The $t\bar{t}$ Total Cross Section at Hadron Colliders at NNLL

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1. Introduction

- We consider the inclusive cross section of pair production of top quarks at the LHC (near) threshold

$$\mathrm{ij}(\mathrm{q\bar{q}}, \mathrm{qg}, \mathrm{gg}) \rightarrow t\bar{t} + X$$

$$\sigma_{t\bar{t}}(m_t, s) = \int_{4m_t^2/s}^{1} dx \mathcal{L}(x, \mu) \hat{\sigma}_{t\bar{t}}(xs, m_t, \mu)$$

$$\beta = \sqrt{1 - 4m_t^2/(xs)}$$

- With the rediscovery of the top quark at the LHC, precision studies of its properties will be performed
  $\Longrightarrow$ Accurate theoretical prediction of the cross section phenomenologically important and currently of much interest.

- Top is expected to couple strongly with the fields responsible for electroweak symmetry breaking $\Longrightarrow$ likely to play a key role in new discoveries.

- Partonic cross section known exactly to NLO.

  [Nason, Dawson and Ellis, 1988; Beenakker, Kuijf, van Neerven and Smith, 1989; Czakon and Mitov, 2008.]

- Enhancement of partonic cross section near threshold $\beta \to 0$

$$\begin{aligned}
\hat{\sigma}_{t\bar{t}} &= \hat{\sigma}^{(0)}_{t\bar{t}} \left[1 + \frac{\alpha_s}{4\pi} \left(\hat{\sigma}^{\mathrm{NLO_{sing}}}_{t\bar{t}} + O(\beta^0) \right) + \left(\frac{\alpha_s}{4\pi} \right)^2 \left(\hat{\sigma}^{\mathrm{NNLO_{sing}}}_{t\bar{t}} + O(\beta^0) \right) + \ldots \right] , \\
\hat{\sigma}^{\mathrm{NLO_{sing}}}_{t\bar{t}} &= \underbrace{a \ln^2 \beta + b \ln \beta}_{\text{"Threshold logarithms"}} + \underbrace{\frac{c}{\beta}}_{\text{"Coulomb singularity"}} , \\
\hat{\sigma}^{\mathrm{NNLO_{sing}}}_{t\bar{t}} &= \hat{\sigma}^{\mathrm{NNLO_{sing}}}_{t\bar{t}} \left(\frac{1}{\beta^2} ; \frac{\ln^{(0,1,2)}(\beta)}{\beta} ; \ln^{(1,2,3,4)}(\beta) \right) .
\end{aligned}$$

[Beneke, Czakon, Falgari, Mitov and Schwinn, 2009.]

- Threshold logarithms:
  soft gluon exchange between initial-initial, initial-final and final-final state particles.
  Resummation in Mellin–space e.g. by

  [Sterman, 1987;Catani, Trentadue, 1989; Kidonakis, Sterman, 1997;Bonciani et.al., 1998;...]

- Coulomb corrections:
  static interactions of slowly moving particles [Fadin, Khoze 1987; Strassler, 1990; NRQCD; ...]

- Enhanced terms can spoil convergence of perturbative series $\Longrightarrow$ Resummation
  - Generally observed to reduce dependence on factorization scale.
  - Allows to predict classes of higher order corrections
  - Accelerated convergence of perturbative series
- Recent applications
  - total top quark cross section [Moch,Uwer, 2008; Cacciari et. al., 2008; Kidonakis, Vogt, 2008.]
  - $t\bar{t}$ invariant mass distribution [Kiyo, Kühn, Moch, Steinhauser, Uwer, 2009; Ahrens, Ferroglia, Neubert, Yang, 2009/10.]
  - squark, gluino production [Kulesza, Motyka 2008/09; Langenfeld, Moch, 2009; Beenakker et.al. 2009/10; Beneke, Falgari, Schwinn, 2010.]
  - Bound–state effects on kinematical distributions of top quarks at hadron colliders [Sumino, Yokoya, 2010.]
- Key idea: factorization into hard, soft and Coulomb functions
  $\Longrightarrow$ joint NNLL resummation of soft and Coulomb gluons.
- Effective-theory prediction of pair production near threshold [Beneke, Falgari, Schwinn 2009/10.] using **SCET+ P(NRQCD)**: valid for arbitrary color representations.

