Mariana 12 August 2020

Lagrangian

$$\mathcal{L} = -\frac{n^\alpha n^\mu}{2n^2} g^{\beta\nu}\left(F^A_{\alpha\beta} F^A_{\mu\nu} + F^B_{\alpha\beta} F^B_{\mu\nu}\right) +$$

$$+ \frac{n^\alpha n^\mu}{4n^2} \varepsilon^{\mu\nu\gamma\delta}\left(F^B_{\alpha\nu} F^A_{\gamma\delta} - F^A_{\alpha\nu} F^B_{\gamma\delta}\right) - J_\mu A^\mu - K_\mu B^\mu$$

$$- \frac{n^\alpha n^\mu}{2n^2} g^{\beta\nu}\left(F^{A_D}_{\alpha\beta} F^{A_D}_{\mu\nu} + F^{B_D}_{\alpha\beta} F^{B_D}_{\mu\nu}\right)$$

$$+ \frac{\epsilon e e_D}{n^2} g^{\beta\nu}\left(F^{A_D}_{\alpha\beta} F^A_{\mu\nu} - F^{B_D}_{\alpha\beta} F^B_{\mu\nu}\right)$$

$$+ \frac{n^\alpha n^\mu}{4n^2} \varepsilon^{\mu\nu\gamma\delta}\left(F^{B_D}_{\alpha\nu} F^{A_D}_{\gamma\delta} - F^{A_D}_{\alpha\nu} F^{B_D}_{\gamma\delta}\right) - J^D_\mu A_D^\mu - K^D_\mu B_D^\mu$$

$$F^X_{\mu\nu} = \partial_\mu X^\nu - \partial_\nu X^\mu$$

$$J_\mu = e \bar{\psi}_e \gamma^\mu \psi_e \qquad K_\mu = g \bar{\psi}_g \gamma^\mu \psi_g$$

$$J^D_\mu = e_D \bar{\psi}_{e_D} \gamma^\mu \psi_{e_D} \qquad K^D_\mu = g_D \bar{\psi}_{g_D} \gamma^\mu \psi_{g_D}$$

Note: normalization of Lagrangian is chosen for diagonalization of kinetic terms

g, g_D must match with Terning's or Verhaaren's convention

$\therefore$ It will amount to redifining $F^X_{\mu\nu}$ to include couplings.

Equations of motion

VISIBLE

(1) $$\frac{n_\alpha}{n^2}\left(n^\mu \partial_\nu F_A^{\alpha\nu} - n^\nu \partial_\nu F_A^{\alpha\mu} - \varepsilon^{\mu\nu\alpha}{}_{\beta}\, n_\gamma \partial_\nu F_B^{\gamma\beta}\right)$$

$$+ \frac{\epsilon e e_D}{n^2} n_\alpha \partial_\nu \left(n^\mu F_{A_D}^{\alpha\nu} - n^\nu F_{A_D}^{\alpha\mu}\right) = J^\mu$$

(2) $$\frac{n_\alpha}{n^2}\left(n^\mu \partial_\nu F_B^{\alpha\nu} - n^\nu \partial_\nu F_B^{\alpha\mu} + \varepsilon^{\mu\nu\alpha}{}_{\beta}\, n_\gamma \partial_\nu F_A^{\gamma\beta}\right)$$

$$- \frac{\epsilon e e_D}{n^2} n_\alpha \partial_\nu \left(n^\mu F_{B_D}^{\alpha\nu} - n^\nu F_{B_D}^{\alpha\mu}\right) = K^\mu$$

DARK

(3) $$\frac{n_\alpha}{n^2}\left(n^\nu \partial_\nu F_{A_D}^{\alpha\nu} - n^\nu \partial_\nu F_{A_D}^{\alpha\mu} - \varepsilon^{\mu\nu\alpha}{}_{\beta}\, n_\gamma \partial_\nu F_{B_D}^{\gamma\beta}\right)$$

$$+ \frac{\epsilon e e_D}{n^2} n_\alpha \partial_\nu \left(n^\mu F_A^{\alpha\nu} - n^\nu F_A^{\alpha\mu}\right) = J_D^\mu + M_{A_D}^2 A_D^\mu \cancel{A_{D\mu}}$$

