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PSI

High RR Lecture Heidelberg

# How Neutron EDM-Experiments Really Work I

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# Outline of the nEDM lecture

Motivation & History

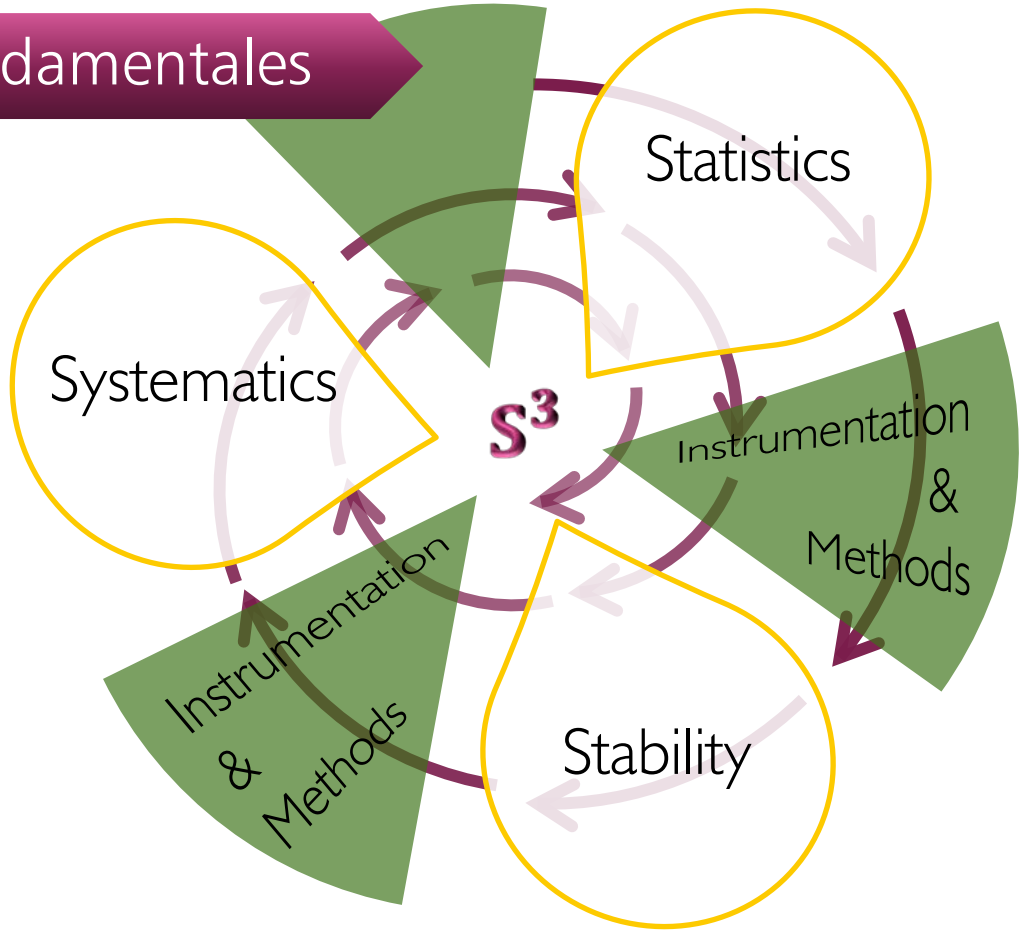
Fundamentales

**Lecture I**:

*Motivation & Fundamentales*

**Lecture II&III:**

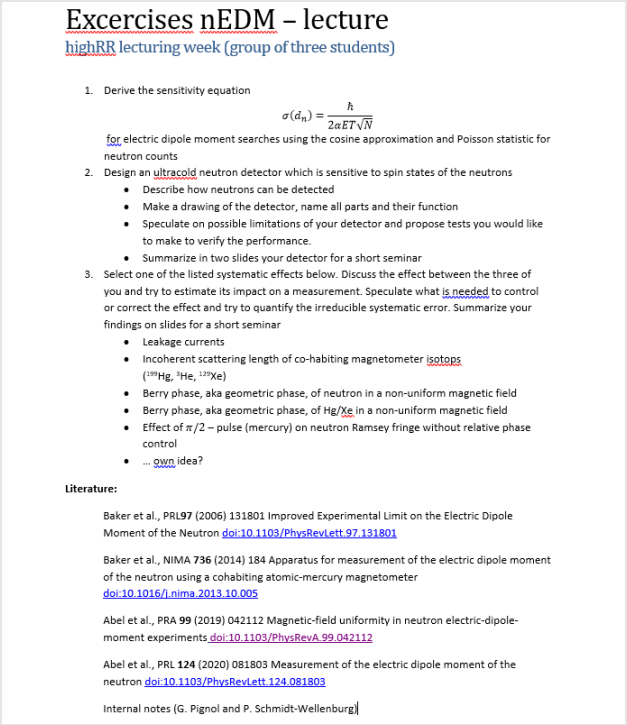
*The spiral to ultimate sensitivity*



# Exercise and questions (tomorrow)

Exercise are meant to deepen the understanding of the lecture and to provoke question.

- Group work (3 students)
- Preparation of short presentation using slides



## Excercises nEDM – lecture

highRR lecturing week (group of three students)

1. Derive the sensitivity equation

$$\sigma(d_n) = \frac{\hbar}{2\alpha E T\sqrt{N}}$$

for electric dipole moment searches using the cosine approximation and Poisson statistic for neutron counts

2. Design an ultracold neutron detector which is sensitive to spin states of the neutrons
   - Describe how neutrons can be detected
   - Make a drawing of the detector, name all parts and their function
   - Speculate on possible limitations of your detector and propose tests you would like to make to verify the performance.
   - Summarize in two slides your detector for a short seminar
3. Select one of the listed systematic effects below. Discuss the effect between the three of you and try to estimate its impact on a measurement. Speculate what is needed to control or correct the effect and try to quantify the irreducible systematic error. Summarize your findings on slides for a short seminar
   - Leakage currents
   - Incoherent scattering length of co-habiting magnetometer isotops ($^{199}$Hg, $^{3}$He, $^{129}$Xe)
   - Berry phase, aka geometric phase, of neutron in a non-uniform magnetic field
   - Berry phase, aka geometric phase, of Hg/Xe in a non-uniform magnetic field
   - Effect of $\pi/2$ – pulse (mercury) on neutron Ramsey fringe without relative phase control
   - … own idea?

**Literature:**


Baker et al., PRL**97** (2006) 131801 Improved Experimental Limit on the Electric Dipole Moment of the Neutron doi:10.1103/PhysRevLett.97.131801

Baker et al., NIMA **736** (2014) 184 Apparatus for measurement of the electric dipole moment of the neutron using a cohabiting atomic-mercury magnetometer doi:10.1016/j.nima.2013.10.005

Abel et al., PRA **99** (2019) 042112 Magnetic-field uniformity in neutron electric-dipole-moment experiments doi:10.1103/PhysRevA.99.042112

Abel et al., PRL **124** (2020) 081803 Measurement of the electric dipole moment of the neutron doi:10.1103/PhysRevLett.124.081803

Internal notes (G. Pignol and P. Schmidt-Wellenburg)

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70 years of measuring zero

# Motivation

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# Motivation to search for an EDM

A nonzero permanent electric dipole moment $\vec{d} = 2d\vec{s}/\hbar$ for a nondegenerate particle with spin $\vec{s}$ implies the violation of time-reversal symmetry. Invoking the $CPT$ theorem [1,2] for quantum field theories, this also indicates the violation of the combined symmetry of charge conjugation and parity ($CP$). The standard model of particle physics (SM) contains two sources of $CP$ violation: the phase of the Cabibbo-Kobayashi-Maskawa matrix, resulting in the observed $CP$ violation in $K$- and $B$-meson decays, and the $\bar{\theta}_{\mathrm{QCD}}$ coefficient of the still-unobserved $CP$-violating term of the QCD Lagrangian [3]. Both are too small to account for the observed baryon asymmetry of the Universe [4], which requires $CP$ violation as one of three essential ingredients [5]. Furthermore, many theories beyond the SM naturally have large $CP$-violating phases [6] that would result in an observable neutron EDM (nEDM). In combination with the limits from searches for the electron [7] and $^{199}$Hg [8] EDM, the limit on the nEDM confirms and complements stringent constraints upon many theoretical models [9]. In particular, the nEDM alone stringently limits $\bar{\theta}_{\mathrm{QCD}}$. This unnaturally small upper limit on $\bar{\theta}_{\mathrm{QCD}}$ is known as the strong $CP$ problem; it gave rise to searches for a Goldstone boson, the axion [10,11], which is also an attractive candidate to solve the dark matter mystery [12].

Just another form factor of the electromagnetic interaction of a fermion

magnetic-dipole

Anapole - moment

$$\langle p' | J_\mu^{\mathrm{EM}} | p \rangle = \bar{\Psi}(p') \left[ F_1 \gamma_\mu + \frac{iF_2}{2M} \sigma_{\mu\nu} q^\nu + \frac{iF_3}{2M} \sigma_{\mu\nu} \gamma_5 q^\nu + \frac{F_4}{M^2} (q^2 \gamma_\mu - \gamma^\mu q_\mu q_\mu) \right] \Psi(p)$$

charge

electric-dipole

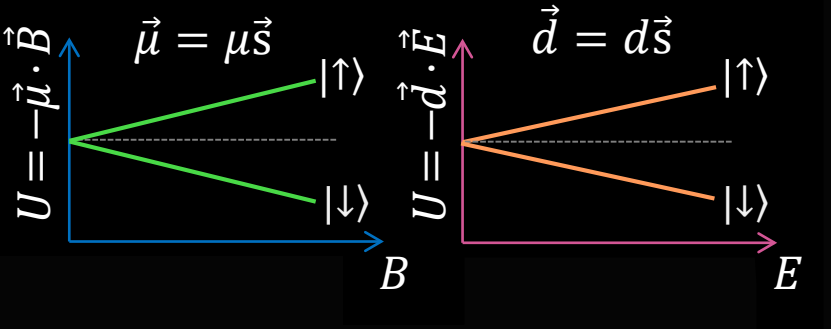
non-relativistic

$$H = -2(\mu \vec{\sigma} \cdot \vec{B} + d \vec{\sigma} \cdot \vec{E})$$

P,T conserving

P,T violating

# CP violation & EDM



# Parity / Charge conjugation / Time Reversal

| | Parity: | Charge conjugation | Time reversal: |
|---|---|---|---|
| Angular Momentum | $\vec{x} \to -\vec{x}$ | $\vec{x} \to \vec{x}$ | $\vec{x} \to \vec{x}$ |
| Spin | $t \to t$ | $t \to t$ | $t \to -t$ |
| Charge | $\vec{p} \to -\vec{p}$ | $\vec{p} \to \vec{p}$ | $\vec{p} \to -\vec{p}$ |
| Electric Dipole Moment | $\vec{L} \to \vec{L} = \vec{x} \times \vec{p}$ | $\vec{L} \to \vec{L} = \vec{x} \times \vec{p}$ | $\vec{L} \to -\vec{L} = \vec{x} \times -\vec{p}$ |
| Magnetic Moment | $\vec{\sigma} \to \vec{\sigma}$ | $\vec{\sigma} \to \vec{\sigma}$ | $\vec{\sigma} \to -\vec{\sigma}$ |
| Vector Potential | $q \to q$ | $q \to -q$ | $q \to q$ |
| Electric Field | $d\vec{\sigma} \to d\vec{\sigma}$ | $d\vec{\sigma} \to -d\vec{\sigma}$ | $d\vec{\sigma} \to -d\vec{\sigma}$ |
| Magnetic Field | $\mu\vec{\sigma} \to \mu\vec{\sigma}$ | $\mu\vec{\sigma} \to -\mu\vec{\sigma}$ | $\mu\vec{\sigma} \to -\mu\vec{\sigma}$ |
| | $\vec{A} \to -\vec{A}$ | $\vec{A} \to -\vec{A}$ | $\vec{A} \to \vec{A}$ |
| | $\vec{E} \to -\vec{E}$ | $\vec{E} \to -\vec{E}$ | $\vec{E} \to \vec{E}$ |
| | $\vec{B} \to \vec{B}$ | $\vec{B} \to -\vec{B}$ | $\vec{B} \to -\vec{B}$ |

# Insufficient CP-violation to explain origin of matter

The CP-violating phase of the CKM matrix cannot explain the observed baryon asymmetry of the Universe.

Additional sources of CP-violation beyond the standard model are needed.

