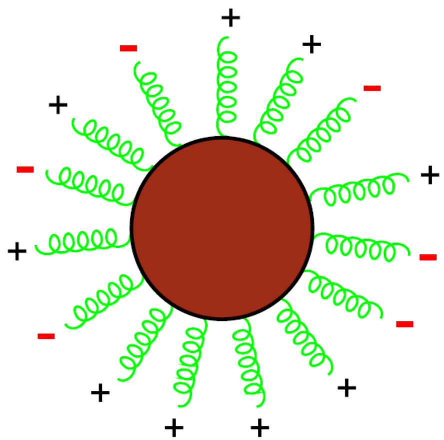
Scattering Amplitudes in Gauge Theory and Gravity

Lecture 4 – N=8 supergravity trees, N=4/8 Loops



Lance Dixon (CERN & SLAC)

CERN Winter School

Jan. 24-28, 2011

Back to trees – for (N=8 super)gravity

$\mathcal{N}=8$ vs. $\mathcal{N}=4$ SYM

2^8 = 256 massless states, ~ expansion of $(x+y)^8$

$\mathcal{N}=8$:	1 ↔	8 ↔	28 ↔	56 ↔	70 ↔	56 ↔	28 ↔	8 ↔	1
helicity :	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
SUSY ↔	h^-	ψ_i^-	v_{ij}^-	χ_{ijk}^-	s_{ijkl}	χ_{ijk}^+	v_{ij}^+	ψ_i^+	h^+

$\mathcal{N}=4$ SYM :	1	4	6	4	1
	g^-	λ_A^-	ϕ_{AB}	λ_A^+	g^+

all in adjoint representation

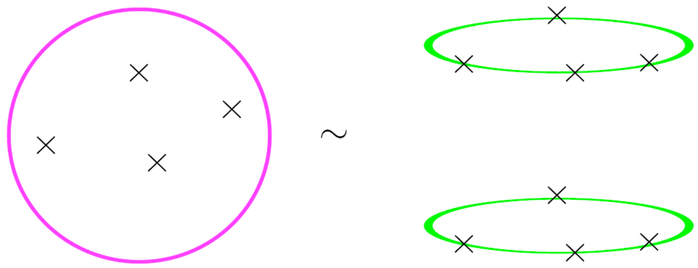
2^4 = 16 states ~ expansion of $(x+y)^4$

$$\Rightarrow \quad [\mathcal{N}=8] = [\mathcal{N}=4] \otimes [\mathcal{N}=4]$$

Kawai-Lewellen-Tye relations

KLT, 1986

N=8 supergravity is the low-energy limit of the type II closed superstring



N=4 super-Yang-Mills is the low-energy limit of the open superstring

KLT found a quadratic relation between open & closed string amplitudes

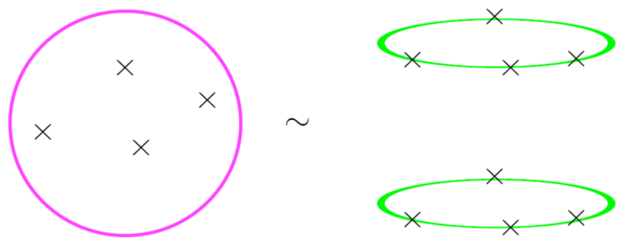
$\Rightarrow$

- Can write (super)gravity tree amplitudes as "squares" of (super-)Yang-Mills tree amplitudes.
- Very useful for doing supergravity loop calculations!

$$\begin{aligned} V^{\text{open}}(z) &\sim \varepsilon_\mu(\partial_z X^\mu + \cdots)e^{ik\cdot X} \\ V^{\text{closed}}(z,\bar{z}) &\sim \varepsilon_{\mu\nu}(\partial_z X^\mu + \cdots)(\partial_{\bar{z}} X^\nu + \cdots)e^{ik\cdot X} \\ &\sim V_L^{\text{open}}(z) \times V_R^{\text{open}}(\bar{z}) \end{aligned}$$

$\varepsilon_{\mu\nu}^{\pm\pm} = \varepsilon_\mu^\pm \times \varepsilon_\nu^\pm$ extends to $[\mathcal{N}=8] = [\mathcal{N}=4] \otimes [\mathcal{N}=4]$

KLT relations (cont.)



$$M_n^{\text{tree}} = \prod_{i\neq 1,2,3}^{n} \int d^2 z_i \; \langle V_1^{\text{closed}}(z_1,\bar{z}_1)\cdots V_n^{\text{closed}}(z_n,\bar{z}_n)\rangle$$

$$= \prod_{i\neq 1,2,3}^{n} \int d^2 z_i \; f_L(z_i) f_R(\bar{z}_i) \prod_{i<j}^{n} |z_i - z_j|^{2k_i\cdot k_j}$$

$$= \prod_{i\neq 1,2,3}^{n} \int d^2 z_i \; \langle V_{L,1}^{\text{open}}(z_1)\cdots V_{L,n}^{\text{open}}(z_n)\rangle$$

$$\times \langle V_{R,1}^{\text{open}}(\bar{z}_1)\cdots V_{R,n}^{\text{open}}(\bar{z}_n)\rangle$$

$$A_{n,L}^{\text{tree}} = \prod_{i\neq 1,2,3}^{n} \int dx_i \; \langle V_{L,1}^{\text{open}}(x_1)\cdots V_{L,n}^{\text{open}}(x_n)\rangle$$

Main difference is integration region: complex plane vs. real line. Write plane integral as product of contour integrals, deform contours back to real axis, picking up phases / sine factors from

$$(z_i - z_j)^{k_i\cdot k_j} \;\rightarrow\; e^{i\pi k_i\cdot k_j} \;\rightarrow \sin(2\pi\alpha' s_{ij})$$

KLT relations (cont.)

H. Kawai et al. / Tree amplitudes of closed and open strings

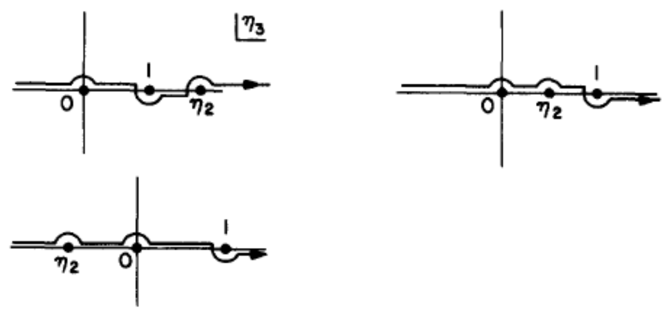


Fig. 5. The η_3 contour integrals in the complex η_3 plane for the case $0 < \xi_2 < \xi_3 < 1$.