- Parametric representation of the partonic cross section near threshold:

$$\hat{\sigma}_{t\bar{t}} \quad \propto \quad \hat{\sigma}^{(0)}_{t\bar{t}} \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{\beta}\right)^k \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{\ln\beta\, g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \alpha_s \underbrace{\ln\beta\, g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}}\Big]$$

$$\times \Big\{1(\text{LL}, \text{NLL}); \alpha_s, \beta(\text{NNLL}); \alpha_s^2, \alpha_s\beta, \beta^2(\text{NNNLL}); \ldots\ldots\Big\}.$$

- Counting: $\alpha_s/\beta, \alpha_s \ln\beta \propto 1$.
- Fixed order expansion contains all terms of the form

$$\text{LL}: \qquad \alpha_s\Big\{\frac{1}{\beta}, \ln^2\beta\Big\}; \alpha_s^2\Big\{\frac{1}{\beta^2}, \frac{\ln^2\beta}{\beta}, \ln^4\beta\Big\}; \ldots.,$$

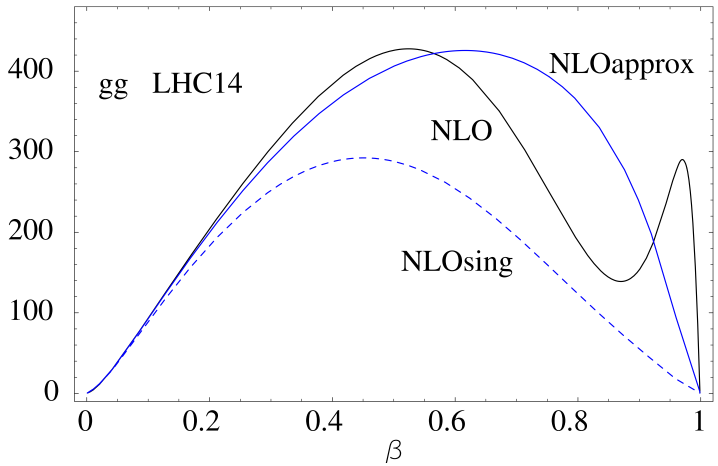
$$\text{NLL}: \qquad \alpha_s \ln\beta; \alpha_s^2\Big\{\frac{\ln\beta}{\beta}, \ln^3\beta\Big\}; \ldots.,$$

$$\text{NNLL}: \qquad \alpha_s\{1, \beta\ln^{2,1}\beta\}; \alpha_s^2\Big\{\frac{1}{\beta}, \ln^{2,1}\beta, \beta\ln^{4,3}\beta\Big\}; \ldots.,$$

- Non–relativistic log summation must be added separately - relevant from NNLL.

Why threshold expansion

- Strictly valid for high masses $2m_t \to s_{\text{had}}$.
- Certainly not for tops at LHC7. Invariant mass distribution peaks at 380 GeV, corresponding to $\beta \approx 0.4$, but the average β is larger.
- Assume that threshold expansion provides a good approximation for the integrals over all β. Works reasonably well for gg at LO and NLO, less well for $q\bar{q}$ and probably better at NNLO, because the average is dominated by smaller β as the order increases.
- multiplying with the exact tree $\hat{\sigma}^{(0)}_{t\bar{t}}$ improves the approximation.



	Tev.	LHC7	LHC14
$\langle\beta\rangle_{gg,\,\text{NLO}}$	0.41	0.49	0.53
LO	5.25	101.9	562.9
NLO	6.50	149.9	842.2
NLO_{sing}	6.76	138.8	751.2
$\text{NLO}_{\text{approx}}$	7.45	159.0	867.6

MSTW2008nnlo PDFs.

2. NNLL Resummation

- Apply NNLL soft resummation and coulomb resummation to total cross section

$$\hat{\sigma}_{t\bar{t}}^{\rm NNLL} = \sum_{i,R_\alpha} H_i(m_t,\mu_h) U_i(m_t,\mu_h,\mu_S,\mu_f) \left(\frac{2m_t}{\mu_S}\right)^{-2\eta} \tilde{s}_i^{R_\alpha}(\partial_\eta,\mu_S) \times \frac{\exp(-2\gamma_E\eta)}{\Gamma(2\eta)} \int_0^\infty \frac{dw}{w} \left(\frac{w}{\mu_S}\right)^{2\eta} J_{R_\alpha}(E-\frac{w}{2},\mu_C)\ ,(E=\sqrt{xs}-m_t)\ .$$

