High-energy resummation for rapidity distributions

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High-energy resummation for rapidity distributions

Resummation of large logs:

Logs must be under control for high precision physics.
Basically two classes of large logs:

- Sudakov logs
- High-energy (or small-x) logs

Sudakov resummation:

Formalism known in all interesting cases:

- Inclusive cross sections
- Rapidity distributions
- k_T distributions...

High-energy resummation for rapidity distributions

High energy resummation:

- Up to now: simple recipe only for the inclusive case [Catani, Ciafaloni, Hautmann (1991)]
  - Corrections can be as large as NNLO
- Not enough! Extension to differential quantities needed:
  - Better resolution of PDFs $x-$dependence
  - “Cure” perturbative instabilities at small-x (e.g. DY)
  - Needed for resummed PDFs fit

In the following:

Resummation formalism for rapidity distributions

Outline:

Resumption for inclusive quantities

- The standard argument [Catani, Ciafaloni, Hautmann (1991)]:
  - k_T-factorization, BFKL and the gluon Green function
- A different perspective:
  - Collinear factorization
  - Iteration *à la* Curci, Furmanski, Petronzio
  - DGLAP-BFKL duality

Extension to rapidity distributions

- Rapidity and kinematics at small-x
- Rapidity evolution along a CFP ladder

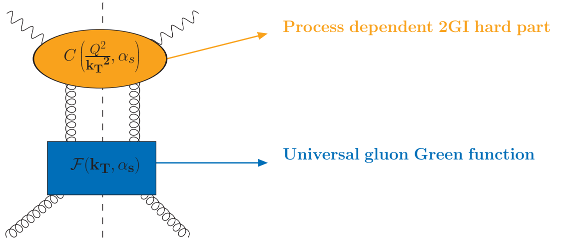
A phenomenological playground: Higgs

- Higgs: dominated by large-x region
- Small-x: control on the $m_t \to \infty$ approximation

Resummation:
Inclusive cross sections

The k_T-factorization theorem (Catani, Ciafaloni, Hautmann (1991)

Power counting plus kinematics at $Q^2 \ll S$:



k_T-factorization formula

$$\sigma = \int \frac{dz}{z} \frac{d\mathbf{k_T}^2}{\mathbf{k_T}^2} C\left(\frac{x}{z}, \frac{Q^2}{\mathbf{k_T}^2}\right) \mathcal{F}(z, \mathbf{k_T}^2)$$

- Process dep. C: off-shell cross section with eikonal gluons
- Universal gluon Green function $\mathcal{F}$: solution of BFKL equation

k_T-factorization and resummation

Factorization in Mellin space

$$\sigma = \int \frac{dz}{z}\frac{d\mathbf{k_T}^2}{\mathbf{k_T}^2} C\left(\frac{x}{z}, \frac{Q^2}{\mathbf{k_T}^2}\right) \mathcal{F}(z, \mathbf{k_T}^2)$$

Undo the convolution in Mellin space $\rightarrow$ define the *impact factor*

$$h(M, N) \equiv M \int_0^1 dx x^{N-1} \int_0^\infty d\mathbf{k_T}^2 (\mathbf{k_T}^2)^{M-1} C(x, \mathbf{k_T}^2)$$

BFKL evolution of $\mathcal{F}$ gives the condition $M = \gamma_s\left(\frac{\alpha_s}{N}\right)$

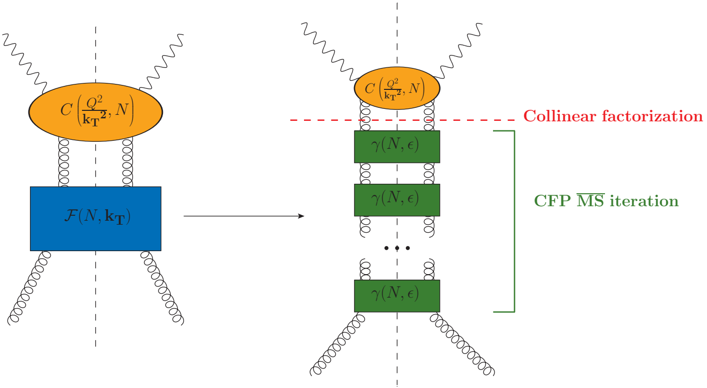
Resummed result:

$$\sigma(N) = h\left(\gamma_s\left(\frac{\alpha_s}{N}\right), N\right) \approx h\left(\gamma_s\left(\frac{\alpha_s}{N}\right), 0\right)$$

Power series in $\frac{\alpha_s}{N} \longrightarrow \alpha_s \ln x$

Inclusive resummation: A new perspective

- Do not solve BFKL for $\mathcal{F}$
- $\mathcal{F}$: CFP $t-$channel iteration of collinear safe kernels γ



- k_T dependence is now trivial (γ: k_T-independent)
- Non trivial information now encoded in γ