Sine factors depend on region of integration, i.e. on color-ordering of open string amplitude.
Let $\alpha' \to 0$ $\quad \sin(2\pi\alpha' s_{ij}) \;\to\; s_{ij}$

Low-energy limit gives N=8 supergravity amplitudes $M_n^{\rm tree}$ as quadratic combinations of N=4 SYM amplitudes $A_n^{\rm tree}$, consistent with product structure of Fock space,

$$[\mathcal{N}=8] \;=\; [\mathcal{N}=4] \otimes [\mathcal{N}=4]$$

$$M_3^{\rm tree}(1,2,3) \;=\; [A_3^{\rm tree}(1,2,3)]^2$$

$$M_4^{\rm tree}(1,2,3,4) \;=\; -i\, s_{12}\, A_4^{\rm tree}(1,2,3,4)\, A_4^{\rm tree}(1,2,4,3)$$

$$M_5^{\rm tree}(1,2,3,4,5) \;=\; i\, s_{12}s_{34}\, A_5^{\rm tree}(1,2,3,4,5)\, A_5^{\rm tree}(2,1,4,3,5) + (2 \leftrightarrow 3)$$

$$M_6^{\rm tree}(1,2,3,4,5,6) \;=\; \cdots$$

AdS/CFT vs. KLT

AdS	=	CFT
gravity	=	gauge theory
weak		strong

KLT		
gravity	=	$(\text{gauge theory})^2$
weak		weak

"KLT copying"

Bern, LD, Dunbar, Perelstein, Rozowsky (1998)

• KLT relations give the N=8 SUGRA cuts – products of N=8 SUGRA trees, summed over all internal states – very simply in terms of:

sums of products of two copies of N=4 SYM cuts

$$[\mathcal{N}=8] \;=\; [\mathcal{N}=4] \otimes [\mathcal{N}=4] \quad\Longrightarrow\quad \sum_{\mathcal{N}=8} \;=\; \sum_{\mathcal{N}=4}\sum_{\mathcal{N}=4}$$

• Need both **planar** (large N_c) and **non-planar** terms in corresponding multi-loop N=4 SYM amplitude

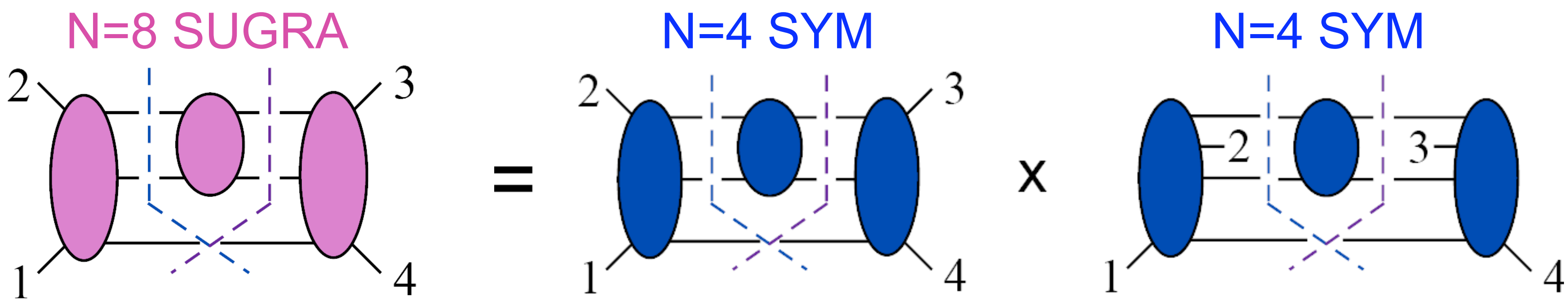
KLT copying at 3 loops

Using

$$M_4^{\rm tree}(1,2,3,4) = -i\,\frac{st}{u}[A_4^{\rm tree}(1,2,3,4)]^2$$

$$M_5^{\rm tree}(1,2,3,4,5) = -i\,s_{51}s_{23}\,A_5^{\rm tree}(1,2,3,4,5)\,A_5^{\rm tree}(1,4,2,3,5) + (1 \leftrightarrow 2)$$

it is easy to see that

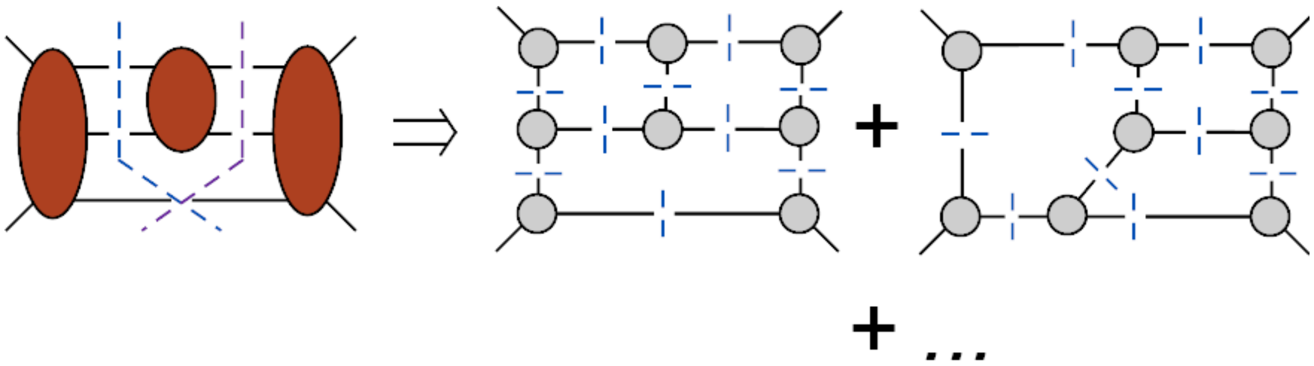


rational function of Lorentz products of external and cut momenta; **all state sums already performed**

+ permutations $(1 \leftrightarrow 2,\ 3 \leftrightarrow 4)$

Method of maximal cuts

Complex cut momenta make sense out of all-massless 3-point kinematics – can chop an amplitude entirely into 3-point trees
→ **maximal cuts**



Maximal cuts are maximally simple,
yet give excellent starting point for constructing full answer

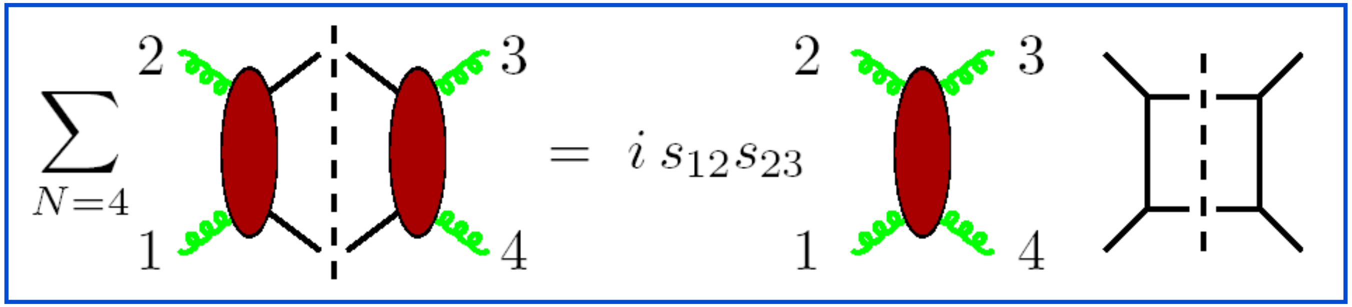
For example, in planar (leading in N_c) N=4 SYM
they find all terms in the complete answer for 1, 2 and 3 loops