- Different contributions:
  - Hard function H_i depends on the specific process, evaluated at hard scale μ_h.
  - Process–independent soft function $W_i^{R_\alpha}(\propto \alpha_s^n \ln^m \beta)$ translates via a Laplace transform into $\tilde{s}_i^{R_\alpha}(\partial_\eta,\mu_S)$. Evaluated at soft scale μ_S.
  - U_i evolution function from solving the RG equations of the hard and soft functions. (for DY:[Becher, Neubert,Xu, 2007.].)
  - Potential function J_{R_α} encodes Coulomb effects, evaluated at Coulomb–scale μ_C.
- Formula valid except for non-Coulomb corrections at $O(\alpha_s^2)$, which are added separately.
- New:
  - full LO and NLO Coulomb effects to all orders, above and below threshold.
  - full NNLL soft resummation (Note: full NNLL soft resummation for invariant mass distribution in [Ahrens, Ferroglia, Neubert, Yang, 2010.]).

Hard and Soft Function

- NLL needs H_i at tree level, NNLL needs H_i at NLO. Known from [Czakon, Mitov, 2008].
- NNLL needs soft function at NLO. It is given by

$$\tilde{s}^{R_\alpha}(\rho, \mu_S) = 1 + \frac{\alpha_s(\mu_S)}{4\pi}\left[(C_r + C_{r'})\left(\rho^2 + \zeta_2\right) - 2C_{R_\alpha}(\rho - 2)\right] .$$

- The evolution function is given by

$$\begin{aligned} U_i(M, \mu_h, \mu_f, \mu_s) &= \left(\frac{4m_t^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu_S)} \left(\frac{\mu_h^2}{\mu_S^2}\right)^{\eta} \times \exp\Big[4(S(\mu_h, \mu_f) - S(\mu_S, \mu_f)) \\ &\quad - 2a_i^V(\mu_h, \mu_S) + 2a^{\phi,r}(\mu_S, \mu_f) + 2a^{\phi,r'}(\mu_S, \mu_f)\Big], \end{aligned}$$

where $a(\mu_1, \mu_2), S(\mu_1, \mu_2)$ denote integrated anomalous dimensions.

- Resummation controlled by cusp and soft anomalous dimensions $\Gamma^r_{\rm cusp}, \gamma_i^V, \gamma^r, \gamma^{R_\alpha}_{H,s}$.

Coulomb effects

- For NNLL: J_{R_α} needed at NLO. Resummation of Coulomb effects well understood from PNRQCD and quarkonia physics. The LO Coulomb function reads

$$J_{R_\alpha} = -\frac{m_t^2}{2\pi}\mathrm{Im}\left(\sqrt{-\frac{E}{m_t}} - D_{R_\alpha}\alpha_s\left[\frac{1}{2}\ln\left(-\frac{4m_tE}{\mu^2}\right) - \frac{1}{2} + \gamma_E + \psi\left(1 + \frac{D_{R_\alpha}\alpha_s}{2\sqrt{-E/m_t}}\right)\right]\right) .$$

- Above threshold, $E > 0$, the potential function evaluates to the Sommerfeldt factor

$$J_{R_\alpha} = \frac{1}{2}\frac{m_t^2 D_{R_\alpha}\alpha_s}{\exp\left(\pi D_{R_\alpha}\alpha_s\sqrt{m_t/E}\right) - 1} .$$

- For an attractive potential, $D_{R_\alpha} < 0$, there is a sum of bound states below threshold:

$$J_{R_\alpha} = -2\sum_{n=1}^{\infty}\left(\frac{m_t D_{R_\alpha}\alpha_s}{2n}\right)^n \delta(E + E_n) \ , E_n = \frac{m_t\alpha_s^2 D_{R_\alpha}^2}{4n^2} .$$

(see also:[Fadin, Kohze 1987; Kiyo et.al. 2009; Hagiwara, Yokoya 2009].)

- Non–Coulomb corrections can be derived from the non–Coulomb potential

$$\hat{\sigma}_{t\bar{t}}^{\mathrm{NC}} = \hat{\sigma}_{t\bar{t}}^{(0)}\alpha_s^2 \ln\beta\left[-2D_{R_\alpha}(1 + v_{\mathrm{spin}}) + D_{R_\alpha}C_A\right] .$$

[Beneke, Signer, Smirnov 1999; Pineda, Signer, 2006; Beneke, Czakon, Falgari, Mitov, Schwinn,2009.].

