Precise Higgs boson masses in the MSSM

Matthias Steinhauser

KIT

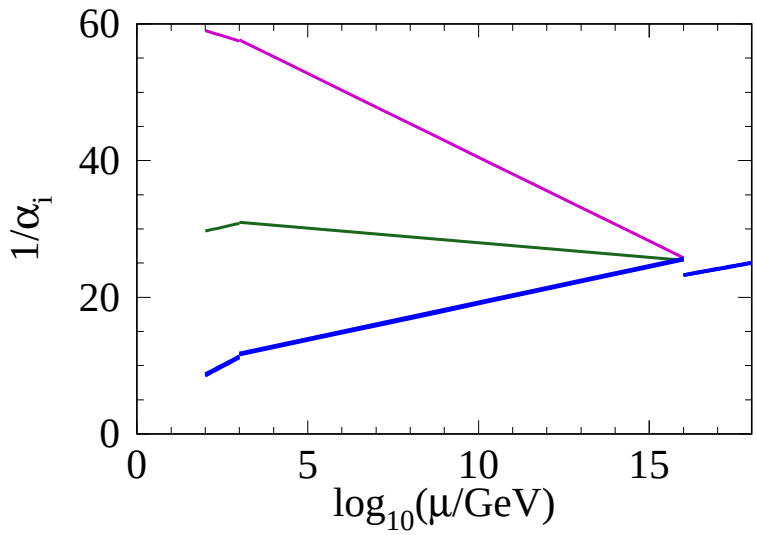
September 2010, HP2.3rd, Florence

in collaboration with Robert Harlander, Philipp Kant, Luminita Mihaila

Outline

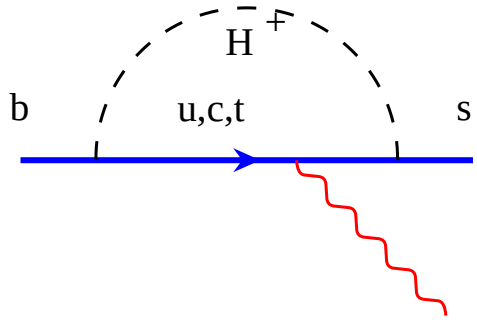
- Motivation
- M_h to 3 loops
- Conclusions

Why many loops (for the MSSM)?

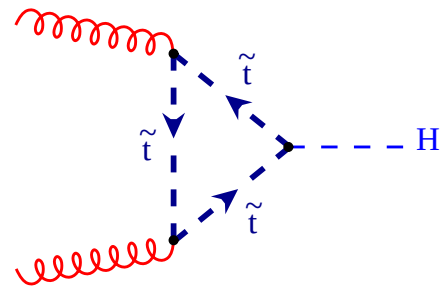


M_h in the MSSM
is a prediction

rare SM processes
e.g.: $\mathcal{B}(\bar{B} \to X_s\gamma)$



$\sigma(gg \to H)$



Higgs boson mass in the MSSM

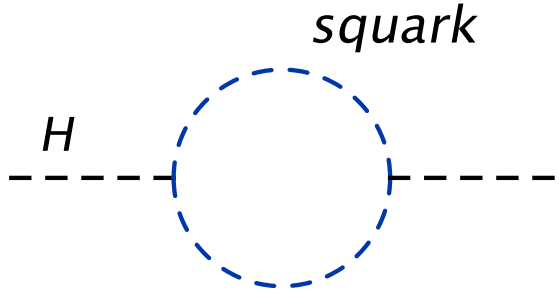
- MSSM: 5 Higgs bosons: h, H, A, $H^\pm$
- prediction of M_h
- tree-level:

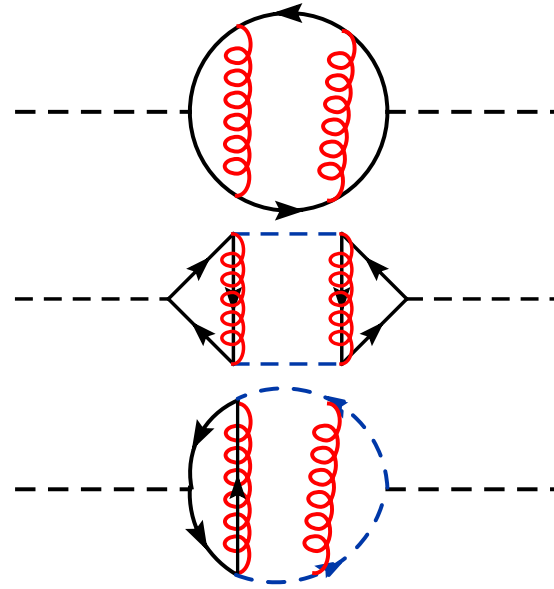
$$\mathcal{M}^2_{H,tree} = \frac{\sin 2\beta}{2} \begin{pmatrix} M_Z^2 \cot\beta + M_A^2 \tan\beta & -M_Z^2 - M_A^2 \\ -M_Z^2 - M_A^2 & M_Z^2 \tan\beta + M_A^2 \cot\beta \end{pmatrix}$$

➪ $M_h \leq M_Z$

- quantum corrections:

$$\mathcal{M}^2_H = \mathcal{M}^2_{H,tree} + \begin{pmatrix} \hat{\Sigma}_{\phi_1} & \hat{\Sigma}_{\phi_1\phi_2} \\ \hat{\Sigma}_{\phi_1\phi_2} & \hat{\Sigma}_{\phi_2} \end{pmatrix}$$





Higgs boson mass in the MSSM (2)

- expected precision:
  LHC: $\delta M_h \sim 0.1 - 0.2$ GeV
  ILC: $\delta M_h \sim 0.050$ GeV
- uncertainty based on 2-loop calculations: 3-5 GeV

[Degrassi et al.'03; Allanach et al.'04]

Known results

- $\Delta M_h^{(1)}$: [Ellis,Ridolfi,Zwirner'91; Okada,Yamaguchi,Yanagida'91; Haber,Hempfling'91]
- $\Delta M_h^{(2)}$: [. . . many authors . . . , Haber et al.'97; Degrassi,Slavich,Zwirner'01; Allanach et al.'04; Heinemeyer'06; Frank et al.'06; Martin'03]
- $\Delta M_h^{(3)}$, leading logs: [Martin'07]

- `FeynHiggs` [Frank,Hahn,Heinemeyer,Hollik,Rzehak,Weiglein]
- `CPSuperH` [Lee,Pilaftsis,Carena,Choi,Drees,Ellis,Wagner]

➪ remaining uncertainty: 3-5 GeV [Degrassi et al.'03; Allanach et al.'04]
2-3 GeV from (unknown) 3-loop result

- $\Delta M_h^{(3)}$: [Kant,Harlander,Mihaila,Steinhauser'08'10]