(4) $$\frac{n_\alpha}{n^2}\left(n^\mu \partial_\nu F_{B_D}^{\alpha\nu} - n^\nu \partial_\nu F_{B_D}^{\alpha\mu} + \varepsilon^{\mu\nu\alpha}{}_{\beta}\, n_\gamma \partial_\nu F_{A_D}^{\gamma\beta}\right)$$

$$- \frac{\epsilon e e_D}{n^2}\left(n_\alpha \partial_\nu n^\mu F_B^{\alpha\nu} - n^\nu_\nu F_B^{\alpha\mu}\right) = K_D^\mu + M_{B_D}^2 B_D^\mu \cancel{B_{D\mu}}$$

Rewrite equations, grouping terms

(1') $\frac{n_\alpha}{n^2}\Big[n^\mu \partial_\nu\left(F_A^{\alpha\nu} + \epsilon e e_D F_{A_D}^{\alpha\nu}\right) - n^\nu \partial_\nu\left(F_A^{\alpha\mu} + \epsilon e e_D F_{A_D}^{\alpha\mu}\right)$

$- n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta\, \partial_\nu F_B^{\gamma\beta}\Big] = J^\mu$

(2') $\frac{n_\alpha}{n^2}\, n^\mu \partial_\nu\left(F_B^{\alpha\nu} - \epsilon e e_D F_{B_D}^{\alpha\nu}\right) - n^\nu \partial_\nu\left(F_B^{\alpha\mu} - \epsilon e e_D F_{B_D}^{\alpha\mu}\right)$

$+ n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta\, \partial_\nu F_A^{\gamma\beta}\Big] = K^\mu$

(3') $\frac{n_\alpha}{n^2}\Big[n^\mu \partial_\nu\left(F_{A_D}^{\alpha\nu} + \epsilon e e_D F_A^{\alpha\nu}\right) - n^\nu \partial_\nu\left(F_{A_D}^{\alpha\mu} + \epsilon e e_D F_A^{\alpha\mu}\right)$

$- n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta\, \partial_\nu F_{B_D}^{\gamma\beta}\Big] = J_D^\mu + M_{A_D}^2 A_D^\mu A_{D\mu}$

(4') $\frac{n_\alpha}{n^2}\, n^\mu \partial_\nu\left(F_{B_D}^{\alpha\nu} - \epsilon e e_D F_B^{\alpha\nu}\right) - n^\nu \partial_\nu\left(F_{B_D}^{\alpha\mu} - \epsilon e e_D F_B^{\alpha\mu}\right)$

$+ n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta\, \partial_\nu F_{A_D}^{\gamma\beta}\Big] = K_D^\mu + M_{B_D}^2 B_D^\mu \cancel{B_{D\mu}}$

?

Rotations

$$A_\mu = (\cos\phi - \epsilon e e_D \sin\phi)\bar{A}_\mu + (-\sin\phi - \epsilon e e_D \cos\phi)\bar{A}_\mu^D$$

$$A_{D\mu} = \sin\phi\, \bar{A}_\mu + \cos\phi\, \bar{A}_{D\mu}$$

$$B_\mu = \cos\phi\, \bar{B}_\mu - \sin\phi\, \bar{B}_{D\mu}$$

$$B_{D\mu} = (\sin\phi + \epsilon e e_D \cos\phi)\bar{B}_\mu + (\cos\phi - \epsilon e e_D \sin\phi)\bar{B}_\mu^D$$

$\phi = 0$ $A_\mu = \bar{A}_\mu - \epsilon e e_D \bar{A}_\mu^D$ $\quad B_\mu = \bar{B}_\mu$

$A_\mu^D = \bar{A}_{\mu D}$ $\quad B_\mu^D = \bar{B}_{\mu D} + \epsilon e e_D \bar{B}_\mu$

→ in (1') $F_A^{\alpha\nu} + \epsilon e e_D F_{A_D}^{\alpha\nu}$

$$= (\cos\phi - \epsilon e e_D \sin\phi)\bar{F}_A^{\alpha\nu} + \epsilon e e_D(\sin\phi\, \bar{F}_A^{\alpha\nu} + \cos\phi\, \bar{F}_{A_D}^{\alpha\nu})$$

$$(-\sin\phi - \epsilon e e_D \cos\phi)\bar{F}_{A_D}^{\alpha\nu} \quad = \quad \cos\phi\, \bar{F}_A^{\alpha\nu} - \sin\phi\, \bar{F}_{A_D}^{\alpha\nu}$$