Scale Choice

- We use $m_t = 173.1\,\mathrm{GeV}$ and set $\mu_f = \mu_R = m_t$.
- Identify hard scale and factorization scale: $\mu_h \propto \mu_f$.
  $\Longrightarrow$ No large logs of the hard scale ($\ln(\mu_h/\mu_f)$).
- The form of the approximate NLO corrections implies that $\mu_S \approx 8 m_t \beta^2$. However, this choice might lead to an ill–defined convolution with the parton luminosity
  [Becher, Neubert, Pecjak, 2007; Becher, Neubert, Xu, 2008.]
  $\Longrightarrow$ Choose μ_S such that one–loop corrections to the hadronic cross section are minimized. This guarantees well-behaved perturbative expansion at the low scale μ_S.
- The choice $\mu_C \propto m_t \beta$ is required to sum correctly all NNLL terms. Additionally, the relevant scale for the bound state effects is set by the inverse Bohr radius of the $t\bar{t}$ bound state and we set

$$\mu_C = \max\{2 m_t \beta, C_F m_t \alpha_S(\mu_C)\} \ .$$

- Note: only the choice $\mu_S \propto m_t \beta^2$, $\mu_C \propto m_t \beta$ reproduces correctly the threshold expansion in β

3. Preliminary Results

- We match the resummed cross section onto the full NLO result

[Zerwas et.al., 1996; Langenfeld, Moch, 2009.]

$$\hat{\sigma}_{t\bar{t}}^{\text{approx}} \propto \hat{\sigma}_{t\bar{t}}^{\text{N(N)LL}} - \hat{\sigma}_{t\bar{t}}^{\text{N(N)LL}}\Big|_{\alpha_s^{1,2}} + \hat{\sigma}_{t\bar{t}}^{\text{NLO}} \; .$$

- For NLL resummation, we previously considered

$$\begin{aligned} \hat{\sigma}_{t\bar{t}}^{\text{NNLO}_{\text{approx}}} &= \hat{\sigma}_{t\bar{t}}^{\text{NLO}} + \hat{\sigma}_{t\bar{t}}^{\text{NNLO}_{\text{sing}}} \; , \\ \hat{\sigma}_{t\bar{t}}^{\text{NNLO}_{\text{approx}}+\text{NLL}} &= \hat{\sigma}_{t\bar{t}}^{\text{NLL}} - \hat{\sigma}_{t\bar{t}}^{\text{NLL}}\Big|_{\alpha_s^2} + \hat{\sigma}_{t\bar{t}}^{\text{NNLO}_{\text{approx}}} \; . \end{aligned}$$

(Note: $\hat{\sigma}_{t\bar{t}}^{\text{NC}}$ included in $\hat{\sigma}_{t\bar{t}}^{\text{NNLO}_{\text{sing}}}$.)

- A natural choice for NNLL resummation would be

$$\hat{\sigma}_{t\bar{t}}^{\text{NNLL}(\alpha_s)} = \hat{\sigma}_{t\bar{t}}^{\text{NNLL}} - \hat{\sigma}_{t\bar{t}}^{\text{NNLL}}\Big|_{\alpha_s} + \hat{\sigma}_{t\bar{t}}^{\text{NC}} + \hat{\sigma}_{t\bar{t}}^{\text{NLO}} \; .$$

- Due to the limitations in choosing the soft scale running, we consider as our best approximation:

$$\hat{\sigma}_{t\bar{t}} \equiv \hat{\sigma}_{t\bar{t}}^{\text{NNLL}(\alpha_s^2)} = \hat{\sigma}_{t\bar{t}}^{\text{NNLL}} - \hat{\sigma}_{t\bar{t}}^{\text{NNLL}}\Big|_{\alpha_s^2} + \hat{\sigma}_{t\bar{t}}^{\text{NNLO}_{\text{approx}}} \; .$$

