Workshop summary and concluding remarks

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thanks to
A. El-Khadra, C. Lehner, D. Giusti, M. Golterman, V. Gülpers, L. Lellouch, C. McNeile, K. Szabo,…

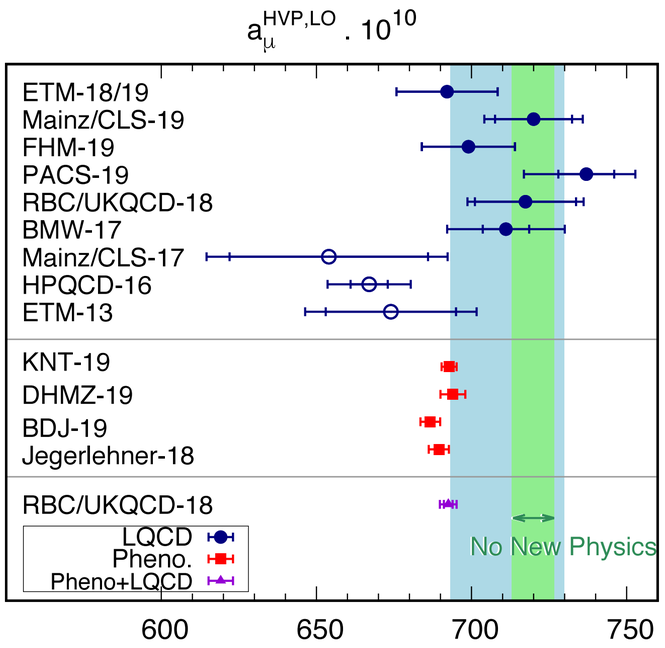
The hadronic vacuum polarization from lattice QCD at high precision
Topical Workshop — Muon g – 2 Theory Initiative
16 – 20 November 2020

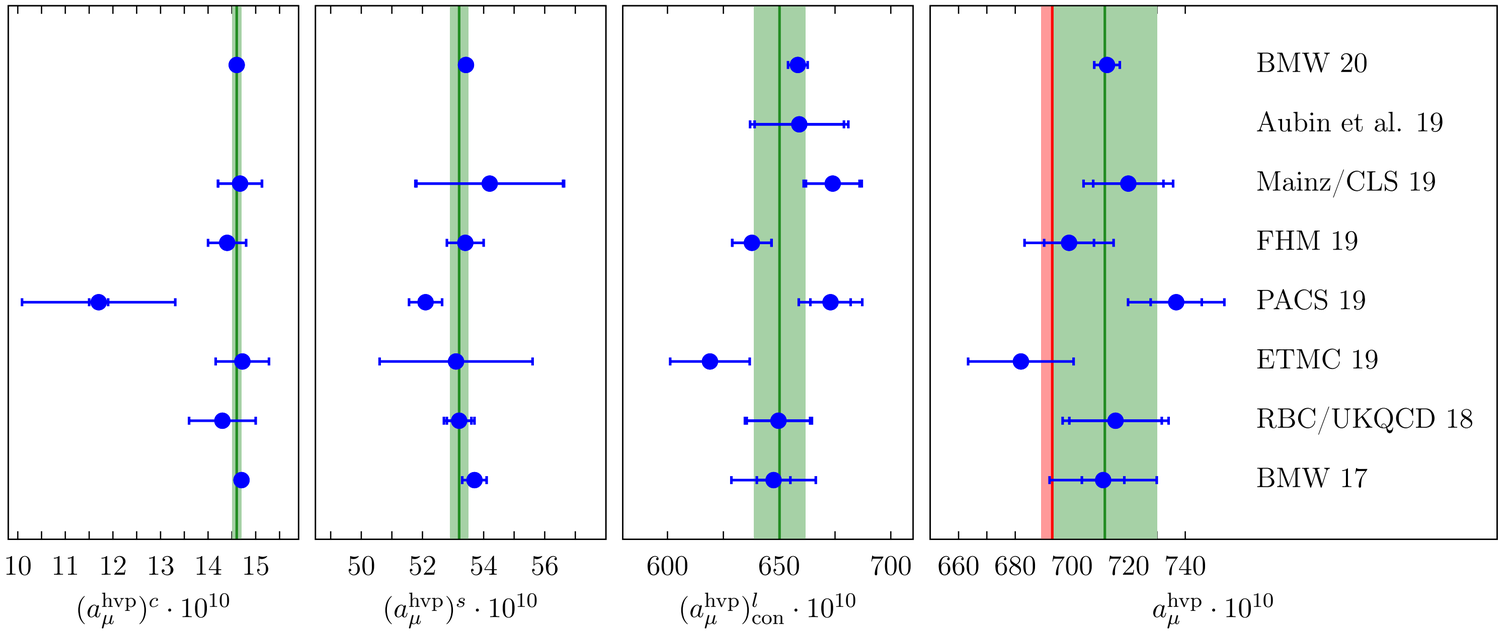
HIM
Helmholtz-Institut Mainz

PRISMA+

SFB 1044
THE LOW-ENERGY FRONTIER
OF THE STANDARD MODEL

The White Paper and its aftermath





Lattice QCD: $a_\mu^{\mathrm{hvp,LO}} = (711.6 \pm 18.4) \cdot 10^{-10}$

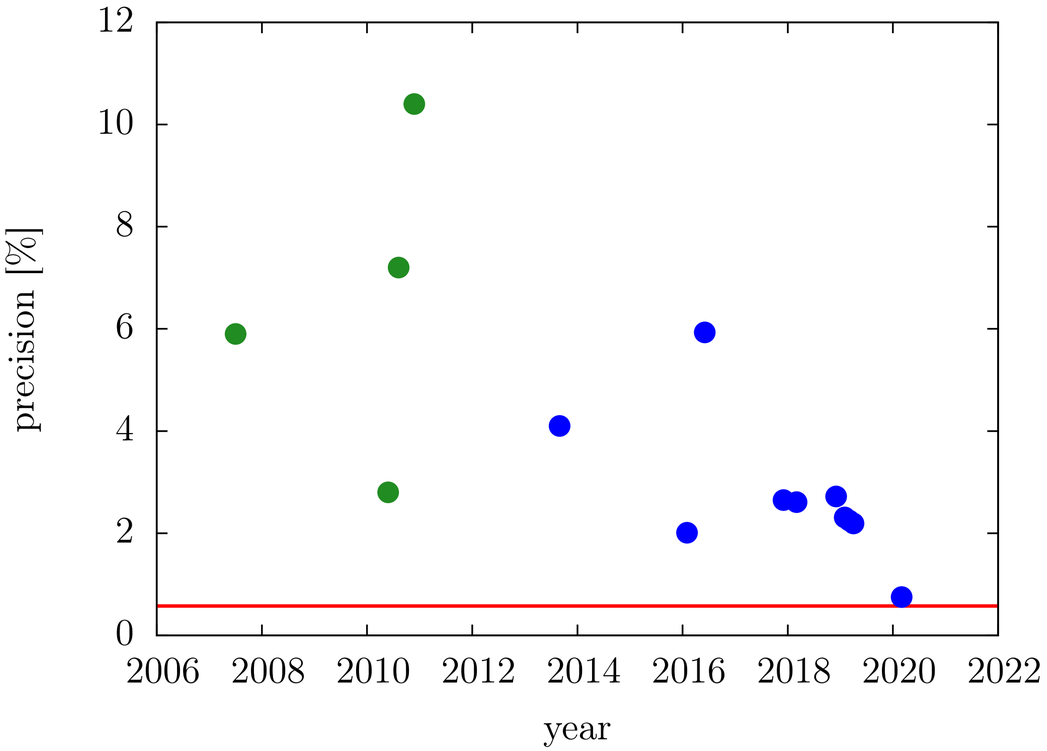
R-ratio: $a_\mu^{\mathrm{hvp,LO}} = (693.1 \pm 4.0) \cdot 10^{-10}$

