Higgs Pseudo-Observables

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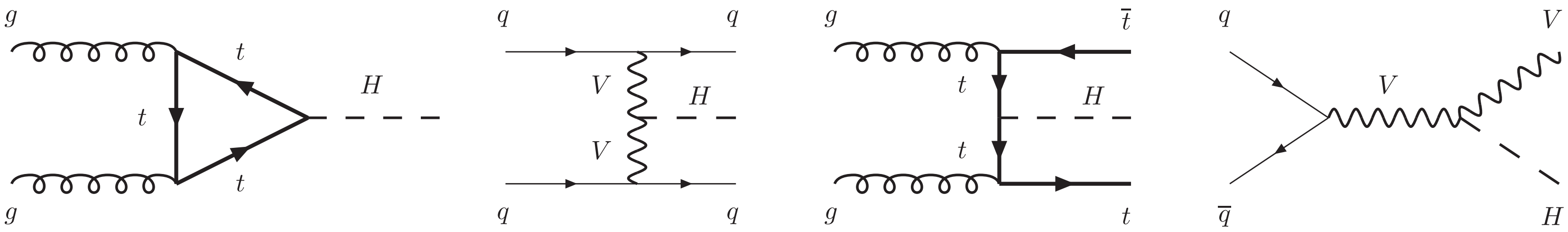
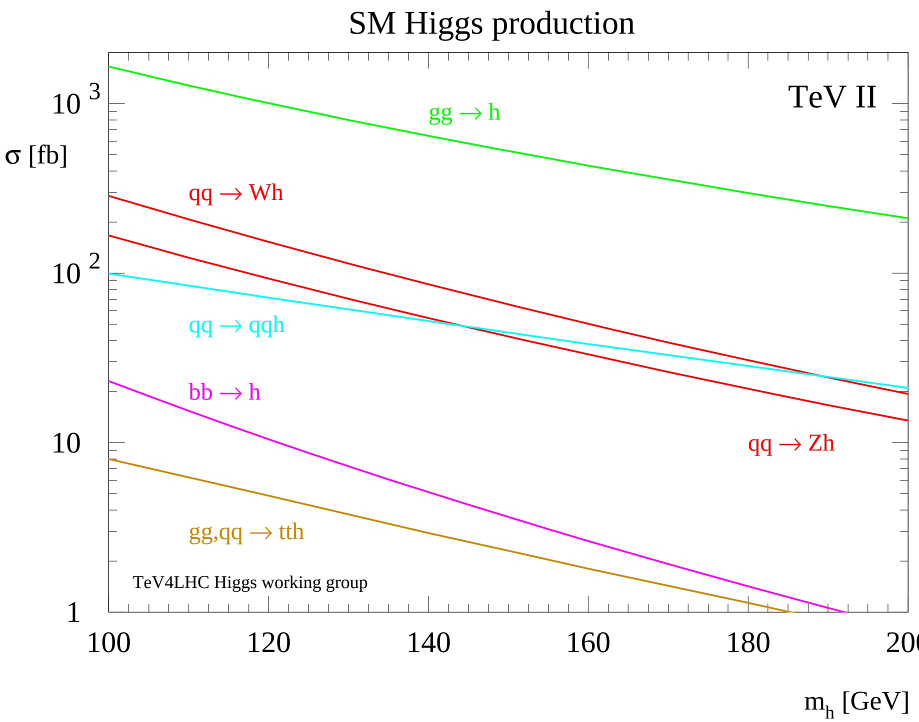
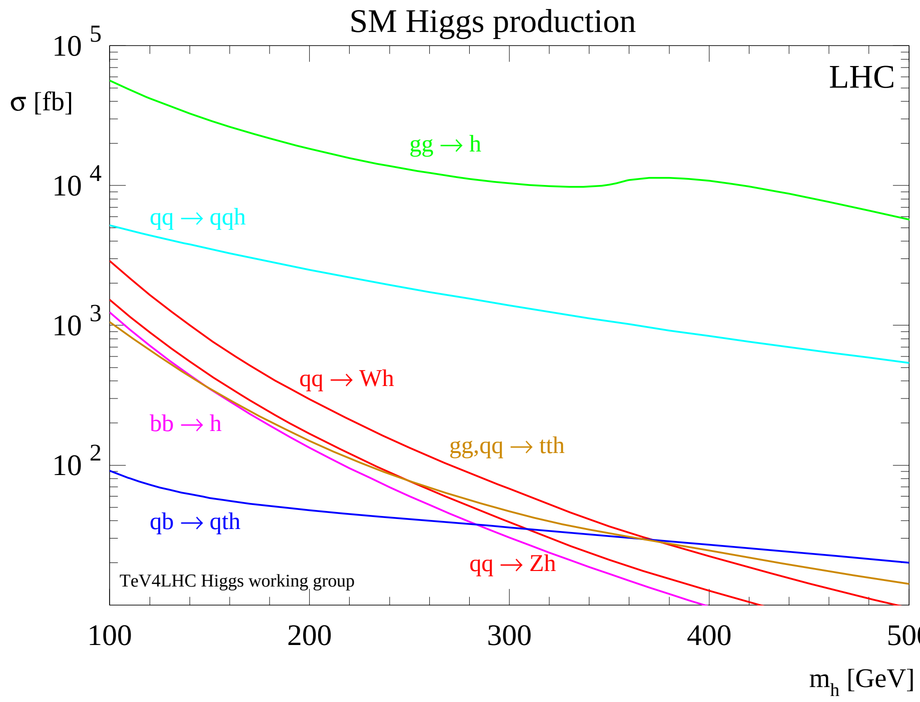
Standard Model hadronic Higgs production channels