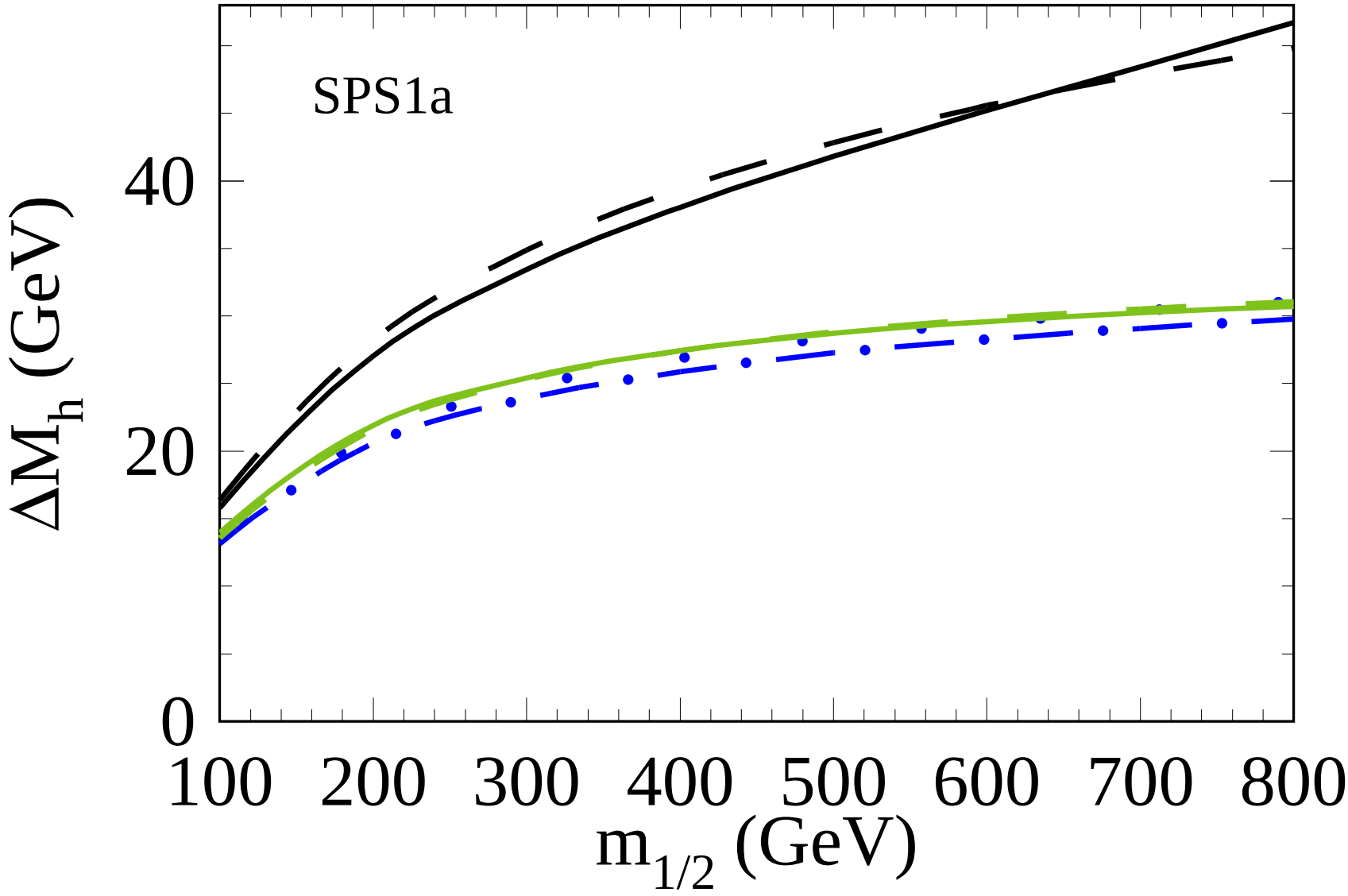
Framework

- g, $\tilde{g}$, t, $\tilde{t}$, q, $\tilde{q}$, c, ϵ; mass scales: m_t, $m_{\tilde{t}1}$, $m_{\tilde{t}2}$, $m_{\tilde{g}}$, $m_{\tilde{q}}$
- DRED; $\overline{\rm DR}$ renormalization [Siegel'79]
- $\hat{\Sigma}(q=0)$, leading contribution: $\sim G_F m_t^4 \alpha_s^2$
- consider hierarchies + asymptotic expansion

$$m_{\tilde{q}} \approx m_{\tilde{t}_1} \approx m_{\tilde{t}_2} \approx m_{\tilde{g}}$$
$$m_{\tilde{q}} \gg m_{\tilde{t}_1} \approx m_{\tilde{t}_2} \approx m_{\tilde{g}}$$
$$m_{\tilde{q}} \gg m_{\tilde{t}_2} \gg m_{\tilde{t}_1} \approx m_{\tilde{g}}$$
$$m_{\tilde{q}} \gg m_{\tilde{t}_2} \approx m_{\tilde{g}} \gg m_{\tilde{t}_1}$$
$$m_{\tilde{q}} \approx m_{\tilde{t}_2} \approx m_{\tilde{g}} \gg m_{\tilde{t}_1}$$
$$m_{\tilde{q}} \approx m_{\tilde{t}_1} \approx m_{\tilde{t}_2} \gg m_{\tilde{g}}$$

- $\sim$ 30 000 diagrams and up to $\sim 100\,000$ subdiagrams
- automated set-up (generation, asymptotic expansion, vacuum integrals): `qgraf` [Nogueira'91], `q2e/exp` [Harlander,Seidelsticker,Steinhauser'97'99], `MATAD` [Steinhauser'96-'00]

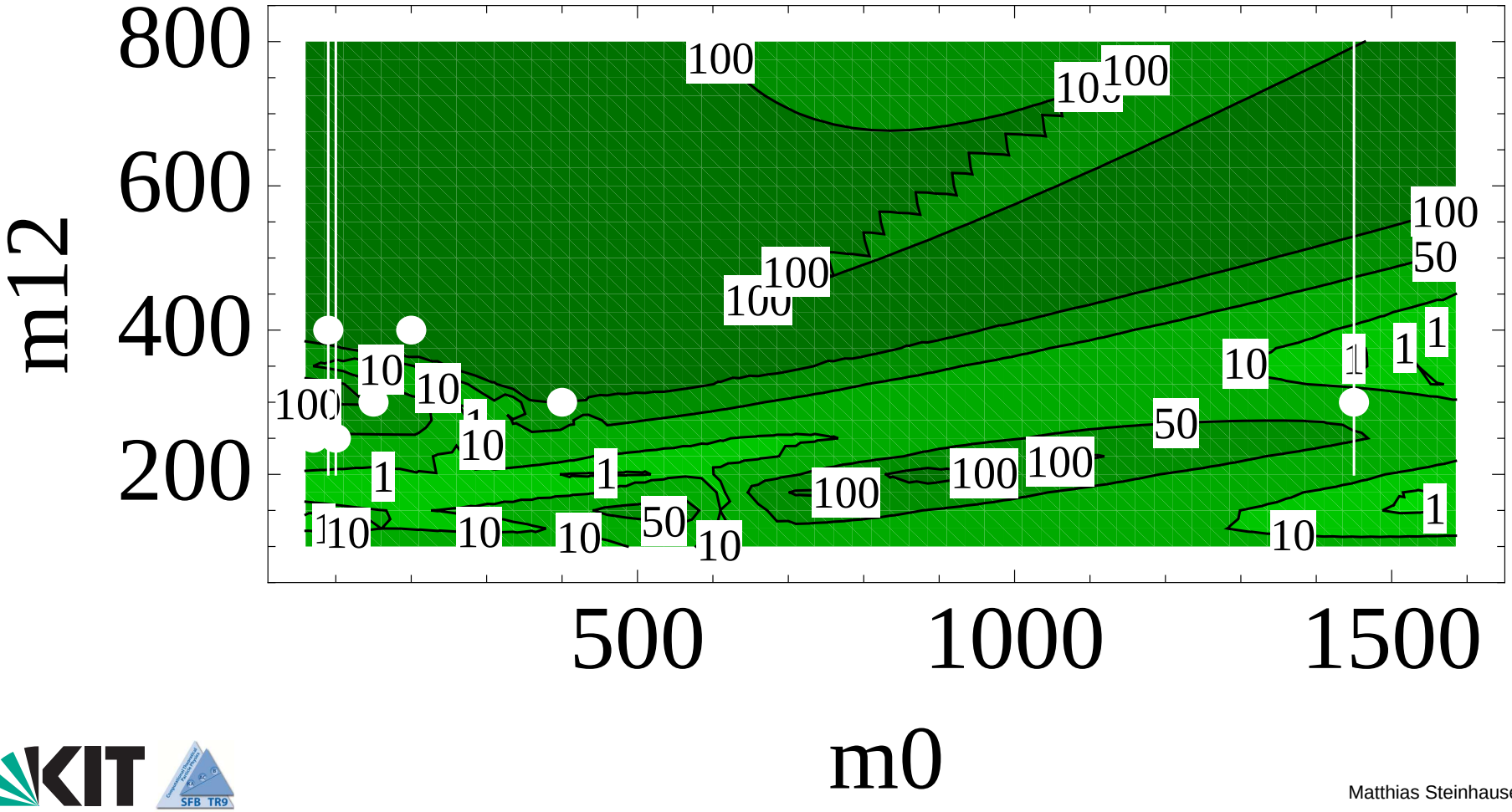
2 loops: complete vs. m_t^4 approximation