Remaining terms found **systematically**: Let 1 or 2 propagators collapse from each maximal cut → **near-maximal cuts**

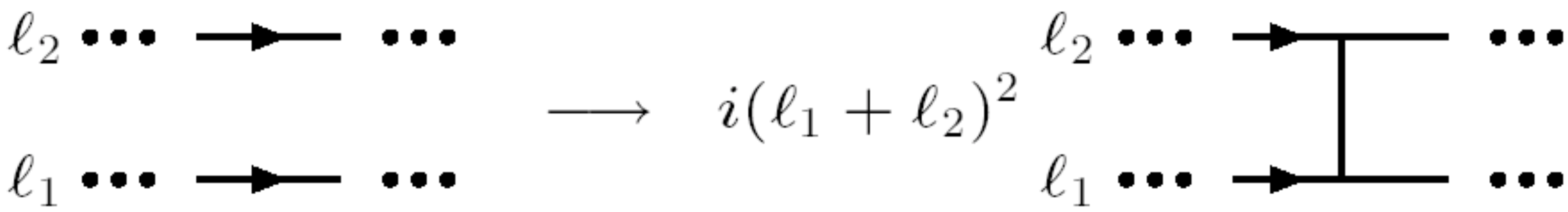
Unitarity and N=4 SYM

- Many higher-loop contributions to *gg -> gg* scattering can be deduced from a simple property of the 2-particle cuts at one loop

Bern, Rozowsky, Yan (1997)

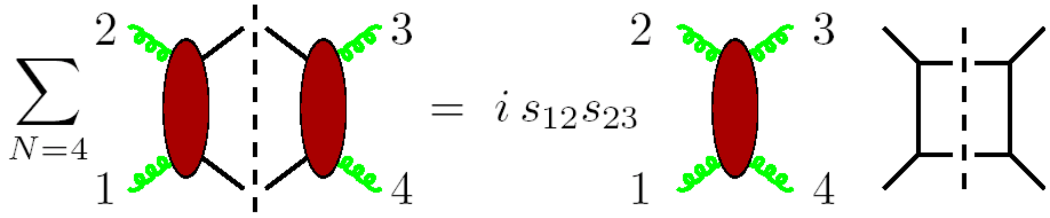


- Leads to "rung rule" for easily computing all contributions which can be built by iterating 2-particle cuts



Unitarity and N=4 SYM (cont.)

Let's show



In most cases we would have to specify the helicity configuration, and the cut would depend on the choice. However, all 4-point amplitudes in N=4 SYM are related by supersymmetry Ward identities.

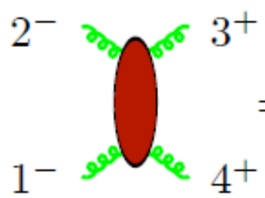
Grisaru, Pendleton, van Nieuwenhuizen

Alternatively, there is a unique MHV superamplitude at L loops

$$\mathcal{A}_n^{\mathrm{MHV}} \propto \frac{\delta^{(4)}(p)\,\delta^{(8)}(q)}{\langle 1\,2\rangle\langle 2\,3\rangle\cdots\langle n\,1\rangle} \qquad q^{\alpha,A} = \sum_{i=1}^{n} \lambda_i^{\alpha}\,\eta_i^{A}$$

$$\propto \langle i\,j\rangle^4 \text{ for gluons}$$

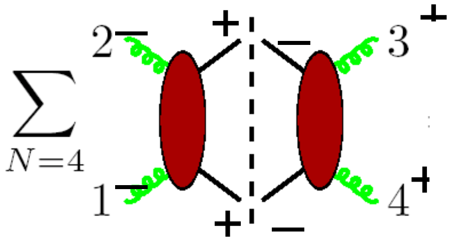
We also have for the 4-gluon color ordered tree amplitude:

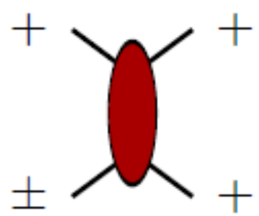


$$= i\,\frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle} = \text{phase} \times \frac{s_{12}}{s_{23}}$$

Unitarity and N=4 SYM (cont.)

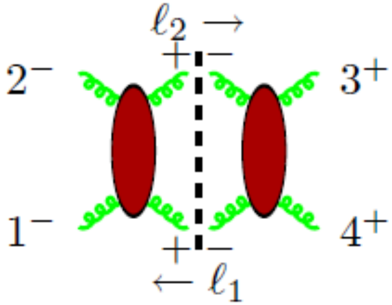
So we can choose the easiest helicity configuration, which can only be satisfied by gluon intermediate states:





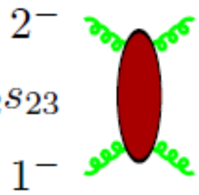
$= 0$

Fermions, scalars require (+,-) across cut

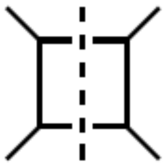


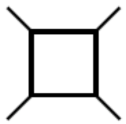
$$= \text{phase} \times \frac{s_{12}}{(\ell_1 - k_1)^2} \times \frac{s_{12}}{(\ell_2 - k_3)^2}$$

$$= i\, s_{12} s_{23}$$



$\times$

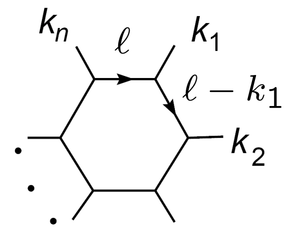




$$= \int \frac{d^{4-2\epsilon}\ell_1}{(2\pi)^{4-2\epsilon}} \frac{1}{\ell_1^2(\ell_1 - k_1)^2(\ell_1 - k_1 - k_2)^2(\ell_1 + k_4)^2}$$

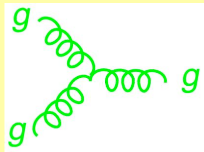
- All loop momenta in numerators come out of integral!

"No triangles" for N=4 SYM



- One-loop N=4 amplitudes contain only boxes, due to SUSY cancellations of loop momenta in numerator:

  Bern, LD, Dunbar, Kosower (1994)



