Introduction to the theory of hadronic collisions, and the physics of the LHC
(part I)

CERN Winter School on Supergravity, Strings, and Gauge Theory 2011

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Few words of wisdom

- Theoretical hadron collider physics is like culinary science:
  - overall framework and rules
  - many ingredients, need to learn their value
  - different ingredients may lead to different, sometimes unexpected, outcomes
  - need to develop a "taste", and an intuition for the impact and interplay of different phenomena
- As in the kitchen, only dedication and experience can put you in the position to fully appreciate the meaning of an experimental result, and to identify the best way to extract useful and reliable information from the data

Outline

- 1. (1hr, Monday) Introduction to the theoretical principles of hadron collisions: factorization, initial state evolution of PDFs

- 2. (2hrs, Tuesday) Introduction to the theoretical principles of hadron collisions: final state evolution, turning quarks and gluons into hadrons, and some examples of key processes (W/Z production, jets, etc)

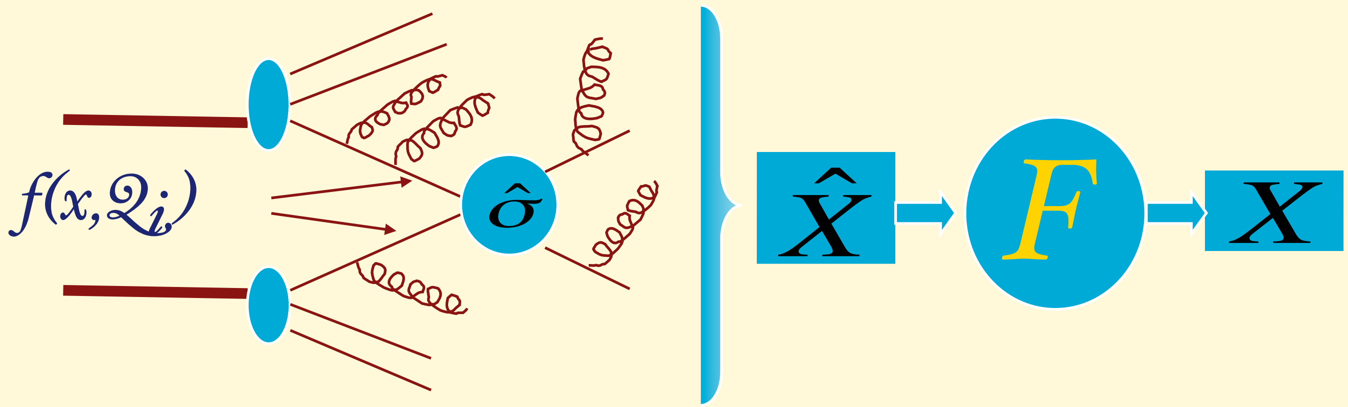
- 3. (1hr, Friday) What have we learned after 1 year of LHC

.. things I'll give for granted you know ..

- quarks and gluons
- mesons and baryons
- asymptotic freedom
- Feynman diagrams and Feynman rules
- basic knowledge of what high-energy hadronic collisions are about:
  - study of jets,
  - production of W/Z bosons,
  - search for Higgs,
  - search for supersymmetry and other new phenomena, etc

Factorization Theorem

$$\frac{d\sigma}{dX} = \sum_{j,k} \int_{\hat{X}} f_j(x_1,Q_i)\, f_k(x_2,Q_i)\, \frac{d\hat{\sigma}_{jk}(Q_i,Q_f)}{d\hat{X}}\, F(\hat{X} \to X; Q_i, Q_f)$$



$f_j(x,Q)$ Parton distribution functions (PDF)

- sum over all initial state histories leading, at the scale Q, to:

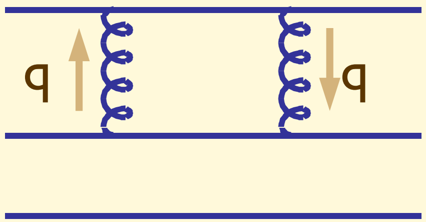
$$\vec{p}_j = x\,\vec{P}_{proton}$$

$F(\hat{X} \to X; Q_i, Q_f)$

- transition from partonic final state to the hadronic observable (hadronization, fragm. function, jet definition, etc)
- Sum over all histories with X in them

Universality of parton densities and factorization, an intuitive view

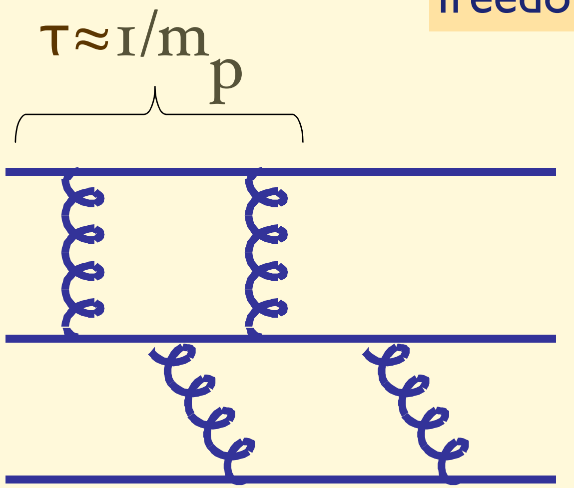
1) Exchange of **hard gluons** among quarks inside the proton is suppressed by powers of $(m_p/Q)^2$



$$\stackrel{q>Q}{\sim} \int_Q^\infty \frac{d^4q}{q^6} \sim \frac{1}{Q^2}$$

Assuming asymptotic freedom!