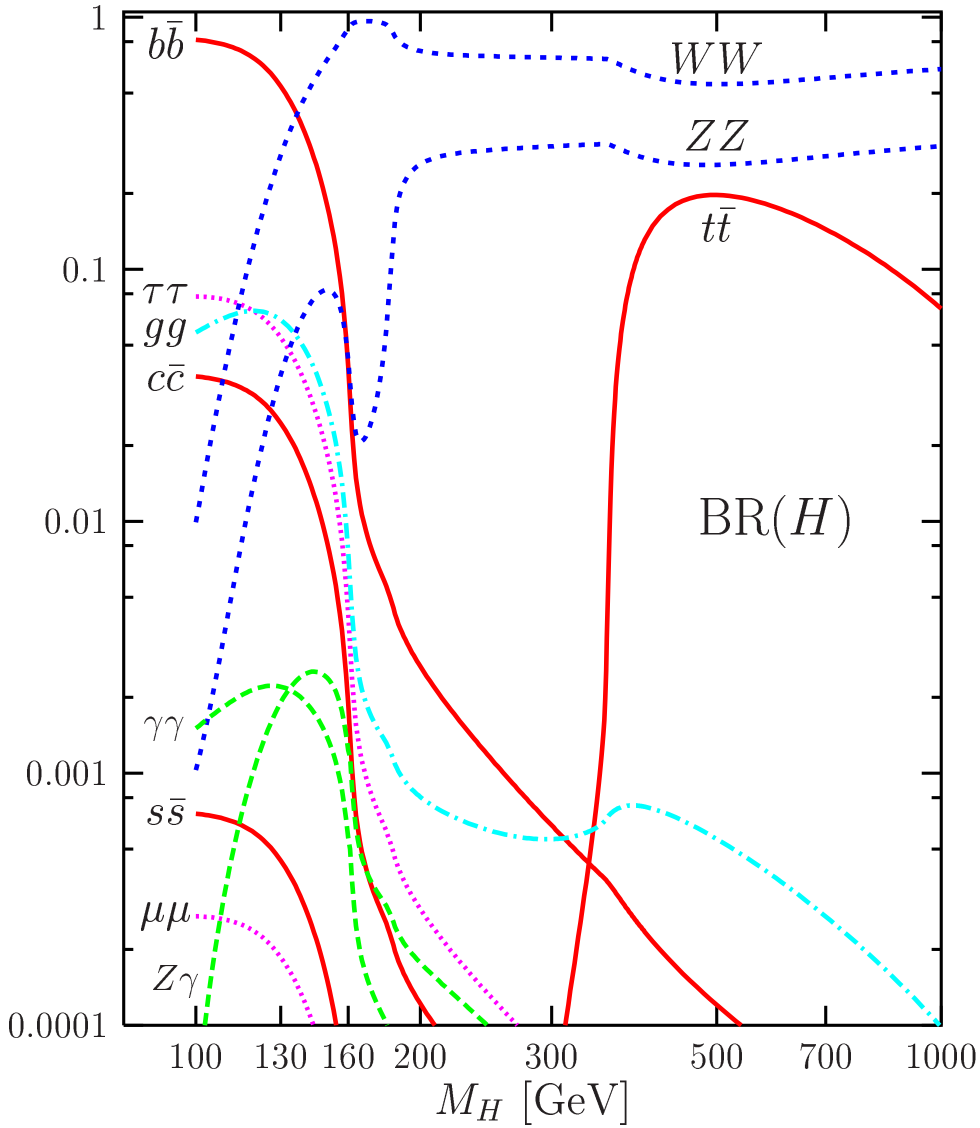




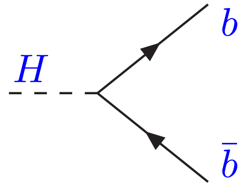
Hahn,Heinemeyer,Maltoni,Weiglein,Willenbrock [hep-ph/0607308]

Gluon-fusion ⇝ largest cross section

Higgs decays in the Standard Model

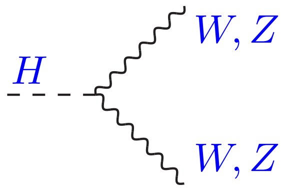


- $H \to b\bar{b}$:



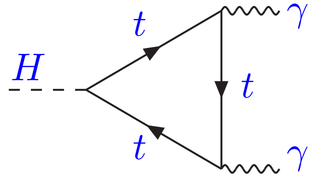
For light Higgs, huge background

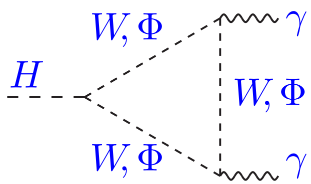
- $H \to WW, ZZ$:

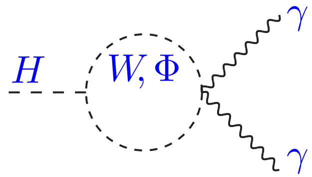


For heavy Higgs

- $H \to \gamma\gamma$:







For Light Higgs: rare, but clean

Problems with gauge invariance: $H(P) \to \gamma(p_1) + \gamma(p_2)$

Amplitude $\quad \to \quad \mathcal{A}^{\mu\nu} = \dfrac{g^3\, s_\theta^2}{16\,\pi^2}\,\left(F_{_D}\,\delta^{\mu\nu} + F_{_P}\,p_2^\mu\,p_1^\nu\right).$

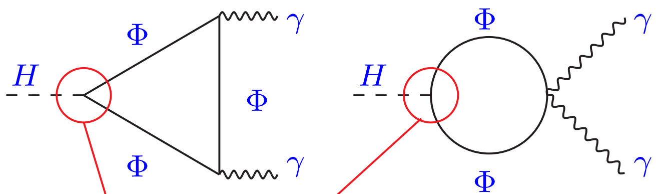
Ward Identity: $\quad F_{_D} + p_1 \cdot p_2\, F_{_P} = 0$

