Towards the next level of string phenomenology using machine learning

Patrick K. S. Vaudrevange

Technische Universität München

string_data – December 15, 2020

Based on collaborations with:
A. Baur, M. Kade, A. Mütter, H. P. Nilles, E. Parr, S. Ramos-Sánchez, A. Trautner, M. Wimmer

Outline

- Part 1: String landscape & machine learning
- Part 2: Modular flavor symmetries

Outline

- Part 1: String landscape & machine learning
- Part 2: Modular flavor symmetries

Big data, machine learning and the string landscape

Big data, machine learning and the string landscape

- Landscape of four-dimensional string models is huge

Big data, machine learning and the string landscape

- Landscape of four-dimensional string models is huge
- Only a tiny fraction is MSSM-like

Big data, machine learning and the string landscape

- Landscape of four-dimensional string models is huge
- Only a tiny fraction is MSSM-like
- Apply techniques from big data & machine learning to string landscape

Big data, machine learning and the string landscape

- Landscape of four-dimensional string models is huge
- Only a tiny fraction is MSSM-like
- Apply techniques from big data & machine learning to string landscape
  We use:

Big data, machine learning and the string landscape

- Landscape of four-dimensional string models is huge
- Only a tiny fraction is MSSM-like
- Apply techniques from big data & machine learning to string landscape
  We use:
  - Autoencoder neural network to draw a chart of the landscape

Big data, machine learning and the string landscape

- Landscape of four-dimensional string models is huge
- Only a tiny fraction is MSSM-like
- Apply techniques from big data & machine learning to string landscape
  We use:
  - Autoencoder neural network to draw a chart of the landscape
  - Contrast patterns to increase probability to find MSSM-like models

Big data, machine learning and the string landscape

- Landscape of four-dimensional string models is huge
- Only a tiny fraction is MSSM-like
- Apply techniques from big data & machine learning to string landscape
  We use:
  - Autoencoder neural network to draw a chart of the landscape
  - Contrast patterns to increase probability to find MSSM-like models
  - Decision trees to predict the stringy origin of the MSSM

Autoencoder neural network for $\mathbb{Z}_6$-II models: preparation & preprocessing

Mütter, Parr, P.V. 1811.05993

Autoencoder neural network for $\mathbb{Z}_6$-II models: preparation & preprocessing

Mütter, Parr, P.V. 1811.05993

- Data set: coarse sample of $\mathcal{O}(700{,}000)$ randomly created, inequivalent $\mathbb{Z}_6$-II models obtained using the

C++ ORBIFOLDER

Nilles, Ramos-Sánchez, P.V., Wingerter 1110.5229

Autoencoder neural network for $\mathbb{Z}_6$-II models: preparation & preprocessing

Mütter, Parr, P.V. 1811.05993

- Data set: coarse sample of $\mathcal{O}$(700,000) randomly created, inequivalent $\mathbb{Z}_6$-II models obtained using the

C++
ORBIFOLDER

Nilles, Ramos-Sánchez, P.V., Wingerter 1110.5229

- Translate data to symmetry-invariant representation

shift & Wilson lines $\rightarrow$ local GUT groups

Autoencoder neural network for $\mathbb{Z}_6$-II models: preparation & preprocessing

Mütter, Parr, P.V. 1811.05993

- Data set: coarse sample of $\mathcal{O}(700{,}000)$ randomly created, inequivalent $\mathbb{Z}_6$-II models obtained using the

C++ ORBIFOLDER

Nilles, Ramos-Sánchez, P.V., Wingerter 1110.5229

- Translate data to symmetry-invariant representation

  shift & Wilson lines $\rightarrow$ local GUT groups

- Use one-hot-encoding: 962-dimensional input vector

Autoencoder neural network for $\mathbb{Z}_6$-II models: preparation & preprocessing

Mütter, Parr, P.V. 1811.05993

- Data set: coarse sample of $\mathcal{O}$(700,000) randomly created, inequivalent $\mathbb{Z}_6$-II models obtained using the

C++ ORBIFOLDER

Nilles, Ramos-Sánchez, P.V., Wingerter 1110.5229

- Translate data to symmetry-invariant representation

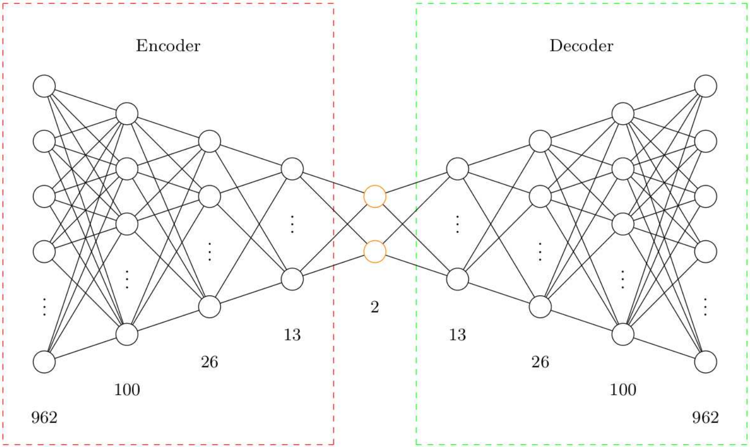
  shift & Wilson lines $\rightarrow$ local GUT groups

- Use one-hot-encoding: 962-dimensional input vector
- Split dataset into

  60% training data and 40% validation data

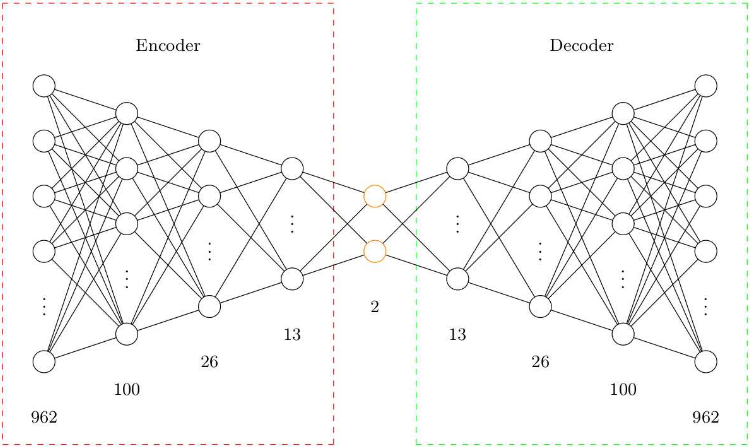
Autoencoder neural network for $\mathbb{Z}_6$-II models: training

Mütter, Parr, P.V. 1811.05993



Autoencoder neural network for $\mathbb{Z}_6$-II models: training

Mütter, Parr, P.V. 1811.05993



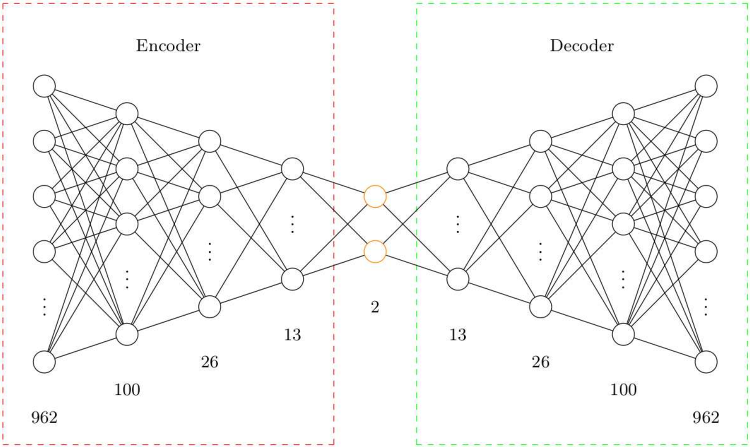
train n.n. (i.e. adjust weights of n.n.) such that

output $\sim$ input

using training set

Autoencoder neural network for $\mathbb{Z}_6$-II models: training

Mütter, Parr, P.V. 1811.05993



train n.n. (i.e. adjust weights of n.n.) such that

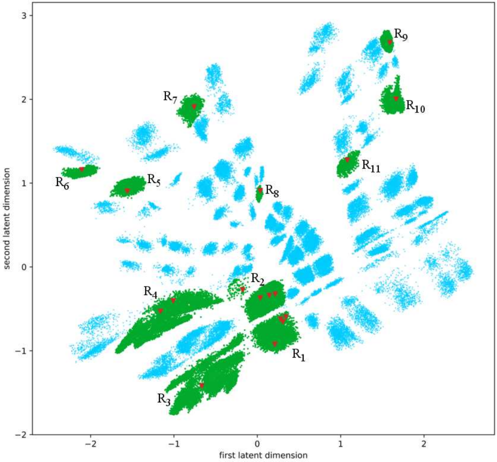
output $\sim$ input

using training set

Read out latent layer for training set

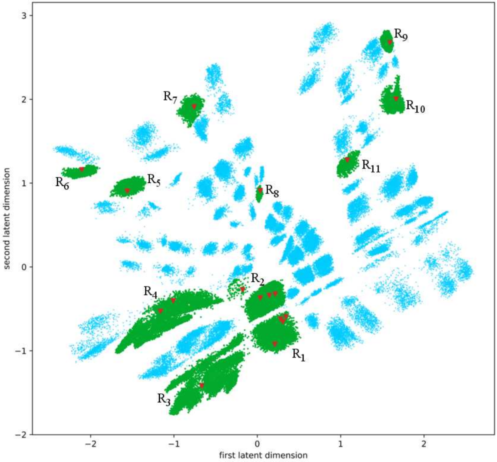
Autoencoder neural network for $\mathbb{Z}_6$-II models: chart of the $\mathbb{Z}_6$-II landscape

Mütter, Parr, P.V. 1811.05993



Autoencoder neural network for $\mathbb{Z}_6$-II models: chart of the $\mathbb{Z}_6$-II landscape

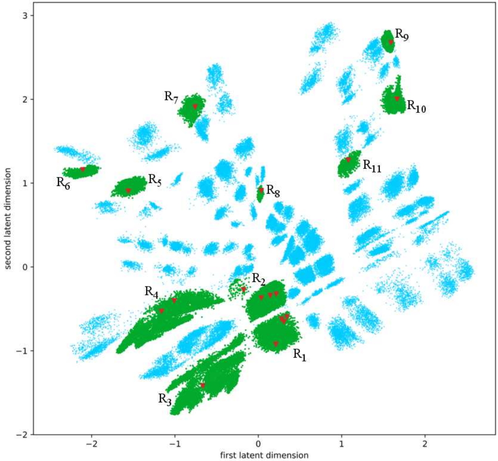
Mütter, Parr, P.V. 1811.05993



- Red dots: ≈ 20 MSSM-like models from training set

Autoencoder neural network for $\mathbb{Z}_6$-II models: chart of the $\mathbb{Z}_6$-II landscape

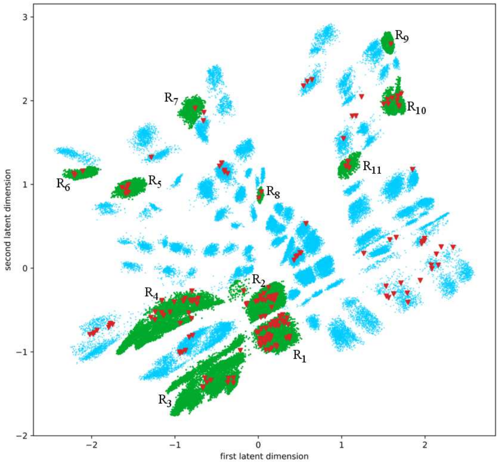
Mütter, Parr, P.V. 1811.05993



- Red dots: ≈ 20 MSSM-like models from training set
- $R_1 - R_{11}$: "fertile islands"

Autoencoder neural network for $\mathbb{Z}_6$-II models: chart of the $\mathbb{Z}_6$-II landscape

Mütter, Parr, P.V. 1811.05993



- Red dots: ≈ 200 MSSM-like models from full dataset
- $R_1 - R_{11}$: "fertile islands"

Extract knowledge from autoencoder neural network using a decision tree

Mütter, Parr, P.V. 1811.05993

Extract knowledge from autoencoder neural network using a decision tree

Mütter, Parr, P.V. 1811.05993

Assign to each model from dataset a region R_i, $i = 0, \ldots, 11$, of the landscape:

Extract knowledge from autoencoder neural network using a decision tree

Mütter, Parr, P.V. 1811.05993

Assign to each model from dataset a region R_i, $i = 0, \dots, 11$, of the landscape:

Classification task

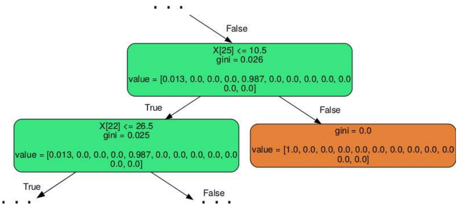
Extract knowledge from autoencoder neural network using a decision tree

Mütter, Parr, P.V. 1811.05993

Assign to each model from dataset a region R_i, $i = 0, \ldots, 11$, of the landscape:

Classification task

Use decision tree:



node labeled by

- a threshold (e.g. is feature no. $X[25] \leq 10.5$?),
- the Gini index (i.e. the impurity),
- the weighted percentages (labeled by "value") of $\mathbb{Z}_6$-II models in this node that belong to the regions R_i – all evaluated on the training set.

Extract knowledge from autoencoder neural network using a decision tree

Mütter, Parr, P.V. 1811.05993

confusion matrix of the decision tree evaluated for the validation set:

true region \ predicted region	R_0	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	R_{11}
R_0	198,994	10	39	10	24	1	7	17	3	16	4	13
R_1	11	3,107	1	2	0	0	0	0	0	0	0	0
R_2	19	3	9,667	2	1	0	0	0	0	0	0	0
R_3	24	2	1	5,256	3	0	0	0	0	0	0	0
R_4	31	2	4	1	6,430	0	0	0	0	0	0	0
R_5	0	0	0	0	0	3,138	0	0	0	0	0	0
R_6	3	0	0	0	0	0	994	0	0	0	0	0
R_7	15	0	0	0	0	0	0	848	0	0	0	0
R_8	0	0	0	0	0	0	0	0	1,139	0	0	0
R_9	10	0	0	0	0	0	0	0	0	1,491	0	0
R_{10}	2	0	0	0	0	0	0	0	0	0	3,333	0
R_{11}	10	0	0	0	0	0	0	0	0	0	0	984

Extract knowledge from autoencoder neural network using a decision tree

Mütter, Parr, P.V. 1811.05993

confusion matrix of the decision tree evaluated for the validation set:

		predicted region											
		R_0	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	R_{11}
true region	R_0	198,994	10	39	10	24	1	7	17	3	16	4	13
	R_1	11	3,107	1	2	0	0	0	0	0	0	0	0
	R_2	19	3	9,667	2	1	0	0	0	0	0	0	0
	R_3	24	2	1	5,256	3	0	0	0	0	0	0	0
	R_4	31	2	4	1	6,430	0	0	0	0	0	0	0
	R_5	0	0	0	0	0	3,138	0	0	0	0	0	0
	R_6	3	0	0	0	0	0	994	0	0	0	0	0
	R_7	15	0	0	0	0	0	0	848	0	0	0	0
	R_8	0	0	0	0	0	0	0	0	1,139	0	0	0
	R_9	10	0	0	0	0	0	0	0	0	1,491	0	0
	R_{10}	2	0	0	0	0	0	0	0	0	0	3,333	0
	R_{11}	10	0	0	0	0	0	0	0	0	0	0	984

But decision tree is large ($\approx 2,000$ nodes)
Still difficult to interpret...