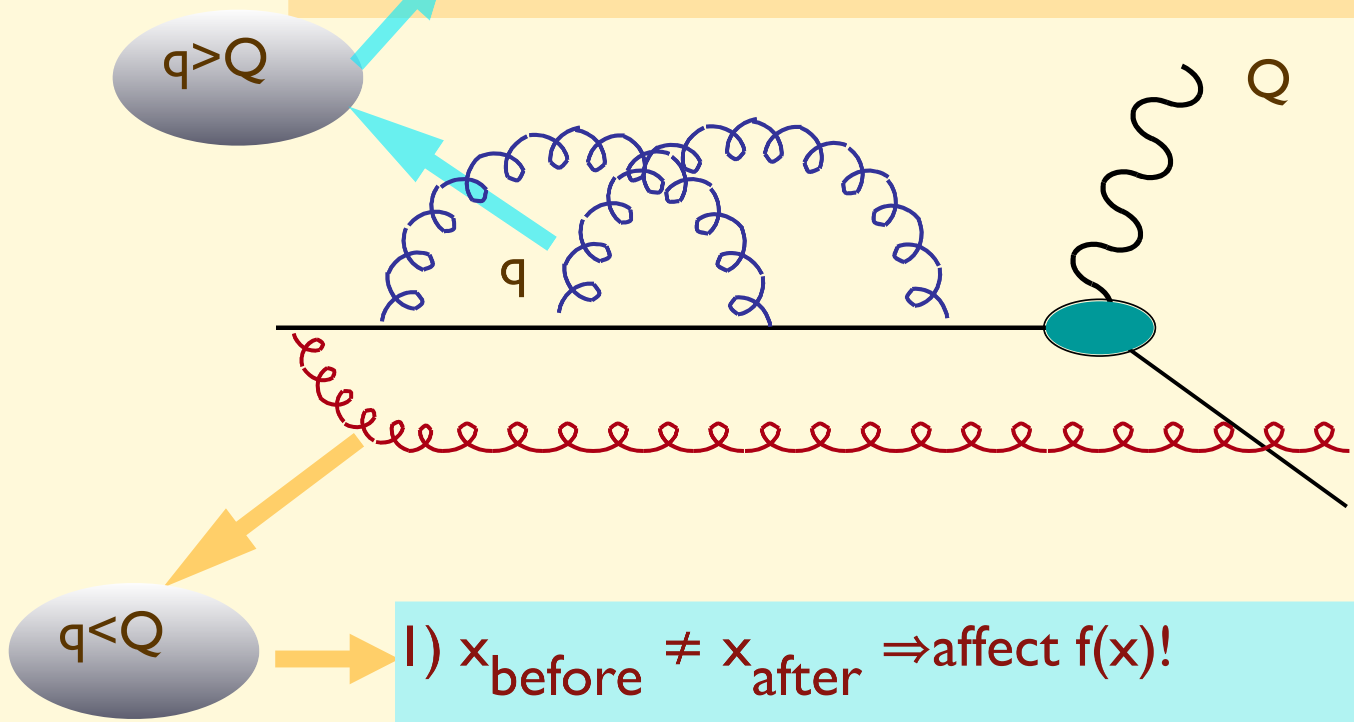
2) Typical time-scale of interactions binding the proton is therefore of $O(1/m_p)$ (in a frame in which the proton has energy E, $\tau=\gamma/m_p = E/m_p^2$)



3) If a hard probe ($Q>>m_p$) hits the proton, on a time scale $=1/Q$, there is no time for quarks to negotiate a coherent response. The struck quark receives no feedback from its pals, and acts as a free particle

As a result, to study inclusive processes at large Q it is sufficient to consider the interactions between the external probe and a single parton:

1) calculable in perturbative QCD (pQCD)
2) do not affect f(x): $x_{before} = x_{after}$



