Vacuum Selection in the Flux Landscape

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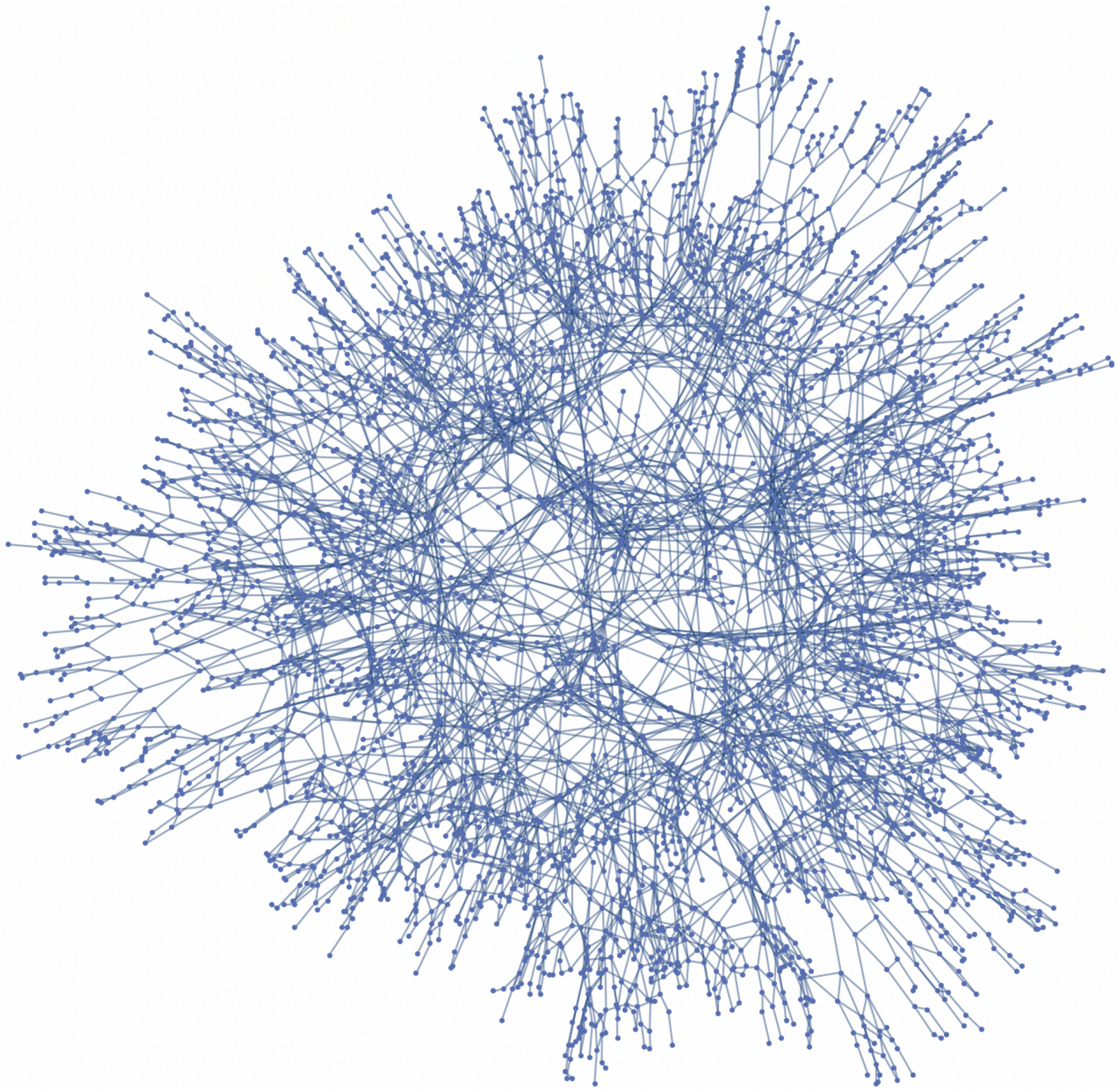
Based on work with Jim Halverson and Justin Khoury, to appear