Contrast patterns: why?

Parr, P.V. 1910.13473

10^7 random (equivalent) string models from the $\mathbb{Z}_6$-II orbifold, ordered by their frequency of occurrence:

ranking	frequency of occurrence	type of model
1	8 008	gauge group $U(1)^{16}$ (with 218 matter fields)
⋮	⋮	
⋮	⋮	
⋮	⋮	
⋮	⋮	
⋮	⋮	

Contrast patterns: why?

Parr, P.V. 1910.13473

10^7 random (equivalent) string models from the $\mathbb{Z}_6$-II orbifold, ordered by their frequency of occurrence:

ranking	frequency of occurrence	type of model
1	8 008	gauge group $U(1)^{16}$ (with 218 matter fields)
⋮	⋮	
19	1 915	first non-Abelian gauge group (gauge group $SU(2) \times U(1)^{15}$)
⋮	⋮	
⋮	⋮	
⋮	⋮	
⋮	⋮	

Contrast patterns: why?

Parr, P.V. 1910.13473

10^7 random (equivalent) string models from the $\mathbb{Z}_6$-II orbifold, ordered by their frequency of occurrence:

ranking	frequency of occurrence	type of model
1	8 008	gauge group $U(1)^{16}$ (with 218 matter fields)
⋮	⋮	
19	1 915	first non-Abelian gauge group (gauge group $SU(2) \times U(1)^{15}$)
⋮	⋮	
635	114	first gauge group $SU(3) \times SU(2)$
⋮	⋮	
⋮	⋮	
⋮	⋮	

Contrast patterns: why?

Parr, P.V. 1910.13473

10^7 random (equivalent) string models from the $\mathbb{Z}_6$-II orbifold, ordered by their frequency of occurrence:

ranking	frequency of occurrence	type of model
1	8 008	gauge group $U(1)^{16}$ (with 218 matter fields)
⋮	⋮	
19	1 915	first non-Abelian gauge group (gauge group $SU(2) \times U(1)^{15}$)
⋮	⋮	
635	114	first gauge group $SU(3) \times SU(2)$
⋮	⋮	
739	10	first $SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times U(1)_Y$ with 1 generation plus vector-like exotics
⋮	⋮	
⋮	⋮	

Contrast patterns: why?

Parr, P.V. 1910.13473

10^7 random (equivalent) string models from the $\mathbb{Z}_6$-II orbifold, ordered by their frequency of occurrence:

ranking	frequency of occurrence	type of model
1	8 008	gauge group $U(1)^{16}$ (with 218 matter fields)
⋮	⋮	
19	1 915	first non-Abelian gauge group (gauge group $SU(2) \times U(1)^{15}$)
⋮	⋮	
635	114	first gauge group $SU(3) \times SU(2)$
⋮	⋮	
739	10	first $SU(3)_C \times SU(2)_L \times U(1)_Y$ with 1 generation plus vector-like exotics
⋮	⋮	
747	2	first $SU(3)_C \times SU(2)_L \times U(1)_Y$ with 2 generations plus vector-like exotics
⋮	⋮	

Contrast patterns: why?

Parr, P.V. 1910.13473

10^7 random (equivalent) string models from the $\mathbb{Z}_6$-II orbifold, ordered by their frequency of occurrence:

ranking	frequency of occurrence	type of model
1	8 008	gauge group $U(1)^{16}$ (with 218 matter fields)
⋮	⋮	
19	1 915	first non-Abelian gauge group (gauge group $SU(2) \times U(1)^{15}$)
⋮	⋮	
635	114	first gauge group $SU(3) \times SU(2)$
⋮	⋮	
739	10	first $SU(3)_C \times SU(2)_L \times U(1)_Y$ with 1 generation plus vector-like exotics
⋮	⋮	
747	2	first $SU(3)_C \times SU(2)_L \times U(1)_Y$ with 2 generations plus vector-like exotics
⋮	⋮	
748	1	first MSSM-like model with 3 generations plus vector-like exotics

Contrast patterns: definitions

Parr, P.V. 1910.13473

- Define pattern c that, if satisfied by a given string model, increases the probability of the model being MSSM-like
- growth rate gr:

$$\mathrm{gr}(c, D_{\text{MSSM-like}}, D_{\cancel{\text{MSSM}}\text{-like}}) := \frac{\mathrm{supp}(c, D_{\text{MSSM-like}})}{\mathrm{supp}(c, D_{\cancel{\text{MSSM}}\text{-like}})}$$

where

$$\mathrm{supp}(c, D) := \frac{|\{M \in D \mid M \text{ satisfies } c\}|}{|D|}$$

Contrast patterns: results

Parr, P.V. 1910.13473

orbifold geometry		$N^{(1)}_{\mathrm{U}_1}$	$N^{(1)}_{\mathrm{U}_2}$	$N^{(1)}_{\mathrm{U}_3}$	$N^{(2)}_{\mathrm{U}_1}$	$N^{(2)}_{\mathrm{U}_2}$	$N^{(2)}_{\mathrm{U}_3}$	gr(c)
$\mathbb{Z}_4$	(2,1)			≥ 4			≤ 1	5.32
	(3,1)			≥ 4			≤ 1	6.92
$\mathbb{Z}_6$-I	(1,1)	≥ 13		≥ 14				2.79
	(2,1)	≥ 13		≥ 14				2.78
$\mathbb{Z}_6$-II	(1,1)		≥ 2				≤ 5	1.81
	(2,1)		≥ 2				≤ 5	1.60
	(3,1)		≥ 2				≤ 5	1.70
	(4,1)		≥ 2				≤ 5	1.86
$\mathbb{Z}_8$-I	(1,1)			≥ 4			≤ 25	1.22
	(2,1)			≥ 4			≤ 25	1.23
	(3,1)			≥ 8				2.21
$\mathbb{Z}_8$-II	(1,1)			≥ 4			≤ 41	1.61
	(2,1)			≤ 3			≤ 1	1.78
				≥ 4			≤ 41	1.01
$\mathbb{Z}_{12}$-I	(1,1)	≤ 10		≥ 2				1.24
	(2,1)	≤ 10		≥ 2				1.24
$\mathbb{Z}_{12}$-II	(1,1)	≥ 2					≤ 5	1.71

For example, for $\mathbb{Z}_4$-(2,1) we find

$$c = ((N^{(1)}_{\mathrm{U}_3} \geq 4) \text{ and } (N^{(2)}_{\mathrm{U}_3} \leq 1))$$

$N^{(1)}_{\mathrm{U}_3}$: number of matter, charged under MSSM from U_3 sector
$N^{(2)}_{\mathrm{U}_3}$: number of matter, charged under hidden E_8 from U_3 sector

Contrast patterns: results

Parr, P.V. 1910.13473

inequivalent MSSM-like orbifold models

orbifold geometry		# MSSM-like from Nilles 2014	# MSSM-like from Olguin-Trejo 2018	# MSSM-like using contrast patterns	# MSSM-like ‘merged’
$\mathbb{Z}_4$	(2,1)	128	138	125	179
	(3,1)	25	26	33	33
$\mathbb{Z}_6$-I	(1,1)	31	30	31	31
	(2,1)	31	30	31	31
$\mathbb{Z}_6$-II	(1,1)	348	363	468	481
	(2,1)	338	349	395	443
	(3,1)	350	351	415	482
	(4,1)	334	354	407	464
$\mathbb{Z}_7$	(1,1)	0	1	1	1
$\mathbb{Z}_8$-I	(1,1)	263	256	248	271
	(2,1)	164	155	144	164
	(3,1)	387	377	408	430
$\mathbb{Z}_8$-II	(1,1)	638	1 833	1 259	2 289
	(2,1)	260	489	349	555
$\mathbb{Z}_{12}$-I	(1,1)	365	556	610	625
	(2,1)	385	554	607	625
$\mathbb{Z}_{12}$-II	(1,1)	211	352	365	435

using contrast patterns: size of hidden gauge group, matter from bulk

Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

- Can we predict the orbifold origin of *the* MSSM?

Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

- Can we predict the orbifold origin of *the* MSSM?
- Examine percentage of MSSM-like orbifold models with certain types of vector-like exotics:

Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

- Can we predict the orbifold origin of *the* MSSM?
- Examine percentage of MSSM-like orbifold models with certain types of vector-like exotics:

	$\mathbb{Z}_4$	$\mathbb{Z}_6$-I	$\mathbb{Z}_6$-II	$\mathbb{Z}_7$	$\mathbb{Z}_8$-I	$\mathbb{Z}_8$-II	$\mathbb{Z}_{12}$-I	$\mathbb{Z}_{12}$-II	...
# MSSM	212	62	1, 870	1	865	2, 844	1, 250	435	...
$(3, 2)_{1/6}$	1.89%	0%	31.60%	0%	4.05%	25.00%	4.00%	24.14%	...
$(\overline{3}, 1)_{-2/3}$	50.47%	0%	28.66%	0%	5.55%	44.13%	3.28%	35.63%	...
$(\overline{3}, 1)_{1/3}$	100%	100%	99.95%	100%	99.54%	100%	100%	100%	...
$(1, 2)_{-1/2}$	96.23%	35.48%	92.19%	100%	93.99%	94.94%	78.96%	91.72%	...
$(1, 1)_1$	1.89%	0%	28.66%	0%	5.55%	44.09%	3.28%	35.63%	...
$(1, 1)_0$	100%	100%	100%	100%	100%	100%	100%	100%	...

Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

- Can we predict the orbifold origin of *the* MSSM?
- Examine percentage of MSSM-like orbifold models with certain types of vector-like exotics:

	$\mathbb{Z}_4$	$\mathbb{Z}_6$-I	$\mathbb{Z}_6$-II	$\mathbb{Z}_7$	$\mathbb{Z}_8$-I	$\mathbb{Z}_8$-II	$\mathbb{Z}_{12}$-I	$\mathbb{Z}_{12}$-II	...
# MSSM	212	62	1, 870	1	865	2, 844	1, 250	435	...
$(3,2)_{1/6}$	1.89%	0%	31.60%	0%	4.05%	25.00%	4.00%	24.14%	...
$(\overline{3},1)_{-2/3}$	50.47%	0%	28.66%	0%	5.55%	44.13%	3.28%	35.63%	...
$(\overline{3},1)_{1/3}$	100%	100%	99.95%	100%	99.54%	100%	100%	100%	...
$(1,2)_{-1/2}$	96.23%	35.48%	92.19%	100%	93.99%	94.94%	78.96%	91.72%	...
$(1,1)_1$	1.89%	0%	28.66%	0%	5.55%	44.09%	3.28%	35.63%	...
$(1,1)_0$	100%	100%	100%	100%	100%	100%	100%	100%	...

where $(3,2)_{1/6}$ means $(3,2)_{1/6} \oplus (\overline{3},2)_{-1/6}$

Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

	$\mathbb{Z}_4$	$\mathbb{Z}_6$-I	$\mathbb{Z}_6$-II	$\mathbb{Z}_7$	$\mathbb{Z}_8$-I	$\mathbb{Z}_8$-II	$\mathbb{Z}_{12}$-I	$\mathbb{Z}_{12}$-II	...
# MSSM	212	62	1, 870	1	865	2, 844	1, 250	435	...
$(3,2)_{-1/2}$	0%	0%	0%	0%	0%	0%	0%	0%	...
$(3,2)_{1/2}$	0%	0%	0%	0%	0%	0%	0%	0%	...
$(3,2)_{-1/3}$	0%	0%	0%	0%	0.23%	1.05%	0.24%	0%	...
$(3,2)_{-1/6}$	0%	0%	0%	0%	0%	0%	0.64%	0%	...
$(3,2)_{-1/12}$	2.83%	0%	0%	0%	0%	0%	0%	0%	...
$(3,1)_{0}$	0%	0%	6.84%	0%	0%	0%	25.12%	0%	...
$(3,1)_{-1/2}$	0%	0%	0%	0%	0%	0%	1.12%	0%	...
$(3,1)_{1/2}$	0%	0%	0%	0%	0%	0%	0.80%	0%	...
$(3,1)_{-2/3}$	0%	0%	0%	0%	0%	0%	0.32%	0%	...
$(3,1)_{1/3}$	0%	0%	4.71%	0%	0%	0%	18.72%	0%	...
$(3,1)_{-5/6}$	0%	0%	2.89%	0%	0%	0.74%	0%	0%	...
$(3,1)_{-1/6}$	0%	0%	0.21%	0%	0%	0%	4.32%	0%	...
$(3,1)_{1/6}$	67.45%	93.55%	66.47%	0%	87.63%	74.30%	69.20%	66.90%	...
$(3,1)_{-7/12}$	2.83%	0%	0%	0%	0%	0%	0.32%	0%	...
$(3,1)_{-1/12}$	41.04%	0%	0%	0%	0.23%	0.56%	1.76%	0%	...
$(3,1)_{5/12}$	4.72%	0%	0%	0%	0%	0.14%	0.80%	0%	...
$(3,1)_{2/21}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(3,1)_{5/21}$	0%	0%	0%	100%	0%	0%	0%	0%	...

Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

	$\mathbb{Z}_4$	$\mathbb{Z}_6$-I	$\mathbb{Z}_6$-II	$\mathbb{Z}_7$	$\mathbb{Z}_8$-I	$\mathbb{Z}_8$-II	$\mathbb{Z}_{12}$-I	$\mathbb{Z}_{12}$-II	...
# MSSM	212	62	1, 870	1	865	2, 844	1, 250	435	...
$(1,2)_0$	82.55%	100%	85.94%	0%	99.31%	93.95%	76.32%	90.34%	...
$(1,2)_{1/3}$	0%	0%	0.43%	0%	0%	0%	4.72%	0%	...
$(1,2)_{2/3}$	0%	0%	0%	0%	0%	0%	0.80%	0%	...
$(1,2)_{1/4}$	41.98%	0%	0%	0%	0.23%	0.70%	2.40%	0%	...
$(1,2)_{3/4}$	2.83%	0%	0%	0%	0%	0%	0%	0%	...
$(1,2)_{1/6}$	0%	0%	8.98%	0%	0%	0%	26.72%	0%	...
$(1,2)_{5/6}$	0%	0%	0%	0%	0%	0%	0%	0%	...
$(1,2)_{1/14}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(1,2)_{5/14}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(1,1)_{1/2}$	100%	100%	92.09%	0%	100%	100%	76.96%	100%	...
$(1,1)_{1/3}$	0%	0%	8.98%	0%	0%	0%	29.20%	0%	...
$(1,1)_{2/3}$	0%	0%	8.98%	0%	0%	0%	28.72%	0%	...
$(1,1)_{1/4}$	52.36%	0%	0%	0%	0.23%	0.95%	3.20%	0%	...
$(1,1)_{3/4}$	42.92%	0%	0%	0%	0.23%	0.42%	1.92%	0%	...
$(1,1)_{1/6}$	0%	0%	1.07%	0%	0%	0%	6.32%	0%	...
$(1,1)_{5/6}$	0%	0%	0%	0%	0%	0%	1.12%	0%	...
$(1,1)_{1/7}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(1,1)_{2/7}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(1,1)_{3/7}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(1,1)_{4/7}$	0%	0%	0%	100%	0%	0%	0%	0%	...

Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

	$\mathbb{Z}_4$	$\mathbb{Z}_6$-I	$\mathbb{Z}_6$-II	$\mathbb{Z}_7$	$\mathbb{Z}_8$-I	$\mathbb{Z}_8$-II	$\mathbb{Z}_{12}$-I	$\mathbb{Z}_{12}$-II	...
# MSSM	212	62	1,870	1	865	2,844	1,250	435	...
$(1,2)_0$	82.55%	100%	85.94%	0%	99.31%	93.95%	76.32%	90.34%	...
$(1,2)_{1/3}$	0%	0%	0.43%	0%	0%	0%	4.72%	0%	...
$(1,2)_{2/3}$	0%	0%	0%	0%	0%	0%	0.80%	0%	...
$(1,2)_{1/4}$	41.98%	0%	0%	0%	0.23%	0.70%	2.40%	0%	...
$(1,2)_{3/4}$	2.83%	0%	0%	0%	0%	0%	0%	0%	...
$(1,2)_{1/6}$	0%	0%	8.98%	0%	0%	0%	26.72%	0%	...
$(1,2)_{5/6}$	0%	0%	0%	0%	0%	0%	0%	0%	...
$(1,2)_{1/14}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(1,2)_{5/14}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(1,1)_{1/2}$	100%	100%	92.09%	0%	100%	100%	76.96%	100%	...
$(1,1)_{1/3}$	0%	0%	8.98%	0%	0%	0%	29.20%	0%	...
$(1,1)_{2/3}$	0%	0%	8.98%	0%	0%	0%	28.72%	0%	...
$(1,1)_{1/4}$	52.36%	0%	0%	0%	0.23%	0.95%	3.20%	0%	...
$(1,1)_{3/4}$	42.92%	0%	0%	0%	0.23%	0.42%	1.92%	0%	...
$(1,1)_{1/6}$	0%	0%	1.07%	0%	0%	0%	6.32%	0%	...
$(1,1)_{5/6}$	0%	0%	0%	0%	0%	0%	1.12%	0%	...
$(1,1)_{1/7}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(1,1)_{2/7}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(1,1)_{3/7}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(1,1)_{4/7}$	0%	0%	0%	100%	0%	0%	0%	0%	...

Also the average numbers of vector-like exotics known.

Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

	$\mathbb{Z}_4$	$\mathbb{Z}_6$-I	$\mathbb{Z}_6$-II	$\mathbb{Z}_7$	$\mathbb{Z}_8$-I	$\mathbb{Z}_8$-II	$\mathbb{Z}_{12}$-I	$\mathbb{Z}_{12}$-II	...
# MSSM	212	62	1, 870	1	865	2, 844	1, 250	435	...
$(1,2)_0$	82.55%	100%	85.94%	0%	99.31%	93.95%	76.32%	90.34%	...
$(1,2)_{1/3}$	0%	0%	0.43%	0%	0%	0%	4.72%	0%	...
$(1,2)_{2/3}$	0%	0%	0%	0%	0%	0%	0.80%	0%	...
$(1,2)_{1/4}$	41.98%	0%	0%	0%	0.23%	0.70%	2.40%	0%	...
$(1,2)_{3/4}$	2.83%	0%	0%	0%	0%	0%	0%	0%	...
$(1,2)_{1/6}$	0%	0%	8.98%	0%	0%	0%	26.72%	0%	...
$(1,2)_{5/6}$	0%	0%	0%	0%	0%	0%	0%	0%	...
$(1,2)_{1/14}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(1,2)_{5/14}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(1,1)_{1/2}$	100%	100%	92.09%	0%	100%	100%	76.96%	100%	...
$(1,1)_{1/3}$	0%	0%	8.98%	0%	0%	0%	29.20%	0%	...
$(1,1)_{2/3}$	0%	0%	8.98%	0%	0%	0%	28.72%	0%	...
$(1,1)_{1/4}$	52.36%	0%	0%	0%	0.23%	0.95%	3.20%	0%	...
$(1,1)_{3/4}$	42.92%	0%	0%	0%	0.23%	0.42%	1.92%	0%	...
$(1,1)_{1/6}$	0%	0%	1.07%	0%	0%	0%	6.32%	0%	...
$(1,1)_{5/6}$	0%	0%	0%	0%	0%	0%	1.12%	0%	...
$(1,1)_{1/7}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(1,1)_{2/7}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(1,1)_{3/7}$	0%	0%	0%	100%	0%	0%	0%	0%	...
$(1,1)_{4/7}$	0%	0%	0%	100%	0%	0%	0%	0%	...

Also the average numbers of vector-like exotics known.
Orbifold geometry leaves imprint in particle spectrum!

Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

- Spectrum of vector-like exotic specific for orbifold geometry

Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

- Spectrum of vector-like exotic specific for orbifold geometry
- Take spectrum of vector-like exotics to predict the orbifold geometry that was used to construct this spectrum

Predicting the orbifold origin of the MSSM

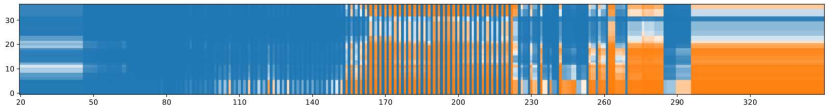
Parr, P.V., Wimmer 2003.01732

- Spectrum of vector-like exotic specific for orbifold geometry
- Take spectrum of vector-like exotics to predict the orbifold geometry that was used to construct this spectrum
- Use (boosted) decision tree

Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

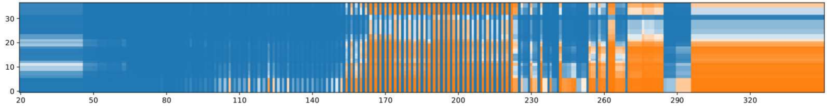
- Spectrum of vector-like exotic specific for orbifold geometry
- Take spectrum of vector-like exotics to predict the orbifold geometry that was used to construct this spectrum
- Use (boosted) decision tree
- Result (assuming heavy top from bulk and non-Abelian flavor symmetry):



Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

- Spectrum of vector-like exotic specific for orbifold geometry
- Take spectrum of vector-like exotics to predict the orbifold geometry that was used to construct this spectrum
- Use (boosted) decision tree
- Result (assuming heavy top from bulk and non-Abelian flavor symmetry):

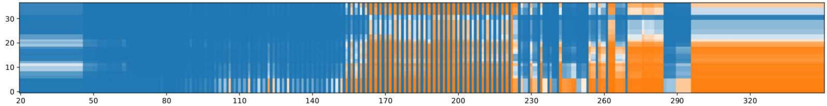


  - x-axis: number of SM singlets

Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

- Spectrum of vector-like exotic specific for orbifold geometry
- Take spectrum of vector-like exotics to predict the orbifold geometry that was used to construct this spectrum
- Use (boosted) decision tree
- Result (assuming heavy top from bulk and non-Abelian flavor symmetry):

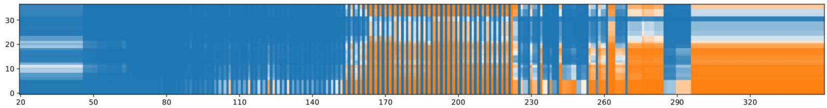


  - x-axis: number of SM singlets
  - y-axis: hidden sector beta function b for SUSY breaking via gaugino condensation ($b \gtrsim 15$)

Predicting the orbifold origin of the MSSM

Parr, P.V., Wimmer 2003.01732

- Spectrum of vector-like exotic specific for orbifold geometry
- Take spectrum of vector-like exotics to predict the orbifold geometry that was used to construct this spectrum
- Use (boosted) decision tree
- Result (assuming heavy top from bulk and non-Abelian flavor symmetry):



- x-axis: number of SM singlets
- y-axis: hidden sector beta function b for SUSY breaking via gaugino condensation ($b \gtrsim 15$)
- color: blue: $\mathbb{Z}_2 \times \mathbb{Z}_4$, orange: $\mathbb{Z}_4 \times \mathbb{Z}_4$, turquoise: $\mathbb{Z}_2 \times \mathbb{Z}_2$

Outline

- Part 1: String landscape & machine learning
- Part 2: Modular flavor symmetries

Outline and Motivation

"Are neutrino masses modular forms?" (1706.08749, F. Feruglio)

Outline and Motivation

"Are neutrino masses modular forms?" (1706.08749, F. Feruglio)
Experimental values:

Parameter	Ordering	Best fit	1σ range
$\delta m^2/10^{-5}$ eV^2	NO	7.34	7.20 – 7.51
	IO	7.34	7.20 – 7.51
$\sin^2\theta_{12}/10^{-1}$	NO	3.05	2.92 – 3.19
	IO	3.03	2.90 – 3.17
$\lvert\Delta m^2\rvert/10^{-3}$ eV^2	NO	2.485	2.453 – 2.514
	IO	2.465	2.434 – 2.495
$\sin^2\theta_{13}/10^{-2}$	NO	2.22	2.14 – 2.28
	IO	2.23	2.17 – 2.30
$\sin^2\theta_{23}/10^{-1}$	NO	5.45	4.98 – 5.65
	IO	5.51	5.17 – 5.67
δ/π	NO	1.28	1.10 – 1.66
	IO	1.52	1.37 – 1.65

$$\Sigma = m_1 + m_2 + m_3 < 0.12 - 0.69\text{eV at } 2\sigma$$

2003.08511 Capozzi et al.

Outline and Motivation

"Are neutrino masses modular forms?" (1706.08749, F. Feruglio)
Experimental values:

Parameter	Ordering	Best fit	1σ range
$\delta m^2/10^{-5}$ eV^2	NO	7.34	7.20 – 7.51
	IO	7.34	7.20 – 7.51
$\sin^2\theta_{12}/10^{-1}$	NO	3.05	2.92 – 3.19
	IO	3.03	2.90 – 3.17
$\lvert\Delta m^2\rvert/10^{-3}$ eV^2	NO	2.485	2.453 – 2.514
	IO	2.465	2.434 – 2.495
$\sin^2\theta_{13}/10^{-2}$	NO	2.22	2.14 – 2.28
	IO	2.23	2.17 – 2.30
$\sin^2\theta_{23}/10^{-1}$	NO	5.45	4.98 – 5.65
	IO	5.51	5.17 – 5.67
δ/π	NO	1.28	1.10 – 1.66
	IO	1.52	1.37 – 1.65

$$\Sigma = m_1 + m_2 + m_3 < 0.12 - 0.69\text{eV at } 2\sigma$$

2003.08511 Capozzi et al.

neutrino mixing $\gg$ quark mixing

Outline and Motivation

"Are neutrino masses modular forms?" (1706.08749, F. Feruglio)
Experimental values:

Parameter	Ordering	Best fit	1σ range
$\delta m^2/10^{-5}$ eV^2	NO	7.34	7.20 – 7.51
	IO	7.34	7.20 – 7.51
$\sin^2\theta_{12}/10^{-1}$	NO	3.05	2.92 – 3.19
	IO	3.03	2.90 – 3.17
$\lvert\Delta m^2\rvert/10^{-3}$ eV^2	NO	2.485	2.453 – 2.514
	IO	2.465	2.434 – 2.495
$\sin^2\theta_{13}/10^{-2}$	NO	2.22	2.14 – 2.28
	IO	2.23	2.17 – 2.30
$\sin^2\theta_{23}/10^{-1}$	NO	5.45	4.98 – 5.65
	IO	5.51	5.17 – 5.67
δ/π	NO	1.28	1.10 – 1.66
	IO	1.52	1.37 – 1.65

$$\Sigma = m_1 + m_2 + m_3 < 0.12 - 0.69\text{eV at } 2\sigma$$

2003.08511 Capozzi et al.

neutrino mixing $\gg$ quark mixing

How can we explain flavor in the lepton sector?

Outline and Motivation

"Are neutrino masses modular forms?" (1706.08749, F. Feruglio)

Experimental values:

Parameter	Ordering	Best fit	1σ range
$\delta m^2/10^{-5}$ eV^2	NO	7.34	7.20 – 7.51
	IO	7.34	7.20 – 7.51
$\sin^2\theta_{12}/10^{-1}$	NO	3.05	2.92 – 3.19
	IO	3.03	2.90 – 3.17
$\lvert\Delta m^2\rvert/10^{-3}$ eV^2	NO	2.485	2.453 – 2.514
	IO	2.465	2.434 – 2.495
$\sin^2\theta_{13}/10^{-2}$	NO	2.22	2.14 – 2.28
	IO	2.23	2.17 – 2.30
$\sin^2\theta_{23}/10^{-1}$	NO	5.45	4.98 – 5.65
	IO	5.51	5.17 – 5.67
δ/π	NO	1.28	1.10 – 1.66
	IO	1.52	1.37 – 1.65

$$\Sigma = m_1 + m_2 + m_3 < 0.12 - 0.69\text{eV at } 2\sigma$$

2003.08511 Capozzi et al.

neutrino mixing $\gg$ quark mixing

How can we explain flavor in the lepton sector?