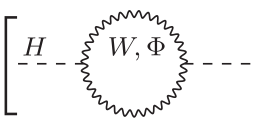
Renormalization (Ren) $\quad \to \quad M^2_{H,0} = M^2_H \left[1 + \dfrac{G_{_F} M^2_W}{2\sqrt{2}\,\pi^2}\,\mathrm{Re}\,\Sigma^{(1)}_{HH}(M^2_H)\right]$

$$F_D = F_D^{(1)} \otimes (1 + \mathrm{Ren}) + F_D^{(2)} \qquad\qquad F_P = F_P^{(1)} \otimes (1 + \mathrm{Ren}) + F_P^{(2)}$$

- 2-loop level

$$\underbrace{F_{_D}^{(2)} + p_1 \cdot p_2\, F_{_P}^{(2)}} + \underbrace{\left(F_{_D}^{(1)} + p_1 \cdot p_2\, F_{_P}^{(1)}\right) \otimes \mathrm{Ren}} \neq \mathbf{0}$$



No "Re" label $\qquad \sim M_{H,0} \to \mathrm{Re}\,\Big[$ 

$\Big]$

- Unstable particles can not be asymptotic states
- Higgs production and decay are not well defined

$$\Downarrow$$

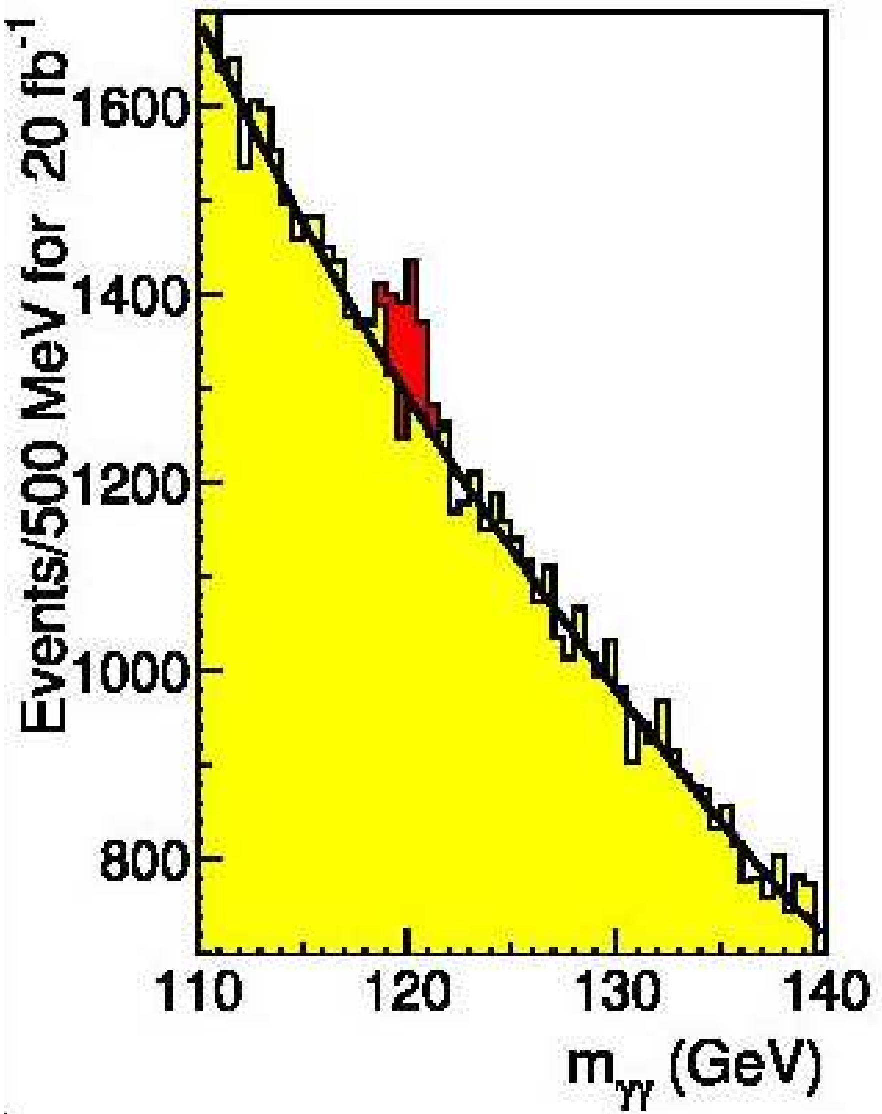
complete process

$$pp \to \gamma\gamma + X$$

which consists of

$$\text{Signal} \left[pp \to (gg \to H \to \gamma\gamma) + X \right]$$

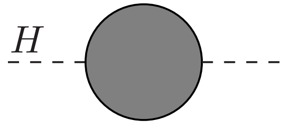
$$+ \quad \text{Background}$$



How to extract a pseudo-observable to be termed *Higgs partial decay width into two photons* which does not violate first principles?

Higgs self-energie

$$\Sigma_H(s, M^2_{H,0}) =$$

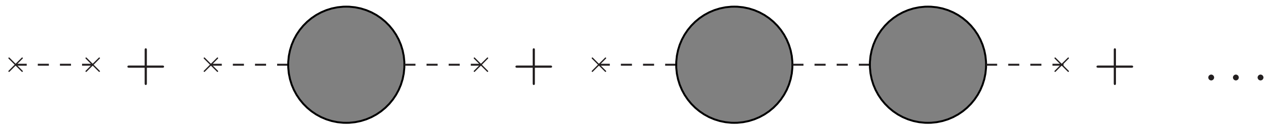


Complex pole: $\quad s_H - M^2_{H,0} + \Sigma_H(s_H, M^2_{H,0}) = 0$

- gauge invariant definition
- $M_{H,0}$ real by construction $\quad \rightsquigarrow \quad s_H = \mu^2_H - i\,\mu_H\,\gamma_H$