## Sakharov criteria*

| | Standard model | beyond SM |
|---|---|---|
| 1. *Baryon number violation* | ✓ | ✓ |
| 2. *C and CP violation* | | ✓ |
| 3. *Thermal non-equilibrium* | | ✓ |

⇨ Search for EDMs

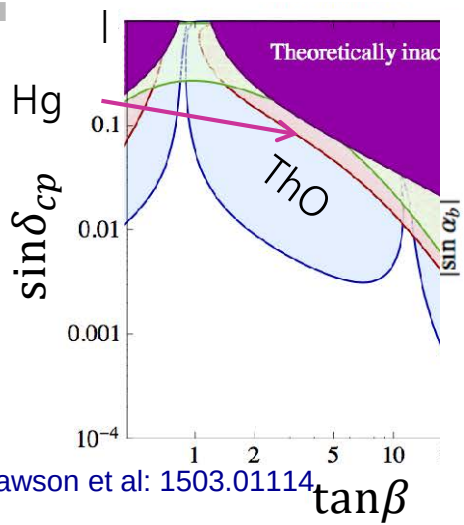
*[A. D. Sakharov, JETP 5 (1967), 32]

# Example of complementarity

## *Higgs Portal CPV (would also lead to strong first order phase transition)*

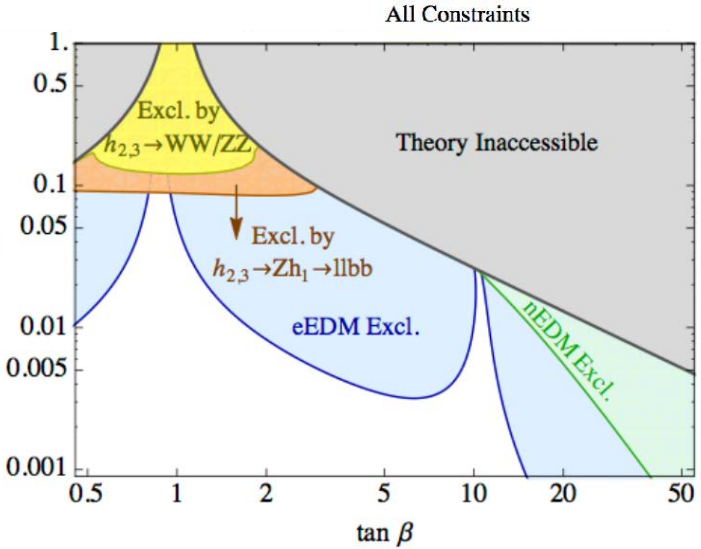
$M_{h2}$ = 400 GeV/$c^2$

+ collider search



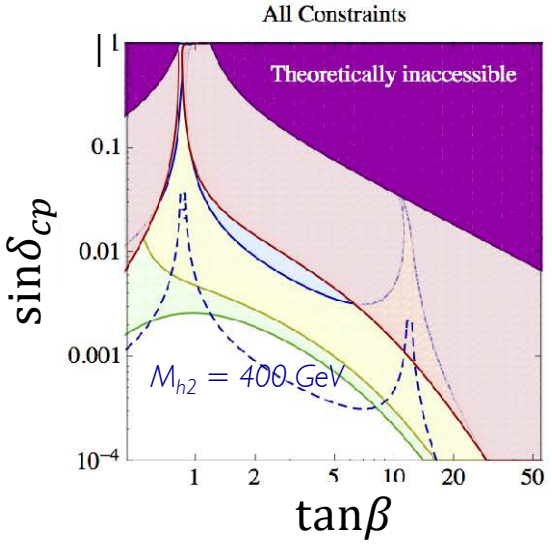

Dawson et al: 1503.01114

*Present (2015)*




*Future (2020)*

$d_n$ x 0.1
$d_A$(Hg) x 0.1
$d_{ThO}$ x 0.1
$d_A$(Ra)

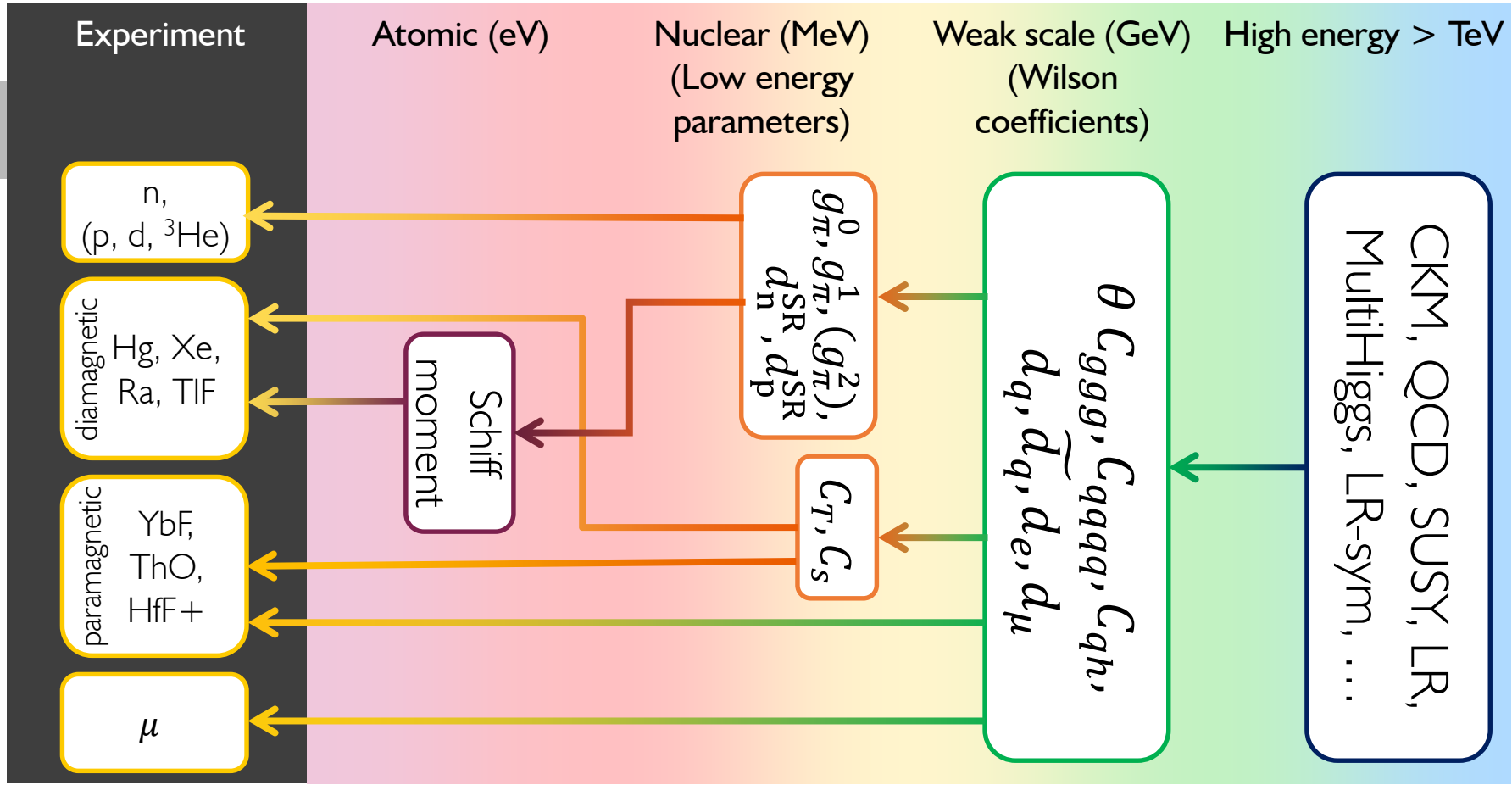



*Farther future:*

dn x 0.01
dA(Hg) x 0.1
dThO x 0.1
dA(Ra)

Inoue *et al.* PRD(2014)115023
Chen *et al.*, JHEP06(2015)056

# Complementarity of different nEDM searches



# nucleon EDM and the QCD $\theta$-term

$$\delta L_{\mathrm{CPV}}^{\mathrm{QCD}} = \theta \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a$$

QCD theta term "natural" in neutron, proton and nuclei

e.g. from chiral perturbation theory*

$$\frac{d_n}{\theta} \approx 2 \times 10^{-16} e\mathrm{cm} \qquad \frac{d_p}{\bar{\theta}} = 2.4 \times 10^{-16} e\mathrm{cm} \qquad \frac{d_d}{\bar{\theta}} = 1.4 \times 10^{-17} e\mathrm{cm}$$

but

$$d_{\mathrm{n}}^{\mathrm{ex}} < 1.8 \times 10^{-26}\ e\mathrm{cm}^{**}$$

$$\theta < 1 \times 10^{-10}\ e\mathrm{cm}$$

## Strong CP problem

Might be solved by introducing a new chiral symmetry $U(1)_{PQ}$***, which is spontaneously broken and gives rise to the **Axion**, a viable DM candidate.

*E. Mereghetti et al., **PLB** 696 (2011) 97
**Abel C. et al., **PRL** 124 (2020)081803
*** R.D. Peccei and H. R. Quinn, **PRL**38 (1977) 1440

# nEDM from CKM

$$d_{\mathrm{n}} = 10^{-16} e\mathrm{cm} \cdot \left(\theta + \delta_{\mathrm{n}}^{\mathrm{BSM}}\right) + d_{\mathrm{n}}^{\mathrm{CKM}} < 3 \times 10^{-26} e\mathrm{cm}^{*}$$

Weak interaction SM contribution:
$1 - 6 \times 10^{-32}$ e$cm^{**}$

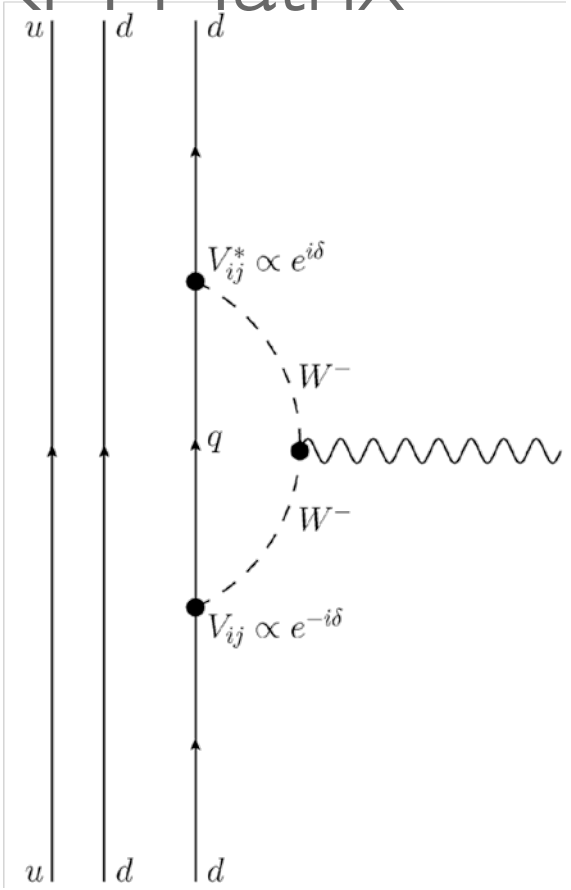
$$V_{\mathrm{KM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{12}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{-i\delta} & s_{23} \\ s_{12}s_{23} - c_{13}c_{23}c_{13}e^{-i\delta} & -c_{12}s_{23}s_{13}e^{-i\delta} & c_{23}c_{13} \end{pmatrix}$$

Known from neutral K and B meson decays

*J.M. Pendlebury et al., PRD 92 (2015) 092003
**C.-Y. Seng,PRC(2015)025502

# nEDM from CKM Matrix

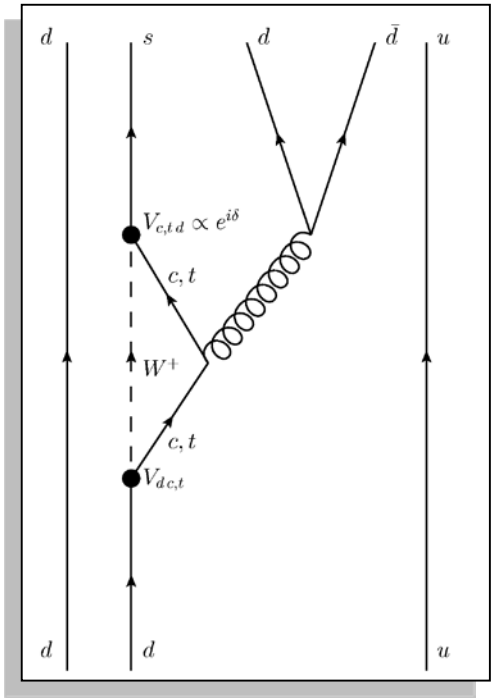
- No tree level contribution
- No first loop contribution
- No pure week interaction two loop contribution
- Only gluonic two loop contribution
  → strongly suppressed

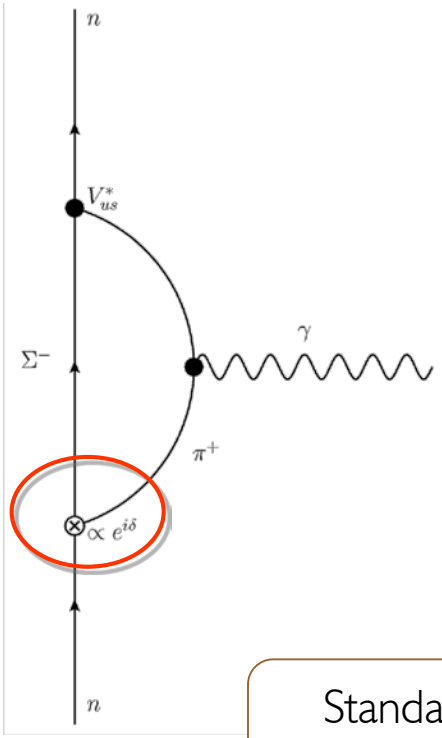



*Khriplovich PLB109(1982) & PLB173 (1986)*

# nEDM from CKM Matrix

CP odd vertex




Standard Model nEDM:
$10^{-30}$ e·cm > $d_n$ > $10^{-32}$ e·cm

*Khriplovich PLB109(1982) & PLB173 (1986)*

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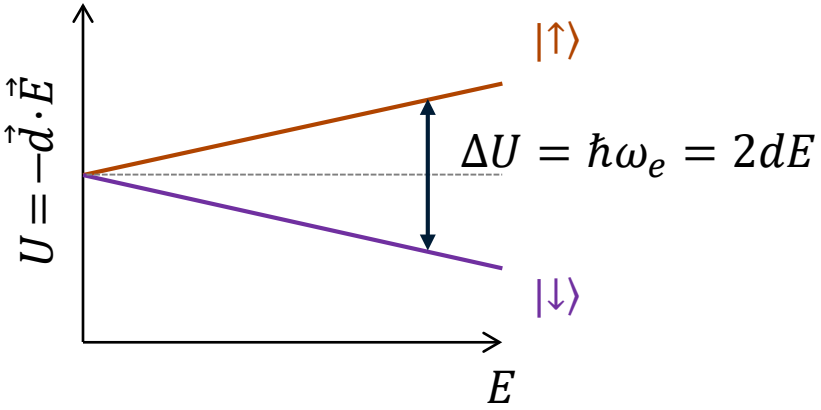
Rabi, Ramsey and spin precession