(1'')
$$\boxed{\frac{n_\alpha}{n^2}\Big[n^\mu \partial_\nu(\cos\phi\, \bar{F}_A^{\alpha\nu} - \sin\phi\, \bar{F}_{A_D}^{\alpha\nu}) - n^\nu \partial_\nu(\cos\phi\, \bar{F}_A^{\alpha\mu} - \sin\phi\, \bar{F}_{A_D}^{\alpha\mu}) - n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta\, \partial_\nu(\cos\phi\, \bar{F}_B^{\gamma\beta} - \sin\phi\, \bar{F}_{B_D}^{\gamma\beta})\Big] = J_\mu}$$

$$\Rightarrow \underline{J_\mu = \cos\phi\, \bar{J}_\mu - \sin\phi\, \bar{J}_\mu^D}$$

$e\cos\phi\, \bar{\psi}_e \gamma^\mu \psi_e + e\sin\phi\, \bar{\psi}_e^D \gamma^\mu \psi_e^D$

→ In (2') $F_B^{\alpha\nu} - \epsilon e e_D F_{B_D}^{\alpha\nu} =$

$$= \cos\phi\, \bar{F}_B^{\alpha\nu} - \sin\phi\, \bar{F}_{B_D}^{\alpha\nu} - \epsilon e e_D(\sin\phi + \epsilon e e_D \cos\phi)\bar{F}_B^{\alpha\nu}$$

$$- \epsilon e e_D(\cos\phi - \epsilon e e_D \sin\phi)\bar{F}_{B_D}^{\alpha\nu}$$

$$\simeq (\cos\phi - \epsilon e e_D \sin\phi)\bar{F}_B^{\alpha\nu} - (\sin\phi + \epsilon e e_D \cos\phi)\bar{F}_{B_D}^{\alpha\nu}$$

(2'')
$$\boxed{\begin{aligned}&\frac{n_\alpha}{n^2}\Big[n^\mu \partial_\nu\{(\cos\phi - \epsilon e e_D \sin\phi)\bar{F}_B^{\alpha\nu} - (\sin\phi + \epsilon e e_D\cos\phi)\bar{F}_{B_D}^{\alpha\nu}\} \\ &\quad - n^\nu \partial_\nu\{(\cos\phi - \epsilon e e_D \sin\phi)\bar{F}_B^{\alpha\mu} - (\sin\phi + \epsilon e e_D \cos\phi)\bar{F}_{B_D}^{\alpha\mu}\} \\ &\quad + n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta\, \partial_\nu\big[(\cos\phi - \epsilon e e_D \sin\phi)\bar{F}_A^{\gamma\beta} - (\sin\phi + \epsilon e e_D \cos\phi)\bar{F}_{A_D}^{\gamma\beta}\big] \\ &\quad = K^\mu\end{aligned}}$$

$$\Rightarrow \underline{K_\mu = (\cos\phi - \epsilon e e_D \sin\phi)\bar{K}_\mu - (\sin\phi + \epsilon e e_D \cos\phi)\bar{K}_\mu^D}$$

$\rightarrow$ In (3') $F_{A_D}^{\alpha\nu} + \epsilon e e_D F_A^{\alpha\nu} =$

$$\sin\phi\,\bar{F}_A^{\alpha\nu} + \cos\phi\,\bar{F}_{A_D}^{\alpha\nu} + \epsilon e e_D(\cos\phi - \epsilon e e_D \sin\phi)\bar{F}_A^{\alpha\nu}$$

$$-\epsilon e e_D(\sin\phi + \epsilon e e_D\cos\phi)\bar{F}_{A_D}^{\alpha\nu}$$

$$\cong (\sin\phi + \epsilon e e_D\cos\phi)\bar{F}_A^{\alpha\nu} + (\cos\phi - \epsilon e e_D\sin\phi)\bar{F}_{A_D}^{\alpha\nu}$$

(3'')
$$\boxed{\begin{aligned}&\frac{n_\alpha}{n^2}\Big\{ n^\mu\partial_\nu\left[(\sin\phi + \epsilon e e_D\cos\phi)\bar{F}_A^{\alpha\nu} + (\cos\phi - \epsilon e e_D\sin\phi)\bar{F}_{A_D}^{\alpha\nu}\right]\\ &\quad - n^\nu\partial_\nu\left[(\sin\phi + \epsilon e e_D\cos\phi)\bar{F}_A^{\alpha\mu} + (\cos\phi - \epsilon e e_D\sin\phi)\bar{F}_{A_D}^{\alpha\mu}\right]\\ &\quad - n_\gamma\epsilon^{\mu\nu\alpha}{}_\beta\,\partial_\nu\left[(\sin\phi + \epsilon e e_D\cos\phi)\bar{F}_B^{\gamma\beta} + (\cos\phi - \epsilon e e_D\sin\phi)\bar{F}_{B_D}^{\gamma\beta}\right]\\ &\quad = J_D^\mu + M_{A_D}^2(\sin^2\phi\,\bar{A}_\mu + \cos^2\phi\,A_{\mu D})\end{aligned}}$$