BMW Collab.: $a_\mu^{\mathrm{hvp,LO}} = (708.7 \pm 2.8 \pm 4.5) \cdot 10^{-10}$

(0.75% uncertainty,
2.3σ tension with R-ratio)

[Borsanyi et al., arXiv:2002.12347]

Precision Issues



BMW: *[Borsanyi et al., arXiv:2002.12347]*

- First lattice calculation to achieve sub-percent precision
- Needs confirmation: other discretisations and methods

This workshop:

- Discuss how to improve control over systematic errors
- Consistency of different lattice calculations
- Comparison with R-ratio
- Benchmark quantities

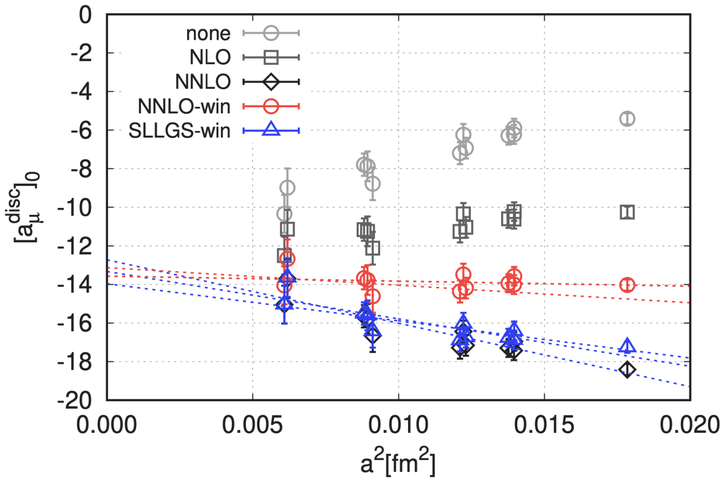
Systematic effects are sizeable or dominant

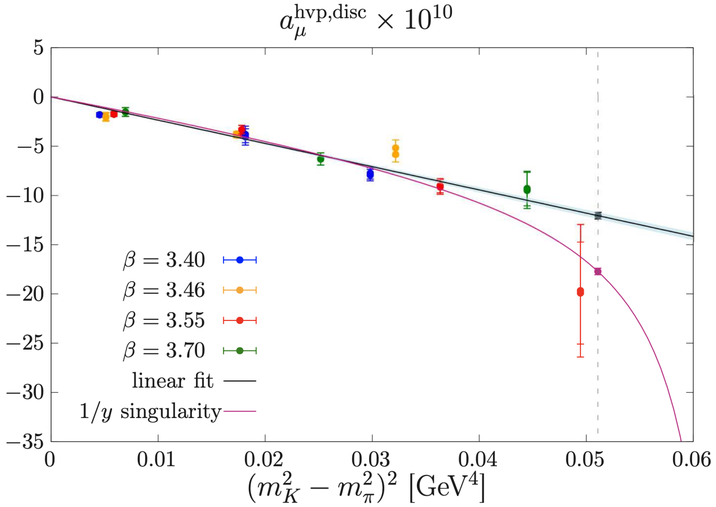
Workshop sessions

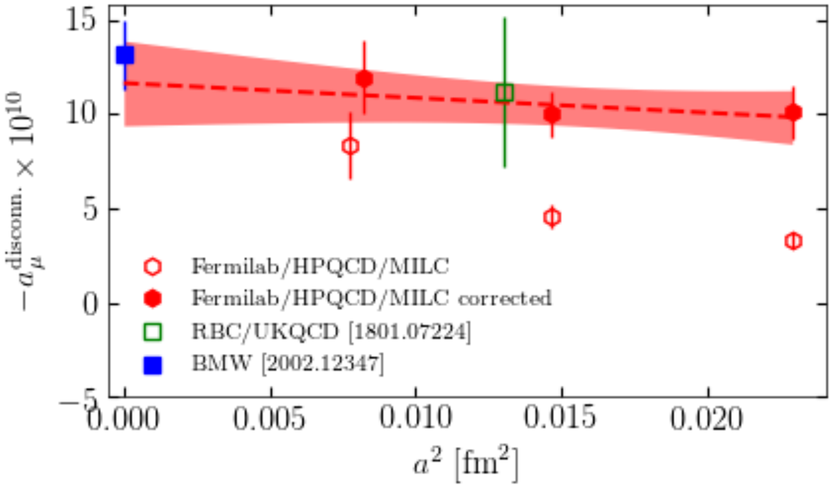
* **Quark-disconnected contributions** (H.B. Meyer)
* **Discretisation effects and continuum extrapolation** (R. Van de Water)
* **Scale setting** (K. Szabo)
* **Isospin breaking effects** (V. Gülpers)
* **Finite-size effects** (M. Golterman)
* **Long-distance regime of the vector correlator** (D. Mohler)
* **Lattice crosschecks** (L. Lellouch)
* **Comparison with e^+e^- / R-ratio data** (M. Hoferichter)

Quark-disconnected contributions

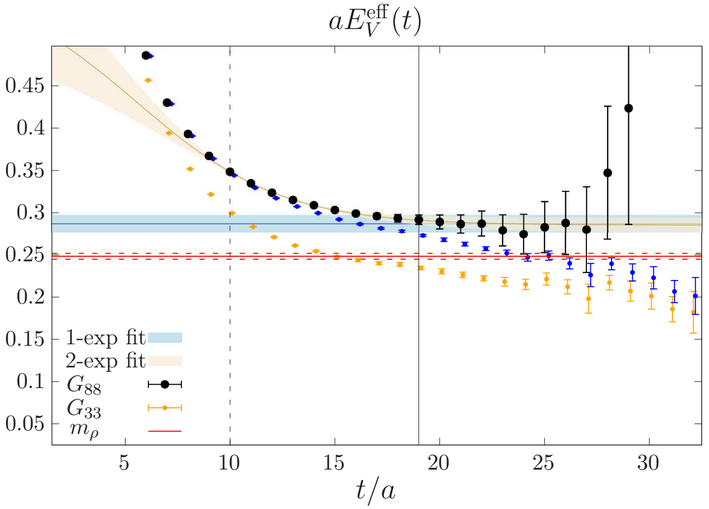
Success story of lattice QCD calculations!







Disconnected contribution do what they are supposed to do!