This gluon cannot be reabsorbed because the quark is gone

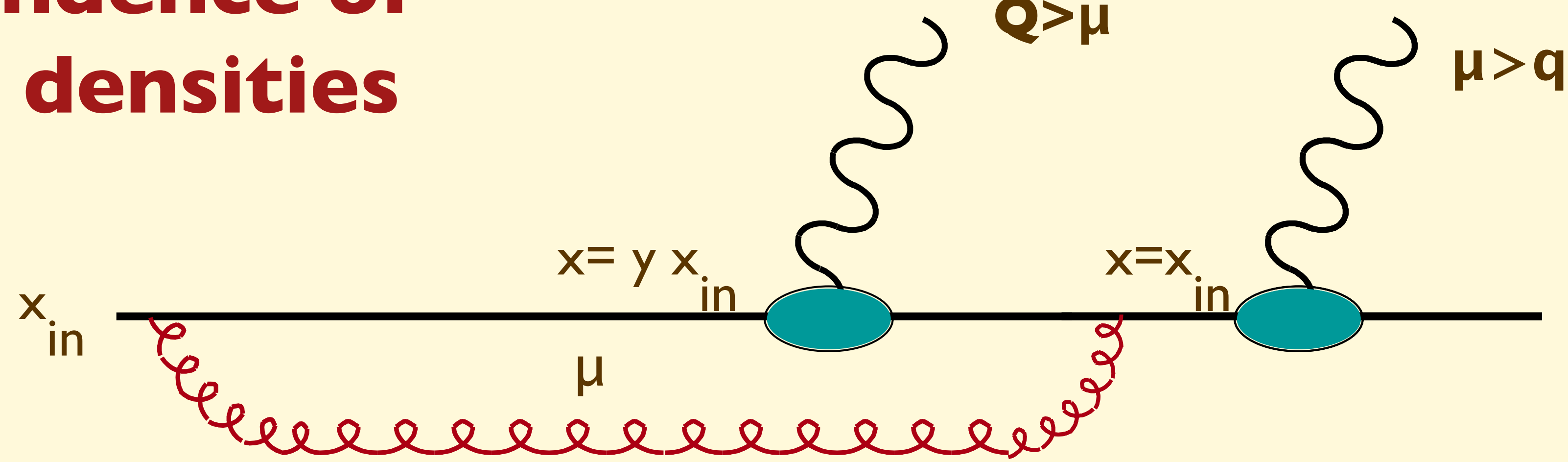
1) $x_{before} \neq x_{after}$ ⇒affect f(x)!

2) for q≈1 GeV **not** calculable in pQCD

However, since τ(q≈1GeV)>>1/Q, the emission of low-virtuality gluons will take place long before the hard collision, and therefore cannot depend on the detailed nature of the hard probe. While it is not calculable in pQCD, f(q<<Q) can be measured using a reference probe, and used elsewhere

➡ Universality of f(x)

Q dependence of parton densities



The larger is Q, the more gluons will **not** have time to be reabsorbed

PDF's depend on Q!

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in}\, f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy\, P(y,q^2)\, \delta(x - y x_{in})$$

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in} f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy P(y,q^2)\,\delta(x - yx_{in})$$

f(x,Q) should be independent of the intermediate scale μ considered:

$$\frac{df(x,Q)}{d\mu^2} = 0 \quad \Rightarrow \frac{df(x,\mu)}{d\mu^2} = \int_x^1 \frac{dy}{y} f(y,\mu)\, P(x/y,\mu^2)$$

One can prove that:

$$P(x,Q^2) = \frac{\alpha_s}{2\pi}\frac{1}{Q^2}P(x)$$

calculable in pQCD

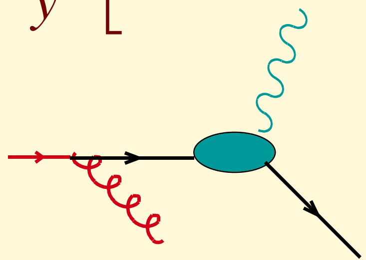
and finally (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi DGLAP equation):

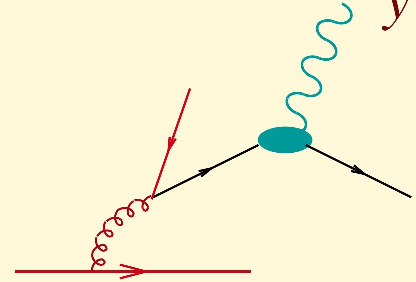
$$\frac{df(x,\mu)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} f(y,\mu)\, P(x/y)$$

More in general, one should consider additional processes which lead to the evolution of partons at high Q ($t=\log Q^2$):

$$[g(x)]_+ : \quad \int_0^1 dx\, f(x)\, g(x)_+ \equiv \int_0^1 [f(x) - f(1)]\, g(x)\, dx$$

$$\frac{dq(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[q(y,Q)\, P_{qq}\left(\frac{x}{y}\right) + g(y,Q)\, P_{qg}\left(\frac{x}{y}\right) \right]$$

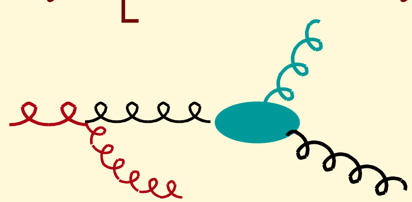


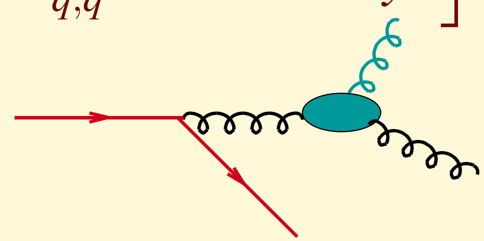


$$P_{qq}(x) = C_F \left(\frac{1+x^2}{1-x} \right)_+$$

$$P_{qg}(x) = \frac{1}{2} \left[x^2 + (1-x)^2 \right]$$

$$\frac{dg(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[g(y,Q)\, P_{gg}\left(\frac{x}{y}\right) + \sum_{q,\bar{q}} q(y,Q)\, P_{gq}\left(\frac{x}{y}\right) \right]$$