# Fundamental principles

# EDM measurement basics

Measure the energy splitting




Best to measure the associated frequency $\omega_e$

$$f_e = \frac{2dE}{h} \approx 53\text{nHz}$$

Corresponds to
1 turn in about 1 year

Systematic effect:
classical Larmor precession

$$f_L = \frac{2\mu B}{h} \approx 0.1\text{mHz}$$

Corresponds to
1 turn in 2.5 h
in the best MSR

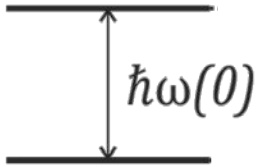
# Use a magnetic field as reference

$\vec{B} \neq 0$

$\vec{E} = 0$

$+\hbar/2$

$-\hbar/2$




I $f^{\uparrow} = \frac{2}{h}(\mu B + dE)$

II $f^{\downarrow} = \frac{2}{h}(\mu B - dE)$

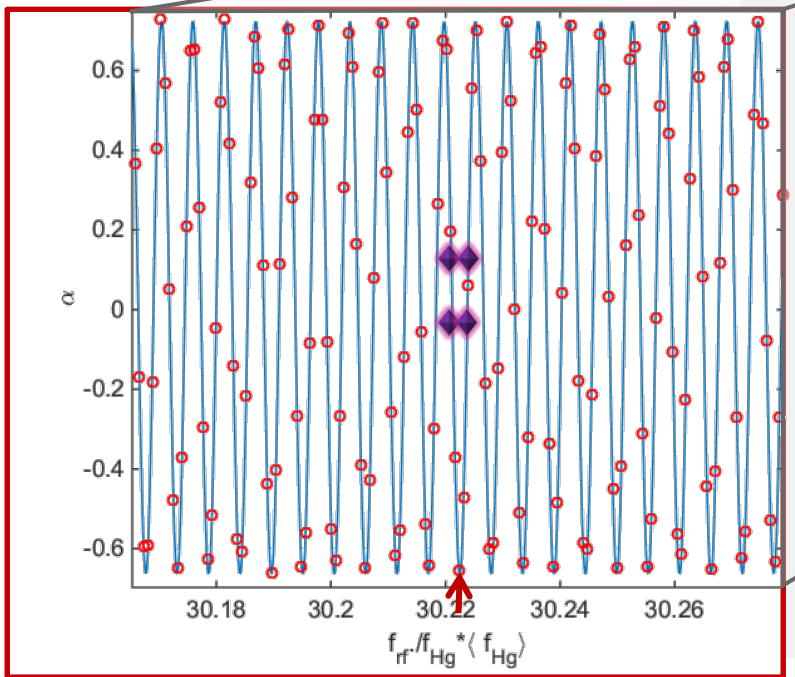
II-I $\Delta f = \frac{2}{h}(\mu \Delta B - 2dE)$

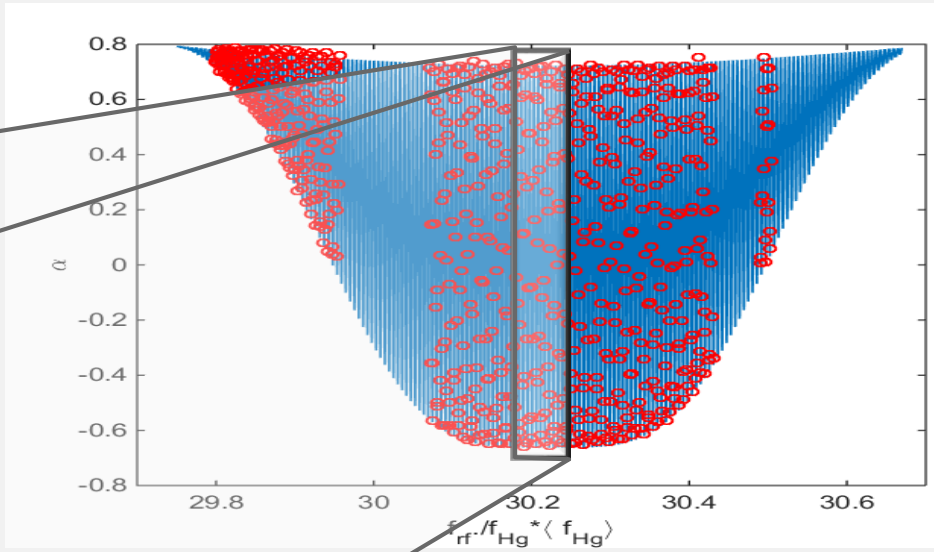
$$d = \frac{h\Delta f}{4E}$$

# Ramsey's technique to measure $f$

*Spin "down" neutron...*

$B_{0\uparrow}$




Sensitivity:

$$\sigma(\omega_n) = \frac{1}{\alpha T \sqrt{N}}$$

| | |
|---|---|
| $\alpha$ | Visibility of resonance |
| $T$ | Time of free precession |
| $N$ | Number of neutrons |

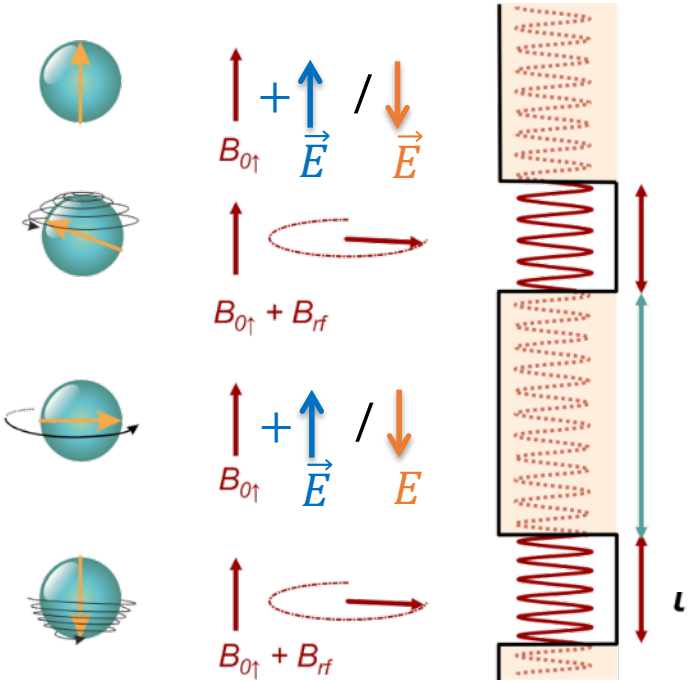
# Coupling of the spin to an electric field

Spin "down" neutron...

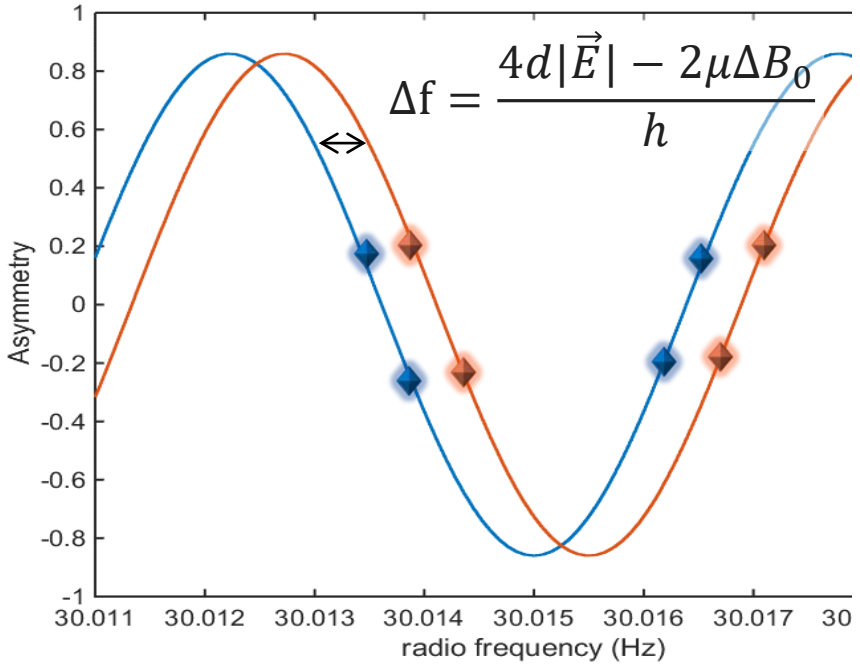
Apply π/2 spin flip pulse...

Free precession at $\omega_L$

Second π/2 spin flip pulse.






$$\Delta f = \frac{4d|\vec{E}| - 2\mu\Delta B_0}{h}$$

$$\sigma(d_n) = \frac{\hbar}{2\alpha E T\sqrt{N}}$$

# Sensitivity of the frequency measurment:

- Central fringe can be approximated by:

$$\mathrm{N}_{\downarrow} = \frac{\langle N \rangle}{2}\left[1 + \alpha \cos\left((\omega_{\mathrm{rf}} - \omega_{\mathrm{L}}) \cdot T\right)\right]$$

- Calculate statistical sensitivity (Poisson):

$$\sigma_{\omega}(t) = \frac{1}{\alpha T\sqrt{\dot{N}t}} \text{ for EDM } \sigma_{d_{\mathrm{n}}}(t) = \frac{\hbar}{2\alpha E T\sqrt{\dot{N}t}}$$

# Sensitivity for an nEDM

$$\sigma(d) \propto \frac{1}{PE\sqrt{NT}A}$$

*P*: Initial polarization

*E*: Electric field strength

*N*: Number of neutrons

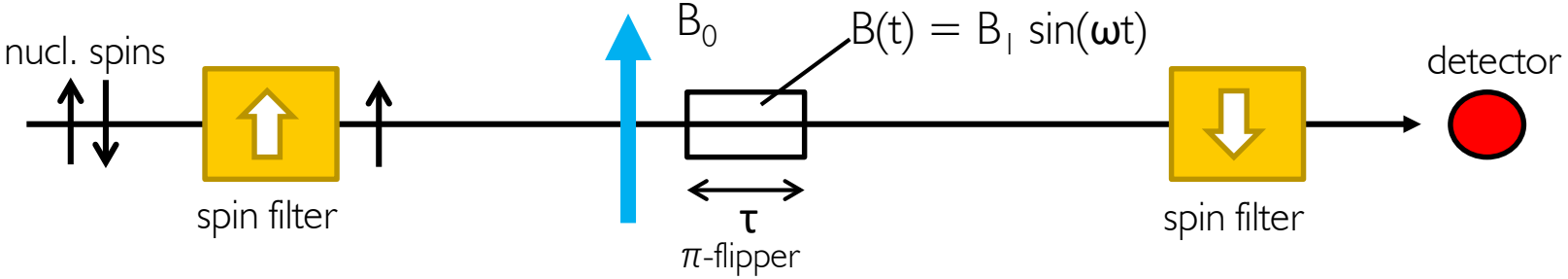
*T*: *Observation time*

*A*: *Analyzing power*

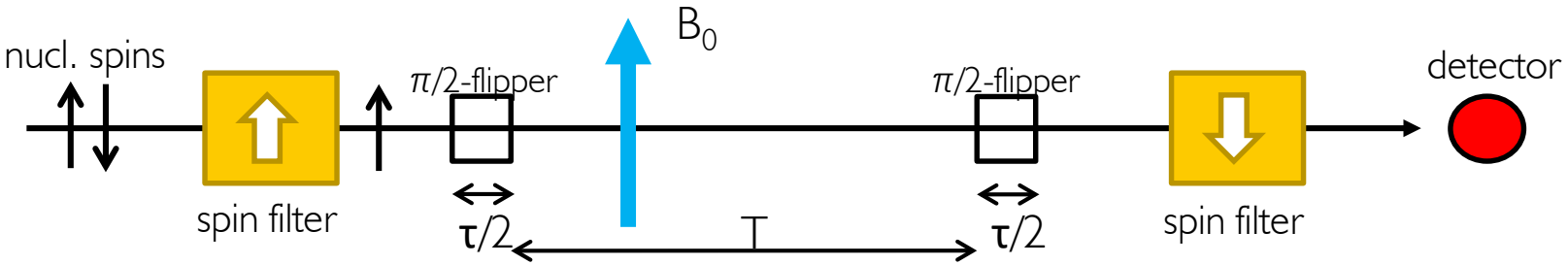
c.f. previous slide: $\alpha = PA$

# Rabi's and Ramsey's method

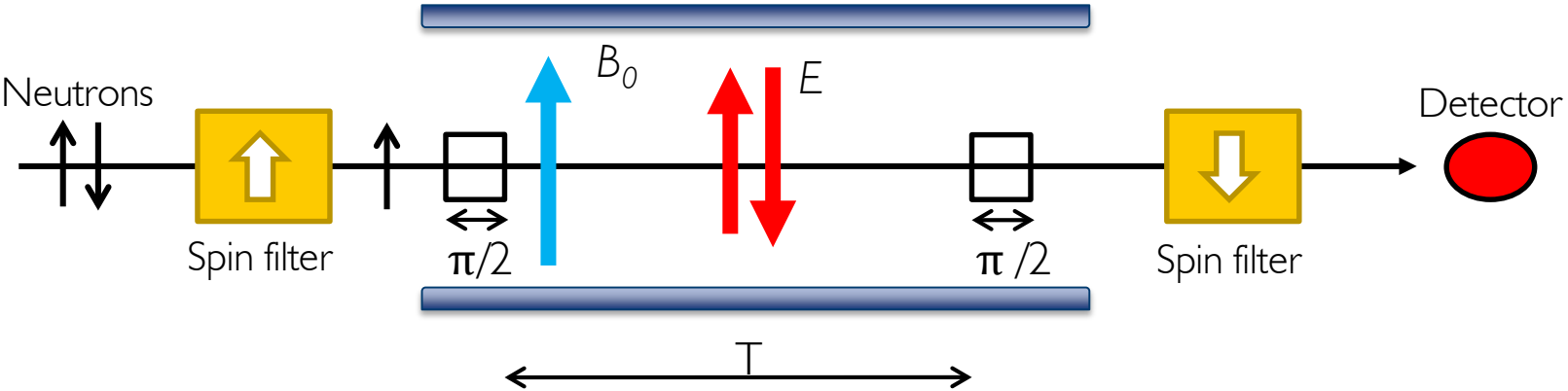
- **Rabi's method to measure magn. moments** (Nobel Prize 1944):