Finite flavor symmetries!

Traditional flavor symmetries

(Supersymmetric case)

Traditional flavor symmetries

(Supersymmetric case)

- Quarks, leptons, Higgs Φ_i transform in (higher-dim.) representations $\boldsymbol{r}_i$ of (non-Abelian) finite group G ("flavor symmetry"):

$$\Phi_i \stackrel{g}{\to} \rho_{\boldsymbol{r}_i}(g)\,\Phi_i \qquad \text{for} \qquad g \in G$$

Traditional flavor symmetries

(Supersymmetric case)

- Quarks, leptons, Higgs Φ_i transform in (higher-dim.) representations $\boldsymbol{r}_i$ of (non-Abelian) finite group G ("flavor symmetry"):

$$\Phi_i \stackrel{g}{\longrightarrow} \rho_{\boldsymbol{r}_i}(g)\,\Phi_i \qquad \text{for} \qquad g \in G$$

- Many examples: $G \in \{S_3, S_4, A_4, T', A_5, \Delta(27), \ldots\}$

Traditional flavor symmetries

(Supersymmetric case)

- Quarks, leptons, Higgs Φ_i transform in (higher-dim.) representations $\boldsymbol{r}_i$ of (non-Abelian) finite group G ("flavor symmetry"):

$$\Phi_i \stackrel{g}{\to} \rho_{\boldsymbol{r}_i}(g)\,\Phi_i \qquad \text{for} \qquad g \in G$$

- Many examples: $G \in \{S_3, S_4, A_4, T', A_5, \Delta(27), \ldots\}$
- Superpotential $\mathcal{W}$ must be invariant under $g \in G$:

$$\mathcal{W} \;\supset\; y_{1\ldots k}\,\Phi_1 \ldots \Phi_k \qquad \text{if} \qquad \boldsymbol{r}_1 \otimes \ldots \otimes \boldsymbol{r}_k \;\supset\; \mathbf{1}$$

Traditional flavor symmetries

(Supersymmetric case)

- Quarks, leptons, Higgs Φ_i transform in (higher-dim.) representations $\boldsymbol{r}_i$ of (non-Abelian) finite group G ("flavor symmetry"):

$$\Phi_i \stackrel{g}{\rightarrow} \rho_{\boldsymbol{r}_i}(g)\,\Phi_i \qquad \text{for} \qquad g \in G$$

- Many examples: $G \in \{S_3, S_4, A_4, T', A_5, \Delta(27), \ldots\}$
- Superpotential $\mathcal{W}$ must be invariant under $g \in G$:

$$\mathcal{W} \supset y_{1\ldots k}\,\Phi_1 \ldots \Phi_k \qquad \text{if} \qquad \boldsymbol{r}_1 \otimes \ldots \otimes \boldsymbol{r}_k \supset 1$$

- Kähler potential K must be invariant under $g \in G$:

$$K \supset |\Phi_i|^2 \qquad \text{is invariant because} \qquad \rho_{\boldsymbol{r}_i}(g) \text{ is unitary}$$

Traditional flavor symmetries

(Supersymmetric case)

- Quarks, leptons, Higgs Φ_i transform in (higher-dim.) representations $\boldsymbol{r}_i$ of (non-Abelian) finite group G ("flavor symmetry"):

$$\Phi_i \stackrel{g}{\rightarrow} \rho_{\boldsymbol{r}_i}(g)\,\Phi_i \qquad \text{for} \qquad g \in G$$

- Many examples: $G \in \{S_3, S_4, A_4, T', A_5, \Delta(27), \ldots\}$
- Superpotential $\mathcal{W}$ must be invariant under $g \in G$:

$$\mathcal{W} \;\supset\; y_{1\ldots k}\,\Phi_1 \ldots \Phi_k \qquad \text{if} \qquad \boldsymbol{r}_1 \otimes \ldots \otimes \boldsymbol{r}_k \;\supset\; 1$$

- Kähler potential K must be invariant under $g \in G$:

$$K \;\supset\; |\Phi_i|^2 \qquad \text{is invariant because} \qquad \rho_{\boldsymbol{r}_i}(g) \text{ is unitary}$$

- Flavor symmetry must be broken (spontaneously)

Flavon Ψ : d Standard Model singlets with $|\boldsymbol{r}_\Psi| = d$ and $\langle\Psi\rangle = \begin{pmatrix} v_1 \\ \vdots \\ v_d \end{pmatrix} \neq 0$

Traditional flavor symmetries

(Supersymmetric case)

- Quarks, leptons, Higgs Φ_i transform in (higher-dim.) representations $\boldsymbol{r}_i$ of (non-Abelian) finite group G ("flavor symmetry"):

$$\Phi_i \stackrel{g}{\rightarrow} \rho_{\boldsymbol{r}_i}(g)\,\Phi_i \qquad \text{for} \qquad g \in G$$

- Many examples: $G \in \{S_3, S_4, A_4, T', A_5, \Delta(27), \ldots\}$
- Superpotential $\mathcal{W}$ must be invariant under $g \in G$:

$$\mathcal{W} \;\supset\; y_{1\ldots k}\,\Phi_1 \ldots \Phi_k \qquad \text{if} \qquad \boldsymbol{r}_1 \otimes \ldots \otimes \boldsymbol{r}_k \;\supset\; 1$$

- Kähler potential K must be invariant under $g \in G$:

$$K \;\supset\; |\Phi_i|^2 \qquad \text{is invariant because} \qquad \rho_{\boldsymbol{r}_i}(g) \text{ is unitary}$$

- Flavor symmetry must be broken (spontaneously)

Flavon Ψ : d Standard Model singlets with $|\boldsymbol{r}_\Psi| = d$ and $\langle\Psi\rangle = \begin{pmatrix} v_1 \\ \vdots \\ v_d \end{pmatrix} \neq 0$

- Difficult: alignment of $\langle\Psi\rangle$ in d-dim. flavor space

Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

(Supersymmetric case)

Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

(Supersymmetric case)

- Flavon replaced by single field: modulus T

Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

(Supersymmetric case)

- Flavon replaced by single field: modulus T
- Symmetry: $\mathrm{SL}(2,\mathbb{Z})$ where

$$T \stackrel{\gamma}{\to} \frac{a\,T+b}{c\,T+d} \qquad \text{for} \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})$$

Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

(Supersymmetric case)

- Flavon replaced by single field: modulus T
- Symmetry: $\mathrm{SL}(2,\mathbb{Z})$ where

$$T \overset{\gamma}{\rightarrow} \frac{a\,T+b}{c\,T+d} \qquad \text{for} \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})$$

- Superpotential: $\mathcal{W} \supset Y(T)\,\Phi_1 \ldots \Phi_k$

Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

(Supersymmetric case)

- Flavon replaced by single field: modulus T
- Symmetry: $\mathrm{SL}(2,\mathbb{Z})$ where

$$T \stackrel{\gamma}{\rightarrow} \frac{a\,T+b}{c\,T+d} \qquad \text{for} \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})$$

- Superpotential: $\mathcal{W} \supset Y(T)\,\Phi_1 \ldots \Phi_k$
- Couplings $Y(T)$ are modular forms:

$$Y_{\boldsymbol{r_Y}}^{(n_Y)}(T) \stackrel{\gamma}{\rightarrow} Y_{\boldsymbol{r_Y}}^{(n_Y)}\left(\frac{a\,T+b}{c\,T+d}\right)$$

Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

(Supersymmetric case)

- Flavon replaced by single field: modulus T
- Symmetry: $\mathrm{SL}(2,\mathbb{Z})$ where

$$T \stackrel{\gamma}{\rightarrow} \frac{a\,T+b}{c\,T+d} \qquad \text{for} \qquad \gamma \;=\; \begin{pmatrix} a & b \\ c & d \end{pmatrix} \;\in\; \mathrm{SL}(2,\mathbb{Z})$$

- Superpotential: $\mathcal{W} \;\supset\; Y(T)\,\Phi_1 \ldots \Phi_k$
- Couplings $Y(T)$ are modular forms:

$$Y_{\boldsymbol{r_Y}}^{(n_Y)}(T) \;\stackrel{\gamma}{\rightarrow}\; Y_{\boldsymbol{r_Y}}^{(n_Y)}\left(\frac{a\,T+b}{c\,T+d}\right) \;=\; \qquad\qquad Y_{\boldsymbol{r_Y}}^{(n_Y)}(T)$$

Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

(Supersymmetric case)

- Flavon replaced by single field: modulus T
- Symmetry: $\mathrm{SL}(2,\mathbb{Z})$ where

$$T \xrightarrow{\gamma} \frac{a\,T+b}{c\,T+d} \qquad \text{for} \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})$$

- Superpotential: $\mathcal{W} \supset Y(T)\,\Phi_1 \ldots \Phi_k$
- Couplings $Y(T)$ are modular forms:

$$Y_{\boldsymbol{r_Y}}^{(n_Y)}(T) \xrightarrow{\gamma} Y_{\boldsymbol{r_Y}}^{(n_Y)}\left(\frac{a\,T+b}{c\,T+d}\right) = \rho_{\boldsymbol{r_Y}}(\gamma)\, Y_{\boldsymbol{r_Y}}^{(n_Y)}(T)$$

where:

- $\rho_{\boldsymbol{r_Y}}(\gamma)$: unitary representation $\boldsymbol{r_Y}$ of finite modular group Γ_N or Γ'_N

Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

(Supersymmetric case)

- Flavon replaced by single field: modulus T
- Symmetry: $\mathrm{SL}(2,\mathbb{Z})$ where

$$T \stackrel{\gamma}{\rightarrow} \frac{a\,T+b}{c\,T+d} \qquad \text{for} \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})$$

- Superpotential: $\mathcal{W} \supset Y(T)\,\Phi_1 \ldots \Phi_k$
- Couplings $Y(T)$ are modular forms:

$$Y_{\boldsymbol{r_Y}}^{(n_Y)}(T) \stackrel{\gamma}{\rightarrow} Y_{\boldsymbol{r_Y}}^{(n_Y)}\left(\frac{a\,T+b}{c\,T+d}\right) = \underbrace{(c\,T+d)^{n_Y}}_{\text{automorphy factor}} \rho_{\boldsymbol{r_Y}}(\gamma)\, Y_{\boldsymbol{r_Y}}^{(n_Y)}(T)$$

where:

- $n_Y \in \{1,2,3,\ldots\}$: (modular) weight
- $\rho_{\boldsymbol{r_Y}}(\gamma)$: unitary representation $\boldsymbol{r_Y}$ of finite modular group Γ_N or Γ'_N

Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

(Supersymmetric case)

- Flavon replaced by single field: modulus T
- Symmetry: $\mathrm{SL}(2,\mathbb{Z})$ where

$$T \overset{\gamma}{\to} \frac{a\,T+b}{c\,T+d} \qquad \text{for} \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})$$

- Superpotential: $\mathcal{W} \supset Y(T)\,\Phi_1\ldots\Phi_k$
- Couplings $Y(T)$ are modular forms:

$$Y_{\boldsymbol{r_Y}}^{(n_Y)}(T) \overset{\gamma}{\to} Y_{\boldsymbol{r_Y}}^{(n_Y)}\left(\frac{a\,T+b}{c\,T+d}\right) = \underbrace{(c\,T+d)^{n_Y}}_{\text{automorphy factor}}\rho_{\boldsymbol{r_Y}}(\gamma)\,Y_{\boldsymbol{r_Y}}^{(n_Y)}(T)$$

where:

  - $n_Y \in \{1,2,3,\ldots\}$: (modular) weight
  - $\rho_{\boldsymbol{r_Y}}(\gamma)$: unitary representation $\boldsymbol{r_Y}$ of finite modular group Γ_N or Γ'_N

- Modular forms of level N and weight k:
  linear space $\mathcal{M}_k(\Gamma(N)) \Leftrightarrow Y_{\boldsymbol{r_Y}}^{(n_Y)}(T)$ of Γ_N

Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

(Supersymmetric case)

- Flavon replaced by single field: modulus T
- Symmetry: $\mathrm{SL}(2,\mathbb{Z})$ where

$$T \stackrel{\gamma}{\to} \frac{a\,T+b}{c\,T+d} \qquad \text{for} \qquad \gamma \;=\; \begin{pmatrix} a & b \\ c & d \end{pmatrix} \;\in\; \mathrm{SL}(2,\mathbb{Z})$$

- Superpotential: $\mathcal{W} \;\supset\; Y(T)\,\Phi_1\ldots\Phi_k$
- Couplings $Y(T)$ are modular forms:

$$Y_{\boldsymbol{r_Y}}^{(n_Y)}(T) \;\stackrel{\gamma}{\to}\; Y_{\boldsymbol{r_Y}}^{(n_Y)}\left(\frac{a\,T+b}{c\,T+d}\right) \;=\; \underbrace{(c\,T+d)^{n_Y}}_{\text{automorphy factor}}\,\rho_{\boldsymbol{r_Y}}(\gamma)\,Y_{\boldsymbol{r_Y}}^{(n_Y)}(T)$$

where:
  - $n_Y \in \{1,2,3,\ldots\}$: (modular) weight
  - $\rho_{\boldsymbol{r_Y}}(\gamma)$: unitary representation $\boldsymbol{r_Y}$ of finite modular group Γ_N or Γ'_N
- Modular forms of level N and weight k:
  linear space $\mathcal{M}_k(\Gamma(N)) \Leftrightarrow Y_{\boldsymbol{r_Y}}^{(n_Y)}(T)$ of Γ_N

All $Y_{\boldsymbol{r_Y}}^{(n_Y)}(T)$ can be constructed explicitly!

Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

(Supersymmetric case)

- Flavon replaced by single field: modulus T
- Symmetry: $\mathrm{SL}(2,\mathbb{Z})$ where

$$T \stackrel{\gamma}{\rightarrow} \frac{a\,T+b}{c\,T+d} \qquad \text{for} \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})$$

- Superpotential: $\mathcal{W} \supset Y(T)\,\Phi_1 \ldots \Phi_k$
- Couplings $Y(T)$ are modular forms:

$$Y^{(n_Y)}_{\boldsymbol{r_Y}}(T) \stackrel{\gamma}{\rightarrow} Y^{(n_Y)}_{\boldsymbol{r_Y}}\left(\frac{a\,T+b}{c\,T+d}\right) = \underbrace{(c\,T+d)^{n_Y}}_{\text{automorphy factor}} \rho_{\boldsymbol{r_Y}}(\gamma)\, Y^{(n_Y)}_{\boldsymbol{r_Y}}(T)$$

  where:
  - $n_Y \in \{1,2,3,\ldots\}$: (modular) weight
  - $\rho_{\boldsymbol{r_Y}}(\gamma)$: unitary representation $\boldsymbol{r_Y}$ of finite modular group Γ_N or Γ'_N
- Modular forms of level N and weight k:
  linear space $\mathcal{M}_k(\Gamma(N)) \Leftrightarrow Y^{(n_Y)}_{\boldsymbol{r_Y}}(T)$ of Γ_N

  All $Y^{(n_Y)}_{\boldsymbol{r_Y}}(T)$ can be constructed explicitly!
- Vev aligns in d-dim. flavor space "automatically"
  (but moduli stabilization!)

Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

- Examples for finite modular groups:

N	$\mathtt{dim}\mathcal{M}_k(\Gamma(N))$	Γ_N (k even)	$\lvert\Gamma_N\rvert$		
2	$k/2+1$ (k even)	S_3	6		
3	$k+1$	A_4	12		
4	$2k+1$	S_4	24		
5	$5k+1$	A_5	60		

1907.01488 Liu and Ding

Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

- Examples for finite modular groups:

N	$\mathtt{dim}\mathcal{M}_k(\Gamma(N))$	Γ_N (k even)	$\lvert\Gamma_N\rvert$	Γ'_N (k even and odd)	$\lvert\Gamma'_N\rvert$
2	$k/2+1\,(k$ even)	S_3	6	S_3	6
3	$k+1$	A_4	12	T'	24
4	$2k+1$	S_4	24	S'_4	48
5	$5k+1$	A_5	60	A'_5	120

1907.01488 Liu and Ding

Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

- Examples for finite modular groups:

N	$\mathtt{dim}\mathcal{M}_k(\Gamma(N))$	Γ_N (k even)	$\lvert\Gamma_N\rvert$	Γ'_N (k even and odd)	$\lvert\Gamma'_N\rvert$
2	$k/2+1\,(k\ \text{even})$	S_3	6	S_3	6
3	$k+1$	A_4	12	T'	24
4	$2k+1$	S_4	24	S'_4	48
5	$5k+1$	A_5	60	A'_5	120

1907.01488 Liu and Ding

- A lot of bottom-up model building activities

Are neutrino masses modular forms? #18

Ferruccio Feruglio (INFN, Padua and Padua U.) (Jun 27, 2017)

e-Print: 1706.08749 [hep-ph]

pdf DOI cite 100 citations

Modular flavor symmetries from string theory

Modular flavor symmetries from string theory

- String theory compactified on two-torus ⇒ two moduli parameterize metric G and B-field B of two-torus:

$$\begin{aligned} \text{Kähler modulus} \qquad T &:= \frac{1}{\alpha'}\left(B_{12} + \mathrm{i}\sqrt{\det G}\right) \\ \text{complex structure modulus} \qquad U &:= \frac{1}{G_{11}}\left(G_{12} + \mathrm{i}\sqrt{\det G}\right) \end{aligned}$$

Modular flavor symmetries from string theory

- String theory compactified on two-torus $\Rightarrow$ two moduli parameterize metric G and B-field B of two-torus:

$$\text{Kähler modulus} \qquad T \;:=\; \frac{1}{\alpha'}\left(B_{12} + \mathrm{i}\sqrt{\det G}\right)$$

$$\text{complex structure modulus} \qquad U \;:=\; \frac{1}{G_{11}}\left(G_{12} + \mathrm{i}\sqrt{\det G}\right)$$

- Modular symmetry originate from outer automorphisms of Narain lattice:
  Baur, Nilles, Trautner, P.V. 1901.03251 & 1908.00805
  - $\mathrm{SL}(2,\mathbb{Z})_T$:
  $$T \xrightarrow{\hat{K}_\mathrm{S}} -\frac{1}{T}\,, \quad U \xrightarrow{\hat{K}_\mathrm{S}} U\,, \qquad \text{and} \qquad T \xrightarrow{\hat{K}_\mathrm{T}} T+1\,, \quad U \xrightarrow{\hat{K}_\mathrm{T}} U$$
  - $\mathrm{SL}(2,\mathbb{Z})_U$:
  $$T \xrightarrow{\hat{C}_\mathrm{S}} T\,, \quad U \xrightarrow{\hat{C}_\mathrm{S}} -\frac{1}{U}\,, \qquad \text{and} \qquad T \xrightarrow{\hat{C}_\mathrm{T}} T\,, \quad U \xrightarrow{\hat{C}_\mathrm{T}} U+1$$
  - mirror transformation $\hat{M}$:
  $$T \xrightarrow{\hat{M}} U \quad \text{and} \quad U \xrightarrow{\hat{M}} T$$
  - $\mathcal{CP}$-like transformation $\hat{\Sigma}_*$:
  $$T \xrightarrow{\hat{\Sigma}_*} -\bar{T} \quad \text{and} \quad U \xrightarrow{\hat{\Sigma}_*} -\bar{U}$$

Modular flavor symmetries from string theory

Baur, Nilles, Trautner, P.V. 1901.03251 & 1908.00805

Modular flavor symmetries from string theory

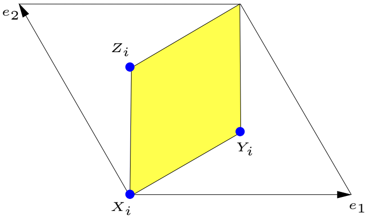
Baur, Nilles, Trautner, P.V. 1901.03251 & 1908.00805

- Consider toroidal orbifold $\mathbb{T}^2/\mathbb{Z}_K$, for example with $K = 3$

Modular flavor symmetries from string theory

Baur, Nilles, Trautner, P.V. 1901.03251 & 1908.00805

- Consider toroidal orbifold $\mathbb{T}^2/\mathbb{Z}_K$, for example with $K = 3$

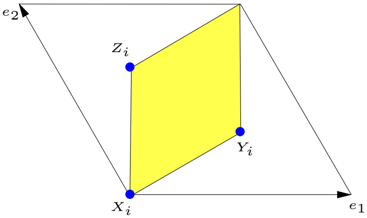


(X_i, Y_i, Z_i): twisted strings, localized in extra dimensions

Modular flavor symmetries from string theory

Baur, Nilles, Trautner, P.V. 1901.03251 & 1908.00805

- Consider toroidal orbifold $\mathbb{T}^2/\mathbb{Z}_K$, for example with $K = 3$



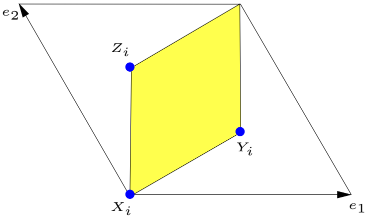
(X_i, Y_i, Z_i): twisted strings, localized in extra dimensions

- $\mathrm{SL}(2,\mathbb{Z})_T$ unbroken $\Rightarrow$ $\Gamma'_3 = T'$ finite modular symmetry (order 24)

Modular flavor symmetries from string theory

Baur, Nilles, Trautner, P.V. 1901.03251 & 1908.00805

- Consider toroidal orbifold $\mathbb{T}^2/\mathbb{Z}_K$, for example with $K = 3$



(X_i, Y_i, Z_i): twisted strings, localized in extra dimensions

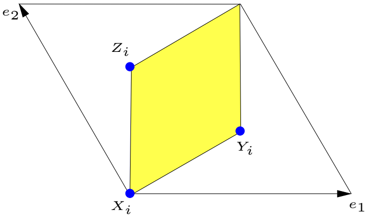
- $\mathrm{SL}(2,\mathbb{Z})_T$ unbroken $\Rightarrow$ $\Gamma'_3 = T'$ finite modular symmetry (order 24)
- Traditional flavor symmetry (relevant for dark matter): $\Delta(27) \cup \mathbb{Z}_2 \Rightarrow \Delta(54)$ R-symmetry

Dark matter from strings: Mütter, P.V. 1912.09909

Modular flavor symmetries from string theory

Baur, Nilles, Trautner, P.V. 1901.03251 & 1908.00805

- Consider toroidal orbifold $\mathbb{T}^2/\mathbb{Z}_K$, for example with $K = 3$



(X_i, Y_i, Z_i): twisted strings, localized in extra dimensions

- $\mathrm{SL}(2,\mathbb{Z})_T$ unbroken $\Rightarrow$ $\Gamma'_3 = T'$ finite modular symmetry (order 24)
- Traditional flavor symmetry (relevant for dark matter): $\Delta(27) \cup \mathbb{Z}_2 \Rightarrow$ $\Delta(54)$ R-symmetry

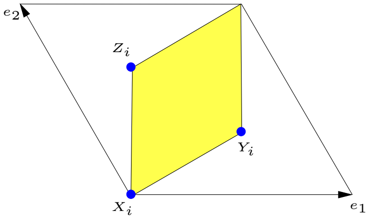
Dark matter from strings: Mütter, P.V. 1912.09909

- Standard $\mathbb{Z}_9$ R-symmetry (for six-dim. orbifolds)

Modular flavor symmetries from string theory

Baur, Nilles, Trautner, P.V. 1901.03251 & 1908.00805

- Consider toroidal orbifold $\mathbb{T}^2/\mathbb{Z}_K$, for example with $K = 3$



(X_i, Y_i, Z_i): twisted strings, localized in extra dimensions

- $\mathrm{SL}(2,\mathbb{Z})_T$ unbroken $\Rightarrow \Gamma_3' = T'$ finite modular symmetry (order 24)
- Traditional flavor symmetry (relevant for dark matter): $\Delta(27) \cup \mathbb{Z}_2 \Rightarrow$ $\Delta(54)$ R-symmetry

Dark matter from strings: Mütter, P.V. 1912.09909

- Standard $\mathbb{Z}_9$ R-symmetry (for six-dim. orbifolds)
- Eclectic flavor group:

$$\Omega(2) \;\cong\; T' \cup \Delta(54) \cup \mathbb{Z}_9$$

group of order $(24 \times 54 \times 9)/(2 \times 3) = 1944$

Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Nilles, Ramos-Sánchez, P.V. 2001.01736, 2004.05200 & 2006.03059

- Eclectic flavor symmetry highly predictive:

Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Nilles, Ramos-Sánchez, P.V. 2001.01736, 2004.05200 & 2006.03059

- Eclectic flavor symmetry highly predictive:
  Kähler and superpotential for (three copies of) localized strings (X_i, Y_i, Z_i) $(i \in \{1, 2, 3\})$

Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Nilles, Ramos-Sánchez, P.V. 2001.01736, 2004.05200 & 2006.03059

- Eclectic flavor symmetry highly predictive:
  Kähler and superpotential for (three copies of) localized strings (X_i, Y_i, Z_i) $(i \in \{1,2,3\})$

$$K(T, X_i, Y_i, Z_i)) \supset (-\mathrm{i}\,T + \mathrm{i}\,\bar{T})^{-2/3} \sum_{i=1}^{3} \left(|X_i|^2 + |Y_i|^2 + |Z_i|^2 \right)$$

  only diagonal terms, and

Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Nilles, Ramos-Sánchez, P.V. 2001.01736, 2004.05200 & 2006.03059

- Eclectic flavor symmetry highly predictive:
  Kähler and superpotential for (three copies of) localized strings (X_i, Y_i, Z_i) $(i \in \{1,2,3\})$

$$K(T, X_i, Y_i, Z_i)) \supset (-\mathrm{i}\,T + \mathrm{i}\,\bar{T})^{-2/3} \sum_{i=1}^{3} \left(|X_i|^2 + |Y_i|^2 + |Z_i|^2\right)$$

only diagonal terms, and

$$\begin{aligned}\mathcal{W}(T, X_i, Y_i, Z_i) \quad \supset \quad c^{(1)} \Big[& \hat{Y}_2(T) \big(X_1\, X_2\, X_3 + Y_1\, Y_2\, Y_3 + Z_1\, Z_2\, Z_3\big) \\ & - \frac{\hat{Y}_1(T)}{\sqrt{2}} \big(X_1\, Y_2\, Z_3 + X_1\, Y_3\, Z_2 + X_2\, Y_1\, Z_3 \\ & + X_3\, Y_1\, Z_2 + X_2\, Y_3\, Z_1 + X_3\, Y_2\, Z_1\big)\Big]\end{aligned}$$

$c^{(1)}$: free parameter

Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Nilles, Ramos-Sánchez, P.V. 2001.01736, 2004.05200 & 2006.03059

- Eclectic flavor symmetry highly predictive:
  Kähler and superpotential for (three copies of) localized strings (X_i, Y_i, Z_i) $(i \in \{1,2,3\})$

$$K(T, X_i, Y_i, Z_i)) \supset (-\mathrm{i}\,T + \mathrm{i}\,\bar{T})^{-2/3} \sum_{i=1}^{3} \left(|X_i|^2 + |Y_i|^2 + |Z_i|^2\right)$$

only diagonal terms, and

$$\begin{aligned}\mathcal{W}(T, X_i, Y_i, Z_i) \quad \supset \quad c^{(1)} \Big[\hat{Y}_2(T)\big(X_1\,X_2\,X_3 + Y_1\,Y_2\,Y_3 + Z_1\,Z_2\,Z_3\big) \\ -\frac{\hat{Y}_1(T)}{\sqrt{2}}\big(X_1\,Y_2\,Z_3 + X_1\,Y_3\,Z_2 + X_2\,Y_1\,Z_3 \\ + X_3\,Y_1\,Z_2 + X_2\,Y_3\,Z_1 + X_3\,Y_2\,Z_1\big)\Big]\end{aligned}$$

$c^{(1)}$: free parameter

- with Yukawa coupling $\hat{Y}^{(1)}_{2''}(T)$: rep. $2''$ of T' and

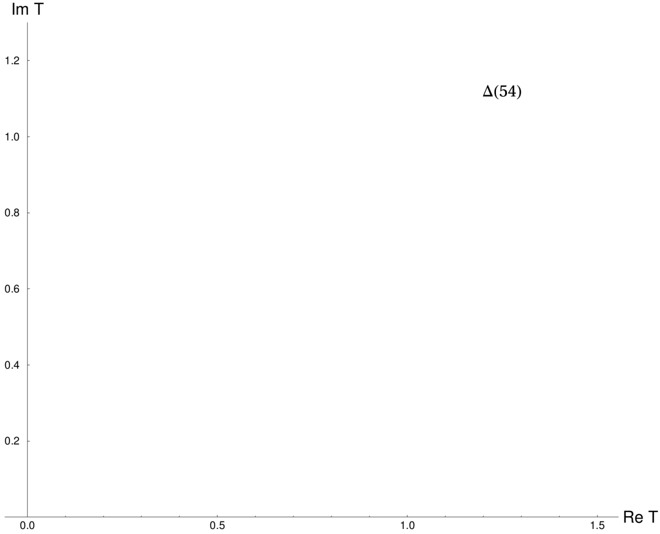
$$\hat{Y}^{(1)}_{2''}(T) \;:=\; \begin{pmatrix}\hat{Y}_1(T)\\ \hat{Y}_2(T)\end{pmatrix} \;:=\; \begin{pmatrix} -3\sqrt{2}\,\frac{\eta^3(3\,T)}{\eta(T)} \\ 3\,\frac{\eta^3(3\,T)}{\eta(T)} + \frac{\eta^3(T/3)}{\eta(T)}\end{pmatrix}$$