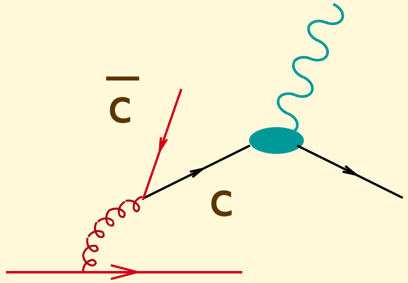




$$P_{gq}(x) = C_F \left(\frac{1+(1-x)^2}{x} \right)$$

$$P_{gg}(x) = 2N_c \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \delta(1-x) \left(\frac{11N_c - 2n_f}{6} \right)$$

Example: charm in the proton



$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q)\, P_{qg}(\frac{x}{y})$$

Assuming a typical behaviour of the gluon density: $g(x,Q) \sim A/x$

and using $P_{qg}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$ we get:

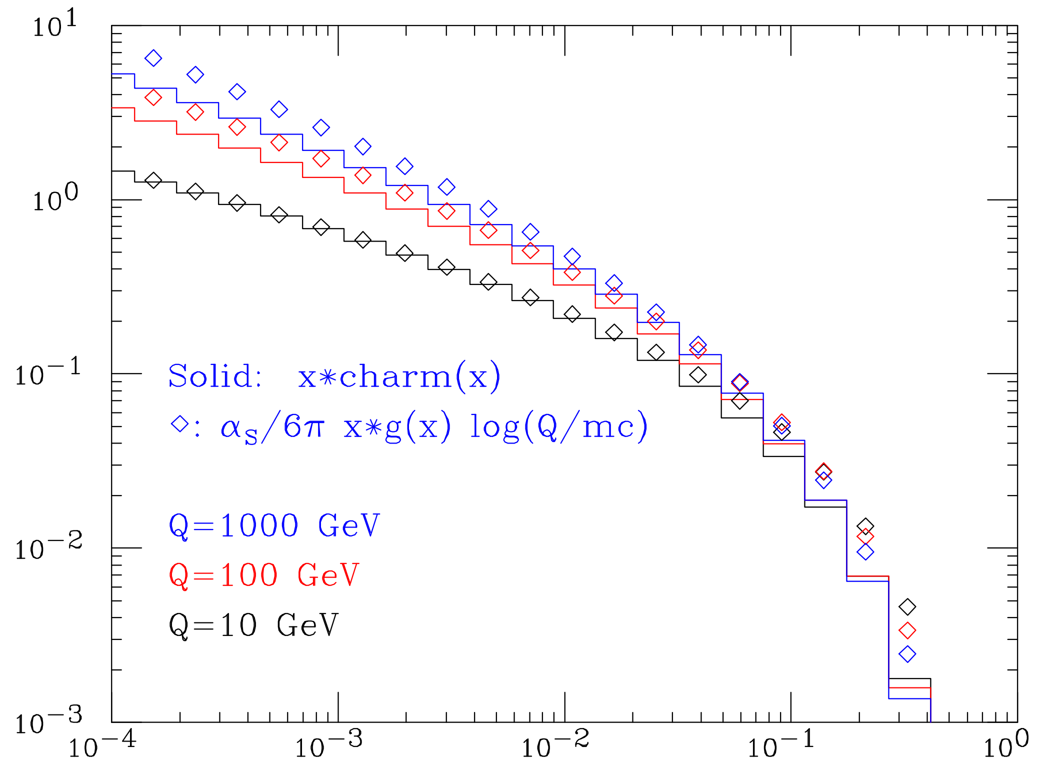
$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(x/y,Q)\, P_{qg}(y) = \frac{\alpha_s}{2\pi}\int_x^1 dy \frac{A}{x}\frac{1}{2}[y^2 + (1-y)^2] = \frac{\alpha_s}{6\pi}\frac{A}{x}$$