- **Ramsey's method of separated oscillating fields** (Nobel Prize 1989):



# nEDM with a beam




$$\Delta\varphi_{edm} = (\omega_{\uparrow\uparrow} - \omega_{\uparrow\downarrow})T = \frac{4\, d_n E}{\hbar} T$$

*Measuring the phase shift corresponds to a measurement of the frequency difference.*
*The accuracy of the nEDM measurement improves with larger times T and electric fields E.*

*Neutron beam T ≈ ms* ⟷ *Stored neutrons: T ≈ 100 sec*

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Neutron liberation

# Neutron sources

# Neutron production

- **Laboratory Sources**:
  - ($\alpha$,n)- sources (e.g. $\alpha$-Be with Po, Am, …) ($\sim 10^{-4}$ n/$\alpha$)
  - ($\gamma$, n)-sources (based on $^{9}\mathrm{Be}(\gamma,\mathrm{n})^{8}\mathrm{Be}$ or $^{2}\mathrm{H}(\gamma,\mathrm{n})^{1}\mathrm{H}$ ) ($\sim 10^{-6}$ n/$\gamma$)
  - $^{252}$Cf – spontaneous fission (about 2.4 x $10^{6}$ n/s per $\mu$g)

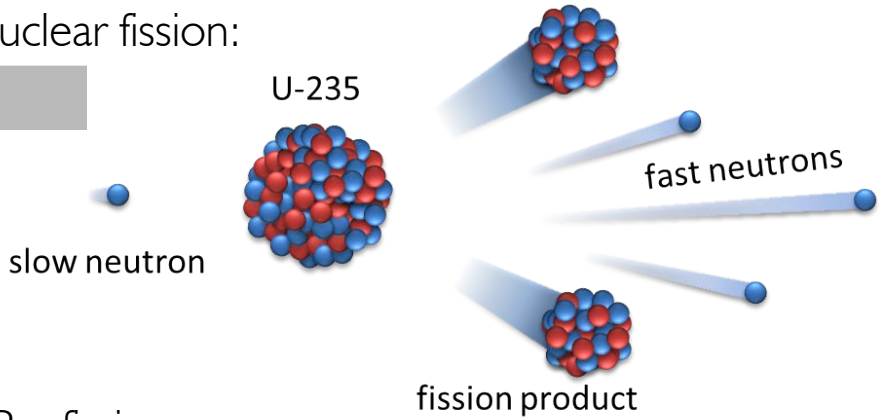
- **Reactor sources**: Induced fission of $^{235}$U (typ. thermal power $<$ 60 MW)
  e.g. FRM2 (Munich), ILL (Grenoble)

- **Spallation sources**: Protons on solid target (Pb)
  'evaporation' of neutrons from the excited nucleus
  e.g. SINQ (PSI), J-Parc (Japan), ISIS (UK), SNS (USA), …

- **Moderation**: Deceleration of the produced neutrons by repeated elastic scattering with nuclei in a cold/hot material.

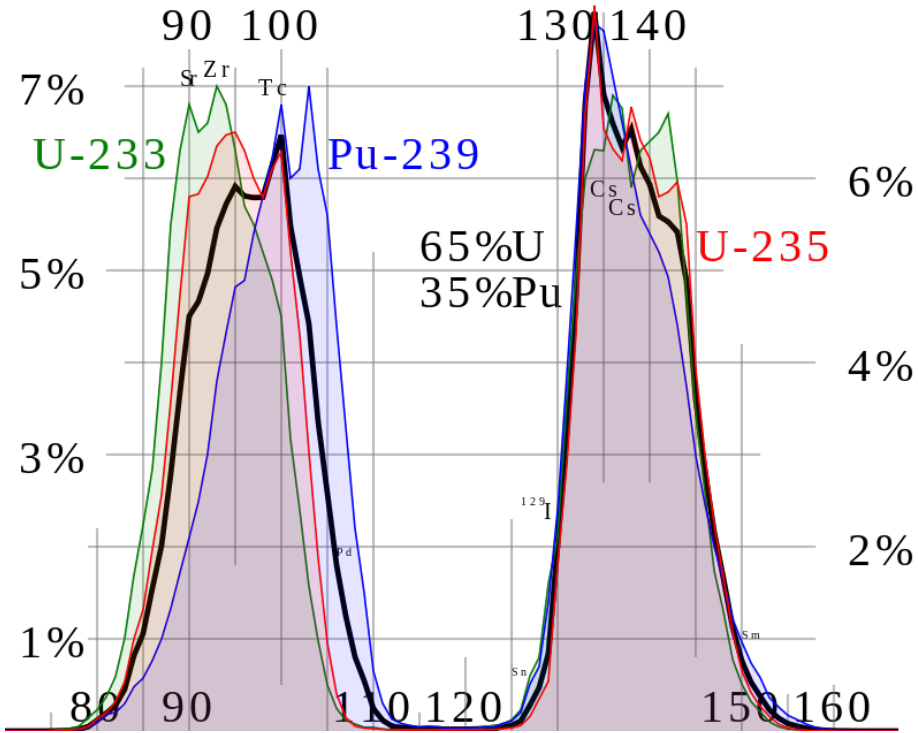
# Nuclear fission

Nuclear fission:




Per fission

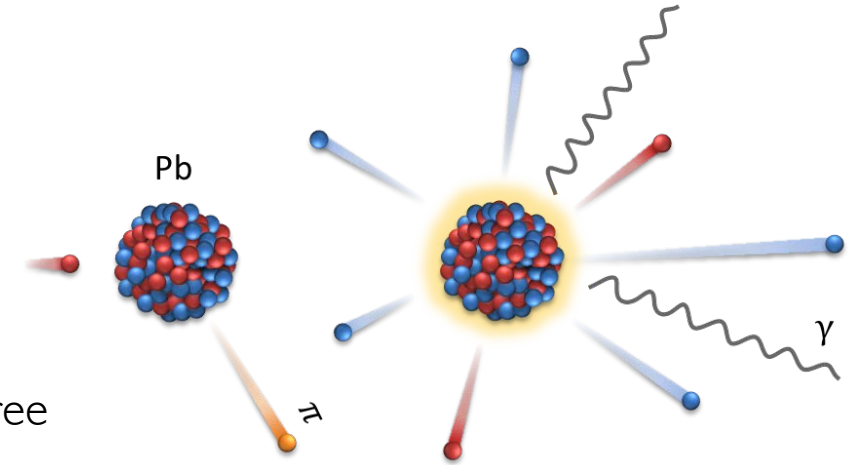
- Mean energy released 200MeV
- Per incident neutron 2.5 fission neutrons
- $^{235}U$ need “slow” neutrons
- Natural Uran: 99.3% $^{238}U$, 0.7% $^{235}U$
  → enrichement



# Fission and spallation-process

Spallation:

- Binding energy per nuclei O(8MeV)
- **Initial phase:** collision between quasi free nucleons and incident particle
- **Final phase:** "evaporation" of nucleons during de-excitation
- Neutron yield per proton: proportional to incident energy and mass number of target
- Limited by heat and radiation damage of target



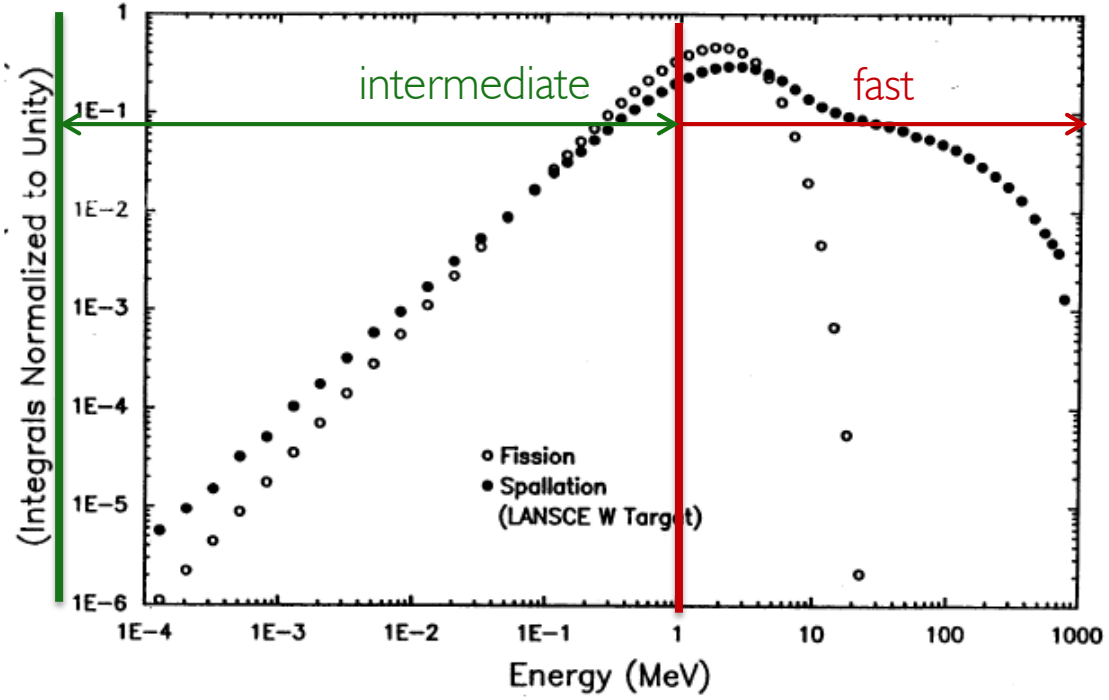

Spallation-process (p + Pb/Hg/…)
(8– 15 neutrons per proton)

# Neutron spectra – fission and spallation

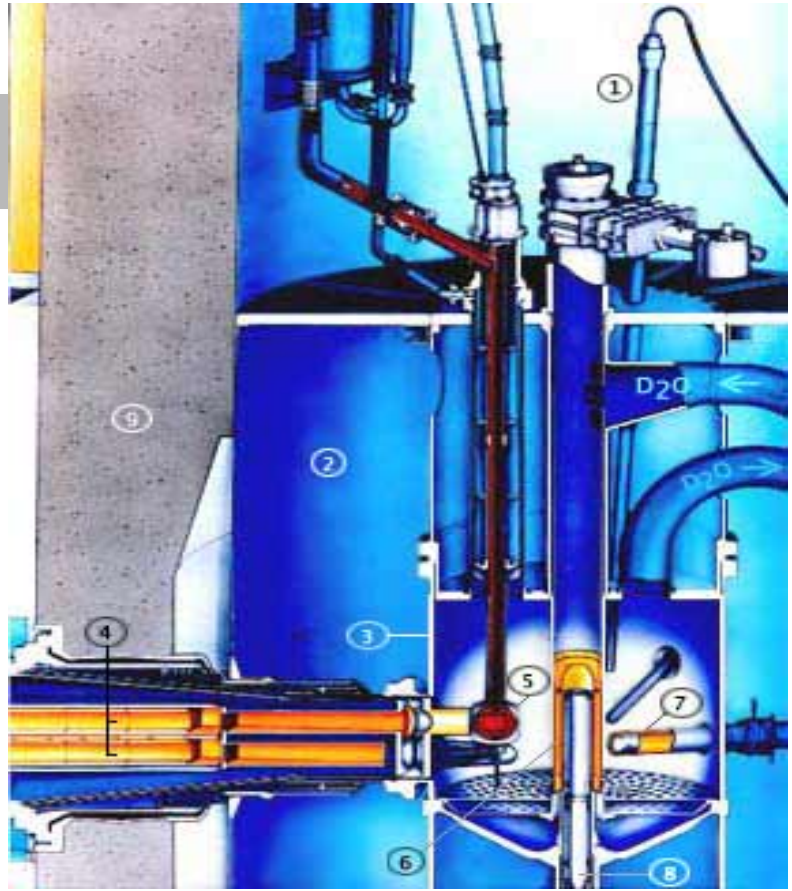
What neutrons do we need?

