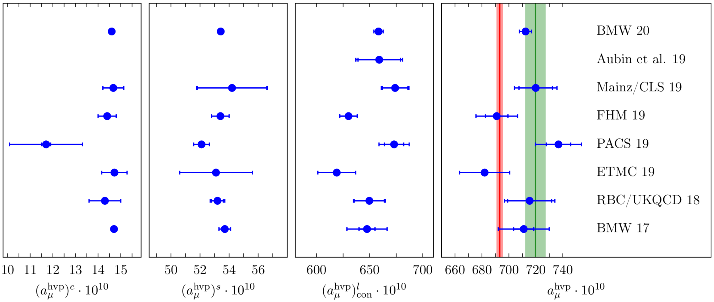
$$j_\mu = \frac{2}{3} \bar{u}\gamma_\mu u - \frac{1}{3} \bar{d}\gamma_\mu d - \frac{1}{3} \bar{s}\gamma_\mu s + \frac{2}{3} \bar{c}\gamma_\mu c + \cdots$$

What to compare?

- The obvious: a_μ, its flavor and isospin-breaking contributions
- Physically motivated quantities: moments of $C(t)$
- Lattice motivated quantities: Euclidean time windows of a_μ
- What else?

The obvious

$$a_\mu = a_\mu^{ud} + a_\mu^{s} + a_\mu^{c} + a_\mu^{\rm disc} + a_\mu^{\rm SIB} + a_\mu^{\rm QED}$$



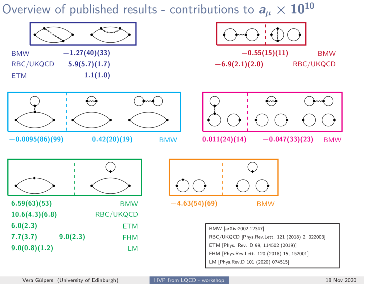
(From A. Gérardin's talk)

- Good agreement on s and c contributions: completely lattice-data driven
- Larger spread on ud contribution: $625 - 675$ [$\pm 4\%$]
- Carries over to total a_μ: $690 - 720$ [$\pm 4\%$] (740 (PACS '19) due to poorly controlled continuum extrapolation?)

The obvious

$$a_\mu = a_\mu^{ud} + a_\mu^{s} + a_\mu^{c} + a_\mu^{\rm disc} + a_\mu^{\rm SIB} + a_\mu^{\rm QED}$$

- $a_\mu^{\rm disc}$: see H. Meyer's talk
  - Typical contribution to a_μ is $\sim [2.0 \pm 0.4]\%$ of a_μ
  - Now have good agreement between collaborations: $a_\mu^{\rm disc} \in\sim [-15.0, -11.2]$
  - Have sufficient precision for now
- $a_\mu^{\rm SIB} + a_\mu^{\rm QED}$: see V. Gülper's talk

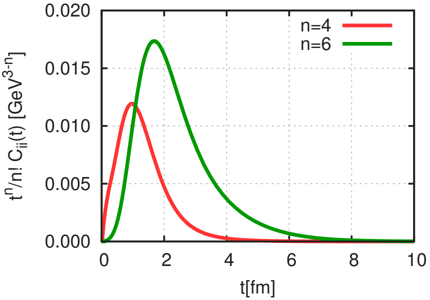


- Scheme depedent, but schemes used are consistent, at least for u and d
- Typically $< 1\%$ of a_μ
- Agreement not bad given that these are first generation calculations
- Caution: large cancellations ; total $a_\mu^{\rm SIB} + a_\mu^{\rm QED} \sim 0.001 a_\mu$
- Caution: SIB contributions have large FV corrections which cancel in sum (Lehner & Meyer '20)

Moments

$$\Pi_n \equiv \frac{1}{n!}\left(\frac{\partial}{\partial Q^2}\right)^n \Pi(Q^2) = \frac{(-1)^{n+1}}{(2n+2)!}\left[\sum_{t/a=1}^{T/(2a)-1} 2t^{2n+2}C(t) + \left(\frac{T}{2}\right)^{2n+2} C\left(\frac{T}{2}\right)\right]$$

- Apply to different flavor components
- Interesting fundamental properties of HVP
- Can be determined precisely from R-ratio
- $[\Pi_n] = \mathrm{GeV}^{-2n}$: higher n have growing scale-setting systematic
- Probe larger and larger distances as n increases



Moments

- Have $a_\mu = \sum_{t/a} K(tm_\mu)C(t)$ w/

$$K(tm_\mu) \xrightarrow{t\ll 1/m_\mu} \frac{(tm_\mu)^4}{9} \qquad \text{and} \qquad K(tm_\mu) \xrightarrow{t\gg 1/m_\mu} \frac{(tm_\mu)^2}{2}$$

- Also have (Bell & De Rafael '69)

$$a_\mu \lesssim \frac{4}{3}\alpha^2 m_\mu^2 \times \Pi_1 \sim 800$$

where result for a_μ/m_μ^2 is exact for $m_\mu \to 0$

- Even Π_1 probes larger distances than a_μ ...