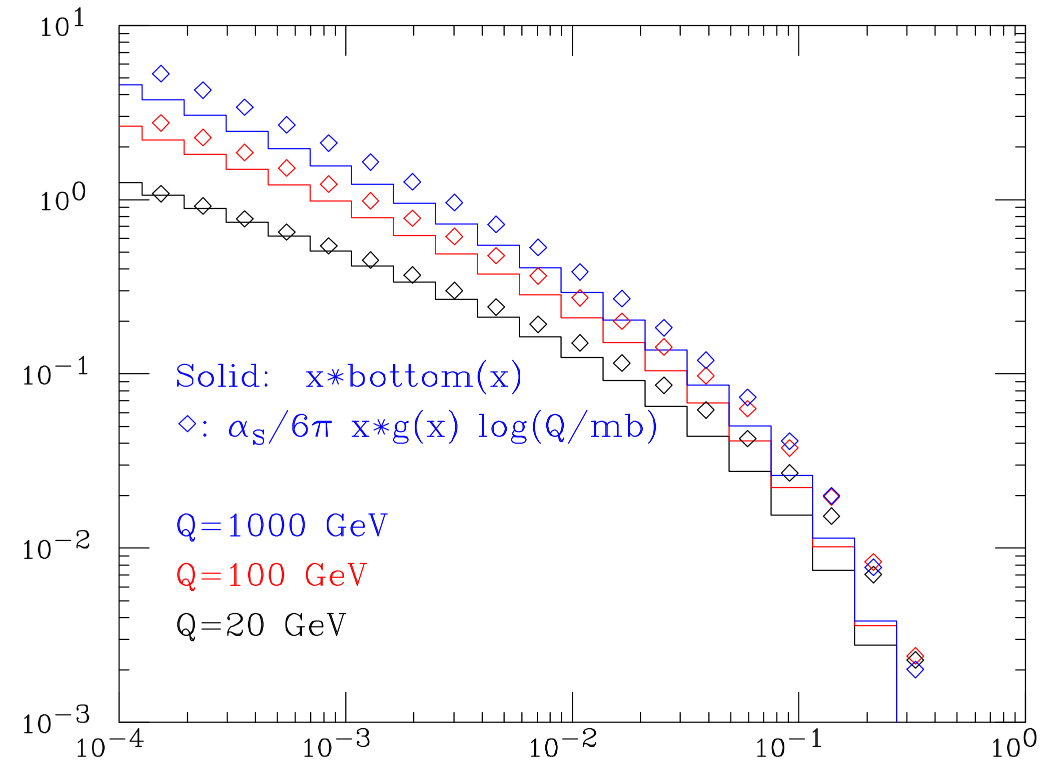
and therefore:

$$c(x,Q) \sim \frac{\alpha_s}{6\pi}\log(\frac{Q^2}{m_c^2})\, g(x,Q)$$

Corrections to this simple formula will arise due to the Q dependence of g(x) and of α_s