- Nuclear reaction: 1-100eV
- Material studies: <1eV
- Properties of the neutron:
  - thermal/cold (1meV<E<25meV)
  - ultracold (E<330 neV

slow



# ILL Reactor

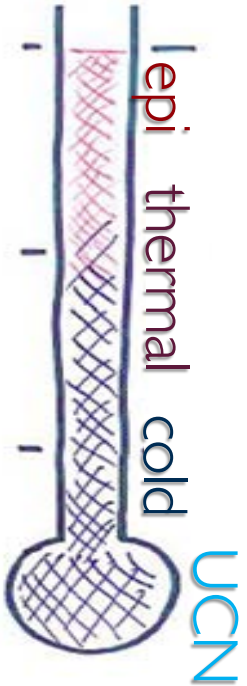



- 1 Safety rod
- 2 Neutron guide pool
- 3 Reflector tank
- 4 Double neutron guide
- 5 Vertical cold source
- 6 Reactor core
- 7 Horizontal cold source
- 8 Control rod

- **Hot source** (10 l of graphite at 2400 K):
  Enhances neutron flux below 0.8 Å
- **Vertical cold source** (20 l of liquid $D_2$ at 25 K):
  Enhances neutron flux above 3 Å ($4.5\ 10^{14}$ n/cm$^2$s)
- **Horizontal cold source** (6 l of liquid $D_2$ at 25 K):
  Enhances neutron flux above 3 Å ($8.0\ 10^{14}$ n/cm$^2$s)

# Categorize Neutrons by velocity/energy

| Name | Moderator | T | $k_B T$ | $v_{peak}$ | $\lambda_{dB}$ |
|---|---|---|---|---|---|
| Intermediate<br>Fast | none | | 1eV< 1MeV<br>> 1MeV | | |
| Epithermal | Graphite | 2000 K | 0.2 eV | 6000 m/s | 0.7 Å |
| Thermal | $D_2O$ | 300 K | 25 meV | 2200 m/s | 1.8 Å |
| Cold | Liquid $D_2$ (25K) | ≈ 50 K | 5 meV | 1000 m/s | 4 Å |
| Very cold | | | | 100 m/s | 40 Å |
| Ultracold* | Solid $D_2$ (6K)<br>Liquid $^4$He (<1K) | (2 mK) | < 250 neV | < 7 m/s | > 600 Å |




$k_B$ = Boltzmann constant 86.3 μeV/K

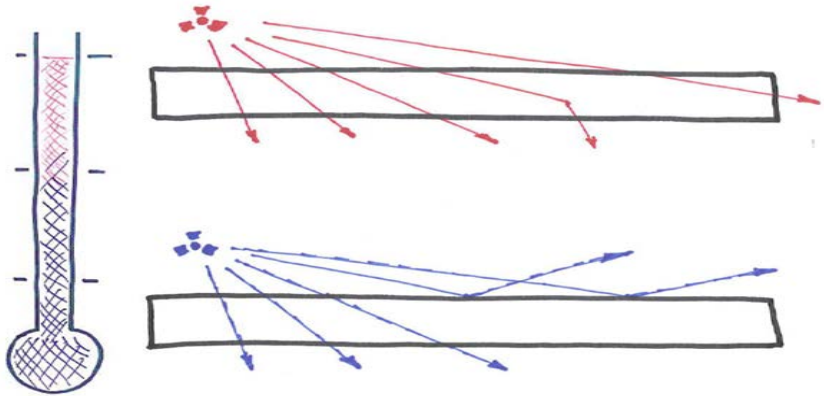
1 Å = $10^{-10}$ m

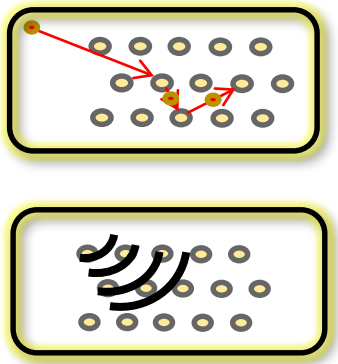
* = superthermal sources - no Maxwell-distribution as there is no thermal equilibrium

Peak velocity of Maxwell-distribution:

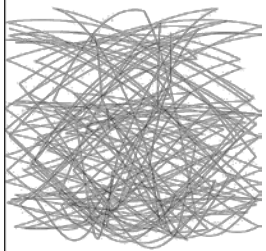
$$v_{\mathrm{peak}} = \sqrt{\frac{2k_{\mathrm{B}}T}{m_{\mathrm{n}}}}$$

# Storable neutrons – UltraCold Neutons (UCN)



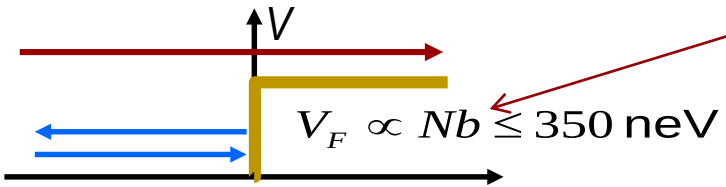


Gravity
102 neV/m



Magnetic
~60 neV/T

Strong
$V_F$




Storage properties are material dependent

350 neV ↔ 8 m/s ↔ 500 Å ↔ 3 mK

E. Fermi & W.H. Zinn(1946),
Y. B. Zeldovich, *Sov. Phys. JETP* (1959)389

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# Spin manipulation

# Polarized neutrons

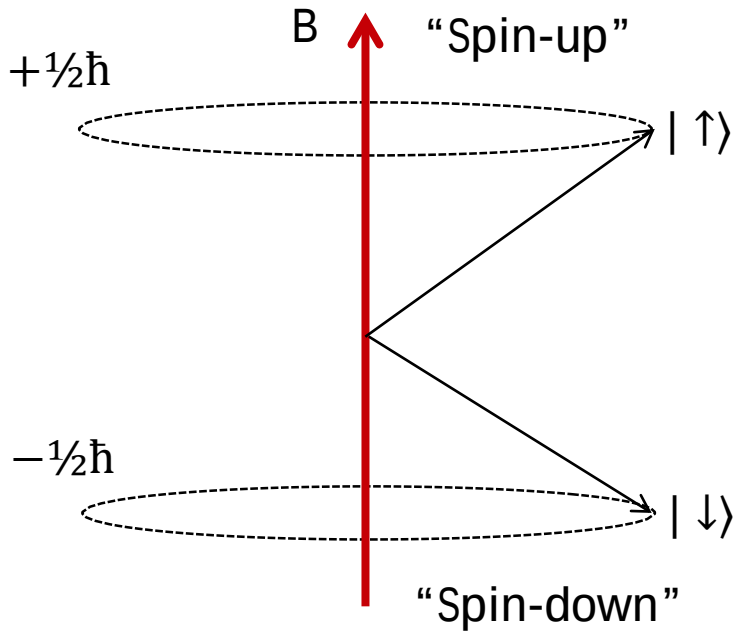
Each individual neutron has spin $s = ½$ and an angular momentum of $\pm ½\hbar$.
Each neutron has a spin vector and we define the polarization of a neutron beam as the ensemble average over all the neutron spin vectors, normalised to their modulus

$$P = \frac{\langle s_z \rangle}{1/2} = 2\langle s_z \rangle$$

If we apply an external field (quantisation axis) then there are only two possible orientations of the neutrons: parallel and anti-parallel to the field. The polarization can then be expressed as a scalar:

$$P = \frac{N_+ - N_-}{N_+ + N_-}$$

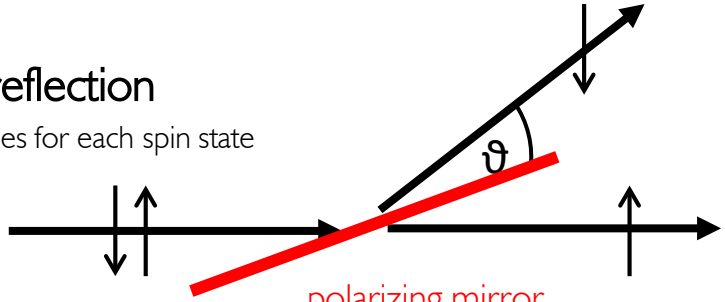
where there are $N_+$ neutrons with spin-up and $N_-$ neutrons with spin-down



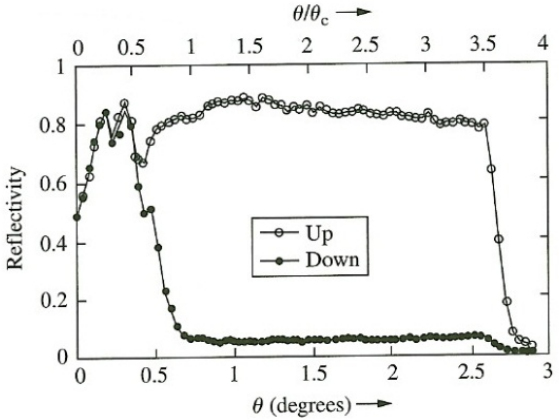
# Production of polarized neutrons

## Spin-dependent reflection

– Different critical angles for each spin state




polarizing mirror
(e.g. FeSi-multilayers)



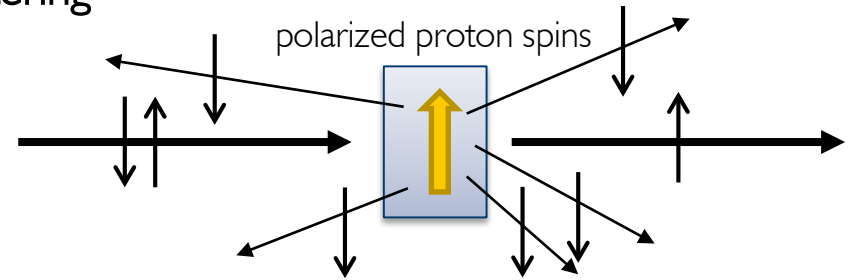

## Spin-dependent absorption

polarized $^3$He gas

Helium gas polarized via optical pumping (SEOP or MEOP).

## Spin-dependent scattering

polarized proton spins



Proton spins polarized using dynamical nuclear polarization methods.

# Efficiency of polarizing filters

The polarizing efficiency of a filter, $P$, is defined in terms of the transmission of the two spin states, $T_+$ and $T_-$:

$$P = \frac{T_+ - T_-}{T_+ + T_-},$$

where the total transmission of the filter is $T = (T_+ + T_-) / 2$.
The filter performance depends upon both $P$ and $T$. The efficiency $P$ can be increased by making the filter thicker, but only at the expense of total transmission, $T$. As a compromise it is the quality factor $P\sqrt{T}$ that is usually optimized.

For a generalised polarizing filter with total absorption cross sections given in terms of the spin-independent absorption cross section, $\sigma_0$ and a spin-dependent absorption cross section $\sigma_p$ we have

$$\sigma = \sigma_0 \pm \sigma_p P_N. \qquad P_N \coloneqq \text{nuclear polarization}$$

From which it can be shown (using $T = \exp(-N\sigma d)$ )

$$P = -\tanh\left(N\sigma_p P_{\mathrm{N}} d\right) \qquad \text{and } T = \exp(-N\sigma_0 d)\cosh(N\sigma_p P_{\mathrm{N}} d)$$

Where $N$ is the number density of nuclei responsible for spin selection and $d$ is the filter thickness.

# $^3$He spin filter

The absorption cross sections of nuclei are dependent upon their polarization and the neutron spin direction

$$\sigma_{\pm} = \sigma_a(1 \pm \rho P_N)$$

where $\sigma_a$ is the mean absorption cross section and $P_N$ is the nuclear polarization,

$$\rho = \frac{I(1-2x)-x}{I+1} \qquad x = \frac{\sigma_{j_-}}{\sigma_{j_+} + \sigma_{j_-}}$$

where $\sigma_{j_-}$ and $\sigma_{j_+}$ ($j_{\pm} = I \pm 1/2$) are the relative absorption cross sections for neutrons with spin down and spin up with respect to the orientation of the nuclear spin $\boldsymbol{I}$.

For $^3$He all the absorption is in the $\sigma_{j_-}$ channel, and therefore, $x = 1$ and $\rho = -1$ giving

$$\sigma_{\pm} = \sigma_a(1 \mp P_N)$$

Therefore, for fully polarized $^3$He ($P_{\mathrm{He}} = 1$), one spin state goes through the filter with zero absorption. The other spin state is almost fully absorbed since $\sigma_a = 5333\ \mathrm{barns}$.

# $^3$He spin filter

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$^3$He

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$\sigma_0 = \sigma_a; \sigma_p = -\sigma_p P$

$$P = \tanh(OP_{\mathrm{He}}) \qquad T = -\exp O \cosh(OP_{\mathrm{He}})$$

where $O$ is the opacity of the spin flipper:

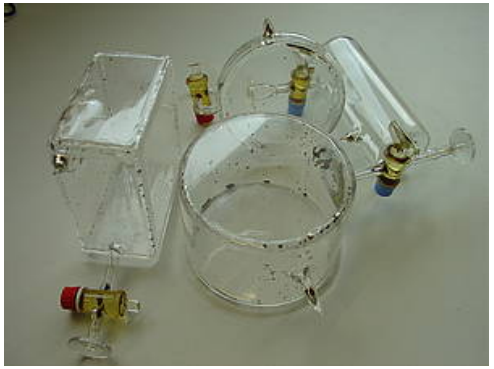
$$O = N\sigma_a d \approx 0.067 \times \lambda[\text{Å}] \times p[\text{bar}] \times d[\text{cm}]$$ at room temperature $T = 300$K

# Helium-3 polarization

3He polarisation facility

*SEOP: spin-exchange optical pumping:*
polarise He at 1-10 bar using spin-exchange with optically pumped Rb vapour.
($P \approx 75\%$)
PRL 5 (1960) 373

*MEOP: metastable exchange opt. pumping:*
polarize with high power lasers at low pressures (1 mbar), afterwards compress gas with a piston up to several bars ($P \approx 80\%$)
PR 132 (1963) 2561


$^3$He gas cells

Transport gas cells in magnetically shielded boxes
Spin-Relaxation due to magnetic-field gradients and wall collisions ($\tau \approx 100$ h);
$^3$He has to be repolarized regularly