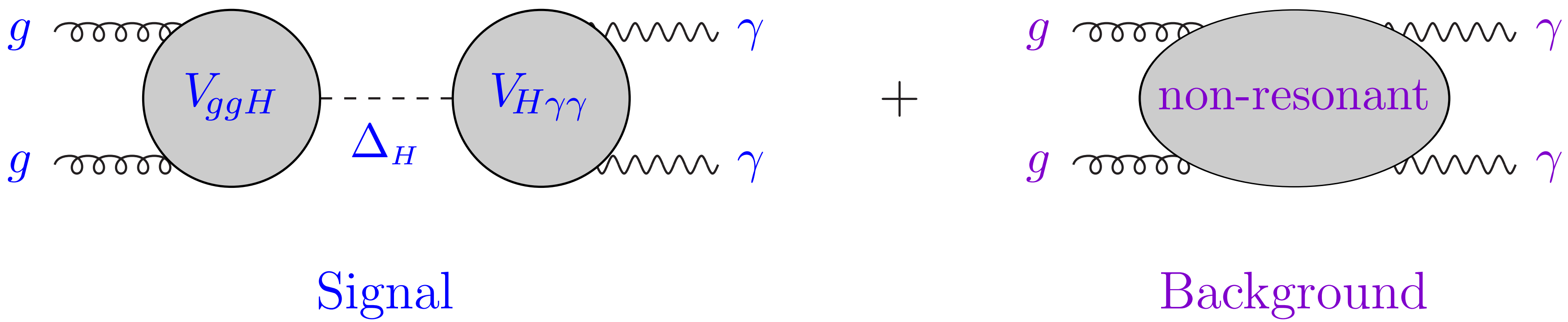
Dyson-resummed Higgs propagator

$$\Delta_H(s) =$$



$$= (s - s_H)^{-1}\Big[1 + \Pi_H(s)\Big]^{-1}, \qquad \Pi_H(s) = \frac{\Sigma_H(s) - \Sigma_H(s_H)}{s - s_H}$$

Amplitude for $gg \to \gamma\gamma$:



In general S-matrix for $i \to f$:

$$S_{fi} = V_i(s)\,\Delta_H(s) V_f(s) + B_{\rm nr}$$
$$= \left[Z_H^{-1/2}(s) V_i(s)\right] \frac{1}{s - s_H} \left[Z_H^{-1/2}(s) V_f(s)\right] + B_{\rm nr},$$

$$Z_H = 1 + \Pi_H \qquad\qquad B_{\rm nr} = \text{non-resonant background}$$

Expand the square brackets around $s = s_H$

$$S_{fi} = \frac{S(i \to H_c)\, S(H_c \to f)}{s - s_H} + \text{non resonant terms.}$$

where

$$\begin{aligned} \text{Production :} \qquad & S(i \to H_c) \;=\; Z_H^{-1/2}(s_H)\, V_i(s_H) \\ \text{Decay :} \qquad & S(H_c \to f) \;=\; Z_H^{-1/2}(s_H)\, V_f(s_H) \end{aligned}$$

- **gauge invariant** order per order in perturbation theory
- Diagrams and renormalization evaluated at the complex pole

$$Z_H(s_H) = 1 + \lim_{s \to s_H} \frac{\Sigma_H(s) - \Sigma_H(s_H)}{s - s_H} = 1 + \frac{\partial \Sigma_H}{\partial s}(s_H)$$

- Universal and well-defined parametrization of experimantal data

⇒ Definition of a gauge-invariant decay width:

$$\Gamma(H_c \to f) \;=\; \frac{(2\pi)^4}{2\,\mu_H} \int d\Phi_f(P_H, \{p_f\}) \sum_{\text{spins}} \left| S(H_c \to f) \right|^2$$

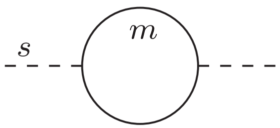
Analytical continuation

- We have diagrams with complex external squared momenta
- We must understand how is defined the physical Riemann sheet

$\downarrow$

$i0^+$ Feynman prescription

Example:



$$= \Delta - \int_0^1 dx \ln\chi, \qquad \chi = -s\,x\,(1-x) + m^2 - i0^+$$

- Complex mass: $m^2 \to \mu^2 - i\mu\gamma$ $\rightsquigarrow$ Imχ does not change sign
- Complex s: $s \to M^2 - iM\Gamma$ $\rightsquigarrow$ Imχ changes sign $\to$ Problem

General rule: $$\lim_{\gamma,\Gamma\to 0} \text{Ampl}\,(s, m) = \text{Ampl}\,(M^2, \mu)$$

If Re$\chi < 0$ and Im$\chi > 0$ (second quadrant):

$$\lim_{\gamma,\Gamma_H\to 0} \text{Im}[\ln\chi] = \pi \quad \neq \quad \begin{array}{l}\text{Feynman prescription for}\\ \text{real masses } (\mu^2 \to \mu^2 - i0)\end{array} = -\pi$$

- If $\mathrm{Re}\chi < 0$ and $\mathrm{Im}\chi > 0$ (second quadrant), we have to change the definition of the log.

  Analytical continuation on the second Riemann sheet:

$$\ln(z) \to \ln^-(z) = \ln(z) - 2\,i\,\pi\,\underbrace{\theta(-\mathrm{Re}z)\,\theta(\mathrm{Im}z)}_{\text{second quadrant}} \quad \Leftrightarrow$$

move the cut on the positive imaginary axis

- This changes the computation of loop functions (analytical continuation for Li_n, HPLs, etc.)

- Change of the integration contour in integral representations:

  - The integration contour ($x \in [0,1]$) never crosses the cut of $\ln\chi$ (negative real axis), but ...