Numerical example

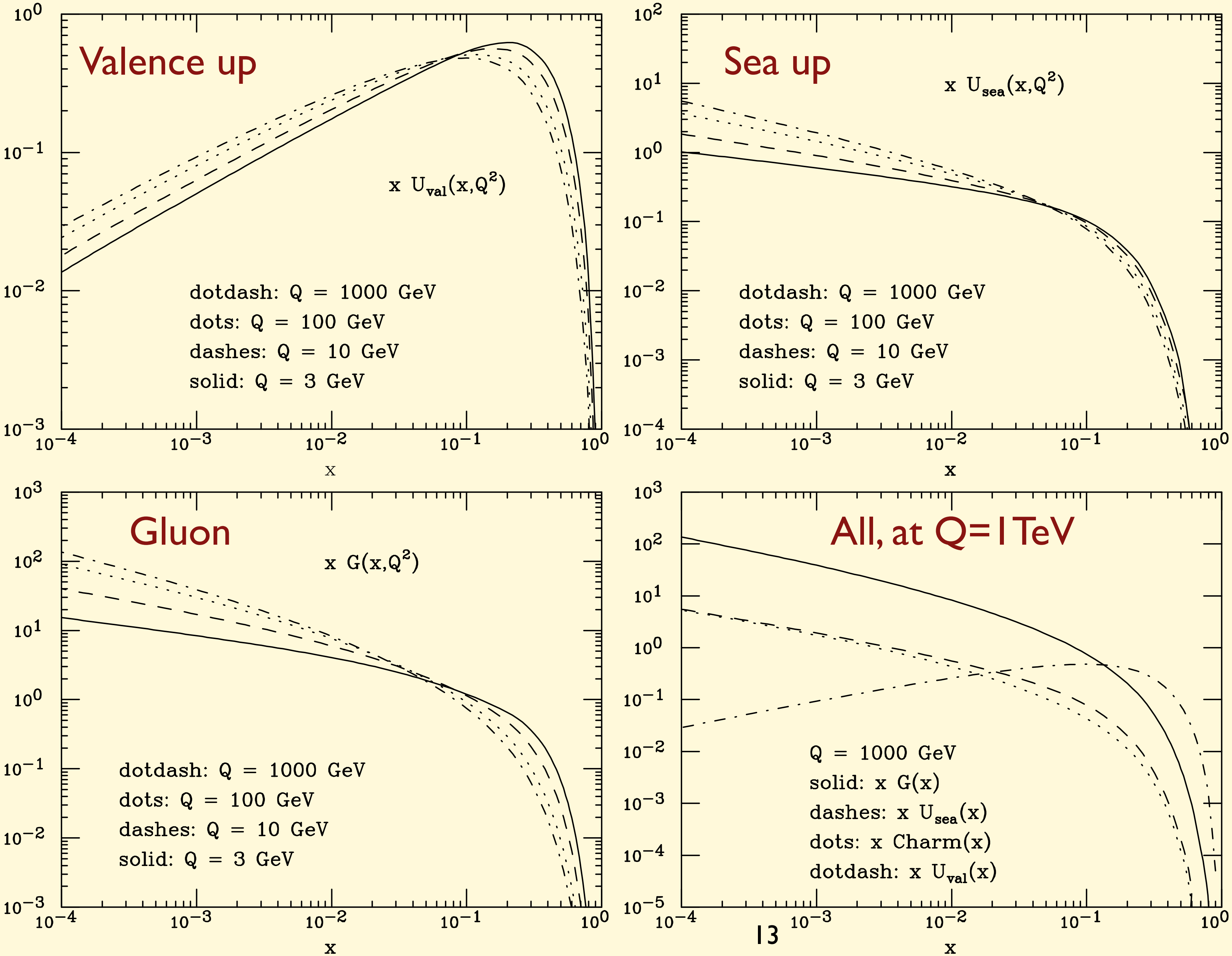




Excellent agreement, given the simplicity of the approximation!

Can be improved by tuning the argument of the log (threshold onset), including a better parameterization of g(x), etc....