string_data 2020

The Landscape is Big Data

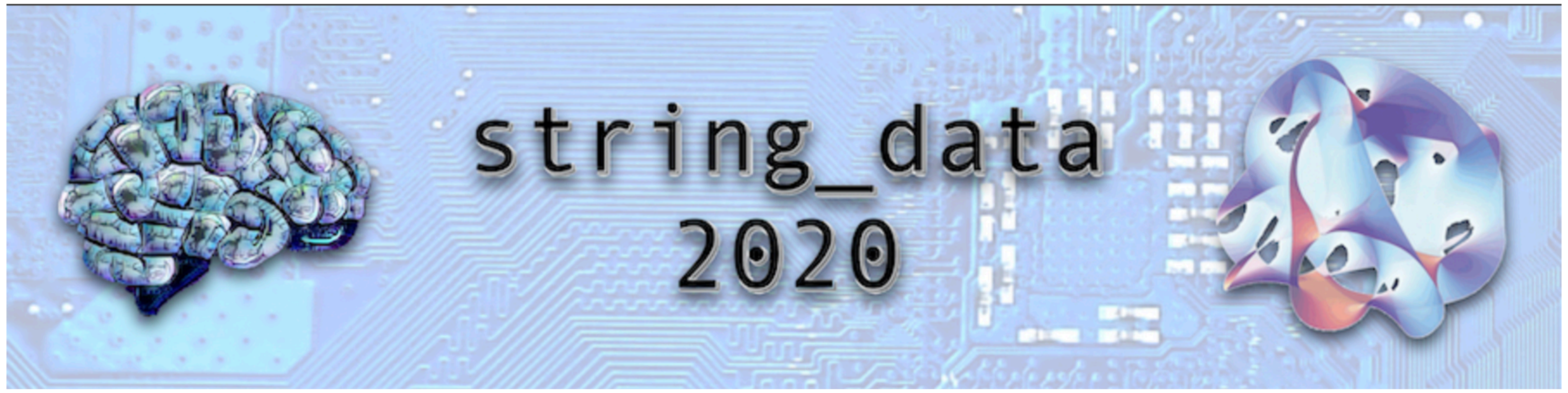
- Number of F-theory/IIb string vacua (effective theories from string theory):
  - number of geometries $\sim 10^{755} - 10^{3000}$ (extra-dimensional configurations)
    Halverson, CL, Sung Taylor, Wang
  - flux per geometry $\sim 10^{500} - 10^{272,000}$ (discrete data within a given geometry)
    Ashok, Denef, Douglas Taylor, Wang
- Vacua all connected in network of topological transitions. Each vacuum corresponds to a distinct physical theory.
- The number of theories is enormous, impossible to enumerate, and increasingly difficult to study as the number of scalar fields n increases, where computational complexity takes over. Halverson, Ruhle
- Physics involves vacua themselves, and the connections between the vacua.

Data Science and String Theory

- Viewing String Theory from a large data and statistical perspective is a <u>necessary</u> perspective, at least to make connections with realistic particle physics and cosmology.