Quark-disconnected contributions

Convergence towards precise & consistent results

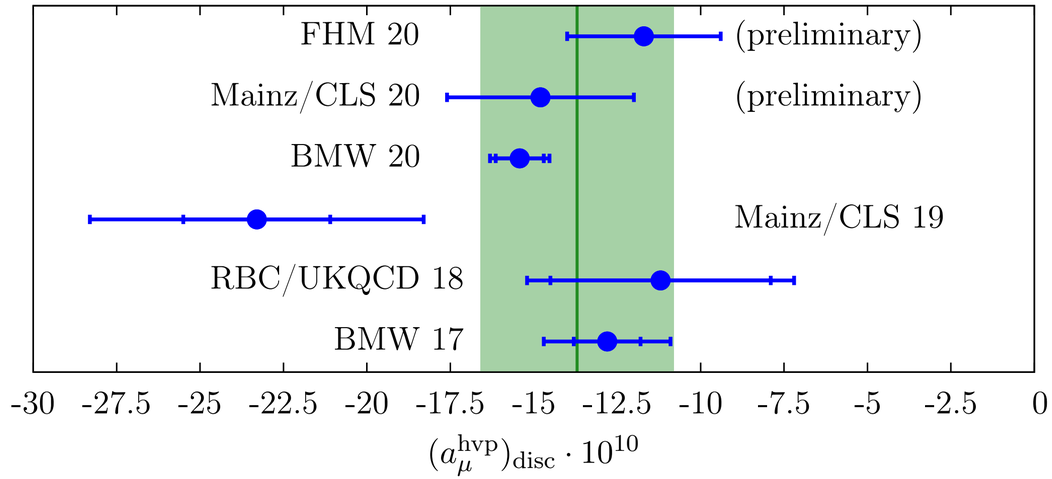
Sufficiently precise for the time being

Remaining issues

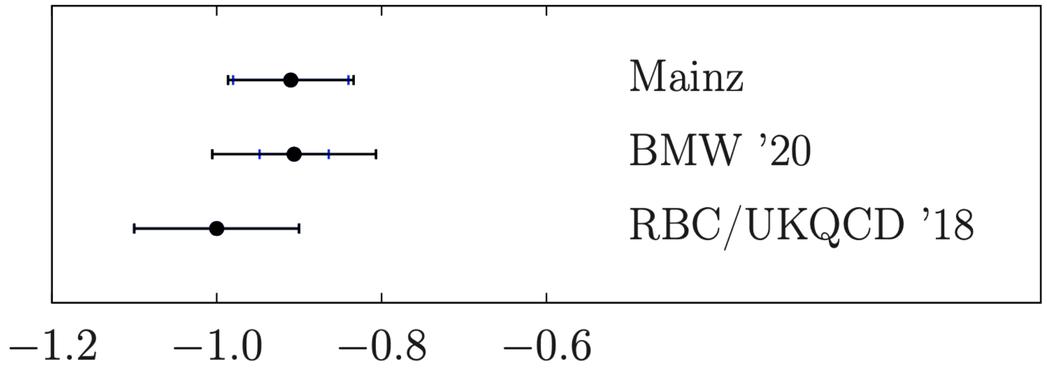
- Long-distance regime and approach to the asymptotic value

$$-\frac{G_{\mathrm{disc}}(x_0)}{G^{\rho\rho}(x_0)} \xrightarrow{x_0\to\infty} -\frac{1}{9}$$

- Computational efficiency:
  stochastic techniques vs. eigenmodes



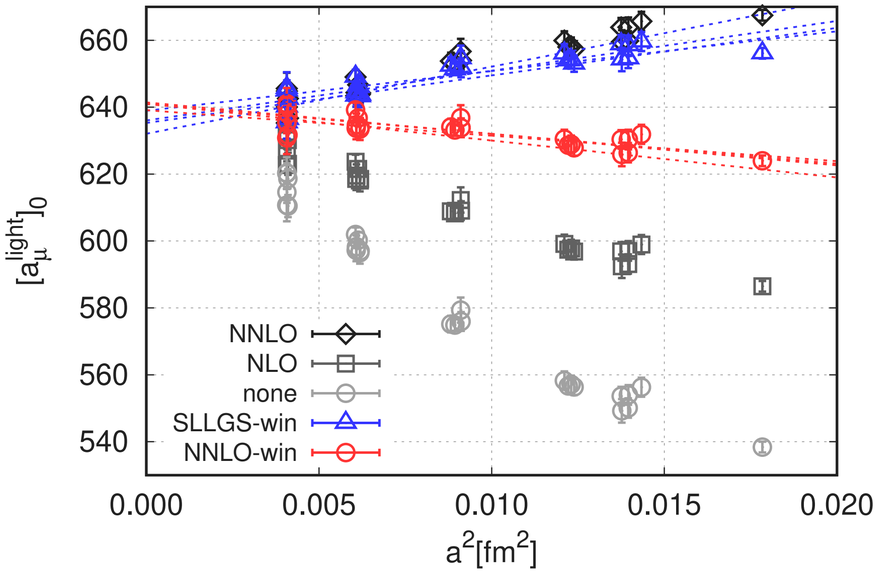
$$\left(a_\mu^{\mathrm{hvp}}\right)^{\mathrm{win,disc}} \times 10^{10}$$

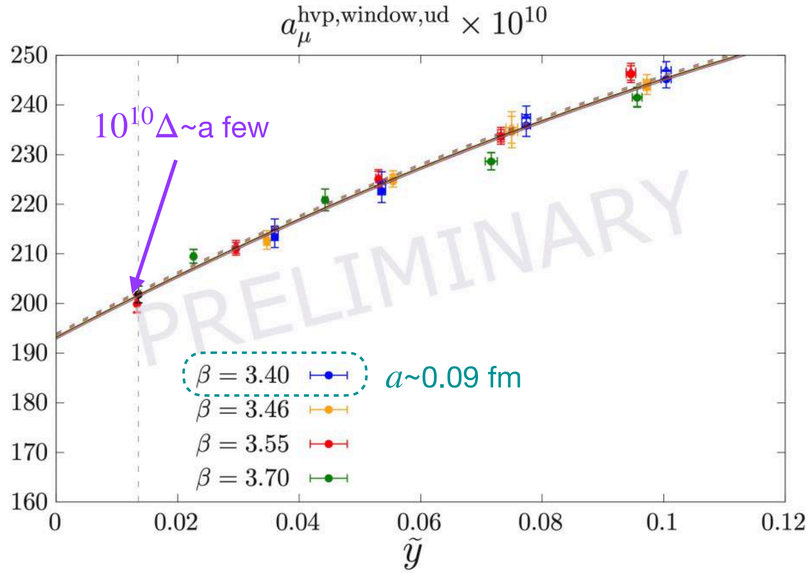


Discretisation errors

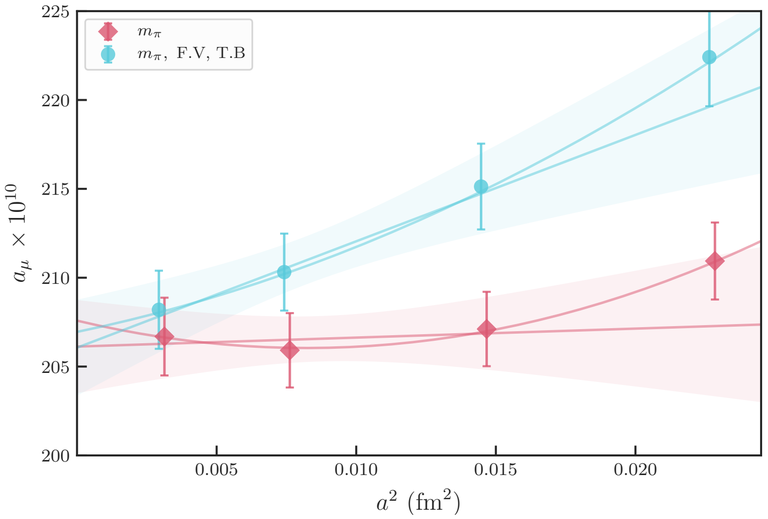
Aspects discussed at the workshop:

- Taste-breaking effects in calculations with staggered quarks *[R. Van de Water, L. Lellouch]*
- Logarithmic corrections to leading discretisation effects *[N. Husung]*
- Rescaling of integration kernel and effect on continuum extrapolation *[A. Gerardin]*
- Intrinsic size of discretisation effects in different calculations





Fermilab/MILC/HPQCD



Scale Setting

(Slides from Kalman Szabo)

Experimental Input quantities

- Pseudoscalar decay constants: still in use but groups are increasingly “turning away”
- Ω-baryon: improved measurements reach 0.1% level precision
  (see talks by C. Lehner [RBC], A. Meyer [FHM], L. Varnhorst [BMW])
- Charmonium splitting: high-precision results (D. Hatton [FHM])

Reference quantities, e.g. w_0

- $w_0 = 0.17236(70)\,\mathrm{fm}$ [0.4%] from M_Ω *[Borsanyi et al., 2020]*
- $w_0 = 0.17236(70)\,\mathrm{fm}$ [0.3%] from charm *[FHM]*

Must compute “bread-and-butter” quantities with sub-percent precision

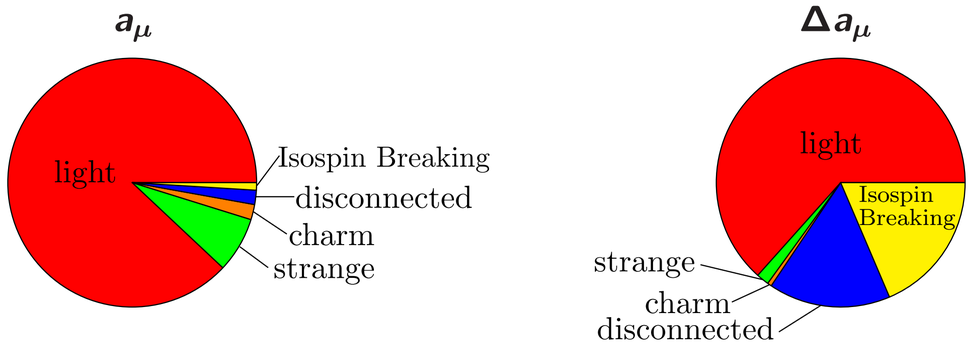
Two comments on crosschecking $(a_\mu^{\mathrm{hvp}})_{\mathrm{win}}$**:**

- Checks using $(a_\mu^{\mathrm{hvp}})_{\mathrm{win}}$ or anything HVP-related could be preceeded by even simpler checks, related to scale setting, e.g. $w_0\, f_\pi$, $w_0\, M_\Omega$, M_Ω / f_π, …
- Isosymmetric point in `crosscheck.pdf` should be made more concrete

Isospin breaking

(Slides from Vera Gülpers)

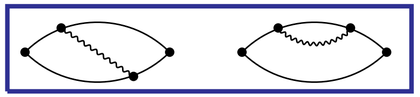
Sizeable contribution to error budget



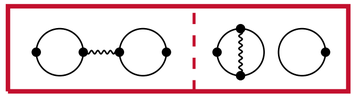
"Agreement not bad given that these are first generation calculations" *[L. Lellouch]*

"More, more precise . . . IB calculations needed" *[T. Blum]*

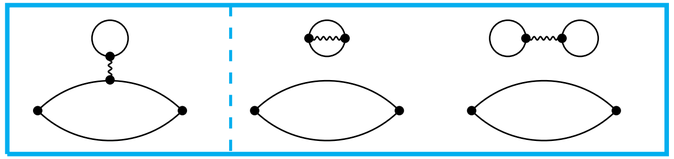
Overview of published results:



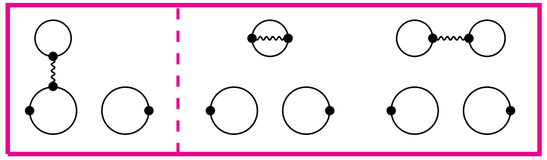
BMW −1.27(40)(33)
RBC/UKQCD 5.9(5.7)(1.7)
ETM 1.1(1.0)



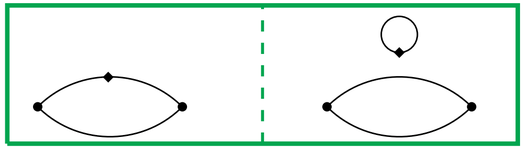
−0.55(15)(11) BMW
−6.9(2.1)(2.0) RBC/UKQCD



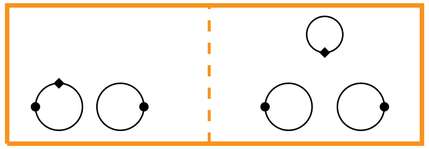
−0.0095(86)(99) 0.42(20)(19) BMW



0.011(24)(14) −0.047(33)(23) BMW



6.59(63)(53) BMW
10.6(4.3)(6.8) RBC/UKQCD
6.0(2.3) ETM
7.7(3.7) 9.0(2.3) FHM
9.0(0.8)(1.2) LM



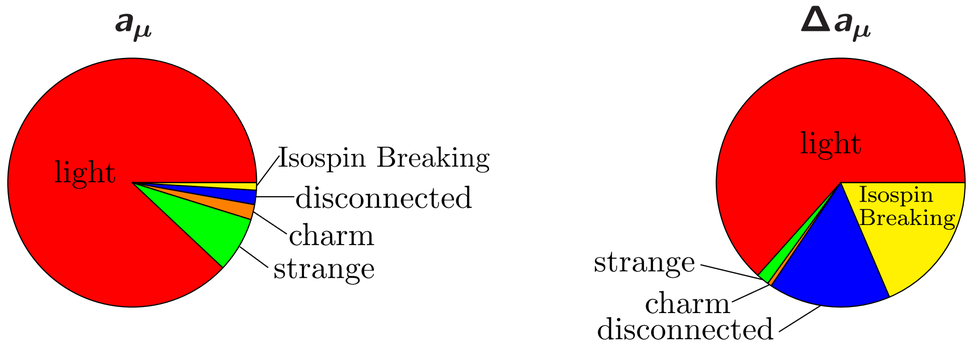
−4.63(54)(69) BMW

BMW [arXiv:2002.12347]
RBC/UKQCD [Phys.Rev.Lett. 121 (2018) 2, 022003]
ETM [Phys. Rev. D 99, 114502 (2019)]
FHM [Phys.Rev.Lett. 120 (2018) 15, 152001]
LM [Phys.Rev.D 101 (2020) 074515]