  - ...it can cross the cut of $\ln^-\chi$ (positive imaginary axis) $\to$ Problem

    In the example this happens for

$$M^2 \geq 4\,\mu^2 \qquad \& \qquad \mu\Gamma - M\gamma \geq 0$$

General strategy in parametric space:

- Diagrams → integrals of polynomial (quadratic in some variables) to negative/non-integer power

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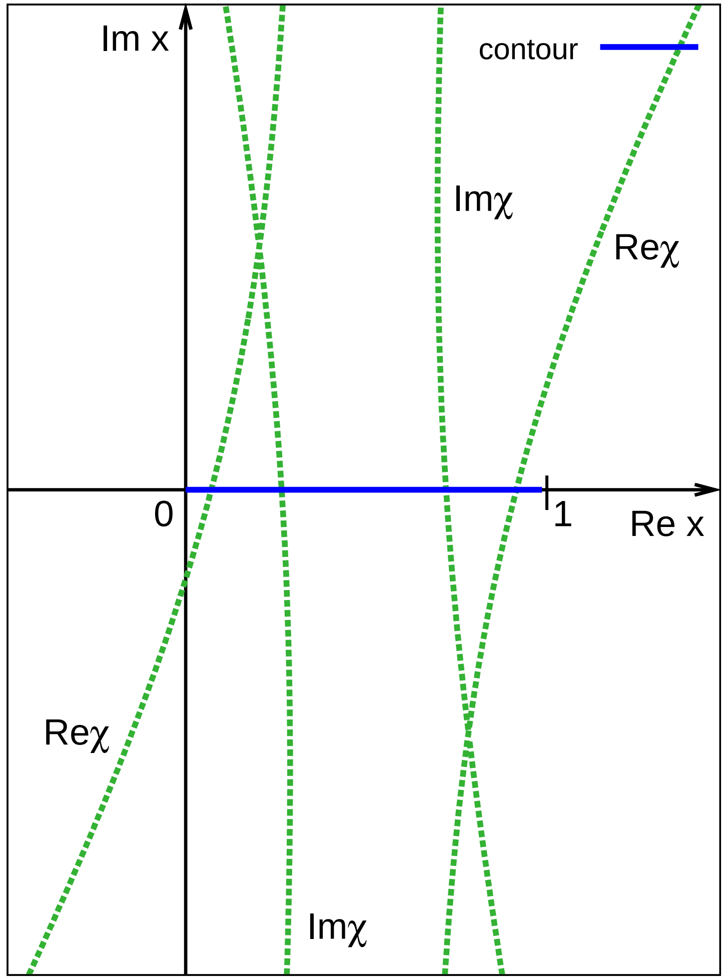
- Pick up one variable x (quadratic):

$$\chi = a\,x^2 + b\,x + c$$

$$\mathrm{Re}\chi = 0, \quad \mathrm{Im}\chi = 0 \quad \rightarrow \quad \text{Hyperbolas}$$



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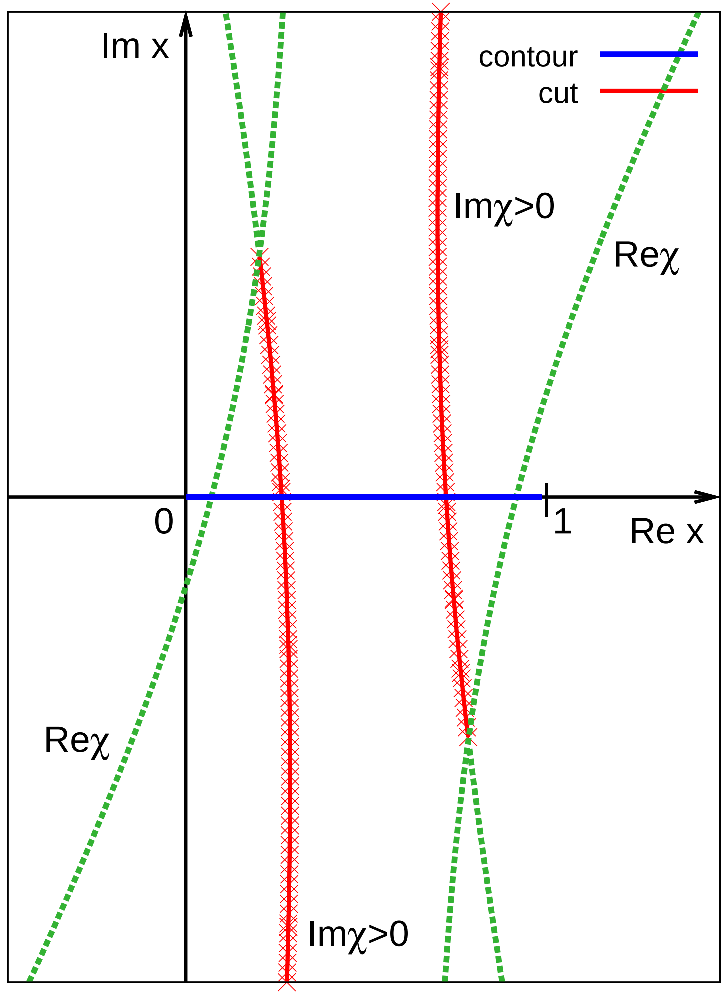
Find the cuts ⇔ study intersections of hyperbolas.



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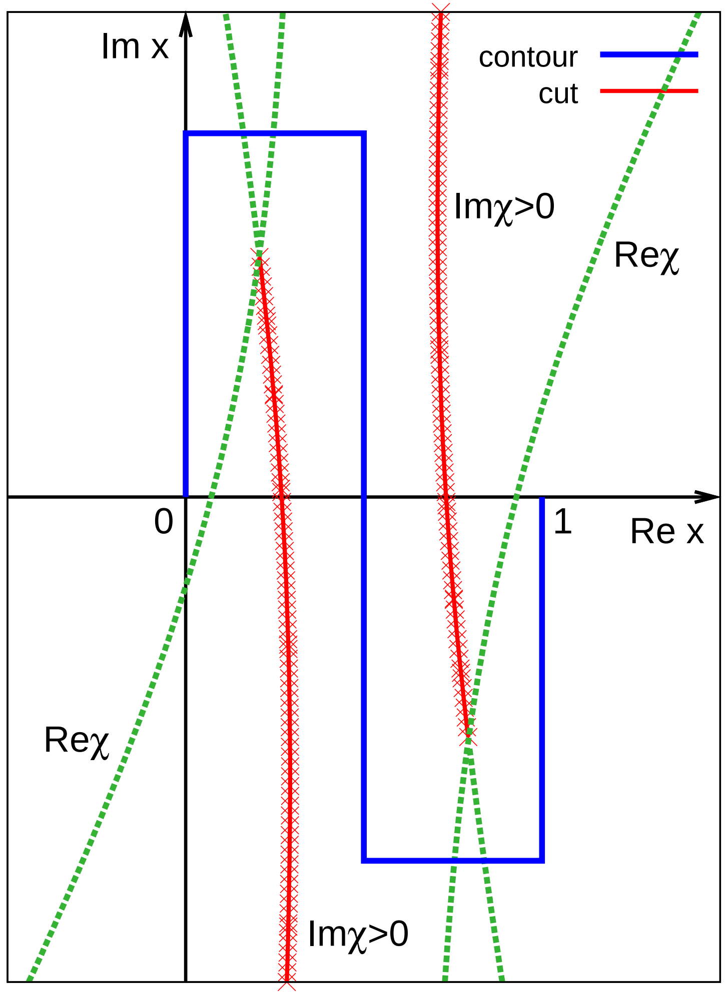