Inclusive resummation: A new perspective

$\overline{\rm MS}$ result for n kernels

$$\sigma_n\left(N, Q^2, \alpha_s\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}, \epsilon\right) = \gamma\left(N, \alpha_s\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}, \epsilon\right)\int_0^\infty \frac{d\xi_n}{\xi_n^{1+\epsilon}} C\left(N, \xi_n, \alpha_s\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}, \epsilon\right) \times$$

$$\times \frac{1}{(n-1)!}\frac{1}{\epsilon^{n-1}}\left[\sum_i \frac{\bar{\alpha}_s^i}{i}\gamma_i(N,0)\left(1-\left(\frac{\mu^2}{Q^2\xi_n}\right)^{i\epsilon}\frac{\gamma_i(N,\epsilon)}{\gamma_i(N,0)}\right)\right]^{n-1}$$

- Small-x information encoded in universal functions $\gamma(N,\epsilon)$
- Control over the factorization scale μ
- Easy to incorporate running coupling effects

Who is γ?

γ is k_T independent $\rightarrow$ reconstructed from collinear limit!

γ is a (generalized) anomalous dimension (residue of a coll. pole)

The full result exponentiates:

$$\sigma(N, Q^2, \alpha_s) = \gamma(N, \alpha_s) \int_0^\infty d\xi \xi^{\gamma(N,\alpha_s)-1} C(N, \xi, Q^2, \alpha_s),$$

with $\xi \equiv \frac{\mathbf{k_T}^2}{Q^2}$

- The small-x anomalous dimension:
  BFKL-DGLAP duality $\longrightarrow \gamma(N, \alpha_s) = \gamma_s\left(\frac{\alpha_s}{N}\right)$

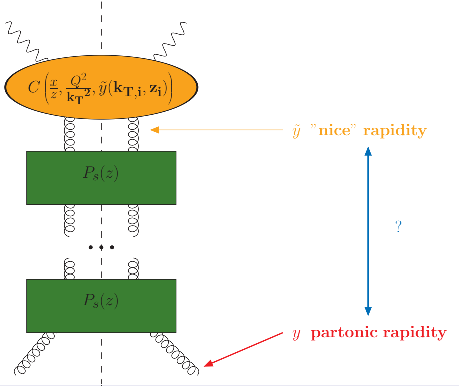
$\Downarrow$

We have recovered the Catani, Ciafaloni, Hautmann result!

Resummation:
Rapidity distributions

Upstairs vs. downstairs rapidity

Towards a factorized formula:



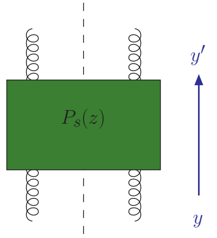
- Process dependent rapidity distribution C: distribution in $\tilde{y}$
- True distribution: partonic rapidity y

Kinematical problem!

How to relate $\tilde{y} \longleftrightarrow y$?

Rapidity evolution along the ladder

Rapidity evolution at small-x



After each kernel:

- The effect of $P_s(z)$ on y: longitudinal boost
- In the small-x limit: $y' = y + \frac{1}{2} \ln z$

Relating $\tilde{y}$ to y

At small-x the relation is very simple!

$$\tilde{y} = y + \frac{1}{2} \ln z_1 z_2 \ldots z_n$$

A resummed formula

The "right" space

This time everything factorizes in Fourier-Mellin space:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y}(N,b) \equiv \int \mathrm{d}x\ x^{N-1} \int \mathrm{d}y\ e^{iby}\frac{\mathrm{d}\sigma}{\mathrm{d}y}(x,y)$$

Not a surprise, see collinear factorization!

The resummed result

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}y}(N,b) &= \int_0^\infty d\xi_1\gamma_s\left(N+\frac{ib}{2}\right)\xi_1^{\gamma_s\left(N+\frac{ib}{2}\right)-1}\times \\ &\times \int_0^\infty d\xi_2\gamma_s\left(N-\frac{ib}{2}\right)\xi_2^{\gamma_s\left(N-\frac{ib}{2}\right)-1} C(N,\xi_1,\xi_2,b)\end{aligned}$$

with $\xi \equiv \frac{\mathbf{k_T}^2}{Q^2}$

A resumed formula

The resummed result

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y}(N,b) = \int_0^\infty d\xi_1 \gamma_s\left(N+\frac{ib}{2}\right)\xi_1^{\gamma_s\left(N+\frac{ib}{2}\right)-1} \times$$
$$\times \int_0^\infty d\xi_2 \gamma_s\left(N-\frac{ib}{2}\right)\xi_2^{\gamma_s\left(N-\frac{ib}{2}\right)-1} C(N,\xi_1,\xi_2,b)$$

Some comments:

- Non universal part: off-shell rapidity distribution $C(N,\xi,b)$ Computed with eikonal off-shell gluons (see inclusive)
- For $b=0 \longrightarrow$ inclusive result OK!
- Full $\overline{\mathrm{MS}}$ computation *ab initio*
- Full μ dependence under control
- Note similarities with collinear factorization!