$\supset \ \ell^\mu \eta^{\nu\rho} + \cdots$

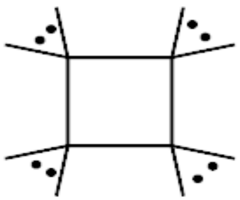
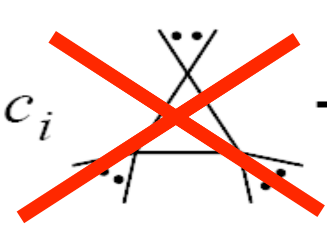
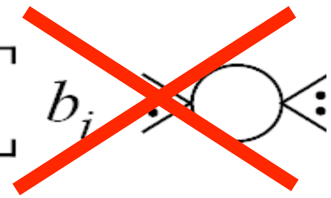
- Use background field method in n-gluon case

$$\Gamma_{\rm eff}^{N=1,chiral}(A) \propto -\ln\det(D^2 + \sigma_{\mu\nu}F^{\mu\nu}) + \ln\det(D^2)$$

$$(\ell^\mu)^n \Rightarrow (\ell^\mu)^{n-2}$$

$$\Gamma_{\rm eff}^{N=4}(A) \propto \ln\det(D^2 + \Sigma_{\mu\nu}F^{\mu\nu}) - 4\ln\det(D^2 + \sigma_{\mu\nu}F^{\mu\nu}) + 3\ln\det(D^2)$$

$$(\ell^\mu)^n \Rightarrow (\ell^\mu)^{n-4}$$

$A_{N=4}^{1-\text{loop}} = \sum_i d_i$  $+ \sum_i c_i$  $+ \sum_i b_i$ 

$+$ ~~R~~ $+ \ \mathcal{O}(\epsilon)$

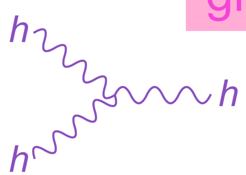
N=8 also obeys “No triangle” property

Bjerrum-Bohr et al., hep-th/0610043; Bern, Carrasco, Forde, Ita, Johansson, 0707.1035 (pure gravity) ; Kallosh, 0711.2108; Bjerrum-Bohr, Vanhove, 0802.0868
Proofs: Bjerrum-Bohr, Vanhove, 0805.3682; Arkani-Hamed, Cachazo, Kaplan, 0808.1446

- Statement about UV behavior of N=8 SUGRA amplitudes at **one loop** but with **arbitrarily many external legs:**

  “N=8 UV behavior no worse than N=4 SYM at one loop”
- Samples arbitrarily many powers of loop momenta
- Necessary but not sufficient for excellent **multi-loop** behavior
- Implies specific **multi-loop** cancellations Bern, LD, Roiban, th/0611086

gravity (spin 2)



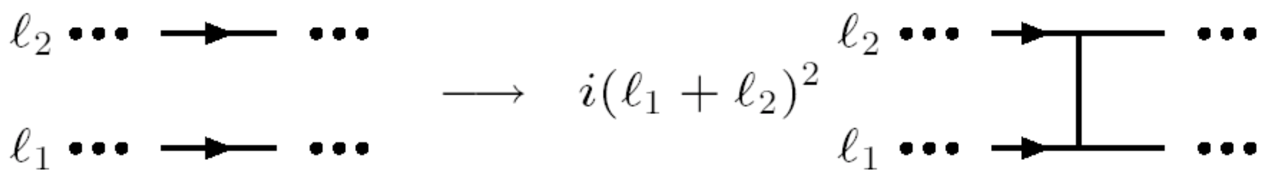
$$\supset \ \ell^{\mu_1}\,\ell^{\mu_2}\,\eta^{\nu_1\rho_1}\,\eta^{\nu_2\rho_2} + \cdots$$

Leading singularities for N=4/8

• More recently, *L*-loop generalization of this property conjectured:
All (important) terms determined by "leading-singularities" –
imposing 4*L* cuts on the *L* loop momenta in D=4
Cachazo, Skinner, 0801.4574; Arkani-Hamed, Cachazo, Kaplan, 0808.1446

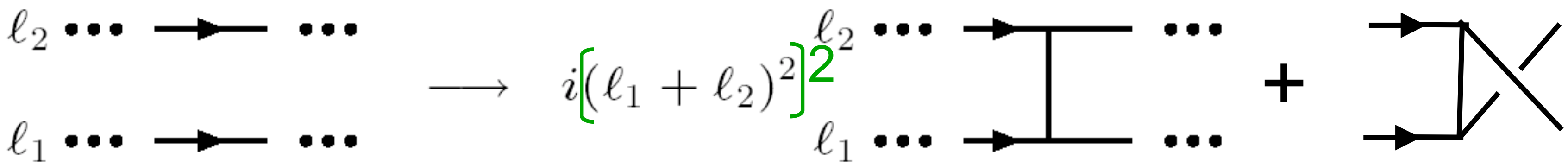
N=4 → N=8 Rung Rule

N=4 SYM rung rule

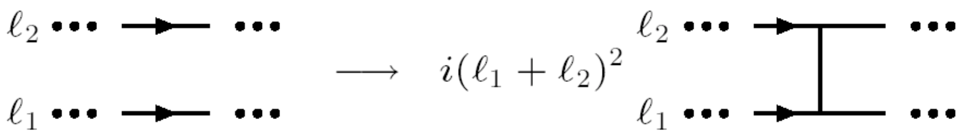


using same algebra “twice”, after using KLT relations:

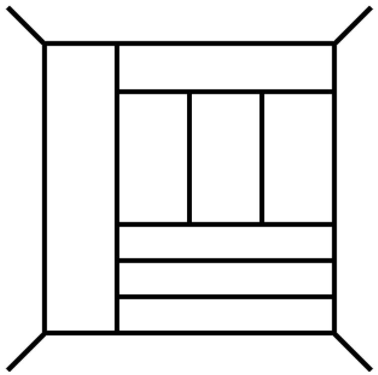
N=8 SUGRA rung rule



Iterated 2-particle cut-constructible contributions all follow from Rung Rule



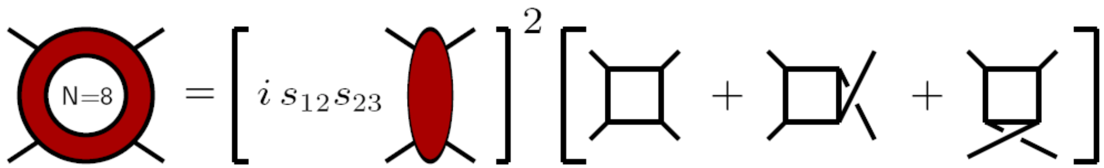
For example, this topology is easily computable



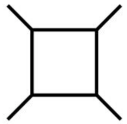
(Does not guarantee absence of “contact” term corrections.)

Rung rule → Extreme simplicity at 1 and 2 loops

- 1 loop:



where

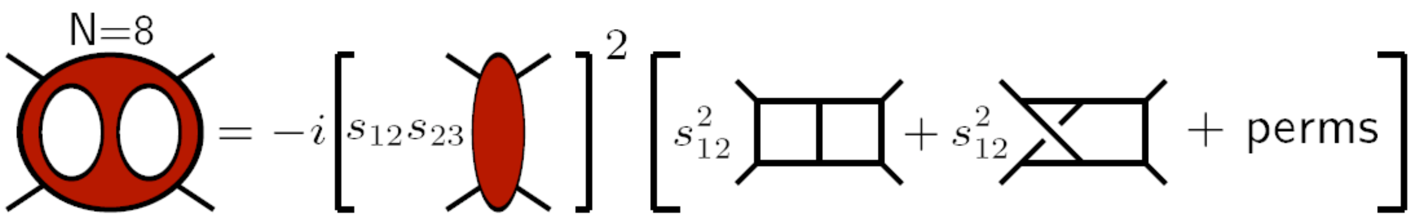


$$= \int \frac{d^{4-2\epsilon}\ell_1}{(2\pi)^{4-2\epsilon}} \frac{1}{\ell_1^2(\ell_1-k_1)^2(\ell_1-k_1-k_2)^2(\ell_1+k_4)^2}$$