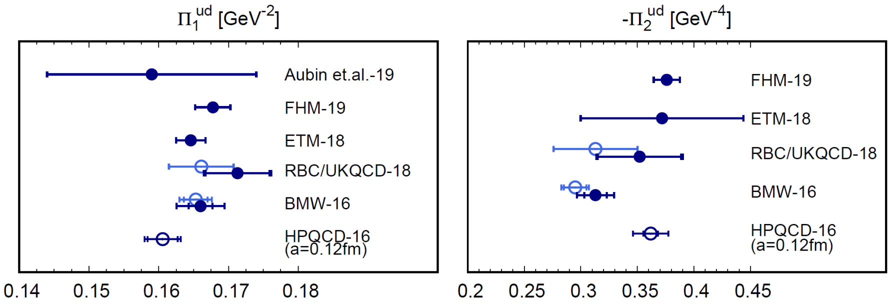
- In a Padé description of $\hat{\Pi}(Q^2)$, h.o.t. act to cancel off excess in tail contribution of $C(t)$ to Π_1 compared to a_μ

$\Rightarrow$ In comparison/tests of lattice calculations for a_μ, not much point in going beyond Π_1, perhaps Π_2

$\longrightarrow$ beyond, probing distances which are not relevant for a_μ and moments become more and more model dependent

Does everyone agree?

Moments



- Good agreement in Π_1, less good in Π_2
- Given same Π_1, larger $|\Pi_2|$ correlates w/ larger a_μ
- $\Pi_2/(\Pi_1)^2$ is dimensionless (i.e. has small dependence on scale setting) and is an interesting measure of HVP shape

Windows

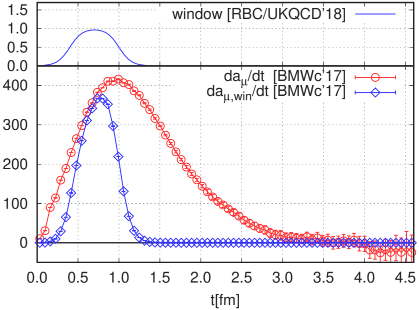
$$a_\mu^{\rm win}(t_0,t_1,\Delta) \equiv \sum_{t/a\geq 0} W(t;t_0,t_1,\Delta)K(tm_\mu)C(t)$$

$$W(t;t_0,t_1,\Delta) \equiv \Theta(t;t_0,\Delta) - \Theta(t;t_1,\Delta), \qquad \Theta(t;t_0,\Delta) \equiv \frac{1}{2}\left[1+\tanh\left(\frac{t-t_0}{\Delta}\right)\right]$$

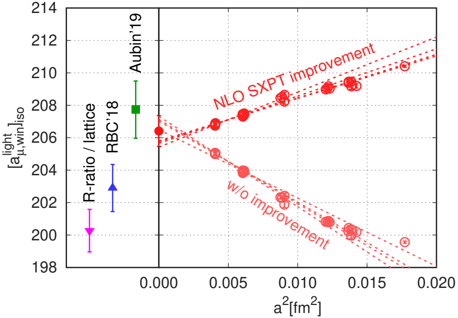
- Introduced by RBC/UQCD '18
- Physics and lattice systematic errors very different for different $[t_0,\ t_1]$
  $\Rightarrow$ windows allow to pick this apart
- Windows also computable w/ R-ratio
- Region with minimal lattice systematics: $0.4 \gtrsim t \gtrsim 1\ \mathrm{fm}$
  $\rightarrow$ we should all agree here!
  $\rightarrow$ should be able to compare to R-ratio to better than $5‰$ very soon
- At larger t, windows can be used to test impact of different assumptions made about long-distances behvior of $C(t)$
- Easier to consider windowed Π_1 than $a_\mu^{\rm win}$
- $a_\mu^{\rm win}(t_0,t_1,\Delta)$ depends on scale setting

Particularly clean:

$W(t; 0.4\,\mathrm{fm}, 1.0\,\mathrm{fm}, 0.15\,\mathrm{fm})$



$(144 \times 96^3,\ a \sim 0.064\,\mathrm{fm},\ M_\pi \sim 135\,\mathrm{MeV})$



- Agreement could be better
- New preliminary numbers presented during workshop, shown in summary talk
- What windows should we agree on?