2 loops, m_t^4: exact vs. approximation

msugra: $\tan\beta = 10$, $A_0 = 0$, $\mu_{\rm SUSY} > 0$,
60 GeV $< m_0 < 1600$ GeV, 100 GeV $< m_{1/2} < 800$ GeV

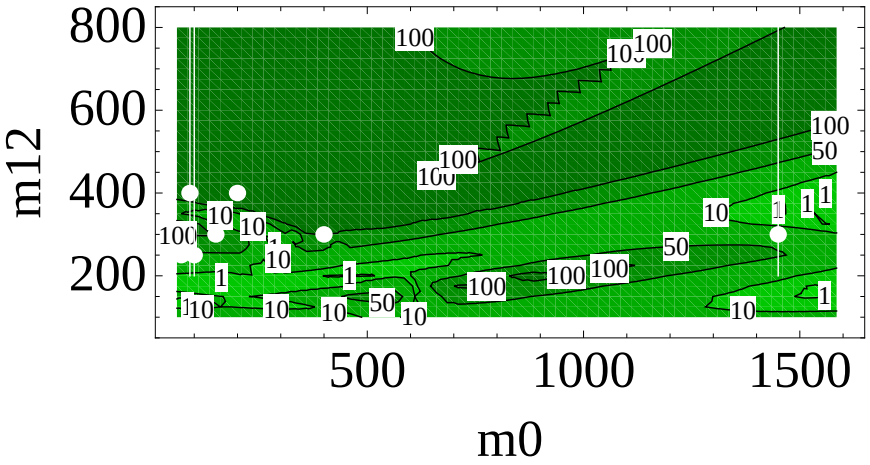
$|M_h^{(2)} - M_h^{(2),\rm appr}|$ (MeV)



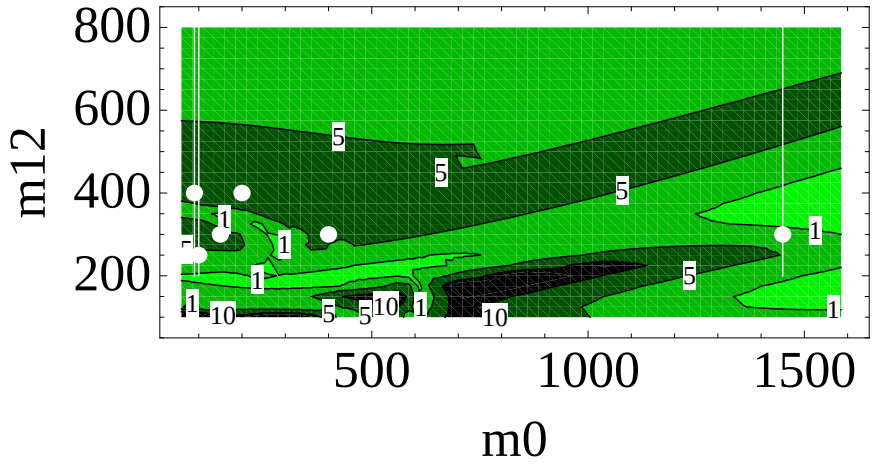
2 loops, m_t^4: exact vs. approximation

msugra: $\tan\beta = 10$, $A_0 = 0$, $\mu_{\rm SUSY} > 0$,
60 GeV $< m_0 < 1600$ GeV, 100 GeV $< m_{1/2} < 800$ GeV

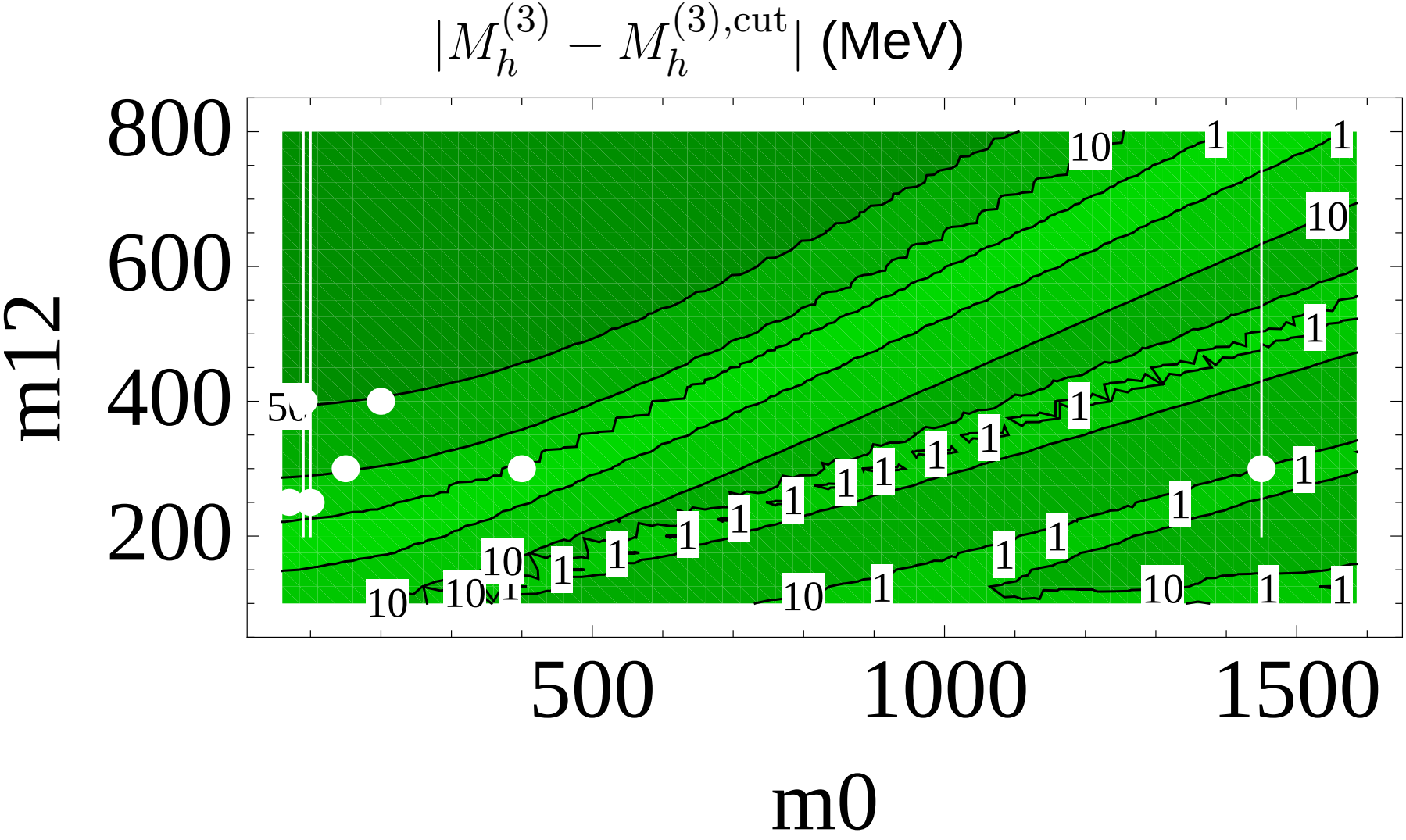
$|M_h^{(2)} - M_h^{(2),\rm appr}|$ (MeV)