Examples of PDFs and their evolution

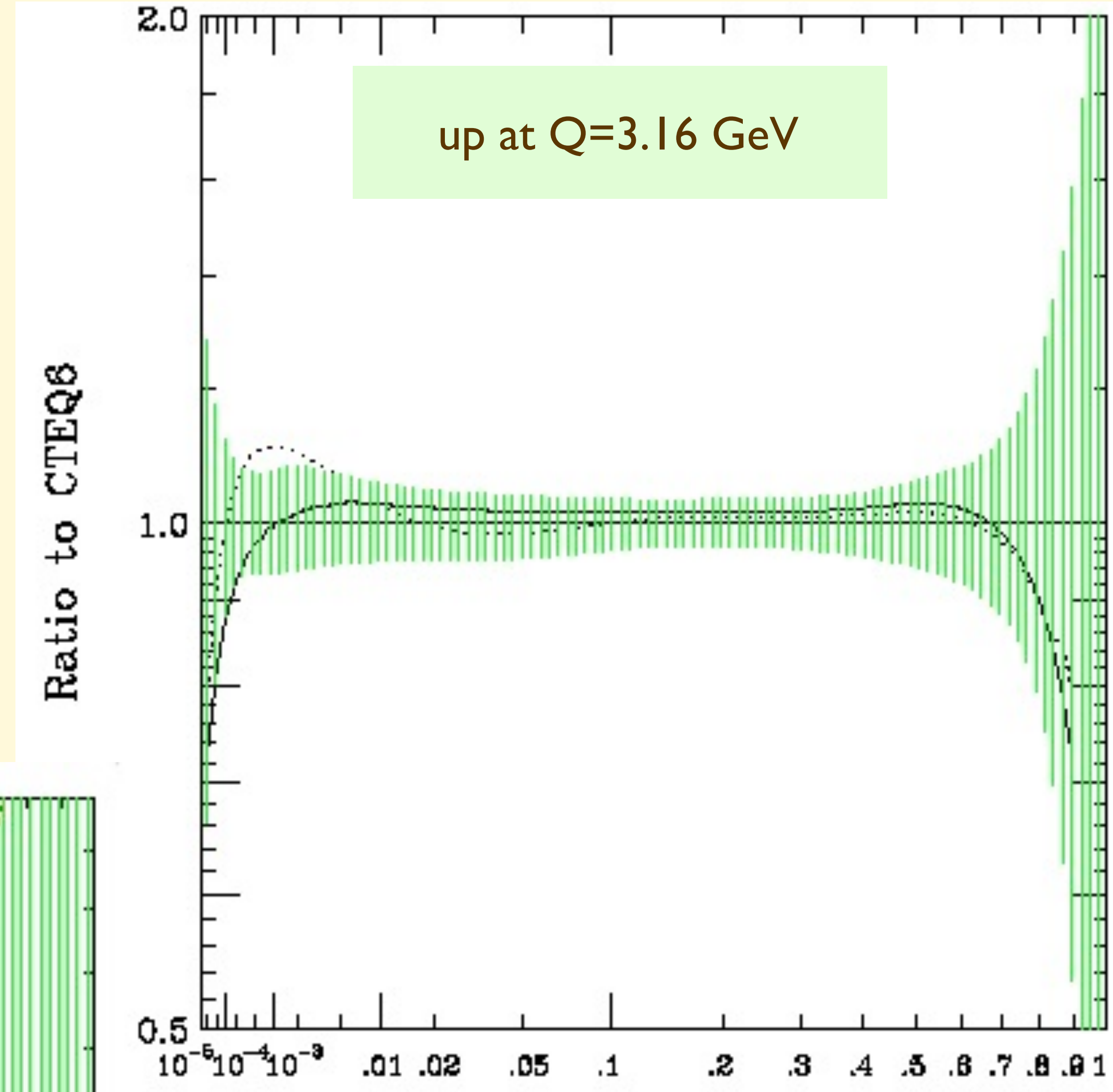


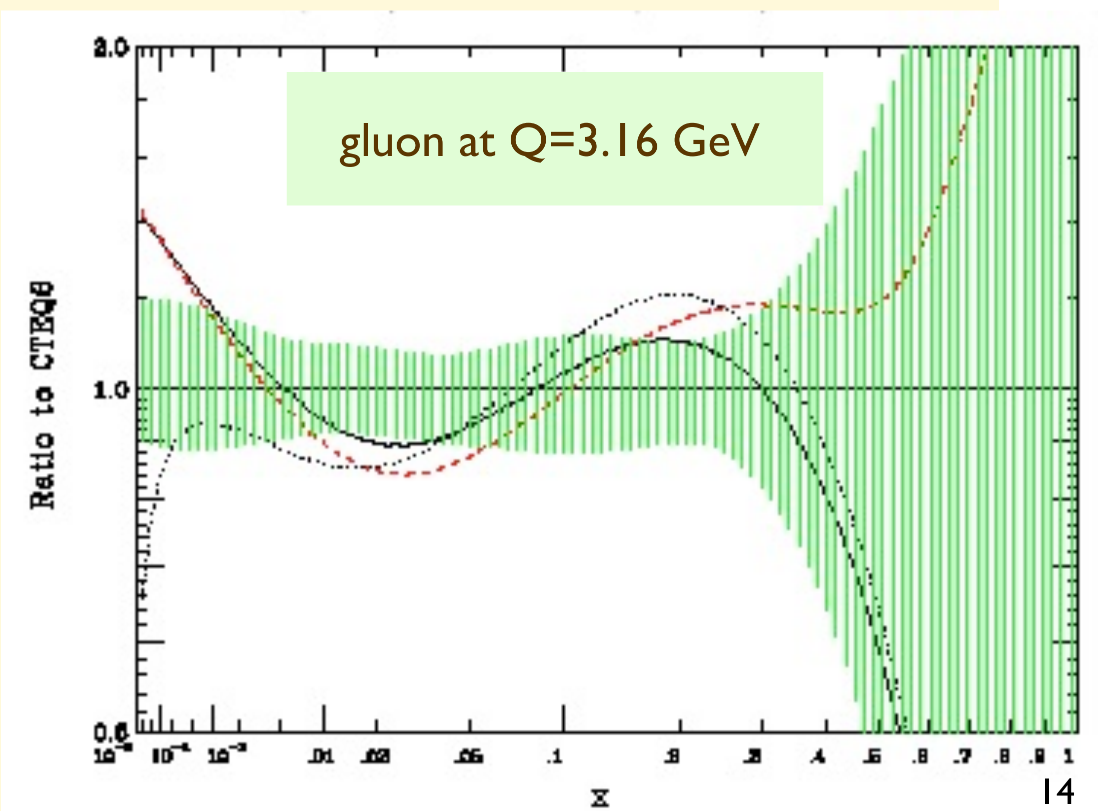
Note:
sea ≈10% glue

Note:
charm≈up at high Q

PDF uncertainties

Green bands represent the convolution of theoretical and experimental systematics in the determination of PDFs