Isospin breaking

(Slides from Vera Gülpers)

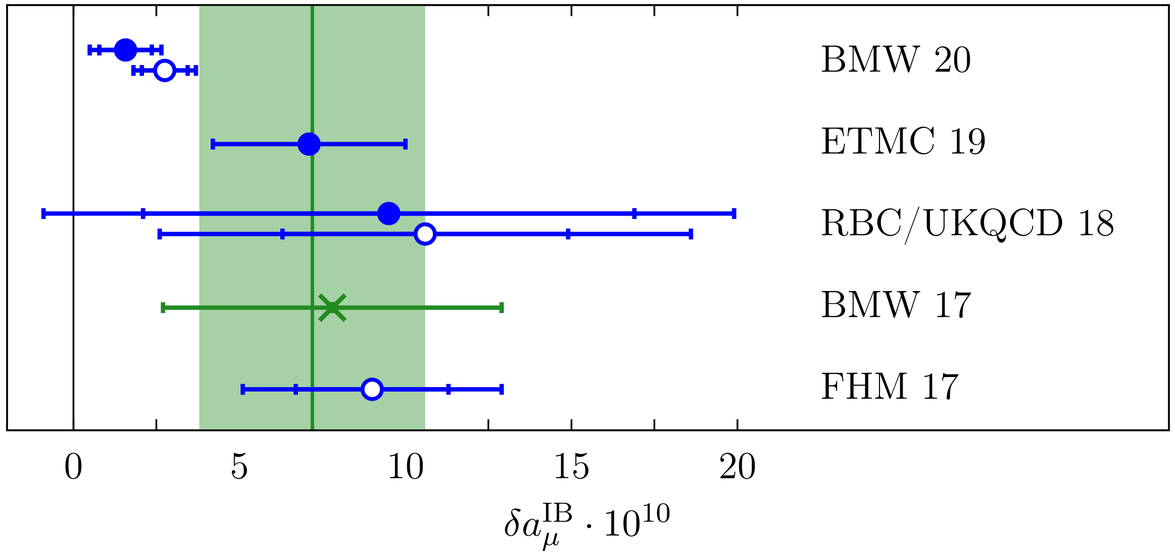
Sizeable contribution to error budget



"Agreement not bad given that these are first generation calculations" *[L. Lellouch]*

"More, more precise . . . IB calculations needed" *[T. Blum]*

Overview of published results:



Small overall value result of large cancellations; large statistical uncertainties:

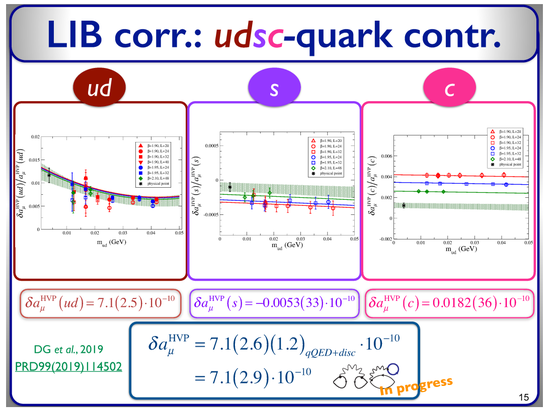
$$a_\mu^{\rm IB} \lesssim 1\,\%\,, \quad \delta a_\mu^{\rm IB} \lesssim 100\,\%$$

Isospin breaking

(Slides from Vera Gülpers)

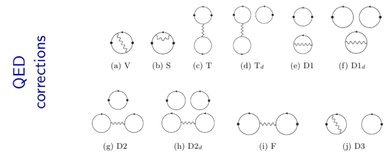
New results at this workshop

ETM (D. Giusti Mon 15:55)



RBC/UKQCD (A. Meyer Mon 17:05)

QED w/ HLbL data



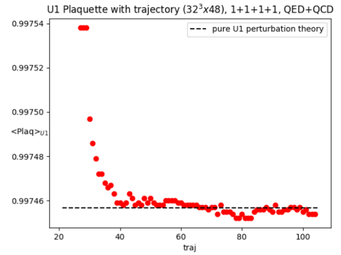
- point-src data reused from HLbL calc
- reduced stat error $\approx \times 5 - \times 10$ on HVP QED (Mattia Bruno, 1812.09562[hep-lat])
- $V \to \infty$, $a \to 0$ limits for diagrams V,S,F
- first results for T,D1,R; other subleading in preparation
- target QED error $O(1 \times 10^{-10})$

Aaron S. Meyer Section: Outlook 37/ 39

FHM (C. McNeile Tue 16:05)

Including QED with the QCD sea quarks

- Just starting (noncompact) QED + QCD HMC simulations.
- 1+1+1+1 sea quarks with QED, lattice spacing a =0.15 fm
- We are testing measurement strategies.



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RC* (M. Marinković Wed 17:15)

Strategy for HVP computation in QCD+QED

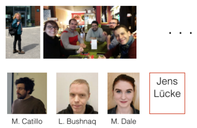
QCD with C* + Pert. QED

- [R123, de Divitiis et al. 1303.4896]
- Two volumes
- Single lattice spacing
- IB effects: electro-quenched + beyond
- Noise reduction techniques [..., Giusti, Harris, Nada, ... Schaefer 1903.10447]

QCD+QED with C*

- Compact QED, Fourier acceleration of QED
- One volume complete, a second one in progress
- Signal to noise comparable to QCD [Hansen, Lucini, Patella, Tantalo 1802.05474]
- Measurements integrated in the openQxD code

HLRN-IV, CSCS, PSNC, Altamira, LRZ , ...



. . .

M. Catillo

L. Bushnaq

M. Dale

Jens Lücke

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Mainz (A. Risch Wed 17:20)

$a_{\mu,\mathrm{HVP}}$ via TMR, single state reconstruction and rescaling $(am_\mu)^{\mathrm{lat}} = \frac{m_\mu^{\mathrm{phys}}}{f_\pi^{\mathrm{phys}}}(af_\pi)^{\mathrm{lat}}$:

	N200	H102
a [fm]	$0.06426[76]_{\mathrm{tot}}$ fm	$0.08636[106]_{\mathrm{tot}}$ fm
m_{π^+} [MeV]	$285.6(5)_{\mathrm{st}}(3.4)_{\mathrm{a}}[3.4]_{\mathrm{tot}}$	$355.2(9)_{\mathrm{st}}(4.4)_{\mathrm{a}}[4.5]_{\mathrm{tot}}$
m_{π^0} [MeV]	$283.4(5)_{\mathrm{st}}(3.4)_{\mathrm{a}}[3.4]_{\mathrm{tot}}$	$353.2(9)_{\mathrm{st}}(4.3)_{\mathrm{a}}[4.4]_{\mathrm{tot}}$
m_{K^+} [MeV]	$460.1(4)_{\mathrm{st}}(5.4)_{\mathrm{a}}[5.5]_{\mathrm{tot}}$	$436.7(7)_{\mathrm{st}}(5.4)_{\mathrm{a}}[5.4]_{\mathrm{tot}}$
m_{K^0} [MeV]	$464.0(4)_{\mathrm{st}}(5.5)_{\mathrm{a}}[5.5]_{\mathrm{tot}}$	$440.6(7)_{\mathrm{st}}(5.4)_{\mathrm{a}}[5.4]_{\mathrm{tot}}$
$(a_{\mu,\mathrm{HVP}})^{(0)}$ $[10^{10}]$	$525[12]_{\mathrm{tot}}$	$490[7]_{\mathrm{tot}}$
$(a_{\mu,\mathrm{HVP}})^{(1)}$ $[10^{10}]$	$5[6]_{\mathrm{tot}}$	$2.5[4]_{\mathrm{tot}}$
$a_{\mu,\mathrm{HVP}}$ $[10^{10}]$	$529[13]_{\mathrm{tot}}$	$493[7]_{\mathrm{tot}}$

- $(a_{\mu,\mathrm{HVP}})^{(0)}$ differs from result in Mainz-2019 [Gérardin et al. 2019: different renormalisation of $Z_{\mathcal{V}_{1,\mathrm{R}}\mathcal{V}_1}$ and no $O(a)$-improvement
- $(a_{\mu,\mathrm{HVP}})^{(1)}$ is $O(1\%)$ of $(a_{\mu,\mathrm{HVP}})^{(0)}$ compatible with [Blum et al. 2018] [Giusti et al. 2019], $(a_{\mu,\mathrm{HVP}})^{(1)}$ dominated by error of $(a_{\mu,\mathrm{HVP}})^{(0)}$

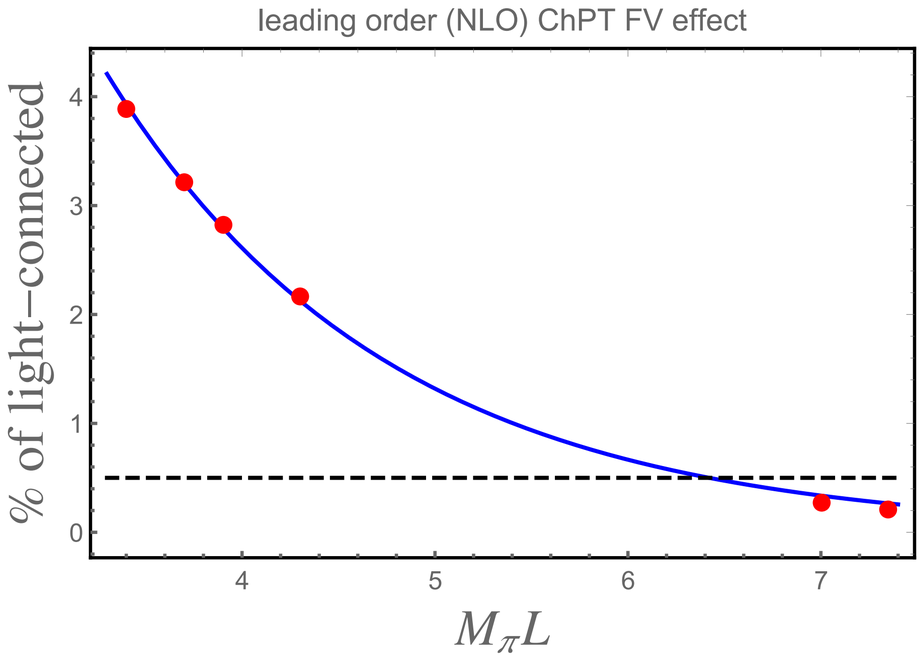
In preparation (with A. Hanlon, P. Madanagopalan and A. Segner):

- Extend list of ensembles
- Scale setting based on baryon masses ⇒ include IB corrections
- Define $Z_{\mathcal{V}_R\mathcal{V}}$ via QCD+QED vector Ward identity ⇒ Setup compatible with Mainz-2019 [Gérardin et al. 2019] with $O(a^2, a\Delta\varepsilon)$ continuum converges

Andreas Risch Isospin breaking on CLS ensembles Wednesday 18th November, 2020 4 /

Finite-volume effects

(Slides from Maarten Golterman)



FV effects exponentially suppressed;
dominated by 2-pion states

Most important:
FV correction of light-quark connected part

Harvey Meyer (Mainz):

The timelike pion form factor and finite-size corrections to the vector correlator

Laurent Lellouch (Marseille):

Correcting finite-size effects in the HVP contribution to the muon g-2 using large-volume lattice QCD simulations

Max Hansen (Edinburgh):

Finite-volume corrections to the vector correlator from space-like observables

Santi Peris (Barcelona):

FV effects in the EFT of a_{mu}^{HVP}

Christoph Lehner (Regensburg):

Finite-volume effects for the strong isospin-breaking contribution

Antonin Portelli (Edinburg):

Electromagnetic finite-size effects on the two-pion contribution to the HVP

Finite-volume effects

(Slides from Maarten Golterman)

Timelike pion form factor and finite-volume quantisation (aka MLLGS)

- finite-volume correlator:

$$G(t,L) = \sum_n |A_n|^2 \, e^{-\omega_n t}$$

- infinite-volume correlator:

$$G(t,\infty) = \int_0^\infty d\omega \, \omega^2 \, \rho(\omega^2) \, e^{-\omega t},$$

$$\rho(\omega^2) = \frac{1}{48\pi^2}\left(1-\frac{4m_\pi^2}{\omega^2}\right)^{\frac{3}{2}} |F_\pi(\omega)|^2 + \text{other channels.}$$

Establish a connection between

$|A_n|^2$, ω_n and pion form factor $F_\pi(\omega)$

Gounaris-Sakurai parameterisation or dedicated lattice study

[Meyer, Lellouch, Lüscher]

Finite-volume corrections from space like observables *[Hansen & Patella]*

$$\Delta G(x_0|T,L) = -\sum_{\boldsymbol{n}\neq\boldsymbol{0}} \int \frac{dp_3}{2\pi} \frac{e^{-|\boldsymbol{n}|L\sqrt{M_\pi^2+p_3^2}}}{24\pi L|\boldsymbol{n}|} \int \frac{dk_3}{2\pi} \cos(x_0 k_3) \, \mathrm{Re}\, T(-k_3^2,-k_3 p_3) + \mathcal{O}(e^{-M_\pi T}, e^{-1.9 M_\pi L})$$

volume dependence — Compton amplitude

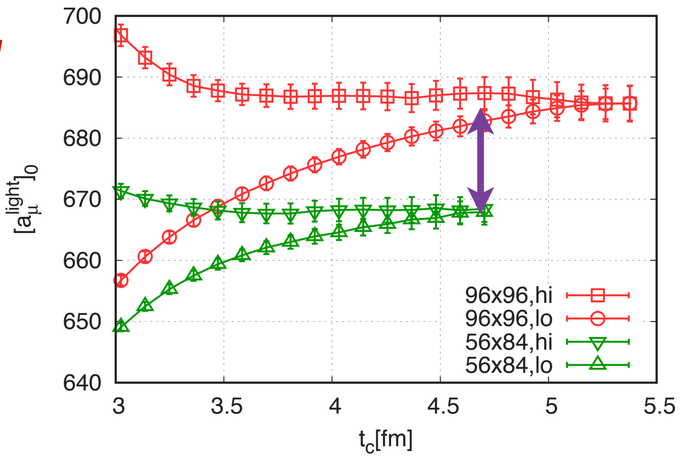
Finite-volume effects

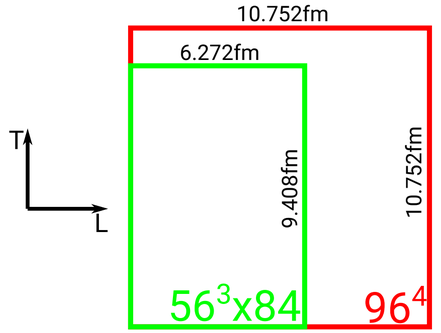
(Slides from Maarten Golterman)