- Deform just the contour of x, for general values of the others
- Deformation for the general case can be easily automatized (numerically)

Numerical effects: Notation

RMRP → Real Masses and Real Momenta.

The usual on-shell scheme where all masses and all Mandelstam invariants are real.

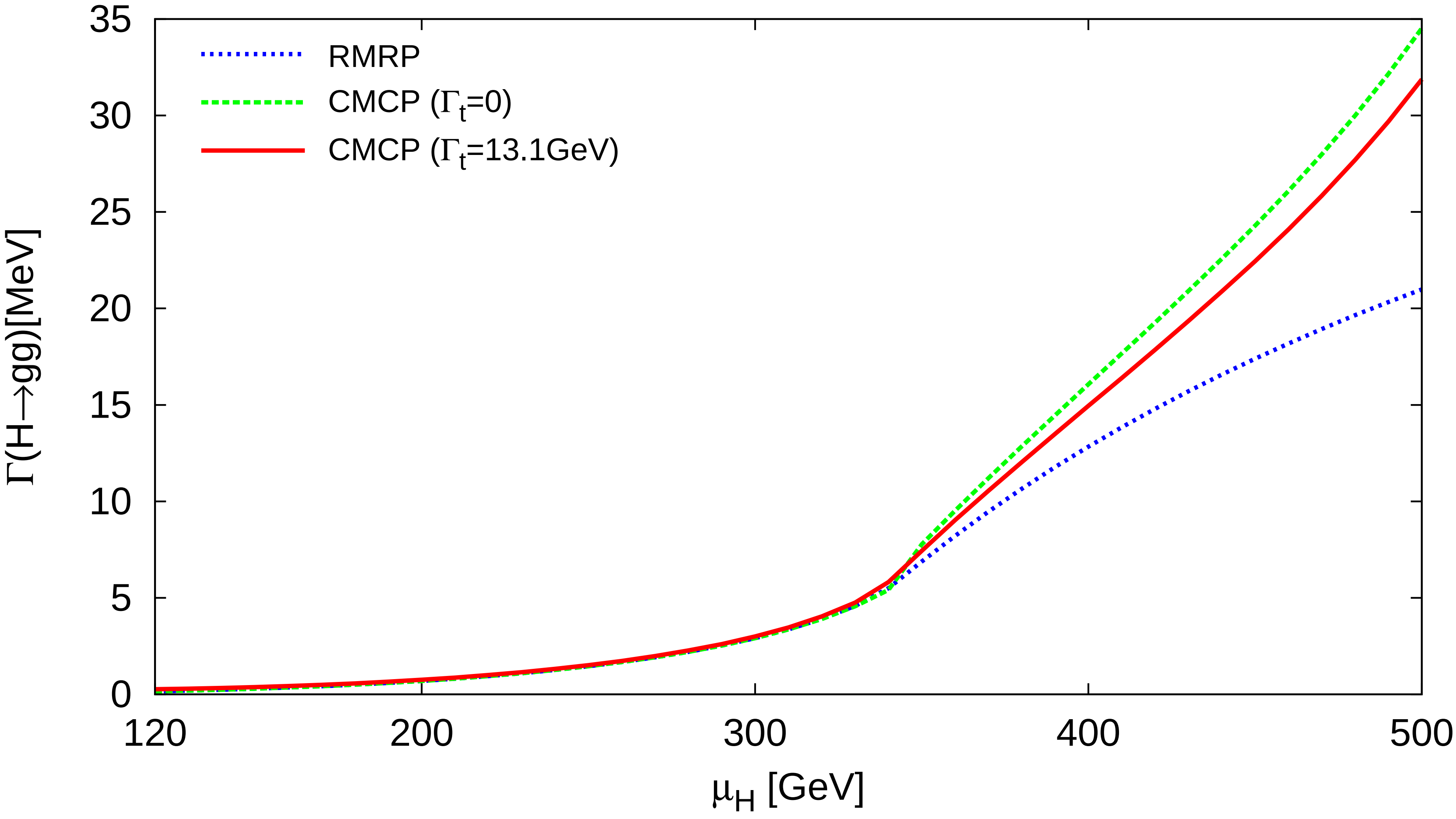
CMRP → Complex Masses and Real Momenta.

The complex mass scheme (Denner-Dittmaier-Roth-Wieders [hep-ph/0505042]) with complex internal W and Z poles (extendable to top complex pole) but with real, external, on-shell Higgs and with the standard LSZ wave-function renormalization.