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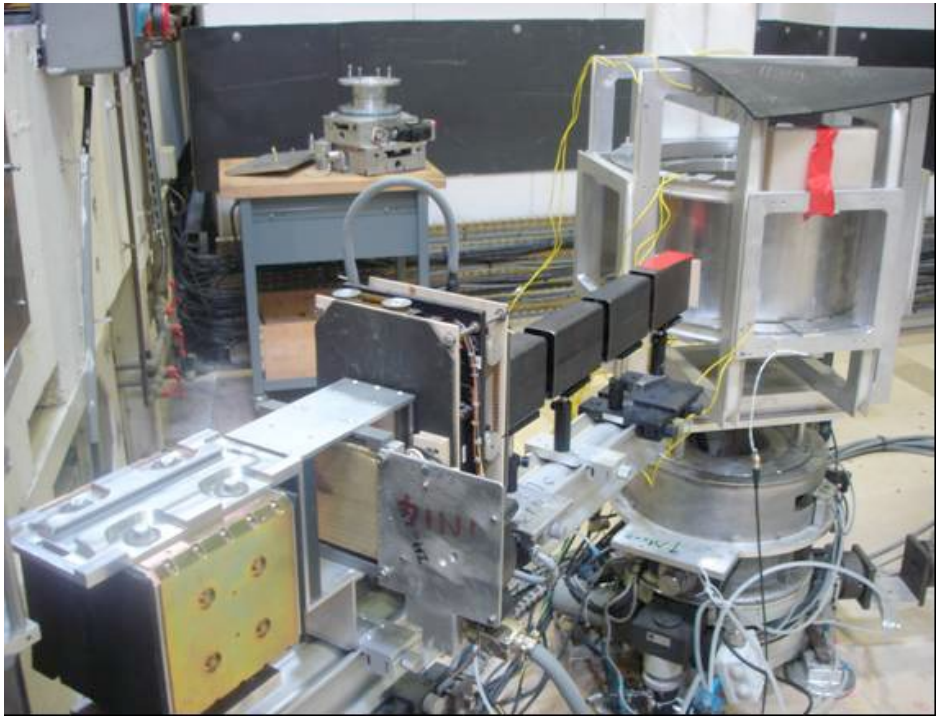
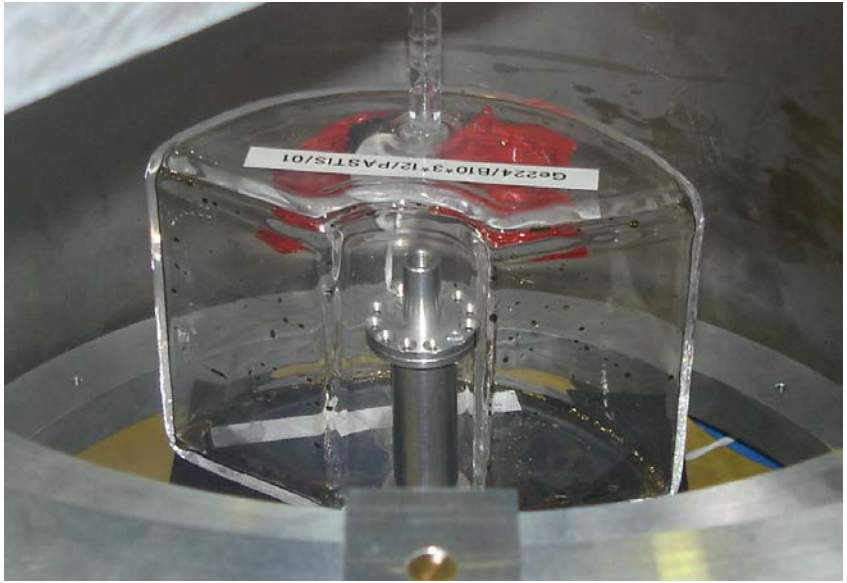
# Helium – 3 in operation

Time-of-flight spectrometers require a neutron spin-filter that covers a wide solid angle (big detector angles) and is efficient at thermal energies - $^3$He spin filter is ideal for this





*Stewart et.al. Physica B* ***385-86*** *(2006)1142-1145*

# Neutron mirror polarizer

The refractive index for neutrons is:

$$n = k_2/k_{\mathrm{vac}}$$

The neutron experiences a change of energy of $V_{\mathrm{opt}}$ when it enters the medium:

$$n = \frac{k_2}{k_{\mathrm{vac}}} = \sqrt{\frac{E - V}{E}} \approx 1 - \frac{V}{2E}$$

The potential $V_{\mathrm{opt}}$ can be a combination of the pseudo Fermi potential and a magnetic potential:

$$V_{\mathrm{opt}} = \frac{2\pi\hbar^2}{m_n} Nb \pm \mu \boldsymbol{B}$$

# Neutron magnetic mirror

Therefore, the spin dependent refractive index of magnetized mirrors for neutrons of wavelength $\boldsymbol{\lambda}$ is:

$$n_{\pm} = 1 - \frac{N\lambda^2}{2\pi}(b \pm p),$$

where $b$ is the mean coherent nuclear scattering length, $N$ is the number density of scattering nuclei, and $p = \mu B \cdot \left(\frac{2\pi\hbar^2 N}{m_n}\right)^{-1}$, with $B$ the flux density applied in the plane of the surface.

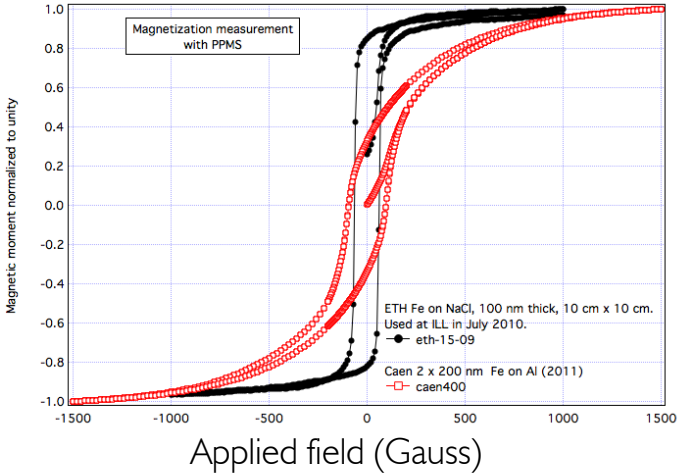
Hence, there are two different critical angles for total reflection:

$$\theta_c = \lambda \sqrt{\frac{Nb}{\pi} \pm \mu B \frac{m_{\mathrm{n}}}{2\pi^2\hbar^2}}.$$

# Magnetized foils and magnetic super mirrors

## Magnetized Fe foil:

Typically used in UCN experiments by magnetizing with permanent magnets, which produce a $H = 90\ \mathrm{mT}$ magnetic field at the center. As a result, the iron layer is magnetized close to saturation, i.e. close to $B = \mu_r \mu_0 H = 2\ \mathrm{T}$.

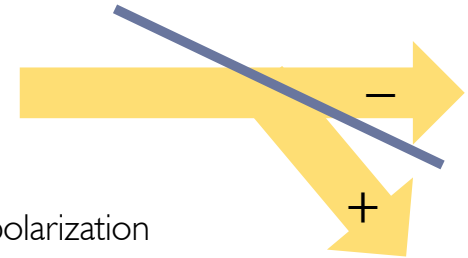



## Polarizing bi-layer mirror

In a polarizing neutron mirror, the basic unit is a non-magnetic/magnetic (A/B) bilayer, which has a reflectivity of the form:

$$R_{\pm} \propto [N_a b_a - N_b (b_b + p)]^2.$$

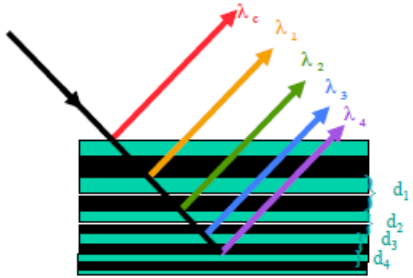
Therefore, with a judicious choice of N and b, it can be arranged that $R_ = 0$ implying perfect beam polarization (but not reflectivity) at all angles of incidence.



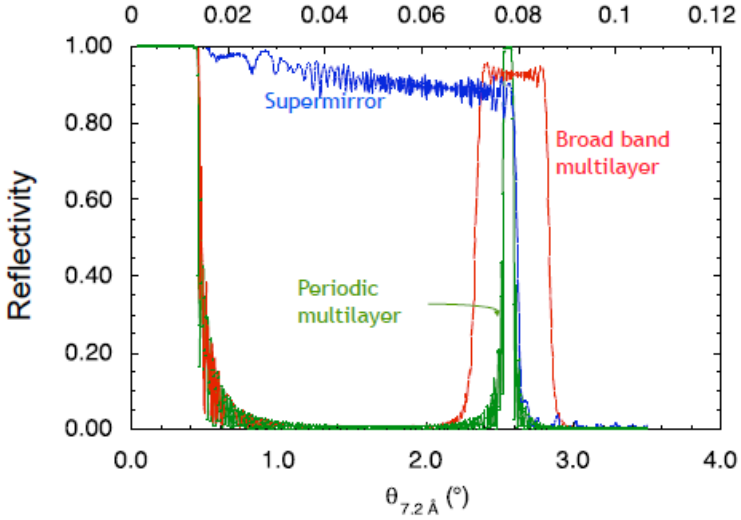

Very good polarization

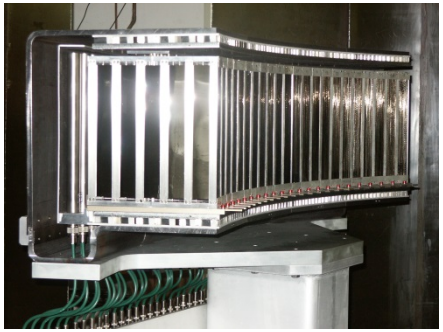
- clean separation of polarization
- only accept limited divergence and one wavelength

# Magnetic super mirrors




*Supermirror systems (eg Co/Ti, Fe/Si etc)*



*Boni et al, (Physica B 267-8(1999) 320 )*

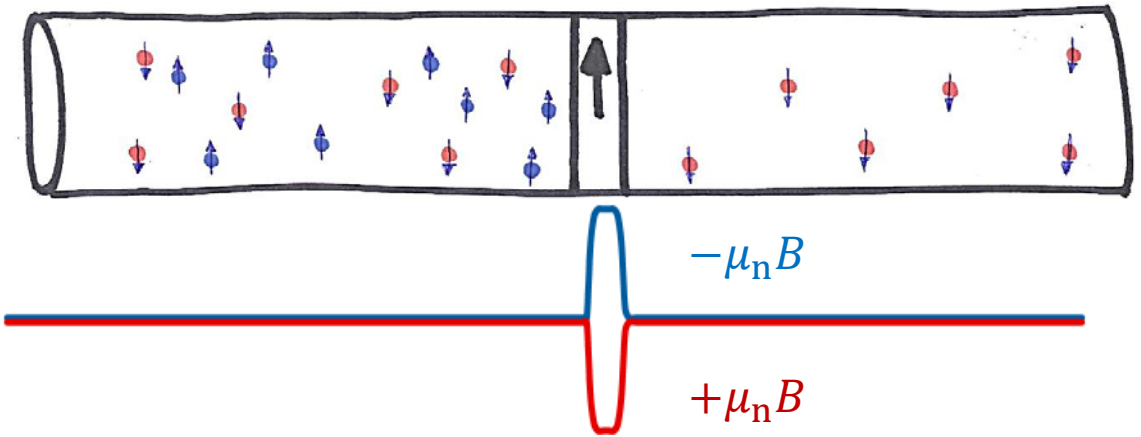
A gradient in the lattice spacing of the bilayers results in a range of effective Bragg angles, and therefore a reflectivity which extends beyond values expected for normal mirror reflections.

Supermirror m-number indicates the range of angles for good reflectivity

For a single (thick) layer of Ni, m = 1 = 0.1°/Å

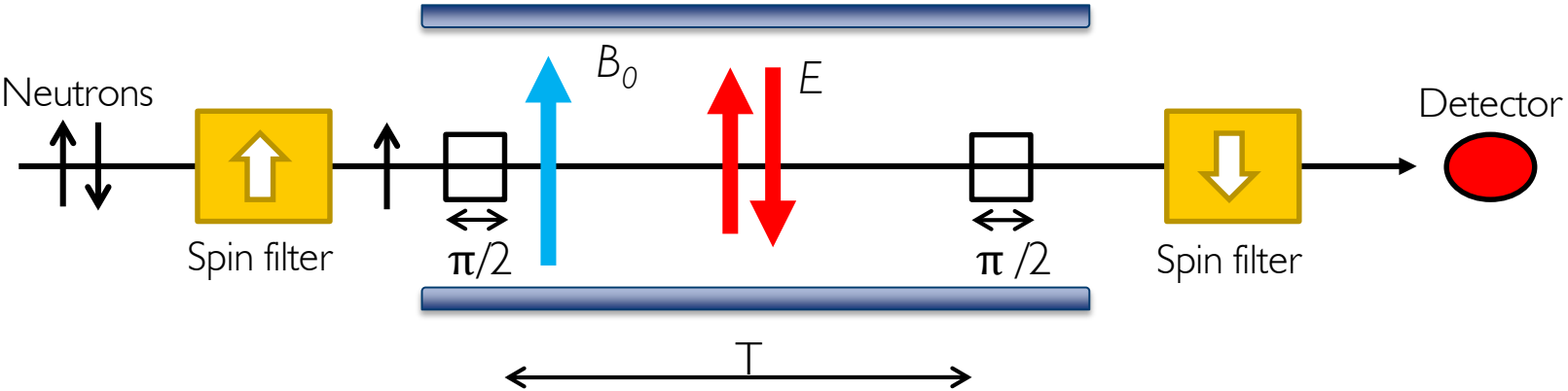
# Magnetic Field as potential barrier for UCN

Neutron magnetic moment: $|\mu_n| = 60\text{neV/T}$




UCN energies below 300neV $\rightarrow$ $B \geq 5\text{T}$

# nEDM with a beam




$$\Delta\varphi_{edm} = (\omega_{\uparrow\uparrow} - \omega_{\uparrow\downarrow})T = \frac{4\, d_n E}{\hbar} T$$

*Measuring the phase shift corresponds to a measurement of the frequency difference.*
*The accuracy of the nEDM measurement improves with larger times T and electric fields E.*

*Neutron beam T ≈ ms* ⟷ *Stored neutrons: T ≈ 100 sec*

UCN

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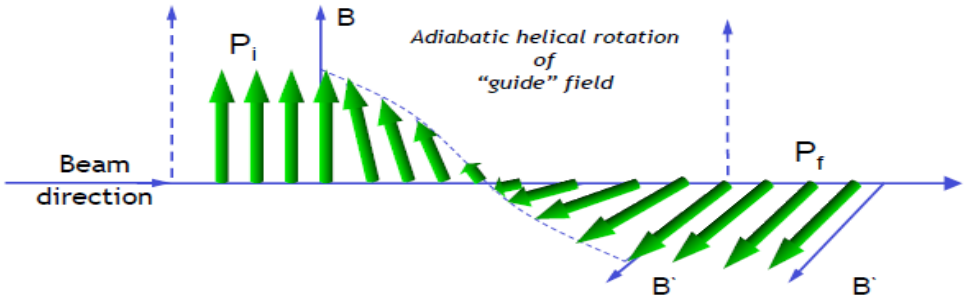
# Guiding and flipping spins

# Guiding neutron spin

If the direction of the magnetic field (quantization axis) remains constant in the rest frame of the neutrons then the direction of the polarization of the neutron beam will be preserved.