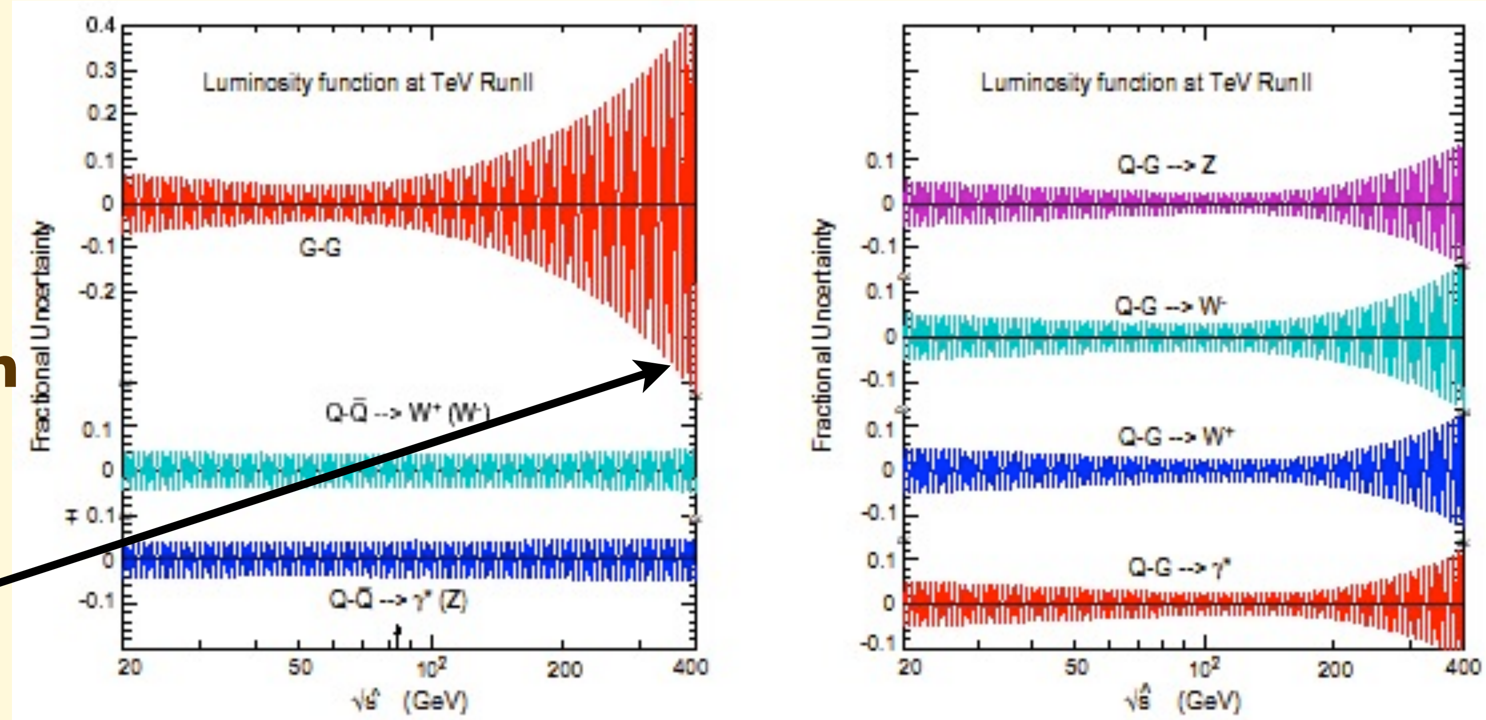




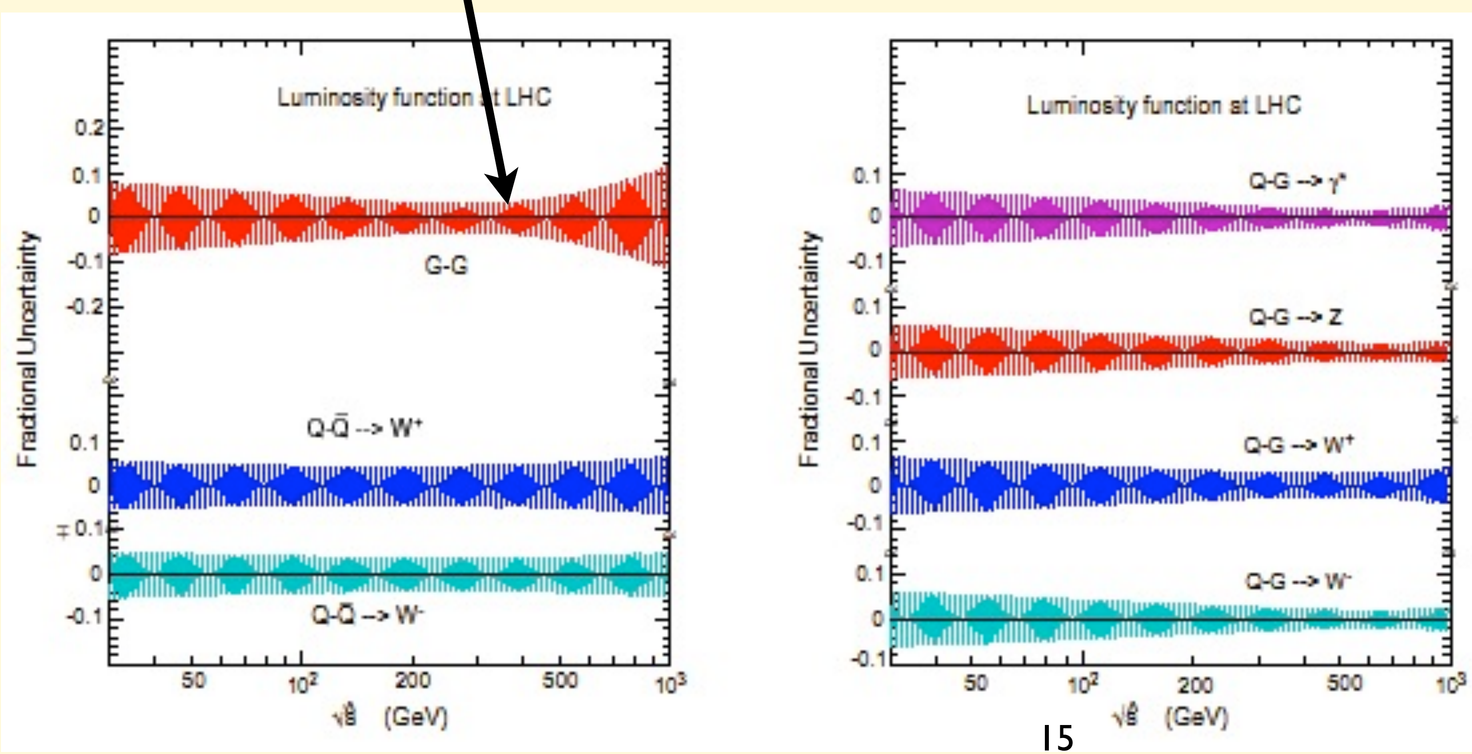
Proton PDFs known to 10-20% for $10^{-3}<x<0.3$, with uncertainties getting smaller at larger Q

PDF luminosity uncertainties

At the Tevatron



tt production, smaller uncertainty at the LHC!



At the LHC