Green, Schwarz, Brink; Grisaru, Siegel (1981)

"color dresses kinematics"

- 2 loops:



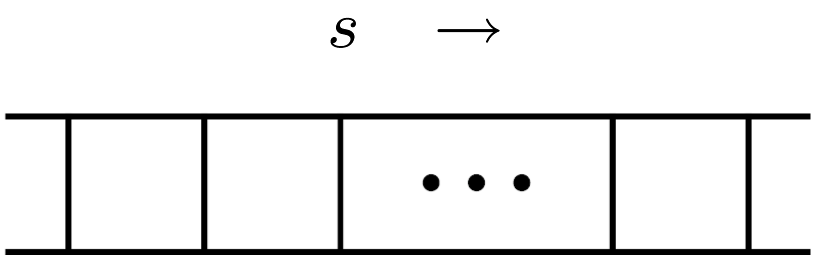
Bern, Rozowsky, Yan (1997); Bern, LD, Dunbar, Perelstein, Rozowsky (1998)

N=8 supergravity: just remove color, square prefactors!
D_c automatically same as for N=4 SYM for $L = 1,2$.

Ladder diagrams (Regge-like)

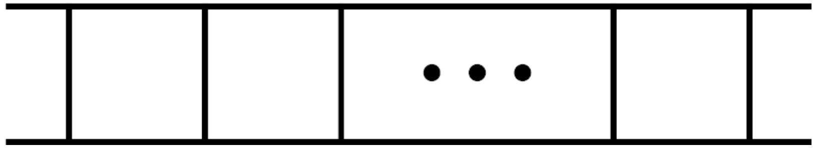
In N=4 SYM

$$st\, A_4^{\text{tree}} \times s^{L-1}$$



In N=8 supergravity

$$stu\, M_4^{\text{tree}} \times s^{2(L-1)}$$



Extra s^L in gravity from "charge" = energy

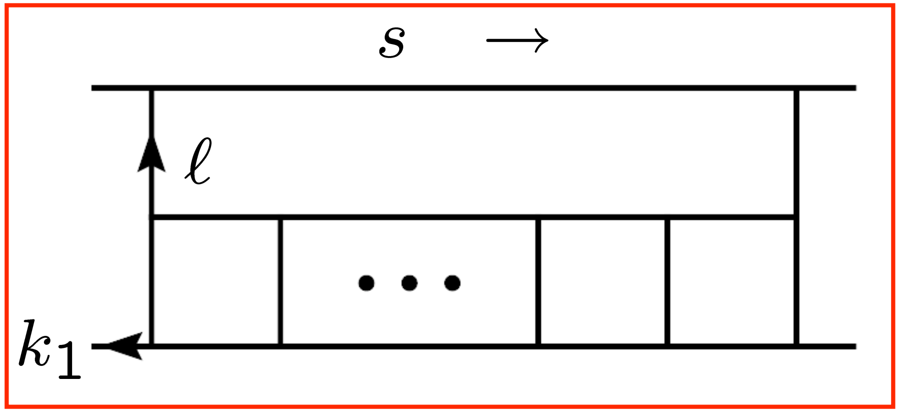
$$G_N E_1 E_2 = G_N\, s \quad \text{per loop}$$

Schnitzer, hep-th/0701217

For $L > 2$, UV behavior of generic rung-rule contributions looks worse in N=8 than N=4

N=4 SYM

$$st\, A_4^{\rm tree} \times t \times [(\ell + k_1)^2]^{L-2}$$



N=8 supergravity

$$stu\, M_4^{\rm tree} \times t^2 \times [(\ell + k_1)^2]^{2(L-2)}$$

← 2 from HE behavior of gravity

Integral in *D* dimensions scales as

$$\mathcal{I} \sim \int d^{DL}\ell \frac{(\ell^2)^{2(L-2)}}{(\ell^2)^{3L+1}}$$

→ Critical dimension D_c for log divergence (if no cancellations) obeys

$$\frac{D_c L}{2} + 2(L-2) = 3L + 1 \quad \Rightarrow$$

$$D_c = 2 + \frac{10}{L} \qquad \text{N=8}$$

$$D_c = 4 + \frac{6}{L} \qquad \text{N=4 SYM}$$

BDDPR (1998)

3 loop amplitude

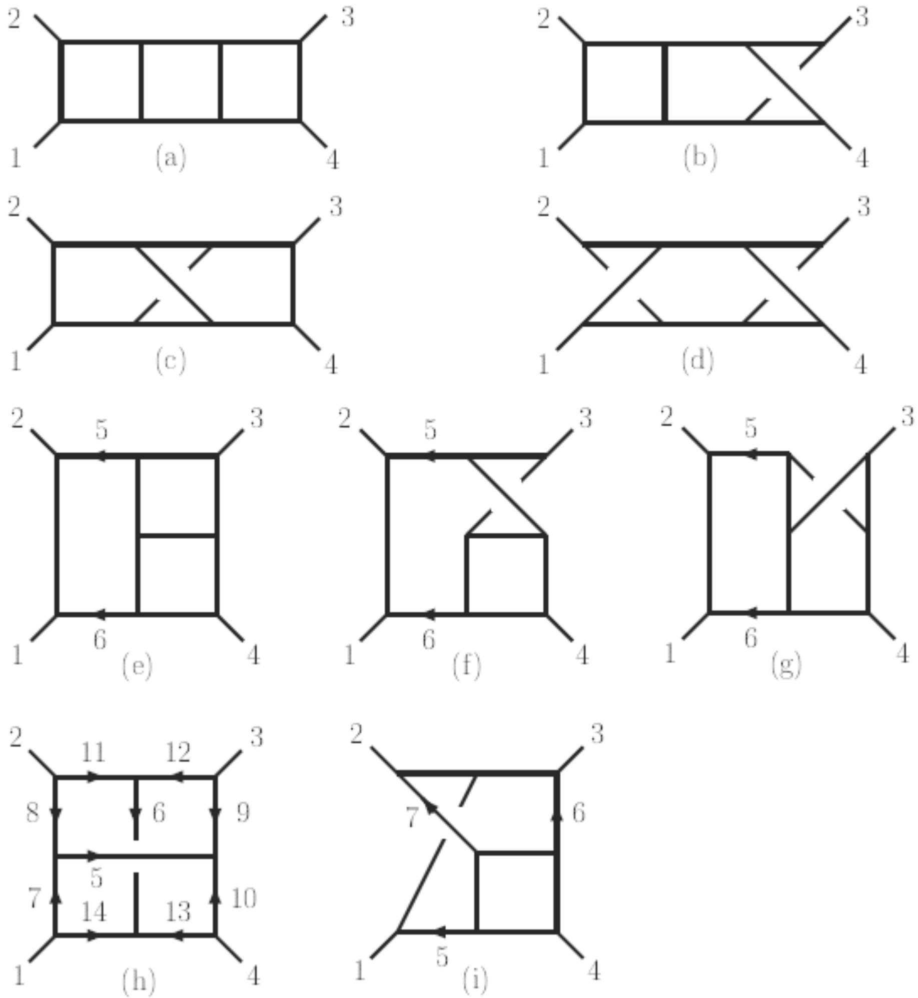
Bern, Carrasco, LD, Johansson, Kosower, Roiban, th/0702112
Bern, Carrasco, LD, Johansson, Roiban, 0808.4112