Using the MSTW2008nnlo and ABKM09 PDF sets, we obtain

		Tevatron	LHC7	LHC10	LHC14
NLO	MSTW08	$6.50\,^{+0.32+0.33}_{-0.70-0.24}$	$150\,^{+18+8}_{-19-8}$	$380\,^{+44+17}_{-46-17}$	$842\,^{+97+30}_{-97-32}$
	ABKM09	$6.43\,^{+0.23+0.15}_{-0.61-0.15}$	$122\,^{+13+7}_{-15-7}$	$322\,^{+36+15}_{-38-15}$	$738\,^{+81+27}_{-83-27}$
$\text{NNLO}_{\text{approx}}$	MSTW08	$7.13\,^{+0.00+0.36}_{-0.33-0.26}$	$162\,^{+3+9}_{-3-9}$	$407\,^{+11+17}_{-5-18}$	$895\,^{+29+31}_{-7-33}$
	ABKM09	$7.01\,^{+0.06+0.18}_{-0.36-0.18}$	$132\,^{+2+8}_{-2-8}$	$345\,^{+8+16}_{-3-16}$	$785\,^{+22+29}_{-6-29}$
$\text{NNLO}_{\text{approx}}$ + NLL	MSTW08	$7.13\,^{+0.08+0.36}_{-0.41-0.26}$	$162\,^{+2+9}_{-1-9}$	$407\,^{+9+17}_{-2-18}$	$895\,^{+23+31}_{-4-33}$
	ABKM09	$7.00\,^{+0.13+0.18}_{-0.44-0.18}$	$132\,^{+1+8}_{-1-8}$	$345\,^{+6+16}_{-1-16}$	$784\,^{+17+29}_{-3-29}$
NNLL(New)	MSTW08	$7.14\,^{+0.13+0.36}_{-0.19-0.26}$	$162\,^{+4+9}_{-2-9}$	$407\,^{+14+17}_{-4-18}$	$896\,^{+36+31}_{-7-33}$
	ABKM09	$7.00\,^{+0.14+0.18}_{-0.21-0.18}$	$132\,^{+3+8}_{-1-8}$	$345\,^{+10+16}_{-3-16}$	$785\,^{+27+29}_{-5-29}$

- First error denotes scale uncertainty, second PDF error.
- We observe an enhancement of the cross section of $5-10\%$.
- Using the expanded tree instead of the full tree changes the result only by less than 1%.
- Scale uncertainty significantly reduced.

- For $\hat{\sigma}_{t\bar{t}}^{\mathrm{NNLL}(\alpha_{\mathrm{s}})}$, we obtain for the central values (MSTW2008nnlo):

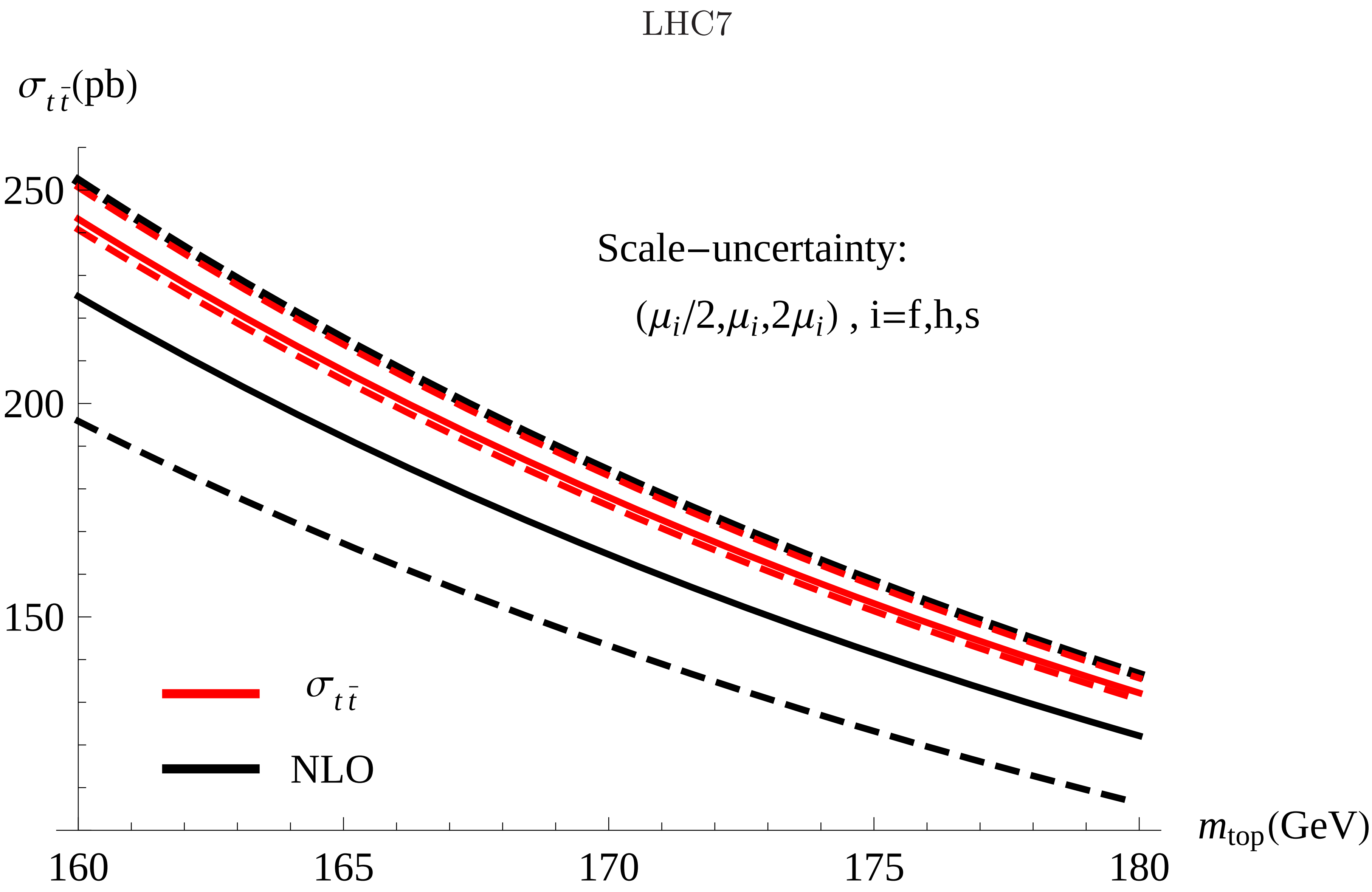
	Tevatron	LHC7	LHC10	LHC14
NNLL(α_s)	6.75	155	392	865