However, if the direction of the field **B** changes **sufficiently slowly** in the rest frame of the neutrons, then the polarization component parallel to **B** is **conserved** – i.e. there is an adiabatic (or reversible) rotation of the polarization.




[rem: An adiabatic process is one in which the system is always "infinitesimally close" to equilibrium. Here the field is changed in such a way that the potential energy of the neutrons is close to its initial value – and returns to this value at the end of the process]

# Larmor precesion

Angular momentum of a spin in a magnetic field equation of motion is

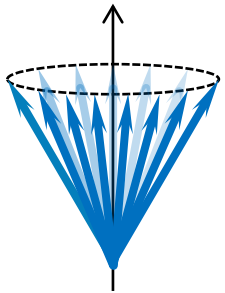
$$\hbar \frac{\mathrm{d}\boldsymbol{I}}{\mathrm{d}t} = \boldsymbol{\mu} \times \boldsymbol{B} \quad \text{or} \quad \frac{\mathrm{d}\mu}{\mathrm{d}t} = \boldsymbol{\mu} \times \gamma \boldsymbol{B},$$

where $\gamma$, the gyromagnetic ratio, is the ratio of angular momentum to magnetic momentum.

This gives an oscillatory solution with

$$\mu_x = |\mu| \cos(\omega_L t) \sin(\theta) \quad \text{and} \quad \mu_x = -|\mu| \sin(\omega_L t) \sin(\theta),$$

With an angular frequency of $\omega_L$, the Larmor frequency.



$\theta$ stays constant if $B$ is constant – rotation on a cone

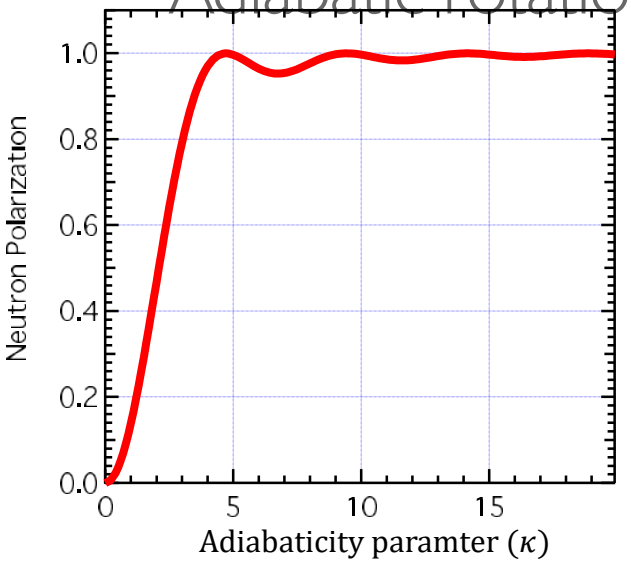
# Adiabatic rotation

The rate of angular rotation, $\omega_B$, of the field along the y-axis in the rest frame of the neutron is:

$$\omega_B = \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\mathrm{d}\theta}{\mathrm{d}y}\frac{\mathrm{d}y}{\mathrm{d}t} = v\frac{\mathrm{d}\theta}{\mathrm{d}y}$$

where v is the neutron velocity. We can therefore define an adiabaticity parameter $\kappa$:

$$\kappa = \frac{\omega_L}{\omega_B} = \frac{\gamma_n B}{v\frac{\mathrm{d}\theta}{\mathrm{d}y}}$$

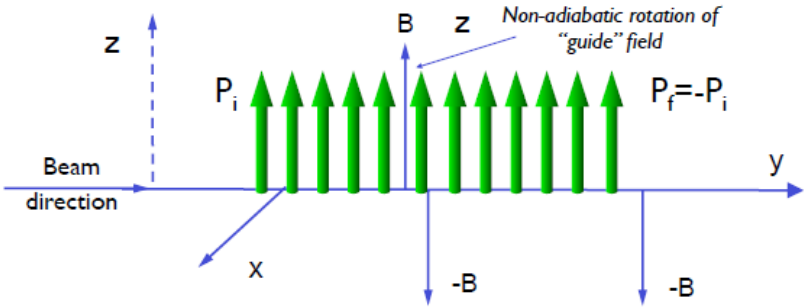



For an adiabatic rotation without loss of polarization we require $\kappa > 10$ (experience)

# Non-adiabatic spin flip (I)

Although we can adiabatically reorient the polarization in the laboratory frame, the polarization of the beam remains constant with respect to the guide field

However, for a non-adiabatic reorientation of the guide field ($\Delta B \neq 0$) the polarization will not re-orient - instead the beam will preserve its initial direction, and begin to precess about the new field direction

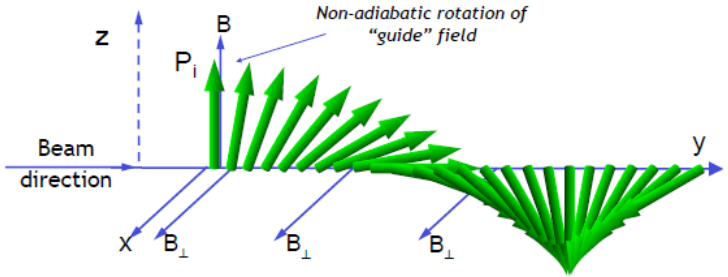



Non-adiabatic rotations enable the beam polarization to be effectively "flipped" withrespect to the guide field

# Non-adiabatic spin flip (II)

Although we can adiabatically reorient the polarization in the laboratory frame, the polarization of the beam remains constant with respect to the guide field

However, for a non-adiabatic reorientation of the guide field ($\Delta B \neq 0$) the polarization will not re-orient - instead the beam will preserve its initial direction, and begin to precess about the new field direction




Non-adiabatic rotations enable the beam polarization to be effectively "flipped" with respect to the guide field.

# Adiabatic spin manipulation

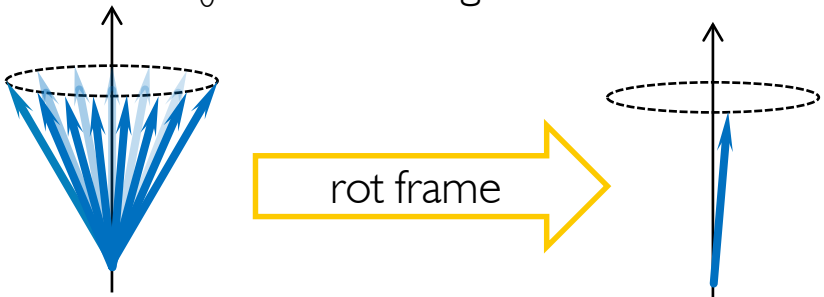
## Rotating frame:

When applying a circular oscillating field it is often easier to transform the problem into the so-called 'Rotating frame' inertial system.

The Bloch equation. becomes:

$$\frac{\mathrm{d}\mu}{\mathrm{d}t} = \boldsymbol{\mu} \times \gamma \boldsymbol{B} \quad \xrightarrow{\text{rot frame } \omega} \quad \frac{\mathrm{d}\vec{\mu}}{\mathrm{d}t} + \vec{\Omega} \times \vec{\mu} = \vec{\mu} \times \gamma \vec{B}(t)$$

Constant $B_0$ in the rotating frame:




$$\frac{\mathrm{d}\vec{\mu}}{\mathrm{d}t} = \vec{\mu} \times (\gamma \vec{B}(t) + \vec{\Omega})$$

$$\vec{B}_{eff} = \vec{B}(t) + \vec{\Omega}/\gamma$$

$$\frac{\delta \vec{\mu}}{\delta t} = 0 \;\Rightarrow\; \vec{B}_{eff} = 0 \;\Rightarrow\; \vec{\Omega} = -\gamma \vec{B}_0$$

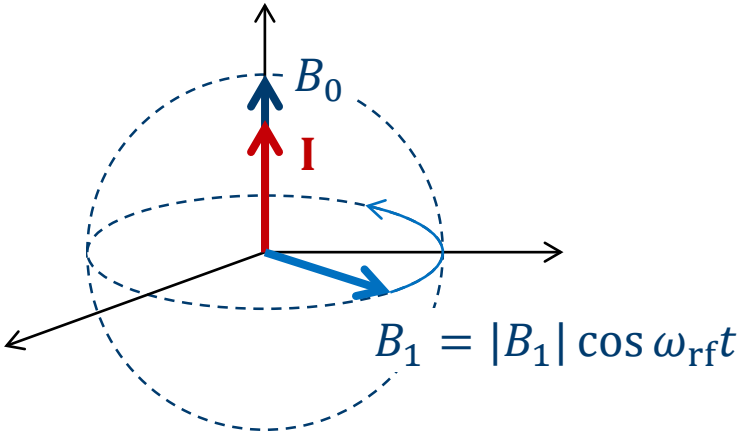
In the rotating frame $\vec{\boldsymbol{\mu}}$ stands still, while in the laboratory frame it rotates with the angular velocity $\vec{\boldsymbol{\Omega}}$.

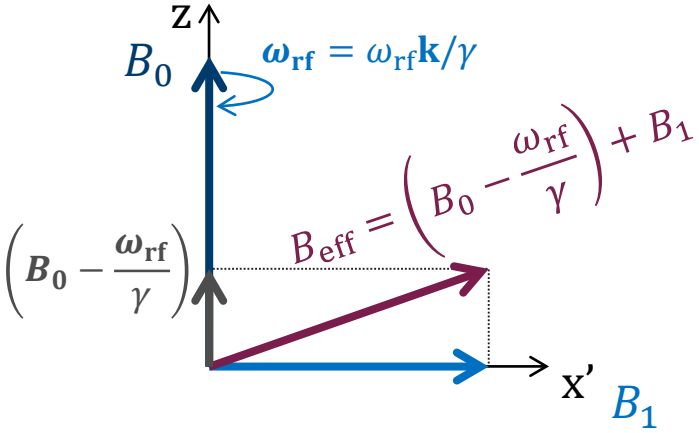
# Adiabatic spin manipulation

Bloch equation in rotating frame

$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times (\gamma \vec{B}(t) + \vec{\Omega})$$

Constant $B_0$ and perpendicular rotating oscillating field $B_1$ with frequency $\omega_{rf}$.




Effective field in rotating coordinate system

# Motion of the spin in effective field

If the frequency

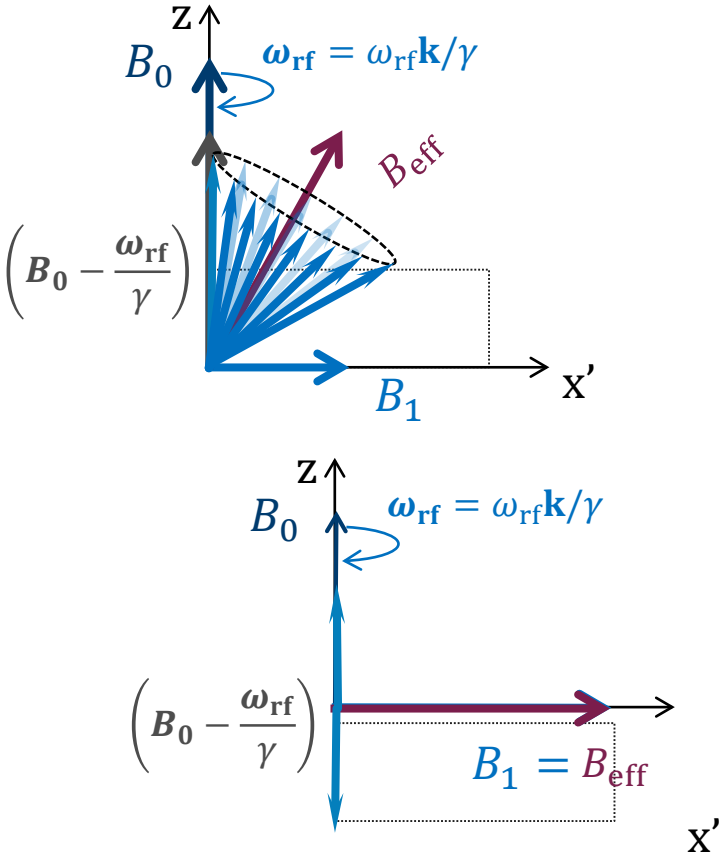
$$\omega_{\mathrm{rf}} = \gamma B_0,$$

exactly, than $B_{\mathrm{eff}} || x'$ and the spin is flipped.