$$\underline{J_{D\mu}^{\mu} = (\sin\phi + \epsilon e e_D\cos\phi)\bar{J}_\mu + (\cos\phi - \epsilon e e_D\sin\phi)\bar{J}_\mu^D}$$

$\rightarrow$ In (4') $F_{B_D}^{\alpha\nu} - \epsilon e e_D F_B^{\alpha\nu} =$

$$(\sin\phi + \epsilon e e_D\cos\phi)\bar{F}_B^{\alpha\nu} + (\cos\phi - \epsilon e e_D\sin\phi)\bar{F}_{B_D}^{\alpha\nu}$$

$$-\varepsilon e e_D\cos\phi\,\bar{F}_B^{\alpha\nu} + \varepsilon e e_D\sin\phi\,\bar{F}_{B_D}^{\alpha\nu}$$

$$= \sin\phi\,\bar{F}_B^{\alpha\nu} + \cos\phi\,\bar{F}_{B_D}^{\alpha\nu}$$

(4'')
$$\boxed{\begin{aligned}&\frac{n_\alpha}{n^2}\Big[n^\mu\partial_\nu(\cos\phi\,\bar{F}_{B_D}^{\alpha\nu} + \sin\phi\,\bar{F}_B^{\alpha\nu}) - n^\nu\partial_\nu(\cos\phi\,\bar{F}_{B_D}^{\alpha\mu} + \sin\phi\,\bar{F}_B^{\alpha\mu})\\ &\quad + n_\gamma\epsilon^{\mu\nu\alpha}{}_\beta\,\partial_\nu(\cos\phi\,\bar{F}_{A_D}^{\gamma\beta} + \sin\phi\,\bar{F}_A^{\gamma\beta})\Big] =\\ &= K_D^\mu + M_{B_D}^2(\sin^2\phi\,\bar{B}_\mu^2 + \cos^2\phi\,\bar{B}_{D\mu}^2)\end{aligned}}$$

$$\underline{K_{D\mu} = \cos\phi\,\bar{K}_{D\mu} + \sin\phi\,\bar{K}_\mu}$$

in (1''), (2''), (3''), (4'') choose $\sin\phi = 0 \Rightarrow \cos\phi = 1$

(1a) $$\frac{n_\alpha}{n^2}\left[\left(n^\mu \partial_\nu \bar{F}_A^{\alpha\nu} - n^\nu \partial_\nu \bar{F}_A^{\alpha\mu}\right) - n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta\, \partial_\nu \bar{F}_B^{\gamma\beta}\right] = \bar{J}_\mu$$

(2a) $$\frac{n_\alpha}{n^2}\Big[n^\mu \partial_\nu\left(\bar{F}_B^{\alpha\nu} - \epsilon e e_D \bar{F}_{BD}^{\alpha\nu}\right) - n^\nu \partial_\nu\left(\bar{F}_B^{\alpha\mu} - \epsilon e e_D \bar{F}_{BD}^{\alpha\mu}\right)$$
$$+ n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta\, \partial_\nu\left(\bar{F}_A^{\gamma\beta} - \epsilon e e_D \bar{F}_{AD}^{\gamma\beta}\right)\Big] = \bar{K}_\mu - \epsilon e e_D \bar{K}_\mu^D$$

(3a) $$\frac{n_\alpha}{n^2}\Big[n^\mu \partial_\nu\left(\epsilon e e_D \bar{F}_A^{\alpha\nu} + \bar{F}_{AD}^{\alpha\nu}\right) - n^\nu \partial_\nu\left(\epsilon e e_D \bar{F}_A^{\alpha\mu} + \bar{F}_{AD}^{\alpha\nu}\right)$$
$$- n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta\, \partial_\nu\left(\epsilon e e_D \bar{F}_B^{\gamma\beta} + \bar{F}_{BD}^{\gamma\beta}\right)\Big] =$$
$$= \epsilon e e_D \bar{J}_\mu + \bar{J}_\mu^D + M_{AD}^2 A_{\mu D}$$