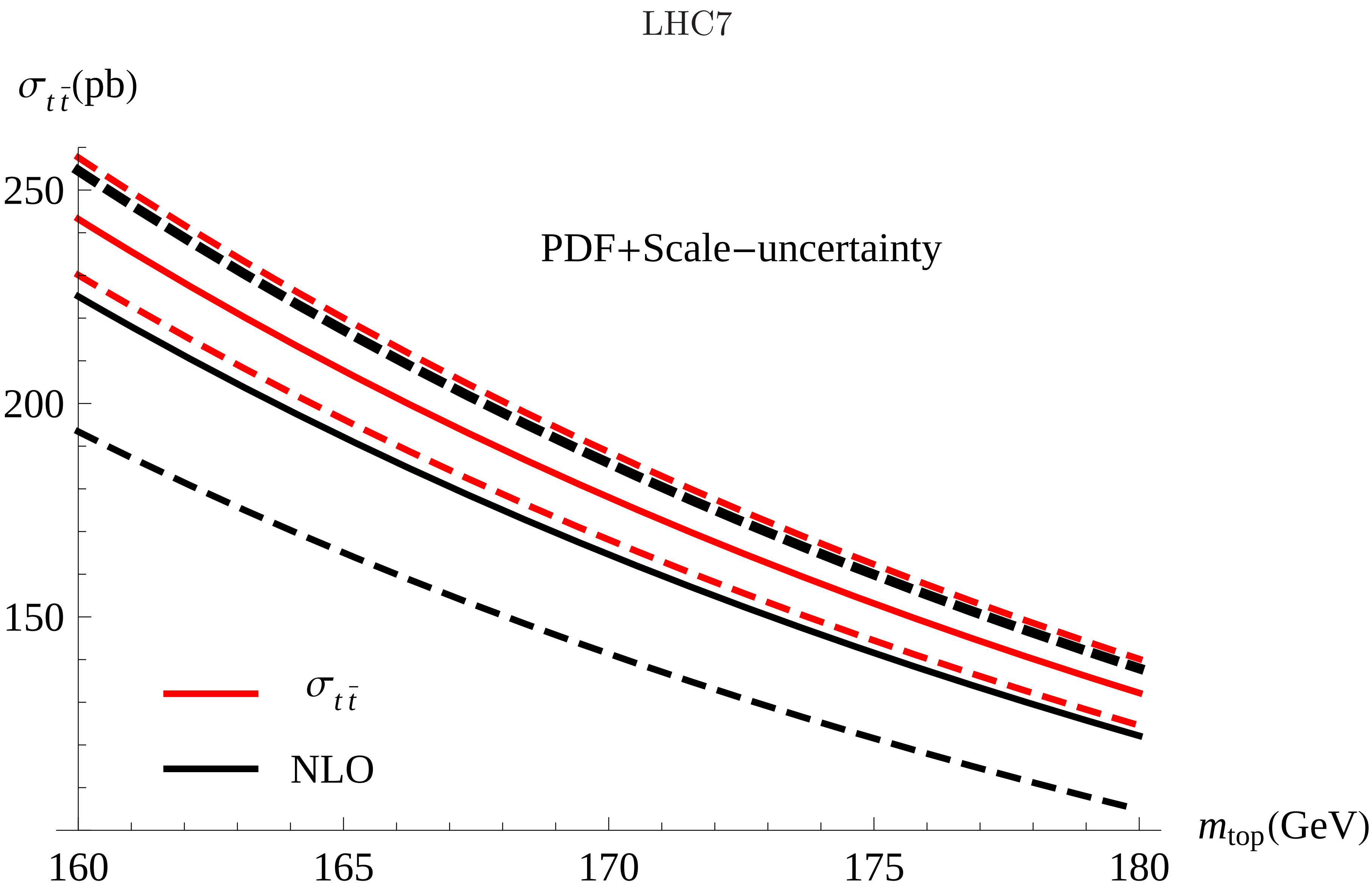
$\Longrightarrow$ Correct description is important. Choose μ_S running?