Nine basic integral topologies

Seven, (a)-(g), have 2-particle cuts → easily determine using rung rule

BDDPR (1998)

Two new ones (h), (i) have no 2-particle cuts

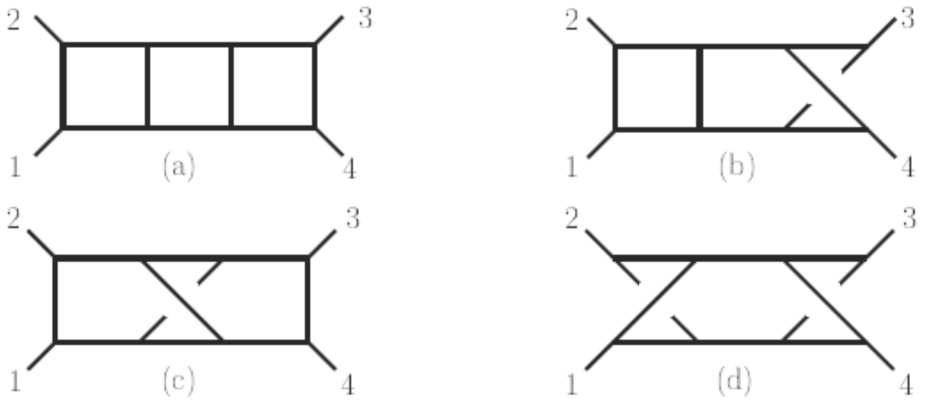


N=4 numerators at 3 loops

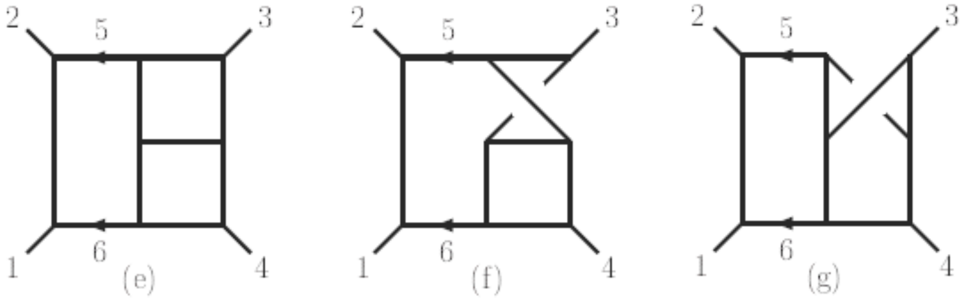
Omit overall $st\, A_4^{\text{tree}}$

$$s_{iM} = (k_i + \ell_M)^2$$
$$\tau_{iM} = 2k_i \cdot \ell_M$$

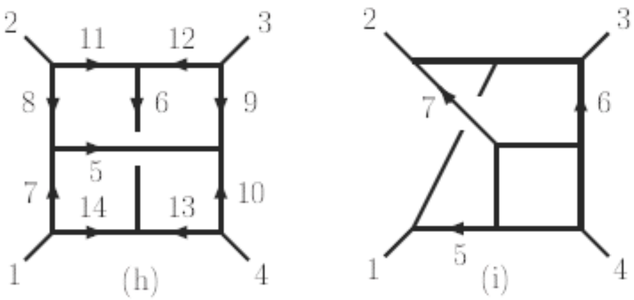
s_{12}^2:



$s_{12}\, s_{46}$:



$$s_{12}(\tau_{26} + \tau_{36}) + s_{14}(\tau_{15} + \tau_{25}) + s_{12}s_{14}$$



$$s_{12}\, s_{45} - s_{14}\, s_{46} - \frac{1}{3}(s_{12} - s_{14})\, \ell_7^2$$

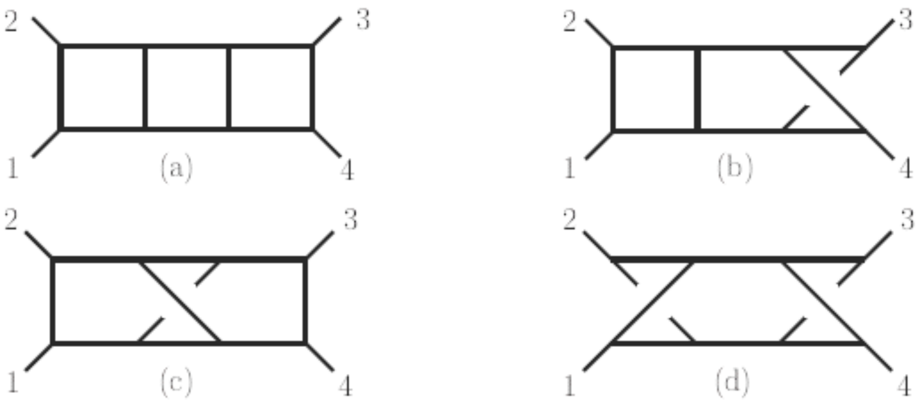
manifestly quadratic in loop momentum ℓ_M

N=8 numerators at 3 loops

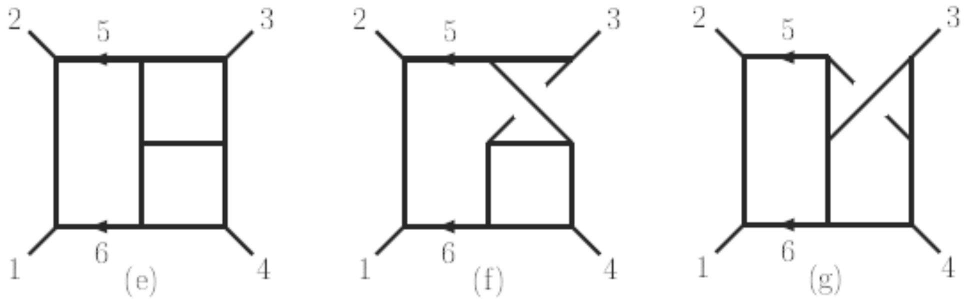
Omit overall
$(stA_4^{\text{tree}})^2 = stu\, M_4^{\text{tree}}$