$|\frac{M_h^{(2)} - M_h^{(2),\rm appr}}{M_h^{(2)} - M_h^{(1)}}|$ (%)



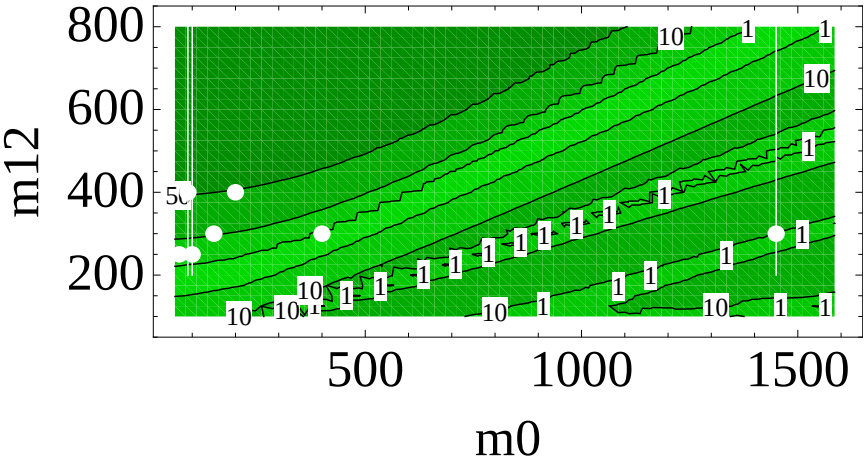
3 loops: check expansion



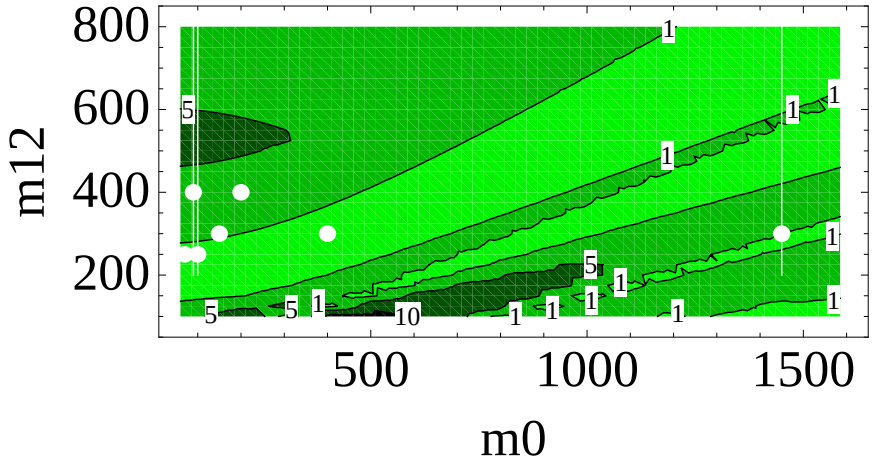
[$M_h^{(3),\mathrm{cut}}$: cut 1 term from expansion]

3 loops: check expansion

$|M_h^{(3)} - M_h^{(3),\mathrm{cut}}|$ (MeV)



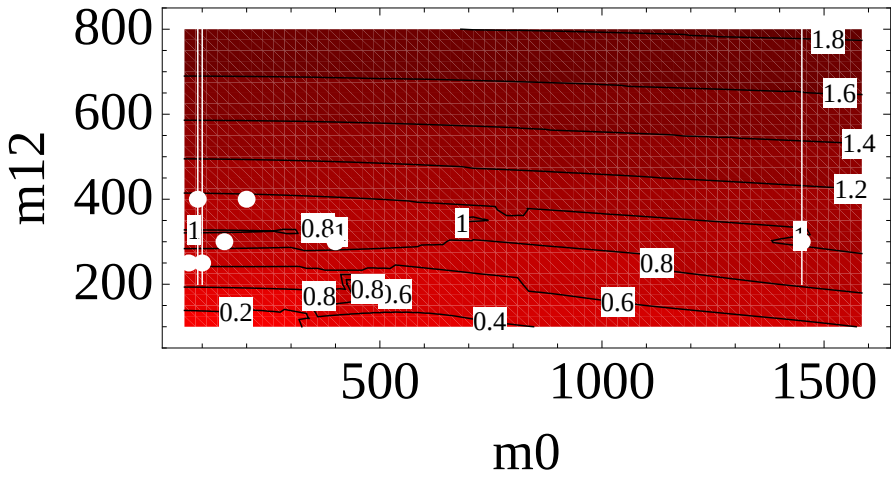
$|\frac{M_h^{(3)} - M_h^{(3),\mathrm{cut}}}{M_h^{(3)} - M_h^{(2)}}|$ (%)



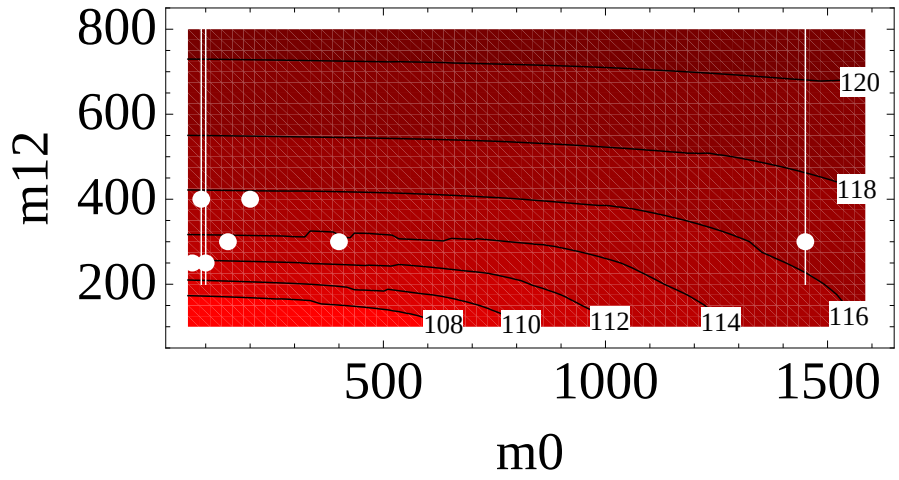
[$M_h^{(3),\mathrm{cut}}$: cut 1 term from expansion]

3 loops: results

$\Delta^{m_t^4} M_h^{(3)}$ (GeV)



M_h (GeV)