$\eta(T)$: Dedekind eta function

Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Baur, Nilles, Trautner, P.V. 1901.03251 & 1908.00805

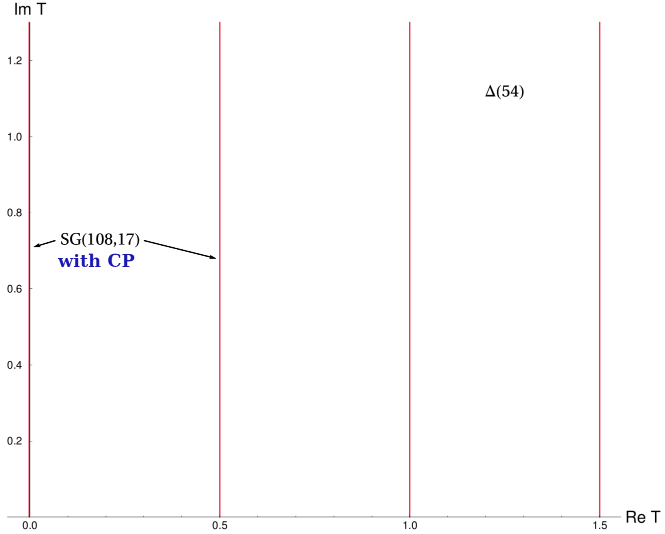
Move in T moduli-space:



Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Baur, Nilles, Trautner, P.V. 1901.03251 & 1908.00805

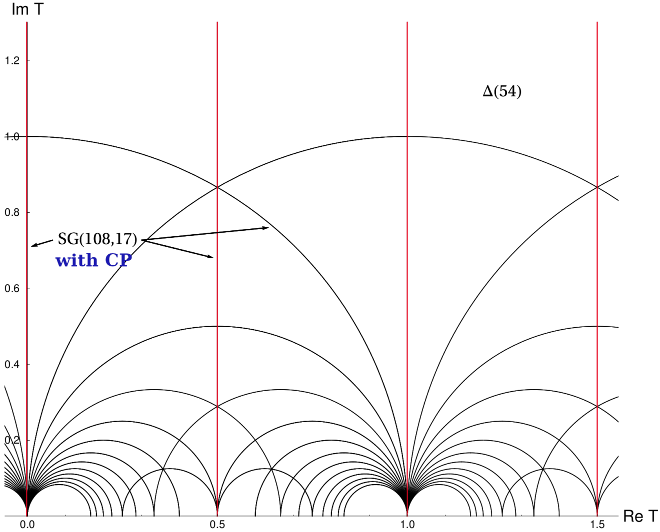
Move in T moduli-space:



Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Baur, Nilles, Trautner, P.V. 1901.03251 & 1908.00805

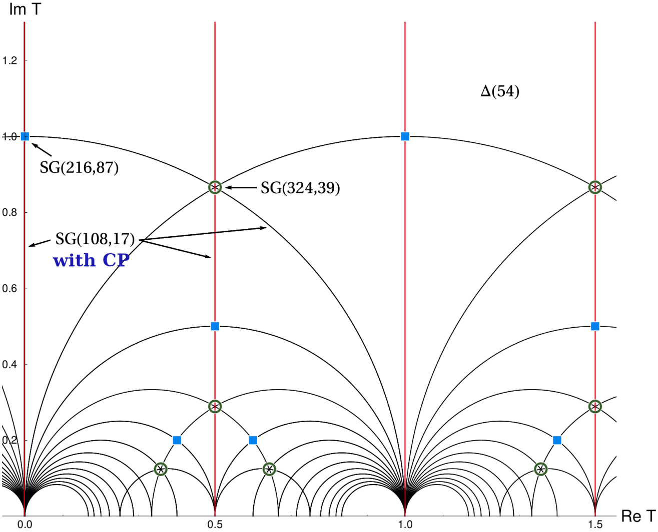
Move in T moduli-space:



Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Baur, Nilles, Trautner, P.V. 1901.03251 & 1908.00805

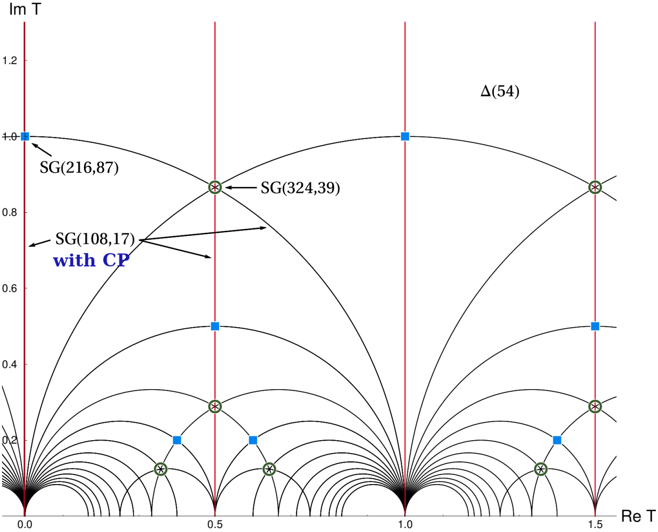
Move in T moduli-space:



Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Baur, Nilles, Trautner, P.V. 1901.03251 & 1908.00805

Move in T moduli-space: spontaneous $\mathcal{CP}$ breaking



Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Nilles, Ramos-Sánchez, P.V. 2010.13798

Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Nilles, Ramos-Sánchez, P.V. 2010.13798

- At $T = \omega := \exp\left(^{2\pi \mathrm{i}}/_{3}\right)$: "accidental" $\mathrm{U}(1) \times \mathrm{U}(1)$ symmetry!

Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Nilles, Ramos-Sánchez, P.V. 2010.13798

- At $T = \omega := \exp\left({}^{2\pi\mathrm{i}}/_{3}\right)$: "accidental" $\mathrm{U}(1) \times \mathrm{U}(1)$ symmetry!
- Perform basis change in field space

$$\begin{pmatrix} X_i^{\mathrm{g}} \\ Y_i^{\mathrm{g}} \\ Z_i^{\mathrm{g}} \end{pmatrix} := M_{\mathrm{B}} \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} \quad \text{where} \quad M_{\mathrm{B}} := \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega^2 & \omega^2 \\ 1 & 1 & \omega \\ 1 & \omega & 1 \end{pmatrix}$$

Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Nilles, Ramos-Sánchez, P.V. 2010.13798

- At $T = \omega := \exp\left({}^{2\pi\mathrm{i}}/{}_3\right)$: "accidental" $\mathrm{U}(1) \times \mathrm{U}(1)$ symmetry!
- Perform basis change in field space

$$\begin{pmatrix} X_i^{\mathrm{g}} \\ Y_i^{\mathrm{g}} \\ Z_i^{\mathrm{g}} \end{pmatrix} := M_{\mathrm{B}} \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} \quad \text{where} \quad M_{\mathrm{B}} := \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega^2 & \omega^2 \\ 1 & 1 & \omega \\ 1 & \omega & 1 \end{pmatrix}$$

- Superpotential in new basis:

$$\mathcal{W}(T, X_i^{\mathrm{g}}, Y_i^{\mathrm{g}}, Z_i^{\mathrm{g}}) = \frac{c^{(1)}}{\sqrt{3}} \Big[\left(\hat{Y}_2(T) - \sqrt{2}\,\omega^2\, \hat{Y}_1(T) \right) \left(X_1^{\mathrm{g}} X_2^{\mathrm{g}} X_3^{\mathrm{g}} + Y_1^{\mathrm{g}} Y_2^{\mathrm{g}} Y_3^{\mathrm{g}} + Z_1^{\mathrm{g}} Z_2^{\mathrm{g}} Z_3^{\mathrm{g}} \right)$$

$$+ \left(\hat{Y}_2(T) + \frac{\omega^2}{\sqrt{2}}\, \hat{Y}_1(T) \right) \left(X_1^{\mathrm{g}}(Y_2^{\mathrm{g}} Z_3^{\mathrm{g}} + Y_3^{\mathrm{g}} Z_2^{\mathrm{g}}) + X_2^{\mathrm{g}}(Y_1^{\mathrm{g}} Z_3^{\mathrm{g}} + Y_3^{\mathrm{g}} Z_1^{\mathrm{g}}) + X_3^{\mathrm{g}}(Y_1^{\mathrm{g}} Z_2^{\mathrm{g}} + Y_2^{\mathrm{g}} Z_1^{\mathrm{g}}) \right) \Big]$$

Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Nilles, Ramos-Sánchez, P.V. 2010.13798

- At $T = \omega := \exp\left({}^{2\pi i}/_3\right)$: "accidental" $\mathrm{U}(1) \times \mathrm{U}(1)$ symmetry!
- Perform basis change in field space

$$\begin{pmatrix} X_i^{\mathrm{g}} \\ Y_i^{\mathrm{g}} \\ Z_i^{\mathrm{g}} \end{pmatrix} \;:=\; M_{\mathrm{B}} \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} \quad \text{where} \quad M_{\mathrm{B}} \;:=\; \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega^2 & \omega^2 \\ 1 & 1 & \omega \\ 1 & \omega & 1 \end{pmatrix}$$

- Superpotential in new basis:

$$\mathcal{W}(T, X_i^{\mathrm{g}}, Y_i^{\mathrm{g}}, Z_i^{\mathrm{g}}) \;=\; \frac{c^{(1)}}{\sqrt{3}} \Big[\left(\hat{Y}_2(T) - \sqrt{2}\,\omega^2\, \hat{Y}_1(T) \right) \left(X_1^{\mathrm{g}} X_2^{\mathrm{g}} X_3^{\mathrm{g}} + Y_1^{\mathrm{g}} Y_2^{\mathrm{g}} Y_3^{\mathrm{g}} + Z_1^{\mathrm{g}} Z_2^{\mathrm{g}} Z_3^{\mathrm{g}} \right)$$
$$+ \left(\hat{Y}_2(T) + \frac{\omega^2}{\sqrt{2}}\, \hat{Y}_1(T) \right) \left(X_1^{\mathrm{g}}(Y_2^{\mathrm{g}} Z_3^{\mathrm{g}} + Y_3^{\mathrm{g}} Z_2^{\mathrm{g}}) + X_2^{\mathrm{g}}(Y_1^{\mathrm{g}} Z_3^{\mathrm{g}} + Y_3^{\mathrm{g}} Z_1^{\mathrm{g}}) + X_3^{\mathrm{g}}(Y_1^{\mathrm{g}} Z_2^{\mathrm{g}} + Y_2^{\mathrm{g}} Z_1^{\mathrm{g}}) \right) \Big]$$

- At $T = \omega$

$$\hat{Y}_1(\omega) \;=\; \frac{\omega}{\sqrt{2}}\, \hat{Y}_2(\omega)$$

Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Nilles, Ramos-Sánchez, P.V. 2010.13798

- At $T = \omega := \exp\left({}^{2\pi\mathrm{i}}/_{3}\right)$: "accidental" $\mathrm{U}(1) \times \mathrm{U}(1)$ symmetry!
- Perform basis change in field space

$$\begin{pmatrix} X_i^{\mathrm{g}} \\ Y_i^{\mathrm{g}} \\ Z_i^{\mathrm{g}} \end{pmatrix} := M_{\mathrm{B}} \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} \quad \text{where} \quad M_{\mathrm{B}} := \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega^2 & \omega^2 \\ 1 & 1 & \omega \\ 1 & \omega & 1 \end{pmatrix}$$

- Superpotential in new basis:

$$\mathcal{W}(T, X_i^{\mathrm{g}}, Y_i^{\mathrm{g}}, Z_i^{\mathrm{g}}) = \frac{c^{(1)}}{\sqrt{3}} \Big[\left(\hat{Y}_2(T) - \sqrt{2}\,\omega^2\, \hat{Y}_1(T)\right) \left(X_1^{\mathrm{g}} X_2^{\mathrm{g}} X_3^{\mathrm{g}} + Y_1^{\mathrm{g}} Y_2^{\mathrm{g}} Y_3^{\mathrm{g}} + Z_1^{\mathrm{g}} Z_2^{\mathrm{g}} Z_3^{\mathrm{g}}\right)$$
$$+ \left(\hat{Y}_2(T) + \frac{\omega^2}{\sqrt{2}}\, \hat{Y}_1(T)\right) \left(X_1^{\mathrm{g}}(Y_2^{\mathrm{g}} Z_3^{\mathrm{g}} + Y_3^{\mathrm{g}} Z_2^{\mathrm{g}}) + X_2^{\mathrm{g}}(Y_1^{\mathrm{g}} Z_3^{\mathrm{g}} + Y_3^{\mathrm{g}} Z_1^{\mathrm{g}}) + X_3^{\mathrm{g}}(Y_1^{\mathrm{g}} Z_2^{\mathrm{g}} + Y_2^{\mathrm{g}} Z_1^{\mathrm{g}})\right) \Big]$$

- At $T = \omega$

$$\hat{Y}_1(\omega) = \frac{\omega}{\sqrt{2}}\, \hat{Y}_2(\omega)$$

- Hence,

$$\mathcal{W}(\omega, X_i^{\mathrm{g}}, Y_i^{\mathrm{g}}, Z_i^{\mathrm{g}}) = c \left(X_1^{\mathrm{g}}(Y_2^{\mathrm{g}} Z_3^{\mathrm{g}} + Y_3^{\mathrm{g}} Z_2^{\mathrm{g}}) + X_2^{\mathrm{g}}(Y_1^{\mathrm{g}} Z_3^{\mathrm{g}} + Y_3^{\mathrm{g}} Z_1^{\mathrm{g}}) + X_3^{\mathrm{g}}(Y_1^{\mathrm{g}} Z_2^{\mathrm{g}} + Y_2^{\mathrm{g}} Z_1^{\mathrm{g}}) \right)$$

Eclectic flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Nilles, Ramos-Sánchez, P.V. 2010.13798