## Resonance Spinflipper (pulse-NMR):

A polarised neutron beam passes a static magnetic field $B_0$ along the z-axis. Along a perpendicular axis a rf-field $B_1$ is irradiated with the Larmor frequency $\omega_0$ (resonance), but only during a short time $\tau$.

| | |
|---|---|
| Resonance condition: | $\omega_{\mathrm{rf}} = \omega_L = -\gamma_n B_0$ |
| $\pi/2$-Flip: | $\pi/2 = -\gamma_n B_1 \tau$ |
| $\pi$-Flip: | $\pi = -\gamma_n B_1 \tau$ |



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# Neutron detection

# Detection of neutrons: nuclear reactions

For slow neutron the only detection method is to use a nuclear reaction to convert a neutron into charged particles which then can be detected using scintillators, Si –pn diodes, or gaseous proportional count chambers.

| | | | |
|---|---|---|---|
| Helium-3: | $\mathrm{n} + {}^{3}\mathrm{He} \rightarrow \mathrm{p} + {}^{3}\mathrm{H}$ | 0.765MeV | 5327(10)b |
| Lithium-6: | $\mathrm{n} + {}^{7}\mathrm{Li} \rightarrow \alpha + {}^{3}\mathrm{H}$ | 4.780MeV | 941(3)b |
| Boron-10: | $\mathrm{n} + {}^{10}\mathrm{B} \rightarrow \alpha + {}^{7}\mathrm{Li}$ | 2.792MeV | 3838(6)b |

1/v –law for slow neutron:
abs cross section increases with 1/v

# Helium-3 gas proportional chamber

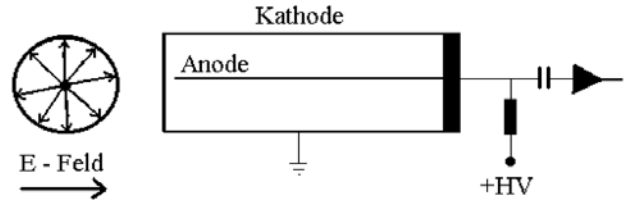
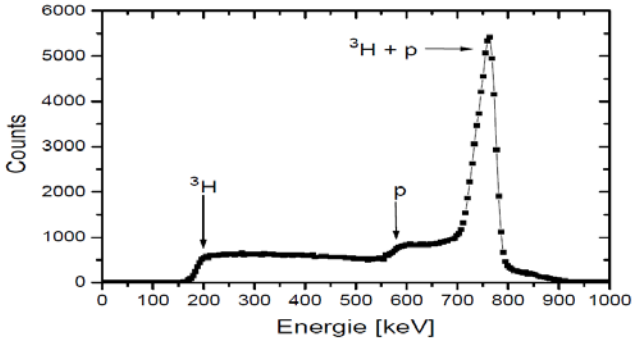
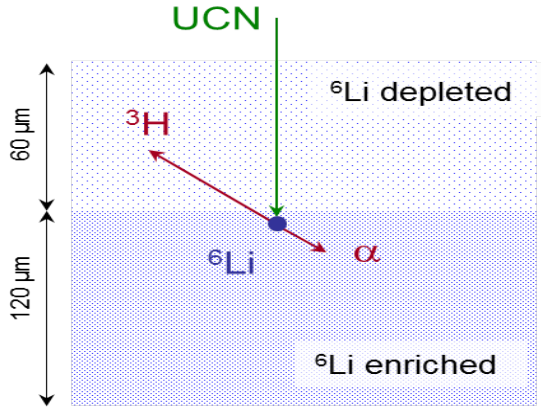


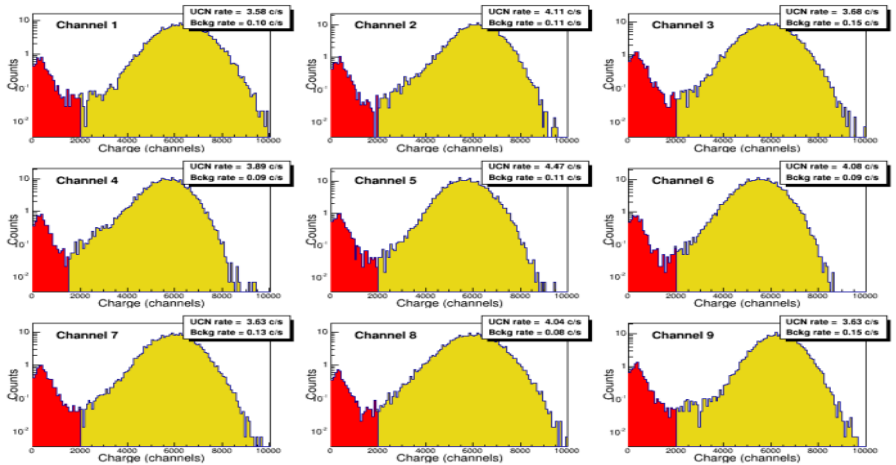

Abbildung 4.5: Schematischer Aufbau eines $^3$He-Proportionalzählers.




- Gas mixture $^3He/CO_2$
- Pressure adjusted for optimal detection efficiency
- HV ($\sim 1kV$) between anode and cathode
- Sudden drop in voltage is proportional to energy of charged particle
- Measured with dedicated electronics

# Neutron detection using $^6$Li



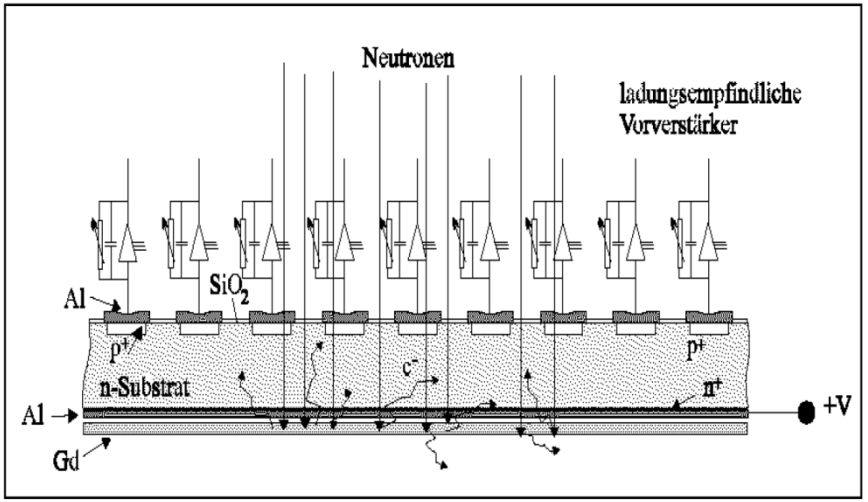



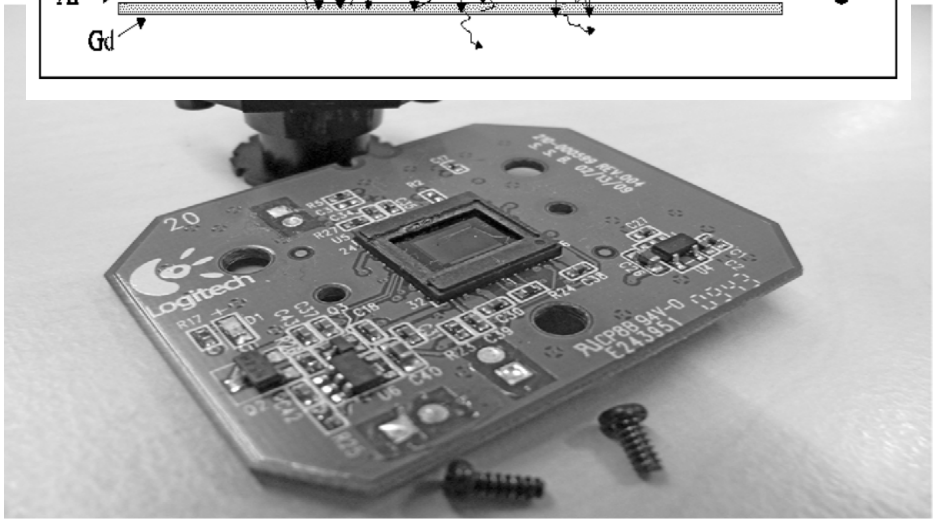

Detection system:

- Two 9-channel scintillation counters
- Reaction:
  $n + {}^6Li \rightarrow {}^3H + alpha$
- High count rate capable

# Boron / Lithium coated CCD pn-diode

- Thin layer of converter on semiconductor
- Charged particles create electron –hole pairs in depleted n-zone
- Applied HV makes them drift to p-zone
- Charge sensitive amplifier counts charges



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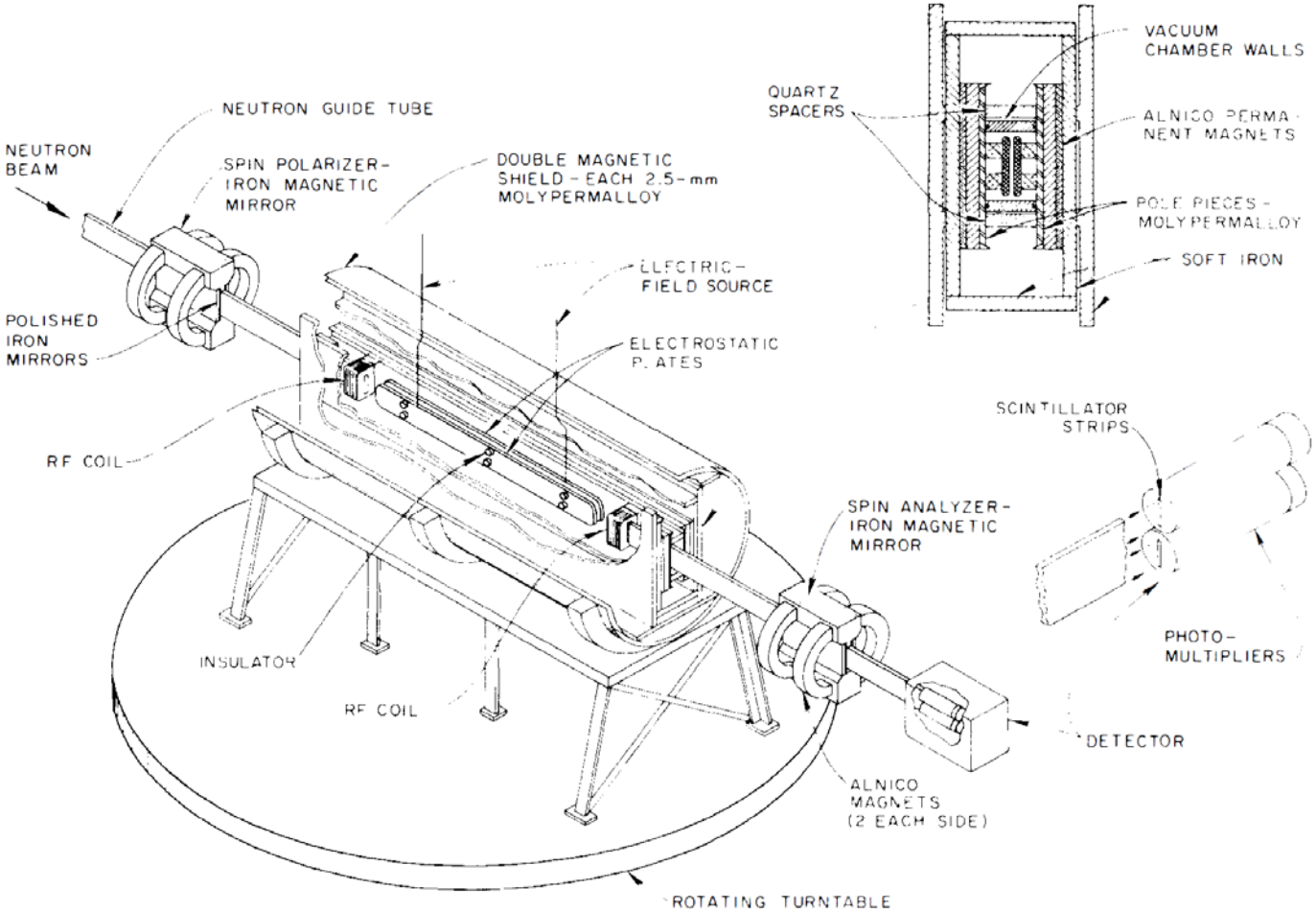
# The last beam result

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# The last beam searches

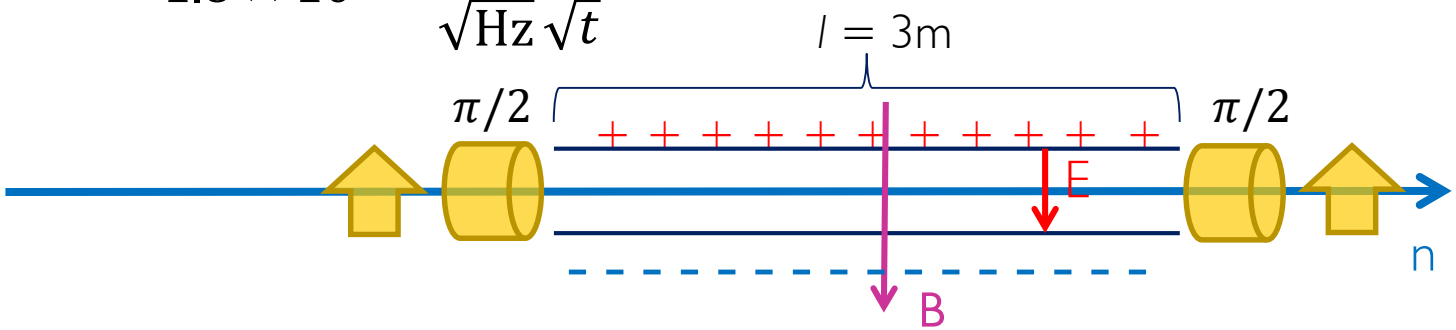
$$T = \frac{l}{v} \approx \frac{3\,\mathrm{m}}{150\mathrm{m/s}};\ \alpha > 0.9;\ E = \frac{100\mathrm{kV}}{\mathrm{cm}};\ \dot{N} = 1 \times 10^{6}\mathrm{s}^{-1}$$

$$\delta(d_\mathrm{n}) = \frac{\hbar}{2\alpha T E\sqrt{\dot{N}}}\frac{1}{\sqrt{t}}$$

$$= 1.8 \times 10^{-22}\,\frac{e\mathrm{cm}}{\sqrt{\mathrm{Hz}}}\frac{1}{\sqrt{t}}$$

1 day

$$\sigma = 1 \times 10^{-24} e\mathrm{cm}$$




Dominant systematic effect:

$$B_v = -\frac{\boldsymbol{v} \times \boldsymbol{E}}{c^2}$$

final result: $\sigma(d_\mathrm{n}) = 1.5 \times 10^{-24} e\mathrm{cm}$
due to misalignment error of 0.1 mrad

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