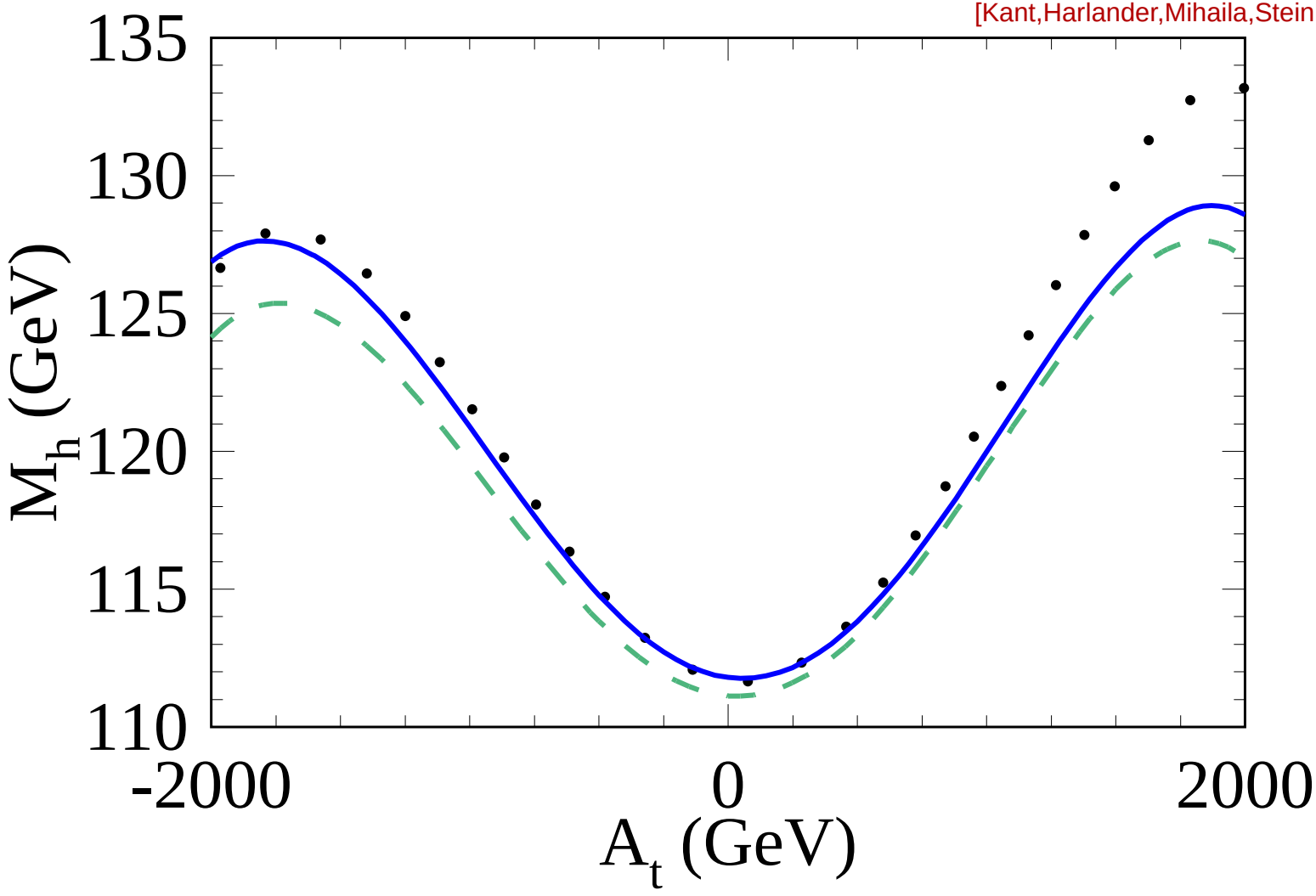
Uncertainties:

theory		parametric	
$\Delta^{\rm rem} M_h^{(3)}$:	100 MeV	$\delta M_t = 1.3$ GeV	$\leq 350 \ldots 1000$ MeV
approx. for $\Delta^{m_t^4} M_h^{(3)}$:	100 MeV		$(m_{1/2} = 100 \ldots 1000$ GeV)
h.o. ($\Delta M_h^{(3)} \approx -\frac{1}{2} \Delta M_h^{(2)}$):	$100 \ldots 1000$ MeV	$\delta \alpha_s = 0.0020$ GeV	$\leq 80 \ldots 600$ MeV
$q^2 \neq 0$, ew, ...	200 MeV	SUSY parameters	few 100 MeV

δ(theory) $\approx \delta$(param.)

M_h to 3 loops

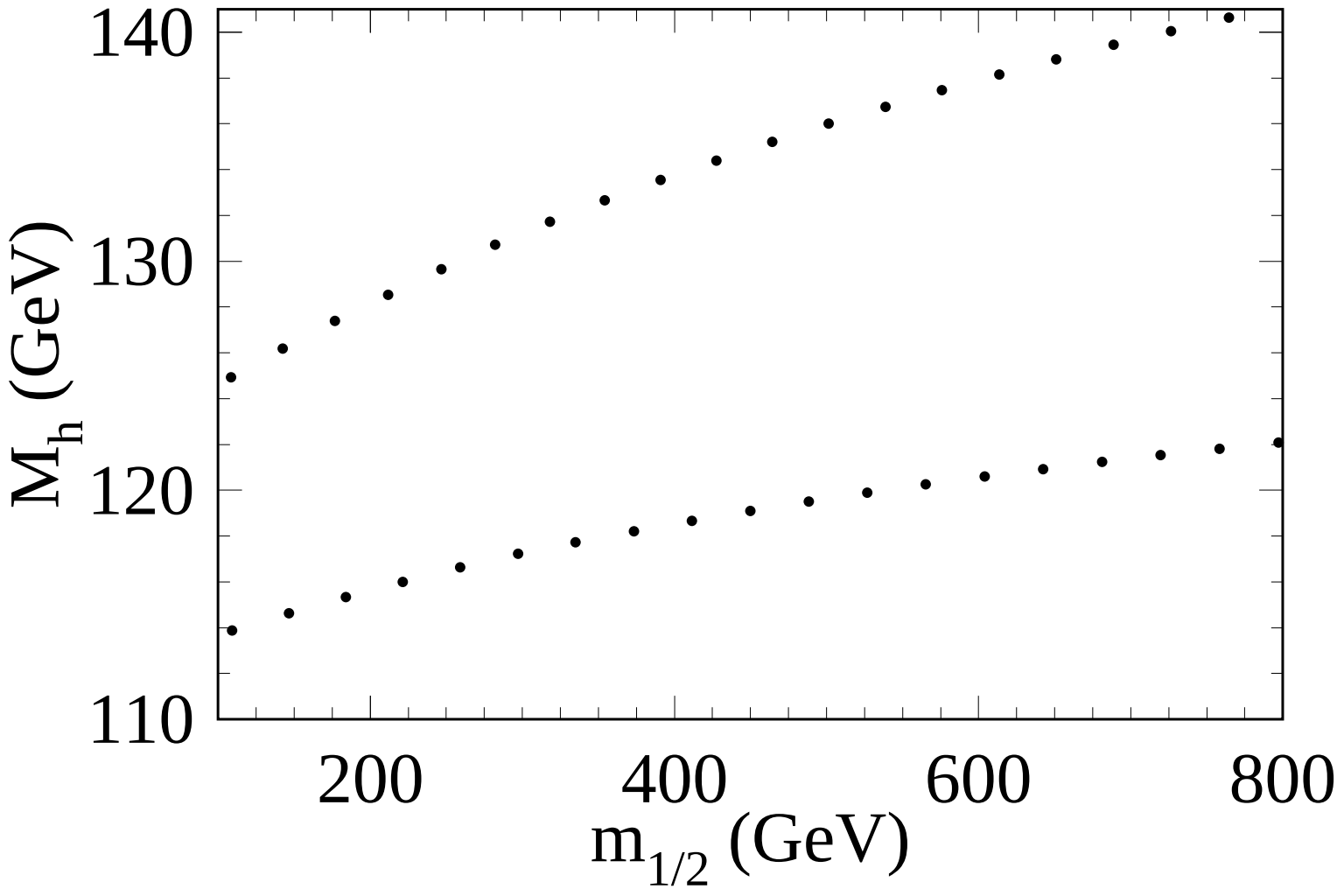
[Kant,Harlander,Mihaila,Steinhauser'10]