(4a) $$\frac{n_\alpha}{n^2}\left[n^\mu \partial_\nu\left(\bar{F}_{BD}^{\alpha\nu} - n^\nu \partial_\nu \bar{F}_{BD}^{\alpha\mu}\right) + n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta\, \partial_\nu \bar{F}_{AD}^{\gamma\beta}\right]$$
$$= \bar{K}_{D\mu} + M_{BD}^2 B_{D\mu}$$

multiplying (1a) by $(-\epsilon e e_D)$ and adding to (3a)

(5a) $$\frac{n_\alpha}{n^2}\left[\left(n^\mu \partial_\nu \bar{F}_{AD}^{\alpha\nu} - n^\nu \partial_\nu \bar{F}_{AD}^{\alpha\mu}\right) - n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta\, \partial_\nu \bar{F}_{BD}^{\gamma\beta}\right] =$$
$$= \bar{J}_\mu^D + M_{AD}^2 A_{\mu D}$$

multiplying (4a) by $(\epsilon e e_D)$ and adding to (2a)

(6a) $$\frac{n_\alpha}{n^2}\left(n^\mu \partial_\nu \bar{F}_B^{\alpha\nu} - n^\nu \partial_\nu \bar{F}_B^{\alpha\mu}\right) + n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta\, \partial_\nu \bar{F}_A^{\gamma\beta} =$$
$$= \bar{K}_\mu + \epsilon e e_D M_{BD}^2 \cdot B_{D\mu}$$

So the (almost) decoupled equations are

(1d) $$\frac{n_\alpha}{n^2}\partial_\nu\left[n^\mu \bar{F}_A^{\alpha\nu} - n^\nu \bar{F}_A^{\alpha\mu} - n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta \bar{F}_B^{\gamma\beta}\right] = \bar{J}^\mu$$

(2d) $$\frac{n_\alpha}{n^2}\partial_\nu\left[n^\mu \bar{F}_B^{\alpha\nu} - n^\nu \bar{F}_B^{\alpha\mu} + n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta \bar{F}_A^{\gamma\beta}\right] = \bar{K}^\mu + e\epsilon e_D M_{B_D}^2 B_D^\mu$$

(3d) $$\frac{n_\alpha}{n^2}\partial_\nu\left[n^\mu \bar{F}_{A_D}^{\alpha\nu} - n^\nu \bar{F}_{A_D}^{\alpha\mu} - n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta \bar{F}_{B_D}^{\gamma\beta}\right] = \bar{J}^{D\mu} + M_{A_D}^2 A_D^\mu$$

(4d) $$\frac{n_\alpha}{n^2}\partial_\nu\left[n^\mu \bar{F}_{B_D}^{\alpha\nu} - n^\nu \bar{F}_{B_D}^{\alpha\mu} + n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta \bar{F}_{A_D}^{\gamma\beta}\right] = \bar{K}^{D\mu} + M_{B_D}^2 B_D^\mu$$

Perhaps we can think of $e\epsilon e_D M_{B_D}^2 B_D^\mu$ as some residual mass term from the dark sector into the visible one.

(unfortunately it depends on B_D^μ :( )

$\bar{J}_\mu = J_\mu$ $\quad\quad$ $\bar{J}_\mu^D = J_\mu^D - e\epsilon e_D J_\mu$

$\bar{K}_\mu = K_\mu + e\epsilon e_D K_\mu^D$ $\quad\quad$ $\bar{K}_\mu^D = K_\mu^D$

20 Oct 2020

continued from pg 7; multiply all equations by $\delta_{\alpha\beta}$ (this gets rid of the $\epsilon^{\mu\nu/\alpha}{}_{\beta}$ terms

$$\frac{n_\beta}{n^2}\partial_\nu\left(n^\mu \bar{F}_A^{\beta\nu} - n^\nu \bar{F}_A^{\beta\mu}\right) = \bar{J}^\mu \delta_{\alpha\beta} = J^\mu \delta_{\alpha\beta} = e\bar{\psi}_e \gamma^\mu \psi_e \ (\delta_{\alpha\beta})$$

$$\frac{n_\beta}{n^2}\partial_\nu\left(n^\mu \bar{F}_B^{\beta\nu} - n^\nu \bar{F}_B^{\beta\mu}\right) = \bar{K}^\mu \delta_{\alpha\beta} = \cancel{K}^\mu \epsilon e e_D M_{B_D}^2 B_D^\mu \delta_{\alpha\beta}$$