$$s_{iM} = (k_i + \ell_M)^2$$
$$\tau_{iM} = 2k_i \cdot \ell_M$$

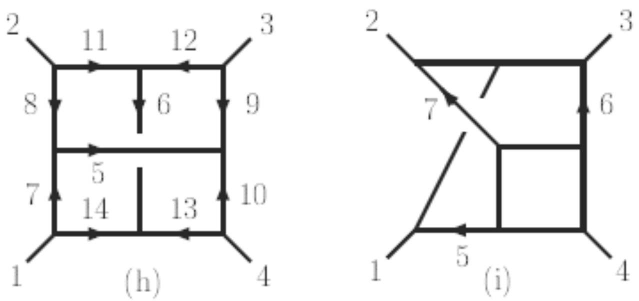
s_{12}^4



$s_{12}^2\, \tau_{35}\, \tau_{46}$



$$\begin{aligned}&(s_{12}(\tau_{26}+\tau_{36}) + s_{14}(\tau_{15}+\tau_{25}) + s_{12}s_{14})^2\\ &+ (s_{12}^2(\tau_{26}+\tau_{36}) - s_{14}^2(\tau_{15}+\tau_{25}))\\ &\times (\tau_{17}+\tau_{28}+\tau_{39}+\tau_{4,10})\\ &+ s_{12}^2(\tau_{17}\tau_{28}+\tau_{39}\tau_{4,10})\\ &+ s_{14}^2(\tau_{28}\tau_{39}+\tau_{17}\tau_{4,10})\\ &+ s_{13}^2(\tau_{17}\tau_{39}+\tau_{28}\tau_{4,10})\end{aligned}$$



$$\begin{aligned}&(s_{12}\tau_{45} - s_{14}\tau_{46})^2\\ &- \tau_{27}(s_{12}^2\tau_{45} + s_{14}^2\tau_{46})\\ &- \tau_{15}(s_{12}^2\tau_{47} + s_{13}^2\tau_{46})\\ &- \tau_{36}(s_{14}^2\tau_{47} + s_{13}^2\tau_{45})\\ &+ l_5^2\, s_{12}^2\, s_{14} + l_6^2\, s_{12}\, s_{14}^2\\ &- \frac{1}{3} l_7^2\, s_{12}\, s_{13}\, s_{14}\end{aligned}$$

also manifestly quadratic in loop momentum ℓ_M

BCDJR (2008)

N=8 ~ N=4 SYM in UV at $L = 3$

Manifest quadratic representation – same behavior as N=4 SYM – implies same critical dimension still for $L = 3$:

$$D_c \leq 4 + \frac{6}{L} = 6$$

- Evaluate UV poles in integrals

→ no further cancellation

- At 3 loops, $D_c = 6$ for N=8 SUGRA as well as N=4 SYM:

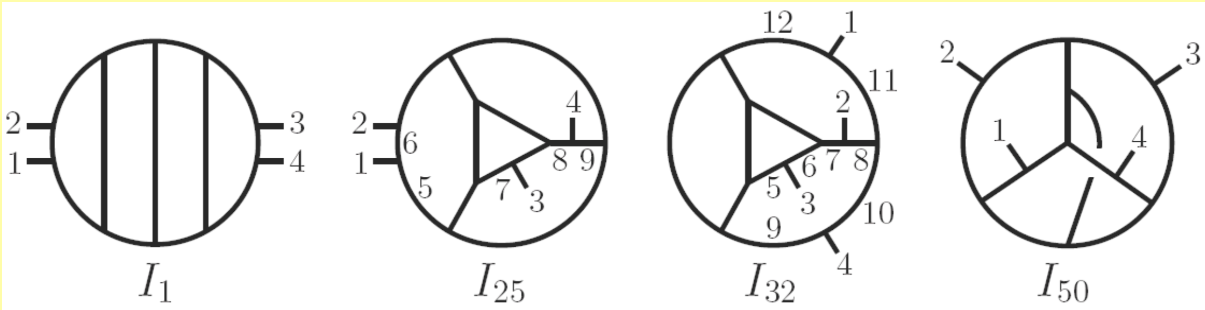
$$M_4^{(3),D=6-2\epsilon}\Big|_{\rm pole} = \frac{1}{\epsilon}\frac{5\zeta_3}{(4\pi)^9}\left(\frac{\kappa}{2}\right)^8 (s_{12}s_{13}s_{14})^2 M_4^{\rm tree}$$

$\mathcal{D}^6 R^4$ counterterm

N=8 ~ N=4 SYM in UV at $L = 4$

Bern, Carrasco, LD, Johansson, Roiban, 0905.2326, 1008.3327

- Now there are, not 9, but 50 nonvanishing cubic 4-point graphs



- Determine the 50 numerator factors, first for N=4 SYM, then, using KLT, for N=8 supergravity.
- Result we found was not manifestly as well behaved as N=4 SYM, but evaluating its UV behavior, many cancellations between the integrals took place, so same critical dimension was obtained:

$$D_c \leq 4 + \frac{6}{L} = 5.5$$

$\mathcal{D}^8 R^4$
"counterterm"

N=8 ~ N=4 SYM in UV at $L = 5$???

- Motivation: Various arguments point to 7-loops as the possible first divergence for N=8 SUGRA in D=4, associated with a D^8R^4 counterterm:

Howe, Lindstrom, NPB181, 487 (1981); Bossard, Howe, Stelle, 0908.3883;
Kallosh, 0903.4630; Green, Russo, Vanhove, 1002.3805;
Bjornsson, Green, 1004.2692; Bossard, Howe, Stelle, 1009.0743;
Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger,1009.1643

- Same D^8R^4 counterterm shows up at $L = 4$ in D = 5.5 (technically we did not show its coefficient is nonzero yet)
- Does 5-loops → $D^{10}R^4$ (same UV as N=4 SYM)
  or → D^8R^4 (worse UV as N=4 SYM)?
- 5-loops would be a very strong indicator for 7 loops
- Now 100s of nonvanishing cubic 4-point graphs!

Conclusions

- On-shell methods at one loop have many practical applications to LHC physics – analytically, but especially in numerical implementations
- On-shell methods in N=4 SYM are especially powerful (particularly in the large-N limit where other hidden symmetries are present). One can construct, for example, the full-color 4 gluon 4 loop amplitude in N=4 SYM, and even more complicated processes in the large-N limit.
- Full-color 4-loop result was also used to construct the 4 graviton 4 loop amplitude in N=8 supergravity, which showed that it is still as well-behaved as N=4 super-Yang-Mills theory through this order. What will happen at 5 loops?
- Wealth of IR information in gauge theory & gravity is also available (once technology is developed for doing non-planar 4-point integrals even numerically in $D = 4 - 2\varepsilon$ at $L = 3,4$)

End of Lecture 4

N=8 allowed

Chart of potential counterterms

Elvang, Freedman, Kiermaier (2010)

More recently ruled out using $E_{7(7)}$

Analytic proofs:

- $D^{2k}R^n$ MHV ∄ for $n > 4$ and $k < 4$.
- $D^{2k}R^n$ NMHV ∄ for $n > 5$ and $k < 2$.

Drummond, Heslop, Howe, Kerstan, th/0305202; Kallosh, 0906.3495

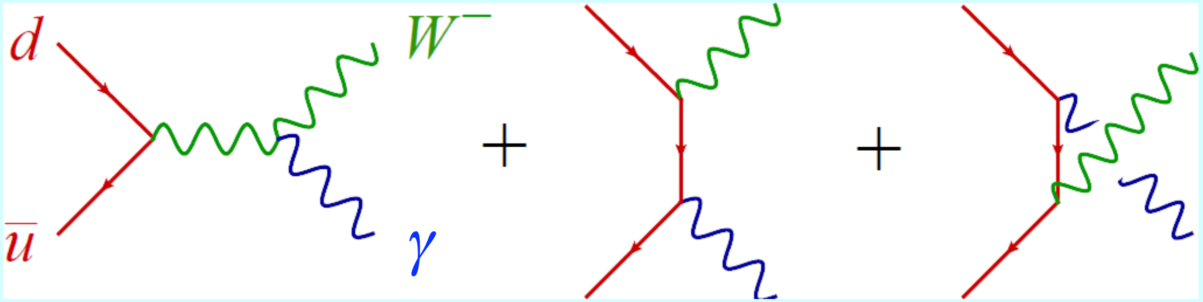
No divergences until at least 7 loops!