A phenomenological playground:
Higgs rapidity distribution

Higgs rapidity distribution

Higgs dominated by large-x

Why Higgs rapidity distribution at small-x?

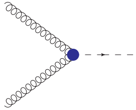
- Higgs: simplest possible case (1 particle in the final state)
- Analytic results exist (Anastasiou, Dixon, Melnikov (2003)) $\longrightarrow$ cross-check of our method!
- Small-x: very hard gluons $\longrightarrow$ sensitive to finite m_t effect
  Match small-x to (N)NLO to asses quality of HEFT
  (At NLO: : Anastasiou, Bucherer, Kunszt (2009))

Higgs Effective Theory

Point-like approximation and small-x

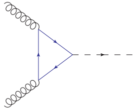
Small-x: high-energy gluon $\longrightarrow$ can resolve the loop

$m_t \longrightarrow \infty$ (HEFT)



$$\sigma \sim \sigma_0 \times \left(\delta(1-x) + \sum_k c_k \alpha_s^k \ln^{2k-1} \frac{1}{x} \right)$$

Finite m_t



$$\sigma \sim \sigma_0 \times \left(\delta(1-x) + \sum_k c_k \alpha_s^k \ln^{k-1} \frac{1}{x} \right)$$

Small-x sensitive to finite m_t effects!

Use (N)NLO + small-x to assess finite m_t corrections

Inclusive: Marzani et al. (2009); Harlander, Ozeren (2009); Pak et al. (2010); Harlander et al. (2010)

The small-x NLO rapidity distribution: LLx contribution

$m_t \to \infty$ approximation

Introduce $u \equiv \exp(-2y)$

$$\frac{d\sigma}{du}(x,u) = 3\sigma_0 \frac{\alpha_s}{\pi}\left[\frac{1}{(u-x)_+} - \delta(u-x)\ln x + \left(u \leftrightarrow \frac{1}{u}\right)\right]$$

- In agreement with Anastasiou, Dixon, Melnikov (2003) ✓
- Note that in rapidity distributions small-$x \neq \ln x$!

Finite m_t

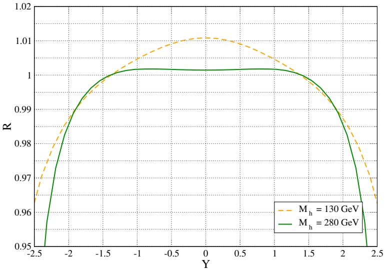
Our result:

$$\frac{d\sigma}{du} = \sigma_0(\tau)c_1(\tau)\delta(u-x) + \left(u \leftrightarrow \frac{1}{u}\right), \quad \tau \equiv \frac{4m_t^2}{m_h^2}$$

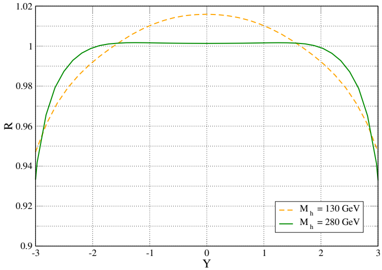
Different partonic rapidity distributions!

Matching and $K-$factor at NLO: Effects at the % level!

$$K = \left(\frac{1}{\sigma_{NLO}} \frac{d\sigma_{NLO}}{dY}\right)_{m_t \to \infty} \Big/ \left(\frac{1}{\sigma_{NLO}} \frac{d\sigma_{NLO}}{dY}\right)$$



$\sqrt{S} = 14$ TeV



- $\sqrt{S} = 7$ TeV
- $\mu_R = \mu_F = m_H$
- NNPDF2.0 central set

Consistent with Anastasiou et al. (2009)

Conclusions

A simple recipe for computing resummed rapidity distributions

- Everything in terms of an off-shell rapidity distribution
- Factorization in Fourier-Mellin space

Application: finite m_t effects in Higgs rapidity distributions

- NLO: effects within 5%, as in Anastasiou et al. (2009)
- NNLO (preliminary): - negligible effects at 7 TeV
  - at most 2% at 14 TeV

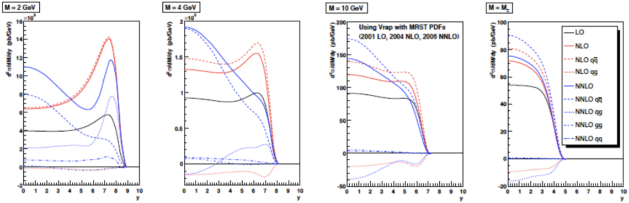
Outlook

Extend to other processes: Drell-Yan!

Systematics

Geometric and kinematic acceptance:

A word about low mass Drell-Yan



LO - NLO - NNLO convergence gets worse as you go to lower masses

Jonathan Anderson, VRAP with MRST PDFs

Extend beyond LL*x* accuracy