$$= \left(g\bar{\psi}_g \gamma^\mu \psi_g + \epsilon e e_D g_D \bar{\psi}_{g_D} \gamma^\mu \psi_{g_D} + \epsilon e e_D M_{B_D}^2 B_D^\mu\right)\delta_{\alpha\beta}$$

$$\frac{n_\beta}{n^2}\partial_\nu\left(n^\mu \bar{F}_{A_D}^{\beta\nu} - n^\nu \bar{F}_{A_D}^{\beta\mu}\right) = \left(\bar{J}^{\mu D} + M_{A_D}^2 A_D^\mu\right)\delta_{\alpha\beta}$$

$$= \left(e_D \bar{\psi}_{e_D} \gamma^\mu \psi_{e_D} - e^2 e_D \epsilon \bar{\psi}_e \gamma^\mu \psi_e + M_{A_D}^2 A_D^\mu\right)\delta_{\alpha\beta}$$

$$\frac{n_\beta}{n^2}\partial_\nu\left(n^\mu \bar{F}_{B_D}^{\beta\nu} - n^\nu \bar{F}_{B_D}^{\beta\mu}\right) = \left(\bar{K}^{D\mu} + M_{B_D}^2 B_D^\mu\right)\delta_{\alpha\beta} =$$

$$\left(K^{D\mu} + M_{B_D}^2 B_D^\mu\right)\delta_{\alpha\beta}$$

$$= \left(g_D \bar{\psi}_{g_D} \gamma^\mu \psi_{g_D} + M_{B_D}^2 B_D^\mu\right)\delta_{\alpha\beta}$$

$A_\alpha + \epsilon e e_D A_{\alpha D}$	$\bar{A}_\alpha \psi_e \psi_e$	e		$\bar{B}_\alpha$	0	B_α
$A_\alpha + \epsilon e e_D A_{\alpha D}$	$\bar{A}_\alpha \psi_{e_D} \psi_{e_D}$	0		$\bar{B}_\alpha$	0	B_α
$A_{\alpha D}$	$\bar{A}_{D\alpha} \psi_e \psi_e$	$-e^2 e_D \epsilon$		$\bar{B}_{D\alpha}$	0	$B_\alpha^D - \epsilon e e_D B_\alpha$
$A_{\alpha D}$	$\bar{A}_{D\alpha} \psi_{e_D} \psi_{e_D}$	e_D		$\bar{B}_{D\alpha}$	0	$B_\alpha^D - \epsilon e e_D B_\alpha$
	$\bar{A}_\alpha \psi_g \psi_g$	0		$\bar{B}_\alpha$	g	
	$\bar{A}_\alpha \psi_{g_D} \psi_{g_D}$	0		$\bar{B}_\alpha$	$\epsilon e e_D g_D$	
	$\bar{A}_{D\alpha} \psi_g \psi_g$	0		$\bar{B}_{D\alpha}$	0	
	$\bar{A}_{D\alpha} \psi_{g_D} \psi_{g_D}$	0		$\bar{B}_{D\alpha}$	g_D	

$\Rightarrow$ to $O(\epsilon)$

$A_\mu \psi_e \psi_e$	e	$B_\mu \psi_e \psi_e$	0
$A_\mu \psi_{eD} \psi_{eD}$	0	$B_\mu \psi_{eD} \psi_{eD}$	0
$A_{D\mu} \psi_e \psi_e$	$-\epsilon e^2 e_D$	$B_{D\mu} \psi_e \psi_e$	0
$A_{D\mu} \psi_{eD} \psi_{eD}$	e_D	$B_{D\mu} \psi_{eD} \psi_{eD}$	0

$A_\mu \psi_g \psi_g$	0	$B_\mu \psi_g \psi_g$	g
$A_\mu \psi_{gD} \psi_{gD}$	0	$B_\mu \psi_{gD} \psi_{gD}$	$\epsilon e e_D g_D$
$A_{D\mu} \psi_g \psi_g$	0	$B_{D\mu} \psi_g \psi_g$	0
$A_{D\mu} \psi_{gD} \psi_{gD}$	0	$B_{D\mu} \psi_{gD} \psi_{gD}$	g_D

using $\bar{A} \to A + \epsilon e e_D A_D$ $\quad$ $\bar{B} \to B$

$\bar{A}_D \to A_D$ $\quad$ $\bar{B}_D \to B_D - \epsilon e e_D B$