Numerical checks on large volumes

[Borsanyi et al.]

$$a_\mu(\infty,\infty) - a_\mu(L_{\rm ref}, T_{\rm ref}) =$$
$$= [a_\mu(L_{\rm big}, T_{\rm big}) - a_\mu(L_{\rm ref}, T_{\rm ref})]_{\rm 4HEX} +$$
$$+ [a_\mu(\infty,\infty) - a_\mu(L_{\rm big}, T_{\rm big})]_{\rm NNLO}$$





$$a_\mu(L_{\rm big}, T_{\rm big}) - a_\mu(L_{\rm ref}, T_{\rm ref}) = 18.1(2.0)_{\rm stat}(1.4)_{\rm cont}$$

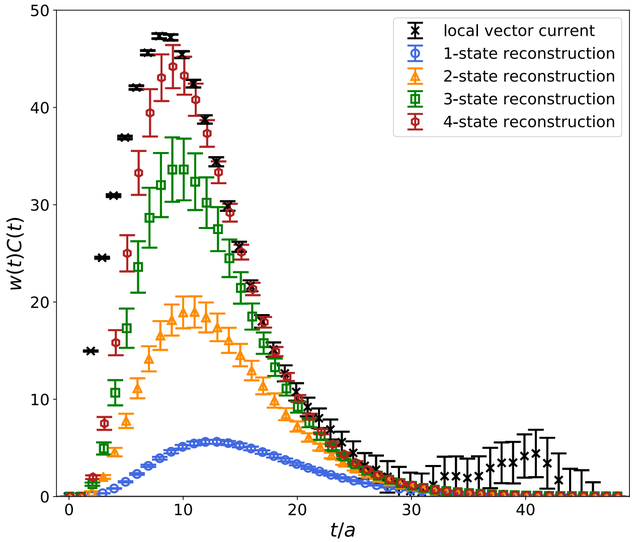
Excellent agreement:

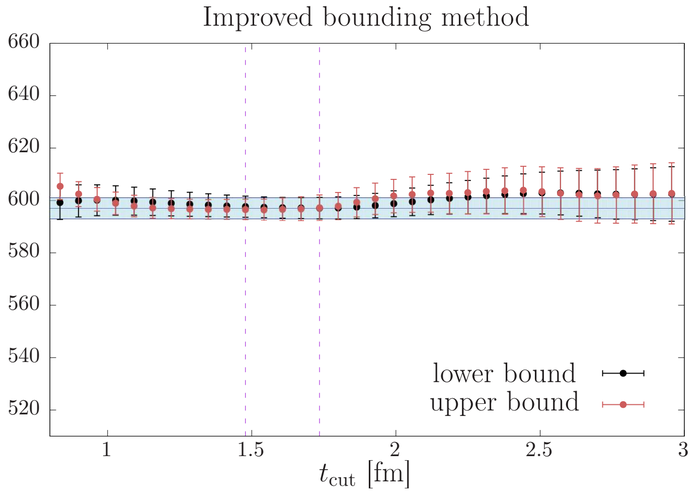
lattice:	$18.1(2.4)$	brute force (but smart) computation
MLLGS:	17.8	pion form factor in finite ⇔ inf. volume
NNLO ChPT:	15.7	knows about rho meson only at NNLO
HP:	16.3	expansion in pion winding number

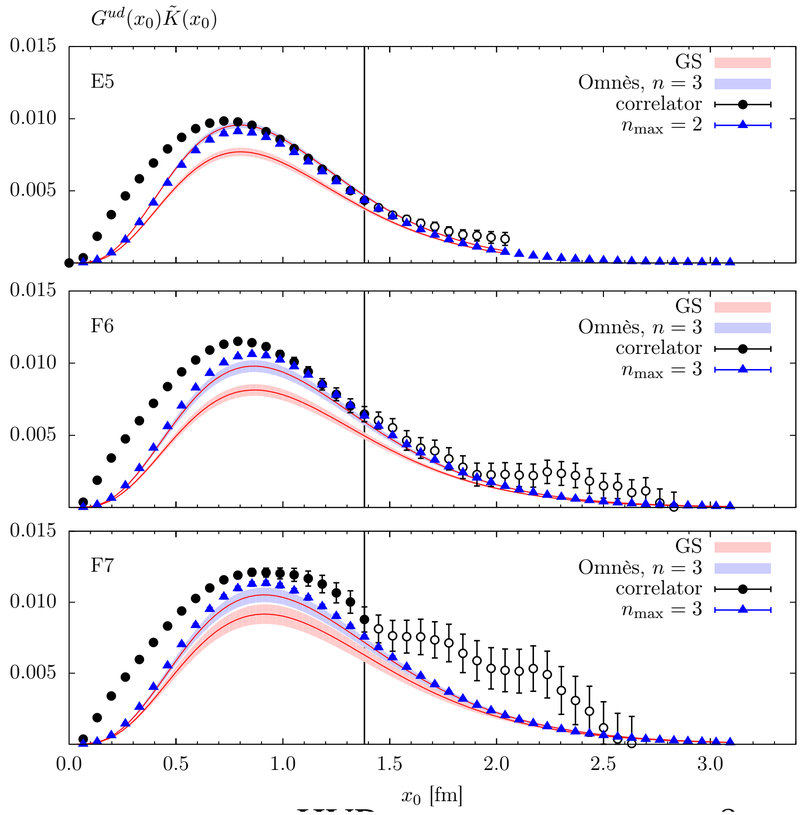
Long-distance regime

Several methods

- Bounding method
- Dedicated spectroscopy studies in isovector channel
- Timelike pion form factor
- Noise reduction: AMA, LMA







Crosschecks

"Window" quantities: Designed to reduce / enhance sensitivity on certain systematics

$$a_\mu^{\rm SD} = \int_0^\infty {\rm d}t\; C(t)\, w(t)\; [1 - \Theta(t, t_0, \Delta)]$$

$$a_\mu^{\rm W} = \int_0^\infty {\rm d}t\; C(t)\, w(t)\; [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]$$

$$a_\mu^{\rm LD} = \int_0^\infty {\rm d}t\; C(t)\, w(t)\, \Theta(t, t_1, \Delta)$$

$$\Theta(t, t', \Delta) \equiv \frac{1}{2}\left(1 + \tanh\left[(t - t')/\Delta\right]\right)$$