L							
3	R^4 MHV ∃!						
4	D^2R^4 MHV ∄	R^5 MHV ∄					
5	D^4R^4 MHV ∃!	D^2R^5 MHV ∄	R^6 (N)MHV ∄				
6	D^6R^4 MHV ∃!	D^4R^5 MHV ∄	D^2R^6 (N)MHV ∄	R^7 (N)MHV ∄			
7	D^8R^4 MHV ∃!	D^6R^5 MHV ∄	D^4R^6 MHV ∄ NMHV	D^2R^7 (N)MHV ∄	R^8 (N)MHV ∄ N^2MHV?		
8	$D^{10}R^4$ MHV ∃!	D^8R^5 MHV ∃!	D^6R^6 MHV ∄ NMHV?	D^4R^7 MHV ∄ NMHV?	D^2R^8 (N)MHV ∄ N^2MHV?	R^9 (N)MHV ∄ N^2MHV?	
9	$D^{12}R^4$ 2×MHV	$D^{10}R^5$?×MHV	D^8R^6 2×MHV NMHV?	D^6R^7 MHV ∄ NMHV?	D^4R^8 MHV ∄ N or N^2MHV?	D^2R^9 (N)MHV ∄ N^2MHV?	R^{10} (N)MHV ∄ N^2 or N^3MHV?

- red: not excluded
- green: ?
- gray: excluded

Radiation Zeroes

- In 1979, Mikaelian, Samuel and Sahdev computed $\frac{d\sigma(d\bar{u}\to W^-\gamma)}{d\cos\theta}$



- They found a "radiation zero" at

$$\cos\theta = -(1+2Q_d) = -1/3$$

- Held independent of (W,γ) helicities
- Implies a connection between "color" (here ~ electric charge Q_d) and kinematics ($\cos\theta$)

From Radiation Zeroes to Color-Kinematic Relations

- MSS result generalized to other 4-point non-Abelian gauge theory amplitudes by Zhu (1980), Goebel, Halzen, Leveille (1981).
- Massless adjoint gauge theory result:

$$\mathcal{A}_4^{\text{tree}} = \frac{n_s C_s}{s} + \frac{n_t C_t}{t} + \frac{n_u C_u}{u}$$

- Group theory → 3 terms are not independent (Jacobi identity):

$$C_t - C_u = C_s$$

- In a suitable "gauge", one finds: $n_t - n_u = n_s$

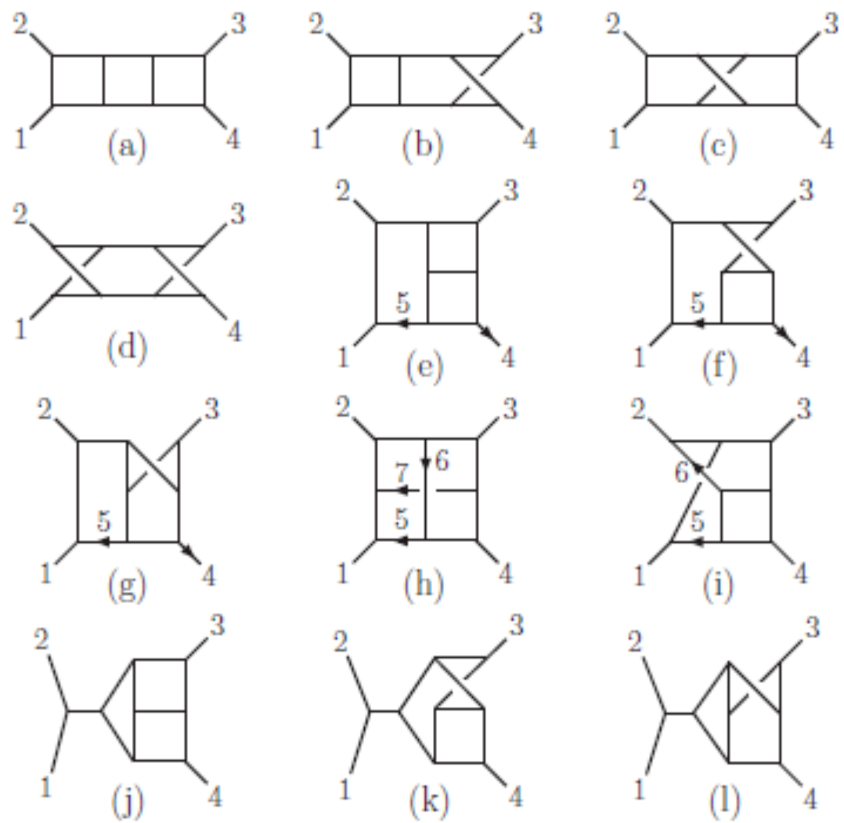
  Same structure can be extended to an arbitrary number of legs and provides a new "KLT-like" relation to gravity:

$$M_4^{\text{tree}} = \frac{n_s^{(L)} n_s^{(R)}}{s} + \frac{n_t^{(L)} n_t^{(R)}}{t} + \frac{n_u^{(L)} n_u^{(R)}}{u}$$

Bern, Carrasco, Johansson, 0805.3993

Multi-loop Color-Kinematic Relations

- Same structure can also be extended to at least 3 loops in N=4 super-Yang-Mills theory BCJ, 1004.0476
- Requires additional cubic topologies, compared to "old form"
- Gravity "squaring" relation works too!



Integral $I^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills ($\sqrt{\mathcal{N}=8}$ supergravity) numerator
(a)–(d)	s^2
(e)–(g)	$\left(s\,(-\tau_{35}+\tau_{45}+t)-t\,(\tau_{25}+\tau_{45})+u\,(\tau_{25}+\tau_{35})-s^2\right)/3$
(h)	$\left(s\,(2\tau_{15}-\tau_{16}+2\tau_{26}-\tau_{27}+2\tau_{35}+\tau_{36}+\tau_{37}-u)\right.$ $\left.+t\,(\tau_{16}+\tau_{26}-\tau_{37}+2\tau_{36}-2\tau_{15}-2\tau_{27}-2\tau_{35}-3\tau_{17})+s^2\right)/3$
(i)	$\left(s\,(-\tau_{25}-\tau_{26}-\tau_{35}+\tau_{36}+\tau_{45}+2t)\right.$ $\left.+t\,(\tau_{26}+\tau_{35}+2\tau_{36}+2\tau_{45}+3\tau_{46})+u\,\tau_{25}+s^2\right)/3$
(j)-(l)	$s(t-u)/3$