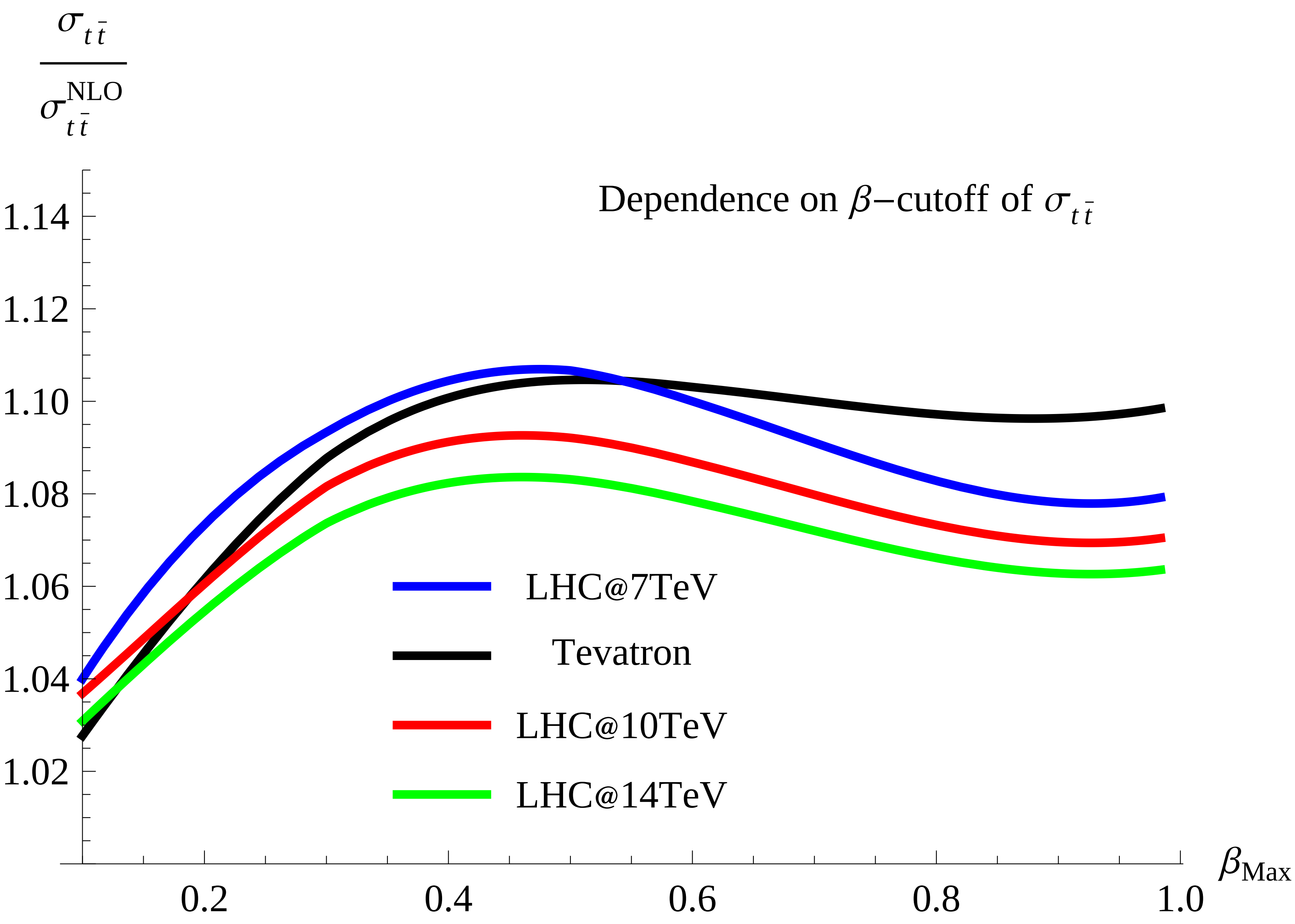
- The NNLL corrections can be split into soft- and Coulomb- corrections. One obtains for LHC@7TeV