3-loop corrections $\approx$ few GeV $\gg$ 100 MeV $\approx$ exp. uncertainty

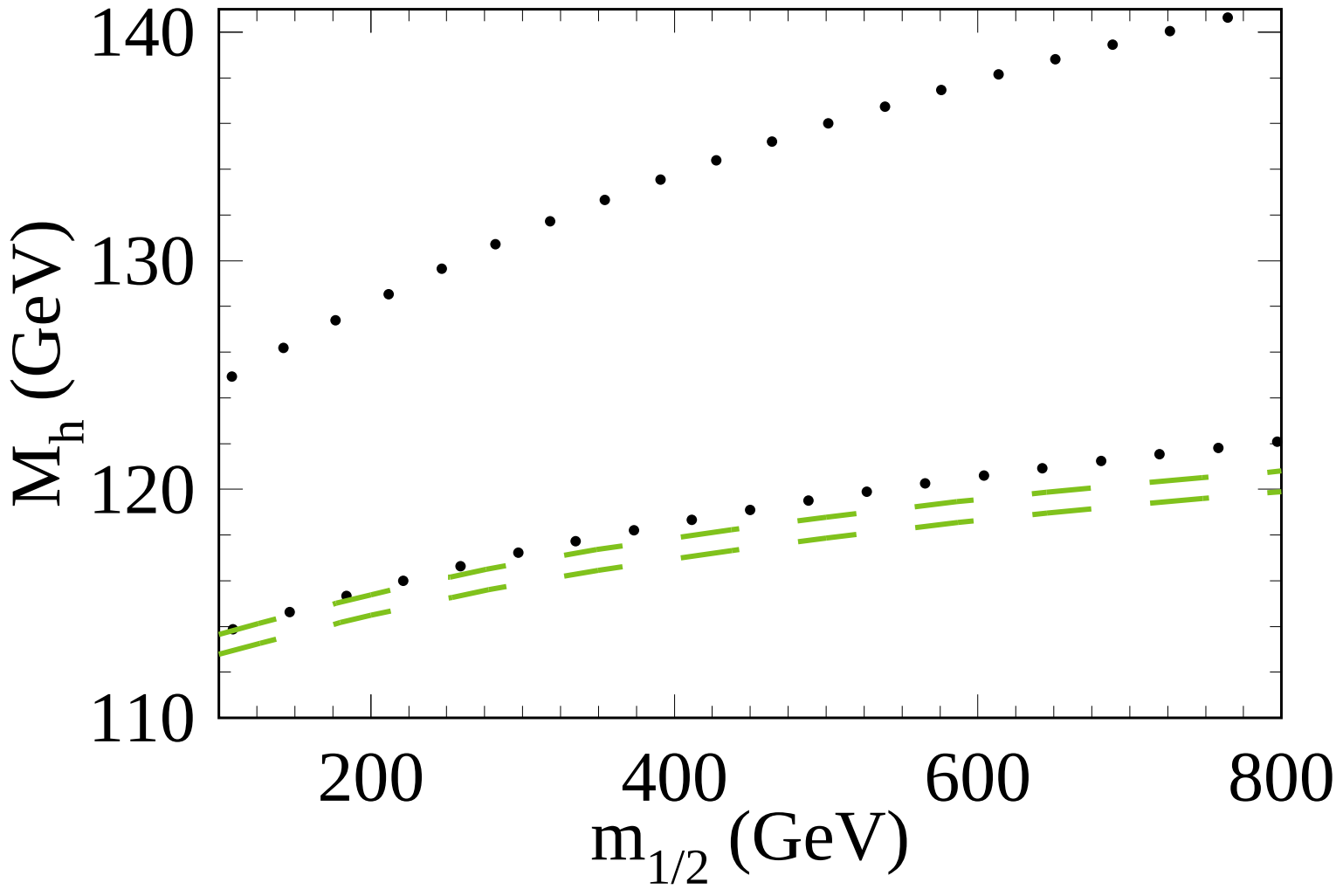
Comparison: OS – $\overline{\mathrm{DR}}$

[Kant,Harlander,Mihaila,Steinhauser'10]



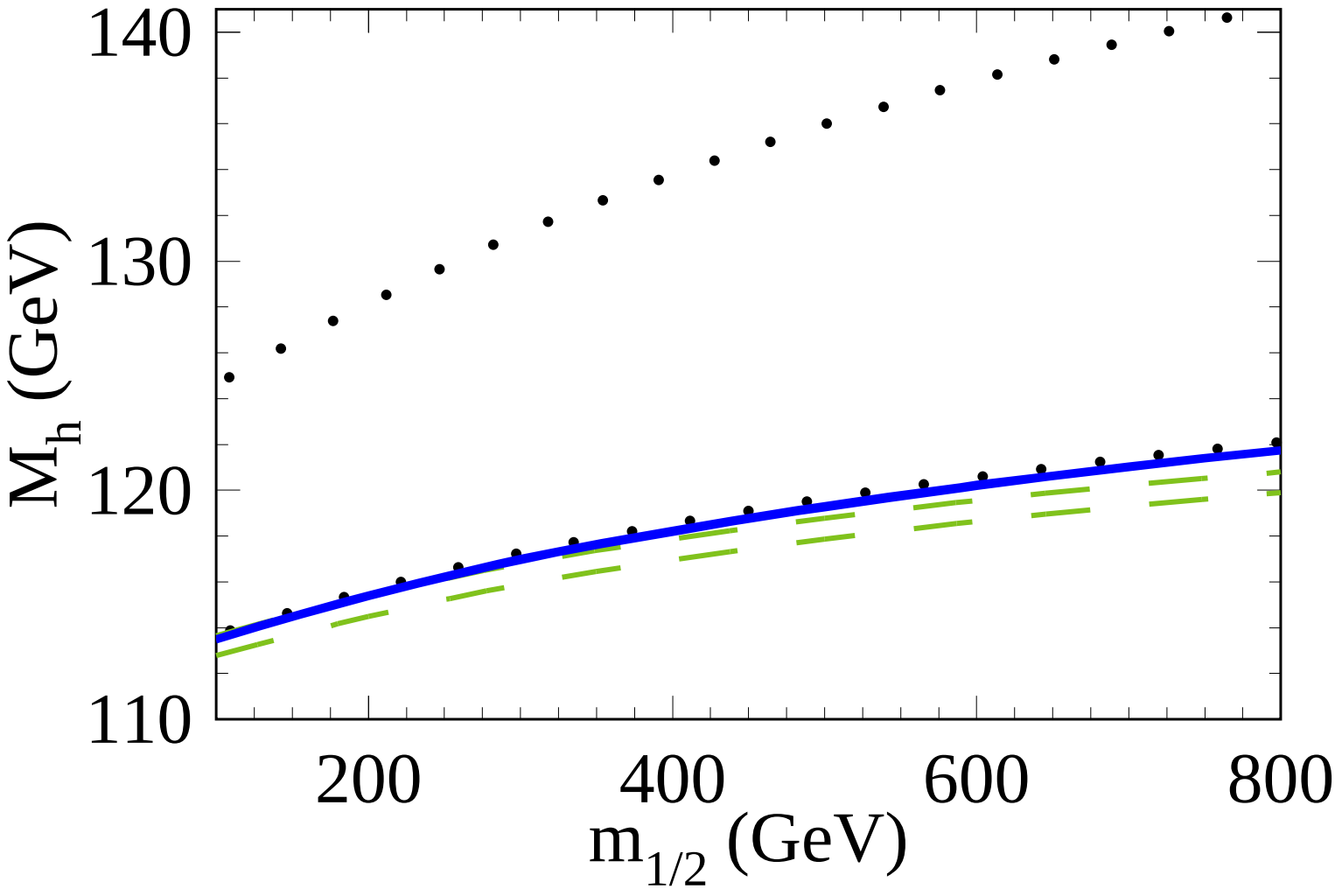
Comparison: OS – $\overline{\mathrm{DR}}$