- At $T = \omega := \exp(2\pi\mathrm{i}/3)$: "accidental" U(1) × U(1) symmetry!
- Perform basis change in field space

$$\begin{pmatrix} X_i^{\mathrm{g}} \\ Y_i^{\mathrm{g}} \\ Z_i^{\mathrm{g}} \end{pmatrix} := M_{\mathrm{B}} \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} \quad \text{where} \quad M_{\mathrm{B}} := \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega^2 & \omega^2 \\ 1 & 1 & \omega \\ 1 & \omega & 1 \end{pmatrix}$$

- Superpotential in new basis:

$$\mathcal{W}(T, X_i^{\mathrm{g}}, Y_i^{\mathrm{g}}, Z_i^{\mathrm{g}}) = \frac{c^{(1)}}{\sqrt{3}} \Big[\left(\hat{Y}_2(T) - \sqrt{2}\,\omega^2\, \hat{Y}_1(T) \right) \left(X_1^{\mathrm{g}} X_2^{\mathrm{g}} X_3^{\mathrm{g}} + Y_1^{\mathrm{g}} Y_2^{\mathrm{g}} Y_3^{\mathrm{g}} + Z_1^{\mathrm{g}} Z_2^{\mathrm{g}} Z_3^{\mathrm{g}} \right)$$
$$+ \left(\hat{Y}_2(T) + \frac{\omega^2}{\sqrt{2}}\, \hat{Y}_1(T) \right) \left(X_1^{\mathrm{g}}(Y_2^{\mathrm{g}} Z_3^{\mathrm{g}} + Y_3^{\mathrm{g}} Z_2^{\mathrm{g}}) + X_2^{\mathrm{g}}(Y_1^{\mathrm{g}} Z_3^{\mathrm{g}} + Y_3^{\mathrm{g}} Z_1^{\mathrm{g}}) + X_3^{\mathrm{g}}(Y_1^{\mathrm{g}} Z_2^{\mathrm{g}} + Y_2^{\mathrm{g}} Z_1^{\mathrm{g}}) \right) \Big]$$

- At $T = \omega$

$$\hat{Y}_1(\omega) = \frac{\omega}{\sqrt{2}}\, \hat{Y}_2(\omega)$$

- Hence,

$$\mathcal{W}(\omega, X_i^{\mathrm{g}}, Y_i^{\mathrm{g}}, Z_i^{\mathrm{g}}) = c \left(X_1^{\mathrm{g}}(Y_2^{\mathrm{g}} Z_3^{\mathrm{g}} + Y_3^{\mathrm{g}} Z_2^{\mathrm{g}}) + X_2^{\mathrm{g}}(Y_1^{\mathrm{g}} Z_3^{\mathrm{g}} + Y_3^{\mathrm{g}} Z_1^{\mathrm{g}}) + X_3^{\mathrm{g}}(Y_1^{\mathrm{g}} Z_2^{\mathrm{g}} + Y_2^{\mathrm{g}} Z_1^{\mathrm{g}}) \right)$$

invariant under U(1) × U(1) with charges $(1, 1, -2)$ and $(0, 1, -1)$

Summary

Summary

- Big data & machine learning to analyze the string landscape & to find *the* MSSM

Summary

- Big data & machine learning to analyze the string landscape & to find *the* MSSM
- (Finite) modular symmetries to address the flavor puzzle

Summary

- Big data & machine learning to analyze the string landscape & to find *the* MSSM
- (Finite) modular symmetries to address the flavor puzzle
- Origin of symmetries from string theory ⇒ eclectic flavor symmetry

More to come!

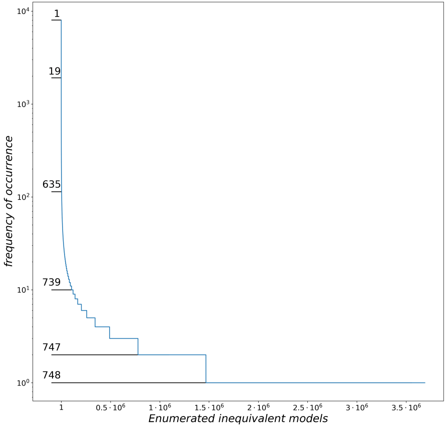
More to come!

Thank you for your attention!

Backup: Contrast patterns: why?

Parr, P.V. 1910.13473

Logarithmic plot of the frequency of occurrence of inequivalent $\mathbb{Z}_6$-II orbifold models:



Horizontal axis: the inequivalent models are enumerated from 1 to 3 690 513
Vertical axis: the corresponding frequency of occurrence, i.e. model # 1 has a frequency of 8 008

Backup: Flavor in lepton sector

Backup: Flavor in lepton sector

- "Are neutrino masses modular forms?" (1706.08749, F. Feruglio)

Backup: Flavor in lepton sector

- "Are neutrino masses modular forms?" (1706.08749, F. Feruglio)
- Flavor in lepton sector:

Backup: Flavor in lepton sector

- "Are neutrino masses modular forms?" (1706.08749, F. Feruglio)
- Flavor in lepton sector:
  - 3 neutrino masses: m_i for $i = 1, 2, 3$

Backup: Flavor in lepton sector

- "Are neutrino masses modular forms?" (1706.08749, F. Feruglio)
- Flavor in lepton sector:
  - 3 neutrino masses: m_i for $i = 1, 2, 3$
    define:

$$\delta m^2 = m_2^2 - m_1^2 > 0 \qquad \text{and} \qquad \Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$$

Backup: Flavor in lepton sector

- "Are neutrino masses modular forms?" (1706.08749, F. Feruglio)
- Flavor in lepton sector:
  - 3 neutrino masses: m_i for $i = 1, 2, 3$
    define:

$$\delta m^2 = m_2^2 - m_1^2 > 0 \qquad \text{and} \qquad \Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$$

    then:
    - $\Delta m^2 > 0$ for normal ordering (NO) and
    - $\Delta m^2 < 0$ for inverted ordering (IO)

Backup: Flavor in lepton sector

- "Are neutrino masses modular forms?" (1706.08749, F. Feruglio)
- Flavor in lepton sector:
  - 3 neutrino masses: m_i for $i = 1, 2, 3$
    define:

    $$\delta m^2 = m_2^2 - m_1^2 > 0 \qquad \text{and} \qquad \Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$$

    then:
    - $\Delta m^2 > 0$ for normal ordering (NO) and
    - $\Delta m^2 < 0$ for inverted ordering (IO)
  - 3 charged lepton masses: m_e, m_μ, m_τ

Backup: Flavor in lepton sector

- "Are neutrino masses modular forms?" (1706.08749, F. Feruglio)
- Flavor in lepton sector:
  - 3 neutrino masses: m_i for $i = 1, 2, 3$
    define:
    $$\delta m^2 = m_2^2 - m_1^2 > 0 \qquad \text{and} \qquad \Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$$
    then:
    - $\Delta m^2 > 0$ for normal ordering (NO) and
    - $\Delta m^2 < 0$ for inverted ordering (IO)
  - 3 charged lepton masses: m_e, m_μ, m_τ
  - 3 mixing angles: θ_{12}, θ_{13} and θ_{23}

Backup: Flavor in lepton sector

- "Are neutrino masses modular forms?" (1706.08749, F. Feruglio)
- Flavor in lepton sector:
  - 3 neutrino masses: m_i for $i = 1, 2, 3$
    define:

    $$\delta m^2 = m_2^2 - m_1^2 > 0 \qquad \text{and} \qquad \Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$$

    then:
    - $\Delta m^2 > 0$ for normal ordering (NO) and
    - $\Delta m^2 < 0$ for inverted ordering (IO)
  - 3 charged lepton masses: m_e, m_μ, m_τ
  - 3 mixing angles: θ_{12}, θ_{13} and θ_{23}
  - 1 CP-phase: δ (plus two extra phases in the case of Majorana neutrinos)

Backup: Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

Backup: Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

- Quarks, leptons, Higgs Φ_i transform in (higher-dim.) representations $\boldsymbol{r}_i$ of (non-Abelian) finite modular group Γ_N (or Γ'_N) with weight n_i:

Backup: Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

- Quarks, leptons, Higgs Φ_i transform in (higher-dim.) representations $\boldsymbol{r}_i$ of (non-Abelian) finite modular group Γ_N (or Γ'_N) with weight n_i:

$$\Phi_i \stackrel{\gamma}{\to} \rho_{\boldsymbol{r}_i}(\gamma)\,\Phi_i \quad \text{for} \quad \gamma \in \mathrm{SL}(2,\mathbb{Z})$$

Backup: Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

- Quarks, leptons, Higgs Φ_i transform in (higher-dim.) representations $\boldsymbol{r}_i$ of (non-Abelian) finite modular group Γ_N (or Γ'_N) with weight n_i:

$$\Phi_i \stackrel{\gamma}{\rightarrow} (c\,T + d)^{n_i}\,\rho_{\boldsymbol{r}_i}(\gamma)\,\Phi_i \qquad \text{for} \qquad \gamma \in \mathrm{SL}(2,\mathbb{Z})$$

Backup: Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

- Quarks, leptons, Higgs Φ_i transform in (higher-dim.) representations $\boldsymbol{r}_i$ of (non-Abelian) finite modular group Γ_N (or Γ'_N) with weight n_i:

$$\Phi_i \;\stackrel{\gamma}{\to}\; (c\,T + d)^{n_i}\,\rho_{\boldsymbol{r}_i}(\gamma)\,\Phi_i \qquad \text{for} \qquad \gamma \in \mathrm{SL}(2,\mathbb{Z})$$

- Superpotential $\mathcal{W}$:

$$\mathcal{W} \;\supset\; Y^{(n_Y)}_{\boldsymbol{r_Y}}(T)\,\Phi_1 \ldots \Phi_k$$

Backup: Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

- Quarks, leptons, Higgs Φ_i transform in (higher-dim.) representations $\boldsymbol{r}_i$ of (non-Abelian) finite modular group Γ_N (or Γ'_N) with weight n_i:

$$\Phi_i \;\stackrel{\gamma}{\to}\; (c\,T + d)^{n_i}\,\rho_{\boldsymbol{r}_i}(\gamma)\,\Phi_i \qquad \text{for} \qquad \gamma \in \mathrm{SL}(2,\mathbb{Z})$$

- Superpotential $\mathcal{W}$:

$$\mathcal{W} \;\supset\; Y^{(n_Y)}_{\boldsymbol{r_Y}}(T)\,\Phi_1\ldots\Phi_k$$

if: $\quad \boldsymbol{r}_Y \otimes \boldsymbol{r}_1 \otimes \ldots \otimes \boldsymbol{r}_k \;\supset\; 1 \quad$ and $\quad n_y + \sum_{i=1}^{k} n_i \;=\; -1$

Backup: Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

- Quarks, leptons, Higgs Φ_i transform in (higher-dim.) representations $\boldsymbol{r}_i$ of (non-Abelian) finite modular group Γ_N (or Γ'_N) with weight n_i:

$$\Phi_i \xrightarrow{\gamma} (c\,T+d)^{n_i}\,\rho_{\boldsymbol{r}_i}(\gamma)\,\Phi_i \qquad \text{for} \qquad \gamma \in \mathrm{SL}(2,\mathbb{Z})$$

- Superpotential $\mathcal{W}$:

$$\mathcal{W} \supset Y_{\boldsymbol{r_Y}}^{(n_Y)}(T)\,\Phi_1\ldots\Phi_k$$

if: $\boldsymbol{r}_Y \otimes \boldsymbol{r}_1 \otimes \ldots \otimes \boldsymbol{r}_k \supset 1$ and $n_y + \sum_{i=1}^{k} n_i = -1$

- Kähler potential K:

$$K \supset -\ln(-\mathrm{i}\,T+\mathrm{i}\,\bar{T}) + (-\mathrm{i}\,T+\mathrm{i}\,\bar{T})^{n_i}|\Phi_i|^2$$

Backup: Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

- Quarks, leptons, Higgs Φ_i transform in (higher-dim.) representations $\boldsymbol{r}_i$ of (non-Abelian) finite modular group Γ_N (or Γ'_N) with weight n_i:

$$\Phi_i \stackrel{\gamma}{\rightarrow} (c\,T+d)^{n_i}\,\rho_{\boldsymbol{r}_i}(\gamma)\,\Phi_i \qquad \text{for} \qquad \gamma\in \mathrm{SL}(2,\mathbb{Z})$$

- Superpotential $\mathcal{W}$:

$$\mathcal{W} \supset Y^{(n_Y)}_{\boldsymbol{r_Y}}(T)\,\Phi_1\ldots\Phi_k$$

if: $\quad \boldsymbol{r}_Y\otimes\boldsymbol{r}_1\otimes\ldots\otimes\boldsymbol{r}_k \supset 1 \quad$ and $\quad n_y+\sum_{i=1}^k n_i = -1$

- Kähler potential K:

$$K \supset -\ln(-\mathrm{i}\,T+\mathrm{i}\,\bar{T}) + (-\mathrm{i}\,T+\mathrm{i}\,\bar{T})^{n_i}|\Phi_i|^2$$

- Invariance under modular transformation $\gamma\in\mathrm{SL}(2,\mathbb{Z})$:

$$\mathcal{W} \stackrel{\gamma}{\rightarrow} (c\,T+d)^{-1}\,\mathcal{W} \stackrel{\text{Kähler}}{\rightarrow} \mathcal{W}$$

$$K \stackrel{\gamma}{\rightarrow} K+\ln(c\,T+d)+\ln(c\,T+d)^* \stackrel{\text{Kähler}}{\rightarrow} K$$

Backup: Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

- Quarks, leptons, Higgs Φ_i transform in (higher-dim.) representations $\boldsymbol{r}_i$ of (non-Abelian) finite modular group Γ_N (or Γ'_N) with weight n_i:

$$\Phi_i \;\xrightarrow{\gamma}\; (c\,T+d)^{n_i}\,\rho_{\boldsymbol{r}_i}(\gamma)\,\Phi_i \qquad \text{for} \qquad \gamma \in \mathrm{SL}(2,\mathbb{Z})$$

- Superpotential $\mathcal{W}$:

$$\mathcal{W} \;\supset\; Y^{(n_Y)}_{\boldsymbol{r_Y}}(T)\,\Phi_1\ldots\Phi_k$$

if: $\quad \boldsymbol{r}_Y \otimes \boldsymbol{r}_1 \otimes \ldots \otimes \boldsymbol{r}_k \;\supset\; 1 \quad$ and $\quad n_y + \sum_{i=1}^k n_i \;=\; -1$

- Kähler potential K:

$$K \;\supset\; -\ln(-\mathrm{i}\,T+\mathrm{i}\,\bar{T}) + (-\mathrm{i}\,T+\mathrm{i}\,\bar{T})^{n_i}|\Phi_i|^2$$