$$
\begin{aligned}
\sigma_{t\bar{t}}^{\mathrm{NNLL}}(\mathrm{LO}_{\mathrm{soft}} \times \mathrm{LO}_{\mathrm{Cb}}) &= 166.7 \text{ pb} , \\
\sigma_{t\bar{t}}^{\mathrm{NNLL}}(\mathrm{LO}_{\mathrm{soft}} \times \mathrm{NLO}_{\mathrm{Cb}}) &= -0.5 \text{ pb} , \\
\sigma_{t\bar{t}}^{\mathrm{NNLL}}(\mathrm{NLO}_{\mathrm{soft}} \times \mathrm{LO}_{\mathrm{Cb}}) &= 29.6 \text{ pb} , \\
\sigma_{t\bar{t}}^{\mathrm{NNLL}}(\mathrm{NLO}_{\mathrm{soft}} \times \mathrm{NLO}_{\mathrm{Cb}}) &= -0.8 \text{ pb} ,
\end{aligned}
$$

$\Longrightarrow$ The soft corrections dominate, as well for the other energies. This can be explained by the observation that the gg–channel is color octet dominated.


LHC7
$\sigma_{t\bar{t}}$(pb)
250
200
150
Scale−uncertainty:
($\mu_i/2, \mu_i, 2\mu_i$) , i=f,h,s
$\sigma_{t\bar{t}}$
NLO
160
165
170
175
180
m_{top}(GeV)


LHC7
$\sigma_{t\bar{t}}$(pb)
250
200
150
PDF+Scale−uncertainty
$\sigma_{t\bar{t}}$
NLO
$m_{\rm top}$(GeV)
160
165
170
175
180



$\Longrightarrow$ From about $\beta \approx 0.4$, the dependence on $\beta_{\max}$ is small.

Theoretical Uncertainties

- PDF–error is of the order of 5%.
- Scale–uncertainty is of the order of $1-5\%$.
- Ambiguity in the resummation prescription: How to choose μ_S?
  Related: use NNLO singular terms?
- For LHC@7, the central values of the different contributions are given by:

$$\begin{aligned} \sigma_{t\bar{t}}^{\rm NNLL} - \sigma_{t\bar{t}}^{\rm NNLL}\Big|_{\alpha_s^2} &= 0.1 \text{ pb} , \quad (\sigma_{t\bar{t}}^{\rm BST} = 0.8 \text{ pb}) , \\ \sigma_{t\bar{t}}^{\rm NNLO_{sing}} &= 12.1 \text{ pb} , \quad (\sigma_{t\bar{t}}^{\rm NC} = 0.5\text{pb}) . \end{aligned}$$

  $\Longrightarrow$ the corrections are mainly dominated by the NNLO–singular terms.

- Estimate the NNLO constant term by comparing the NNLO singular terms to the ratio of the NLO singular and NLO constant term for the average value of $\langle\beta\rangle \approx 0.4$:

$$\Delta\hat{\sigma}_{t\bar{t}}^{\rm NNLO_{const}} \approx \pm 10\text{pb} .$$

  $\Longrightarrow$ Would fit nicely with the $\hat{\sigma}_{t\bar{t}}^{{\rm NNLL}(\alpha_{\rm s})}$- prescription.

Comparisons

- [Moch, Uwer, 2008.]:
  NLL–resummation in Mellin–space and NNLO–singular terms.
  - Different PDFs, NNLO-singular terms
  - Agreement on the level of 1% for the singular terms

- [Ahrens, Ferroglia, Neubert, Yang, 2010.]:
  - NNLL resummation of soft threshold logarithms in x–space of the invariant mass distribution
    $\Longrightarrow$ Coulomb singularities do not appear completely (added "by hand" for comparisons).
  - Subtraction terms determined by setting all scales equal in the resummed result.
  - Their best result for $\mu_f = m_t$ tends to be slightly smaller than the exact NLO result, whereas ours is $5\% - 10\%$ bigger. Still agreement within the error.

4. Conclusions

- First joint NNLL threshold resummation of soft and Coulomb gluons, including non-Coulomb effects, for $t\bar{t}$ production at hadron colliders.
- We presented first preliminary results and observe an enhancement of the total cross section of $5 - 10\%$.
- Significant reduction of the scale dependence.
- In our current implementation, the effect compared to NLL resummation is negligible due to the choice of the soft scale μ_S.
- Work in progress: correct treatment of μ_S.