[Kant,Harlander,Mihaila,Steinhauser'10]



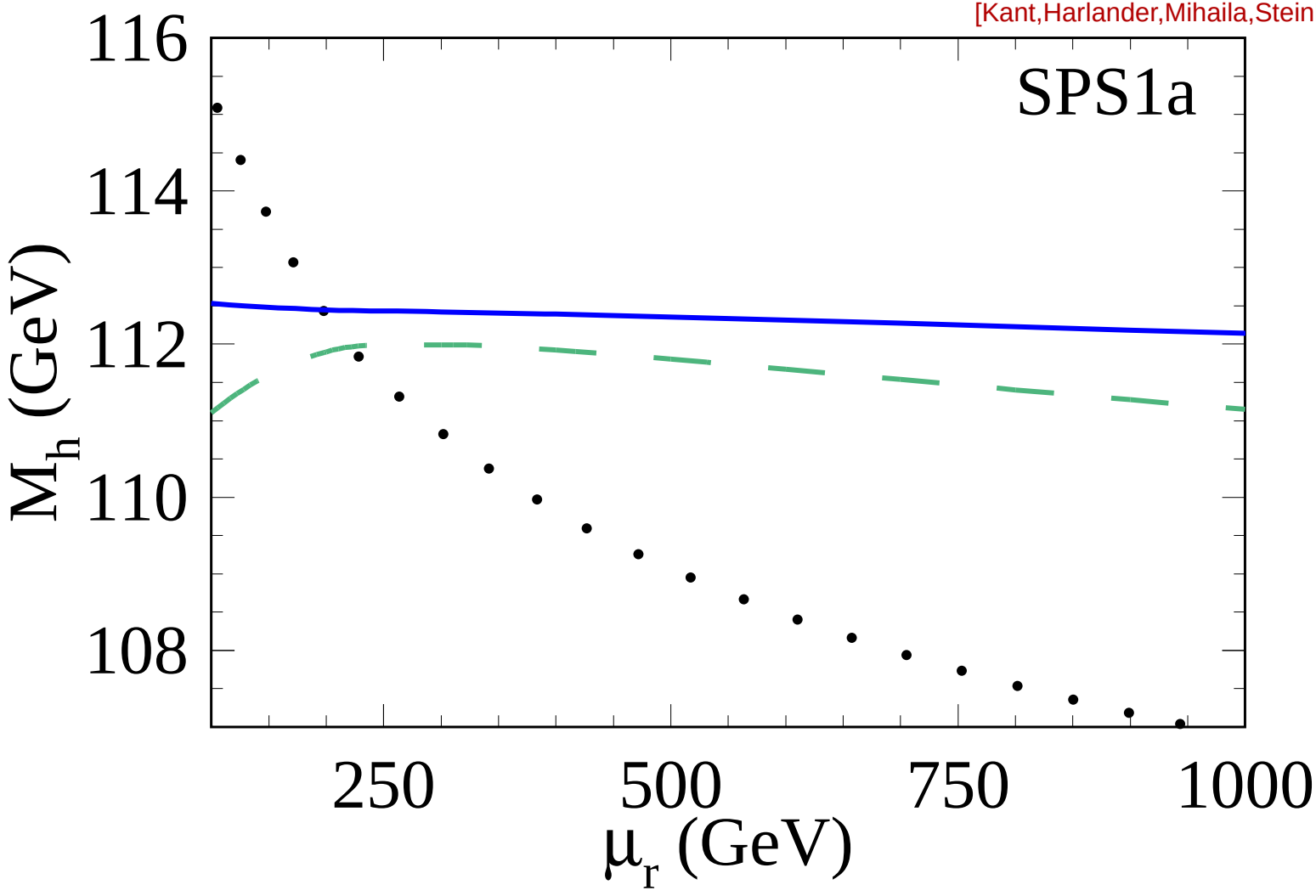
Comparison: OS – $\overline{\text{DR}}$

[Kant,Harlander,Mihaila,Steinhauser'10]