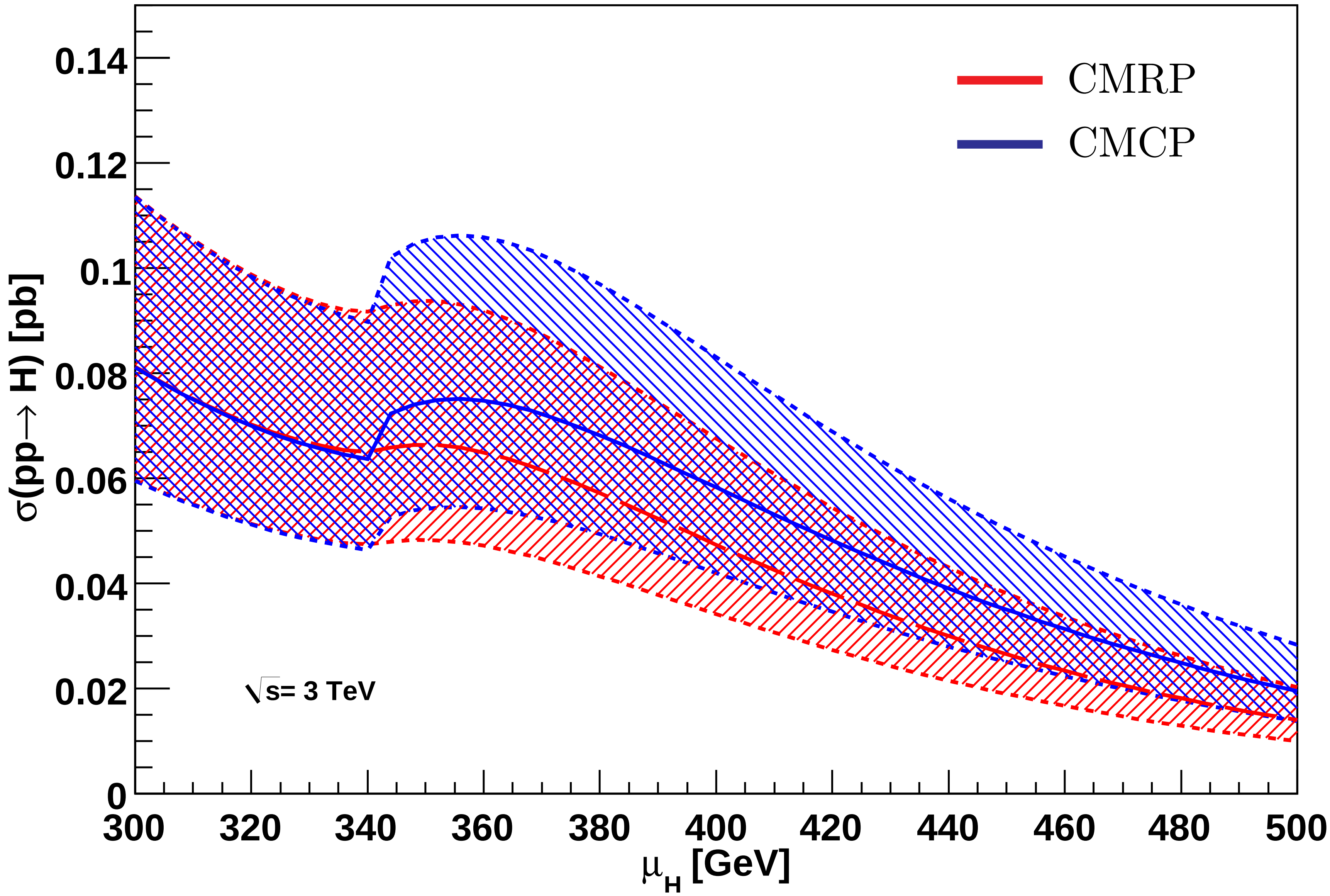
CMCP → Complex Masses and Complex Momenta.

The (complete) complex mass scheme with complex internal W and Z poles and complex, external, Higgs where the LSZ procedure is carried out at the Higgs complex pole (on the second Riemann sheet).