- Test consistency among different lattice calculations
- Check control over systematic effects (without added difficulties)
- Compare with R-ratio

Moments:

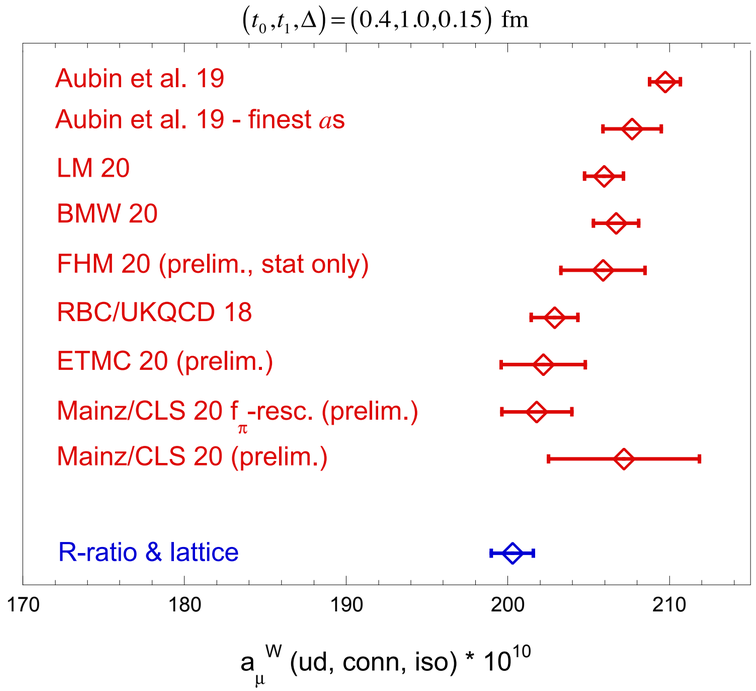
$$G_{2n} = \int_0^\infty {\rm d}t\, t^{2n} C(t) \qquad \Pi_n = \frac{(-1)^{n+1}}{(2n+2)!}\, G_{2n+2}$$

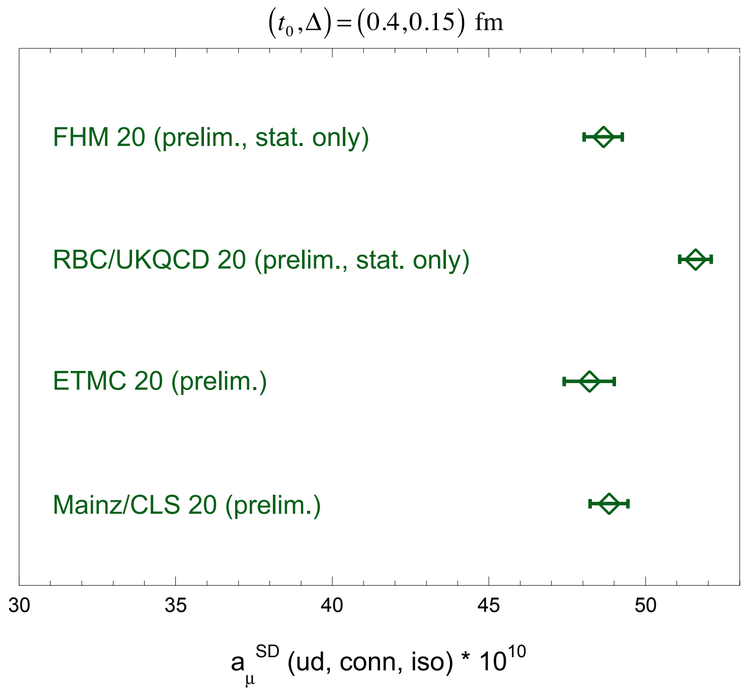
- Dimensionful quantities → scale setting
- Dimensionless ratios more promising, e.g. Π_2/Π_1^2

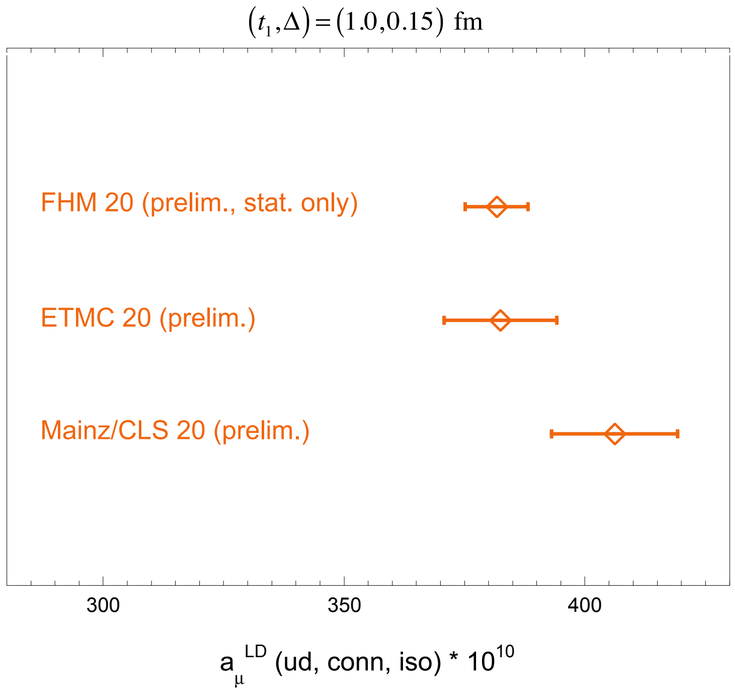
Crosschecks

“Window” quantities

(Plots from Davide Giusti)







- Straightforward reference quantities
- Can be applied to individual contributions (light, strange, charm, disconnected,…)
- Comparison with e^+e^-/ R-ratio may require tuning of the window

Conclusions

Tremendous progress in lattice calculations of HVP (and HLbL!) contributions in Lattice QCD

Interplay between lattice calculations and phenomenology

Sub-percent calculation by BMW must be checked

Prepare for the next edition of the White Paper:

- Investigate discretisation / taste-breaking effects:
  more variants of staggered calculations with similar precision
  extend scaling studies in Wilson-like, DWF calculations
- Improve scale setting: compute “bread-and-butter” quantities with sub-percent precision
- Check and improve calculations of isospin-breaking effects
- Improve the precision of light-connected contribution

Benchmark quantities (windows) crucial for checking the internal consistency of lattice calculations

Extend calculation of window quantities to individual flavour and quark-disconnected contributions

Extend comparisons with phenomenological analyses

(Non-local) Workshop Organizing Committee

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Davide Giusti
(Regensburg)

Vera Gülpers
(Edinburgh)

Kohtaroh Miura
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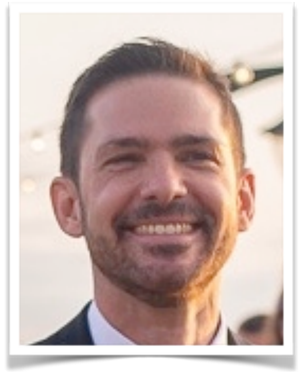

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Many thanks for the excellent organisation!