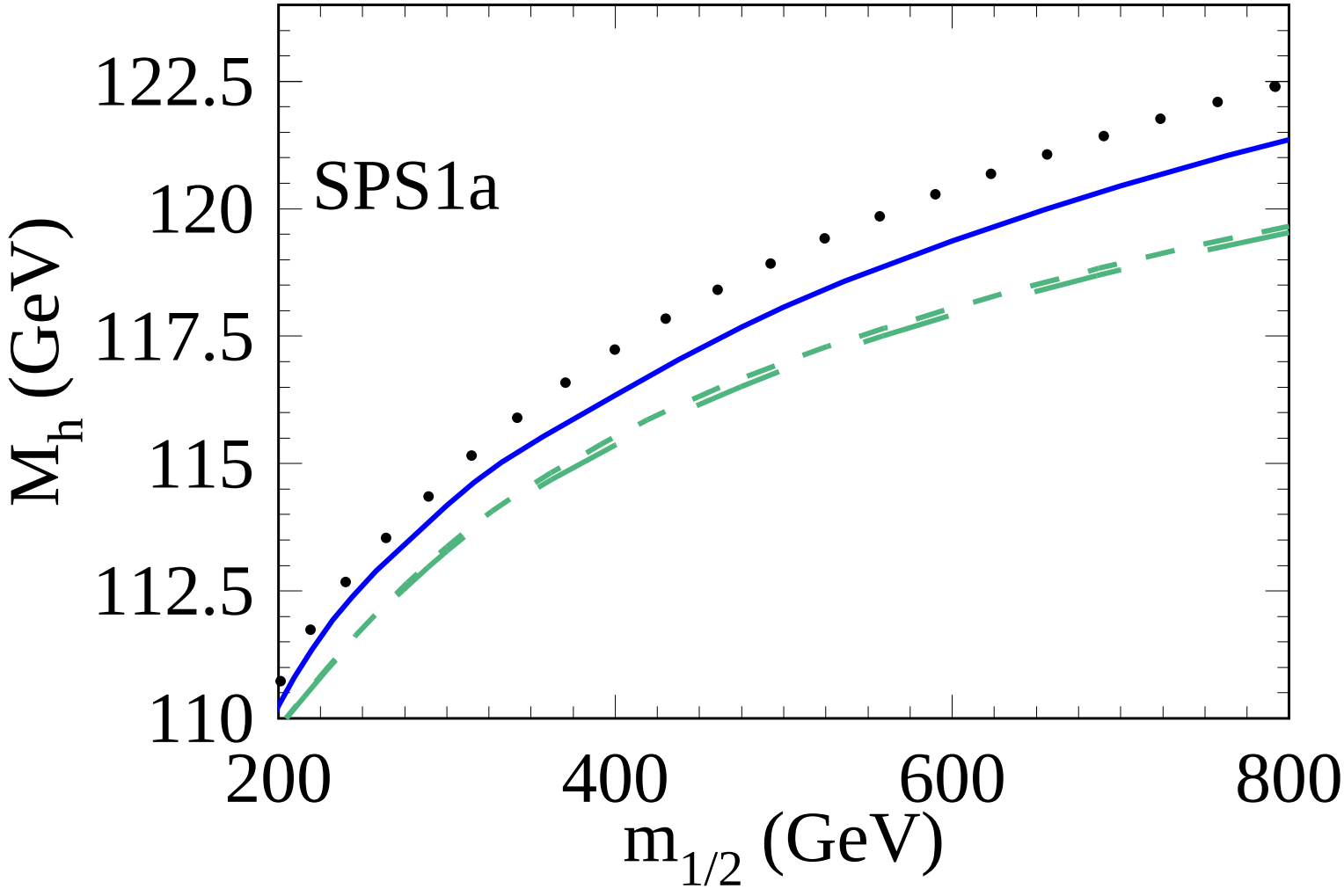
Renormalization scale dependence

[Kant,Harlander,Mihaila,Steinhauser'10]



SPS1a

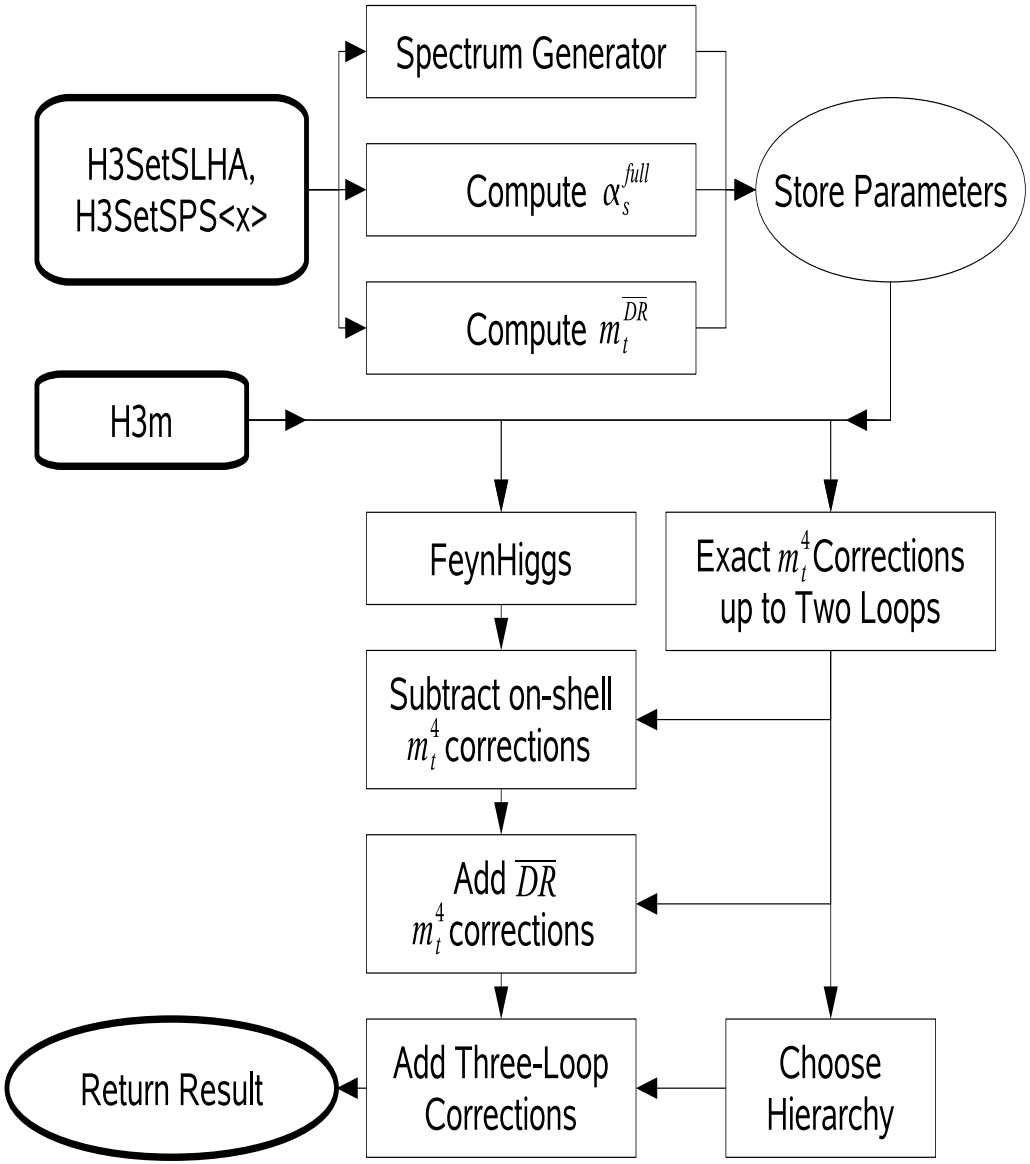
[Kant,Harlander,Mihaila,Steinhauser'10]



SPS at 2 and 3 loops

	$M_h^{(2)}$ (GeV)	$M_h^{(2),\mathrm{appr}}$ (GeV)	$M_h^{(3)}$ (GeV)	$M_h^{(3),\mathrm{cut}}$ (GeV)	optimal hierarchy
SPS1a	111.81	111.84	112.46	112.45	h6b
SPS1a′	113.26	113.27	113.92	113.92	h6b
SPS1b	115.53	115.64	116.49	116.44	h3
SPS2	115.65	115.77	116.67	116.61	h5
SPS3	114.63	114.77	115.59	115.52	h3
SPS4	113.73	113.77	114.82	114.81	h6
SPS5	111.66	111.83	112.02	111.92	h3
SPS7	112.20	112.21	113.04	113.04	h3
SPS8	114.19	114.20	115.03	115.02	h3

H3m




```
Mathematica 7.0 for Linux x86 (64-bit)
Copyright 1988-2008 Wolfram Research, Inc.

In[1]:= Needs["H3‘"];

RunDec: a Mathematica package for running and decoupling
        strong coupling and quark masses
by K.G. Chetyrkin, J.H. Kuhn and M. Steinhauser (January

In[2]:= H3SetSPS1a[ 300.];

H3GetSLHA::TSIL: Using TSIL by S.P. Martin.
 ------------------------------------------------------
 FeynHiggs 2.6.5
 built on Dec 20, 2008
 T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, G. Weigle
 http://www.feynhiggs.de
 ------------------------------------------------------
 FHHiggsCorr contains code by:
 P. Slavich et al. (2-loop rMSSM Higgs self-energies)
Loading Results for hierarchy h3
Loading Results for hierarchy h3
Loading Results for hierarchy h6b2qg2
Loading Results for hierarchy h6b2qg2

In[3]:= H3m[]

Loading Results for hierarchy h6b2qg2

Out[3]= {mh -> 114.176}
```

[exact 2-loop m_t^4 expression from [Degrassi,Slavich,Zwirner'01]; OS-$\overline{\rm DR}$ for M_t from [Martin'05]]

Summary

- Higgs boson mass in the MSSM to 3 loops
- $\Delta M_h^{(\mathrm{3loops})} > \Delta M_h^{(\mathrm{LHC,ILC})}$
- H3m: most precise value for M_h

  `www-ttp.particle.uni-karlsruhe.de/Progdata/ttp10/ttp10-23/`

- $\delta M_h < 1$ GeV