Numerical effects: $H \to gg$



Numerical effects: $pp \to H$



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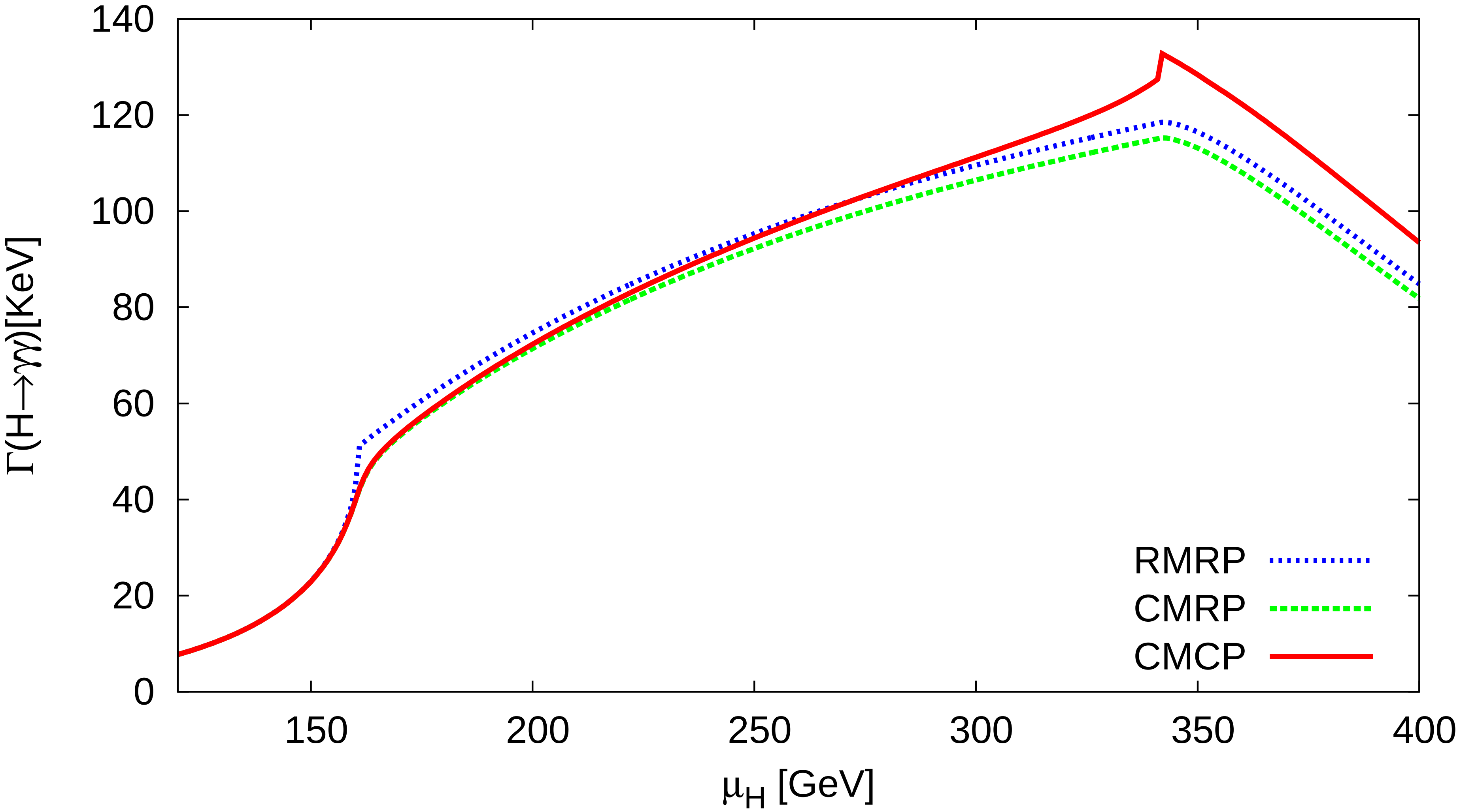
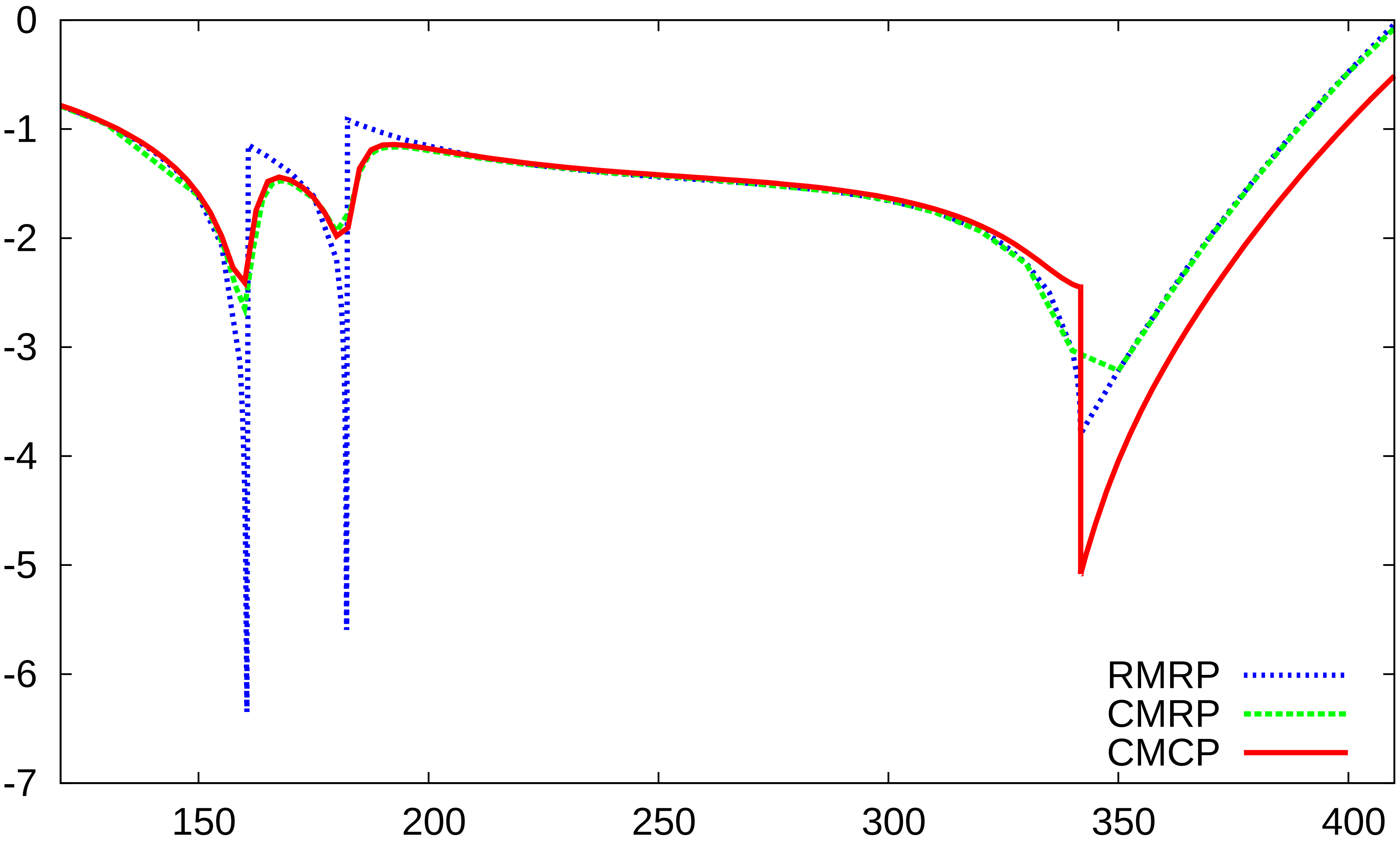


Numerical effects: $H \to \bar{b}b$



Summary

- Proposal for a gauge invariant parametrization of experimental distributions for Higgs physics
- Gauge invariant definition of production cross section and decay width
- Numerical effects: negligible below $t\bar{t}$ threshold, but sizable for large M_H
- **Computational recipe**:

  Analytical continuation and contour distortion for diagrams with complex Mandelstan invariants