- Invariance under modular transformation $\gamma \in \mathrm{SL}(2,\mathbb{Z})$:

$$\mathcal{W} \;\xrightarrow{\gamma}\; (c\,T+d)^{-1}\,\mathcal{W} \;\xrightarrow{\text{Kähler}}\; \mathcal{W}$$

$$K \;\xrightarrow{\gamma}\; K + \ln(c\,T+d) + \ln(c\,T+d)^* \;\xrightarrow{\text{Kähler}}\; K$$

- Modular flavor symmetry broken spontaneously by vev of T:

$$Y^{(n_Y)}_{\boldsymbol{r_Y}}(\langle T\rangle) \;=\; \begin{pmatrix} v_1 \\ \vdots \\ v_d \end{pmatrix} \;\neq\; 0$$

Backup: Modular flavor symmetries

1706.08749, F. Feruglio + many follow-up papers

- Quarks, leptons, Higgs Φ_i transform in (higher-dim.) representations $\boldsymbol{r}_i$ of (non-Abelian) finite modular group Γ_N (or Γ'_N) with weight n_i:

$$\Phi_i \stackrel{\gamma}{\to} (c\,T+d)^{n_i}\,\rho_{\boldsymbol{r}_i}(\gamma)\,\Phi_i \qquad \text{for} \qquad \gamma \in \mathrm{SL}(2,\mathbb{Z})$$

- Superpotential $\mathcal{W}$:

$$\mathcal{W} \supset Y^{(n_Y)}_{\boldsymbol{r}_{\boldsymbol{Y}}}(T)\,\Phi_1\ldots\Phi_k$$

if: $\boldsymbol{r}_Y \otimes \boldsymbol{r}_1 \otimes \ldots \otimes \boldsymbol{r}_k \supset 1$ and $n_y + \sum_{i=1}^k n_i = -1$

- Kähler potential K:

$$K \supset -\ln(-\mathrm{i}T + \mathrm{i}\bar{T}) + (-\mathrm{i}T + \mathrm{i}\bar{T})^{n_i}|\Phi_i|^2$$

- Invariance under modular transformation $\gamma \in \mathrm{SL}(2,\mathbb{Z})$:

$$\mathcal{W} \stackrel{\gamma}{\to} (c\,T+d)^{-1}\,\mathcal{W} \stackrel{\text{Kähler}}{\to} \mathcal{W}$$

$$K \stackrel{\gamma}{\to} K + \ln(c\,T+d) + \ln(c\,T+d)^* \stackrel{\text{Kähler}}{\to} K$$

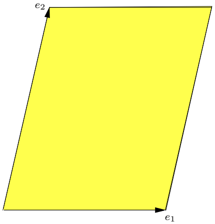
- Modular flavor symmetry broken spontaneously by vev of T:

$$Y^{(n_Y)}_{\boldsymbol{r}_{\boldsymbol{Y}}}(\langle T\rangle) = \begin{pmatrix} v_1 \\ \vdots \\ v_d \end{pmatrix} \neq 0$$

- Vev aligns in d-dim. flavor space "automatically" (but moduli stabilization!)

Backup: Modular flavor symmetries from string theory: $\mathbf{SL}(2,\mathbb{Z})_U$ acts geometrically on $U = e_2/e_1$

e_1 and e_2 span two-torus $\mathbb{T}^2$

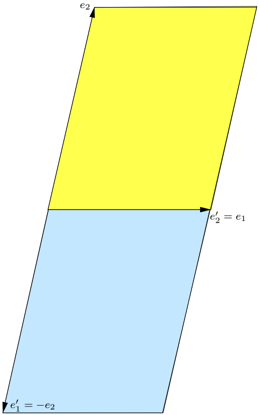


e_1

Backup: Modular flavor symmetries from string theory: $\mathbf{SL}(2,\mathbb{Z})_U$ acts geometrically on $U = e_2/e_1$

e_1 and e_2 span two-torus $\mathbb{T}^2$

modular S-transformation

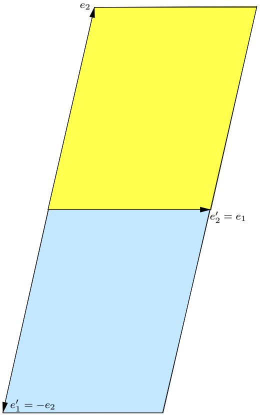


$$e_1 \xrightarrow{\mathrm{S}} e_1' = -e_2$$
$$e_2 \xrightarrow{\mathrm{S}} e_2' = e_1$$

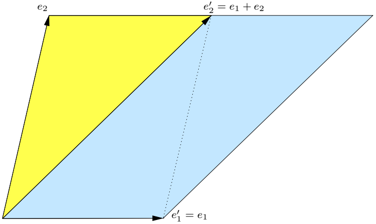
Backup: Modular flavor symmetries from string theory: $\mathbf{SL}(2,\mathbb{Z})_U$ acts geometrically on $U = e_2/e_1$

e_1 and e_2 span two-torus $\mathbb{T}^2$

modular S-transformation



modular T-transformation



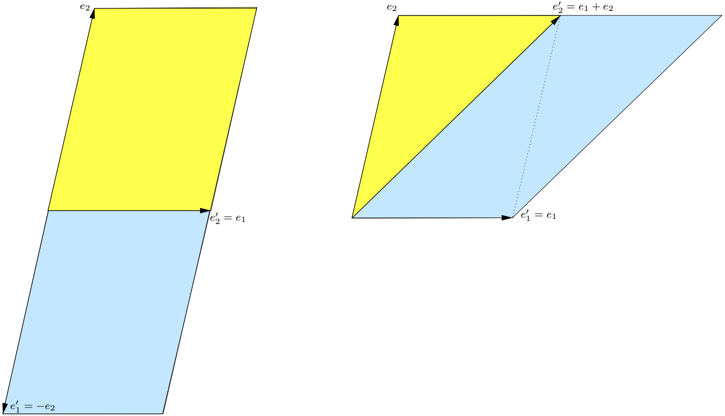
$$e_1 \xrightarrow{\mathrm{S}} e_1' = -e_2$$
$$e_2 \xrightarrow{\mathrm{S}} e_2' = e_1$$

$$e_1 \xrightarrow{\mathrm{T}} e_1' = e_1$$
$$e_2 \xrightarrow{\mathrm{T}} e_2' = e_1 + e_2$$

Backup: Modular flavor symmetries from string theory: $\mathbf{SL}(2,\mathbb{Z})_U$ acts geometrically on $U = e_2/e_1$

e_1 and e_2 span two-torus $\mathbb{T}^2$

modular S-transformation　　　　modular T-transformation



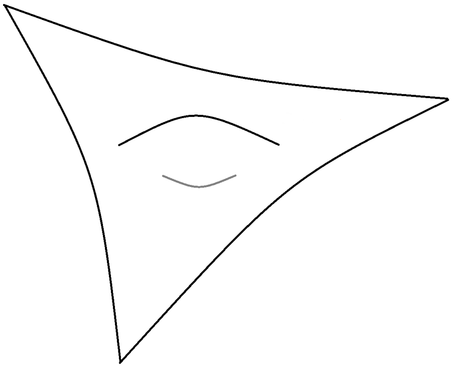
$$e_1 \xrightarrow{\mathrm{S}} e_1' = -e_2$$
$$e_2 \xrightarrow{\mathrm{S}} e_2' = e_1$$

$$e_1 \xrightarrow{\mathrm{T}} e_1' = e_1$$
$$e_2 \xrightarrow{\mathrm{T}} e_2' = e_1 + e_2$$

S and T generate $\mathsf{SL}(2,\mathbb{Z})_U$

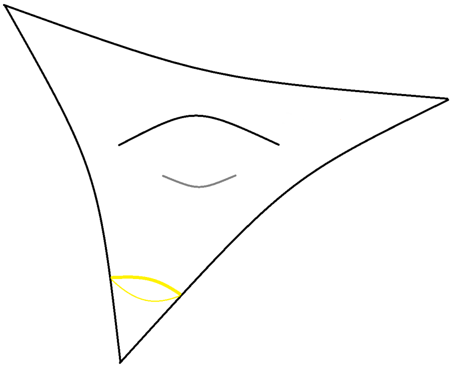
Backup: $\Delta(54)$ traditional flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

$$X_i\, Y_i\, Z_i \subset \mathcal{W}$$



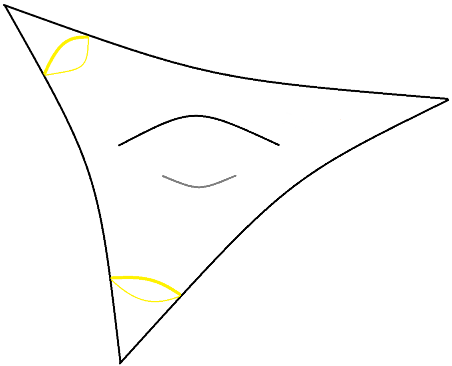
Backup: $\Delta(54)$ traditional flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

$$X_i\, Y_i\, Z_i \subset \mathcal{W}$$



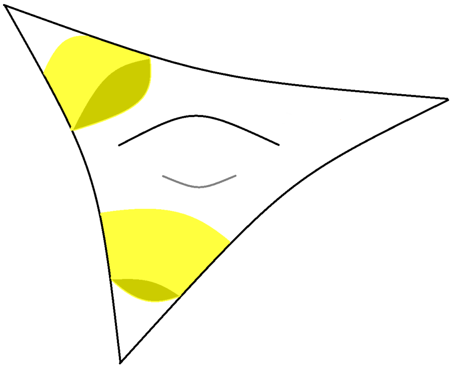
Backup: $\Delta(54)$ traditional flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

$$X_i\, Y_i\, Z_i \subset \mathcal{W}$$



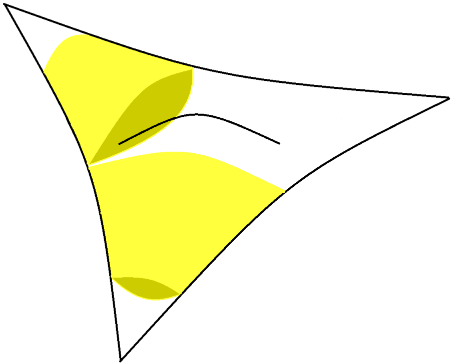
Backup: $\Delta(54)$ traditional flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

$$X_i \; Y_i \; Z_i \subset \mathcal{W}$$



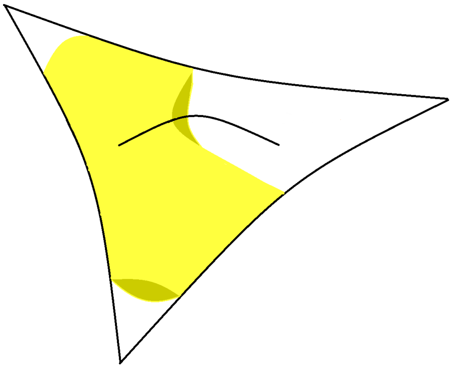
Backup: $\Delta(54)$ traditional flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

$$X_i\, Y_i\, Z_i \subset \mathcal{W}$$



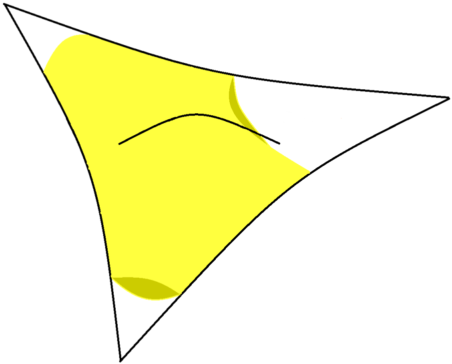
Backup: $\Delta(54)$ traditional flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

$$X_i\, Y_i\, Z_i \subset \mathcal{W}$$



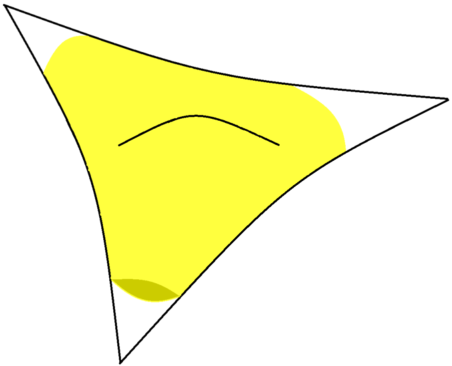
Backup: $\Delta(54)$ traditional flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

$$X_i \; Y_i \; Z_i \subset \mathcal{W}$$



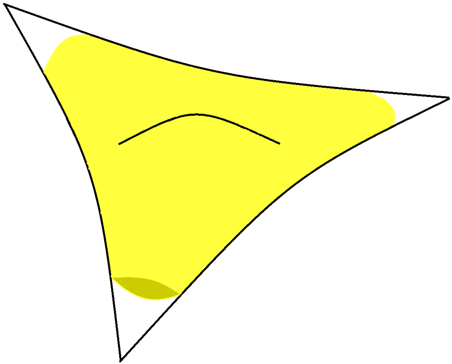
Backup: $\Delta(54)$ traditional flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

$$X_i\, Y_i\, Z_i \subset \mathcal{W}$$



Backup: $\Delta(54)$ traditional flavor symmetry from $\mathbb{T}^2/\mathbb{Z}_3$ orbifold

$$X_i\, Y_i\, Z_i \subset \mathcal{W}$$



Backup: String scale interacting dark matter from π_1

Mütter, P.V. 1912.09909

- Orbifold space group

Backup: String scale interacting dark matter from π_1

Mütter, P.V. 1912.09909

- Orbifold space group $\Rightarrow$ string selection rules realted to topology of orbifold (π_1)

Backup: String scale interacting dark matter from π_1

Mütter, P.V. 1912.09909

- Orbifold space group $\Rightarrow$ string selection rules realted to topology of orbifold (π_1)
- For certain $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold, we find

Application?
Dark matter from massive strings

Observation & main idea: orbifold with space group S

$\mathbb{Z}_4$ from space group selection rule with discrete charge: $n_1 + n_2 + n_3$

$g \in S$

Localization in extra-dimensions/winding

- massless strings: $n_1 + n_2 + n_3 \in \{0, 2\}$
- massive strings: $n_1 + n_2 + n_3 \in \{1, 3\}$

$$|\text{matter}\rangle \longmapsto +|\text{matter}\rangle$$
$$|\text{dark matter}\rangle \longmapsto -|\text{dark matter}\rangle$$

Dark matter strings stable:
they can only be produced and annihilated in pairs