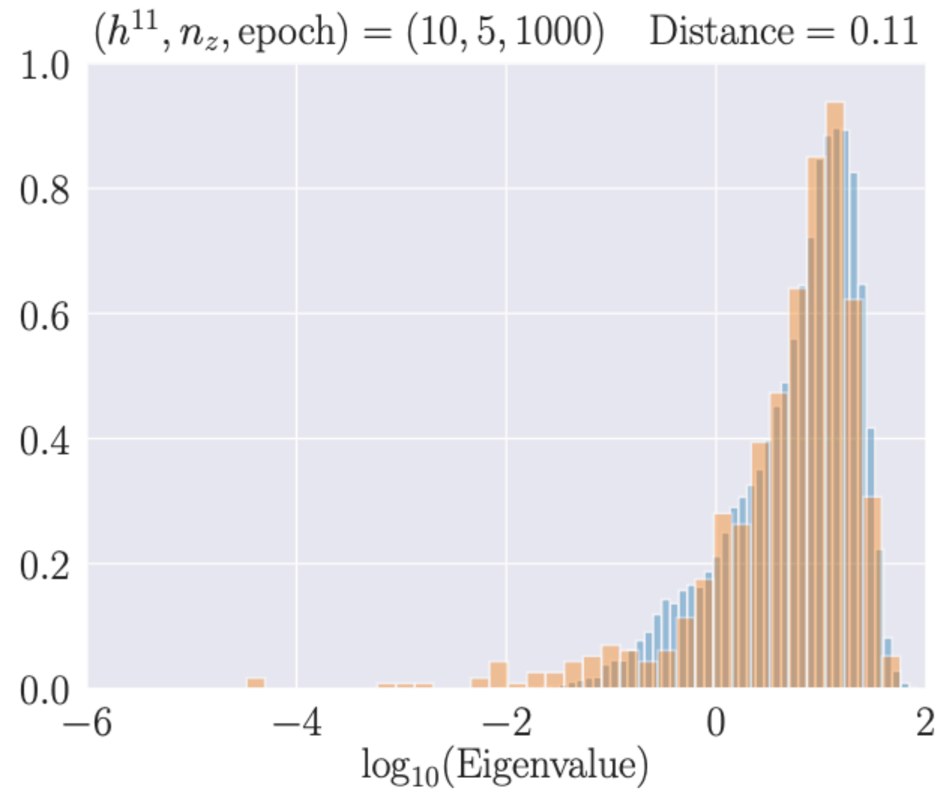
- Due to absolute enormity of the landscape, or set of string theory vacuum solutions.

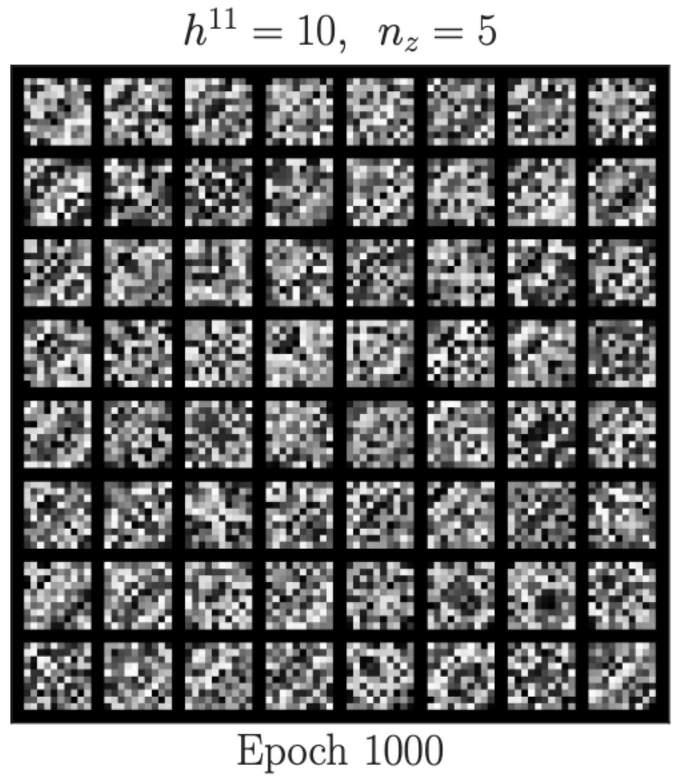
- Data science and computer science techniques (machine learning, complexity, network/graph theory, statistics) have come to the forefront, and have allowed for remarkable progress is many areas.



Progress via exploration and modeling

- At large n we cannot fully enumerate **BUT**
- Efficient construction of physical theories is improving with better sampling and computational techniques (see e.g. talk by McAllister).
- Modeling of vacua as a function of n has been improved by applying ML techniques. Example: Modeling the moduli space metric on Kähler moduli using a Wasserstein GAN constructing of a random matrix theory.





Halverson, CL

- Increasingly optimistic that data science techniques will allow us to further understand string theory vacua as sets, especially at large n.

Assigning importance

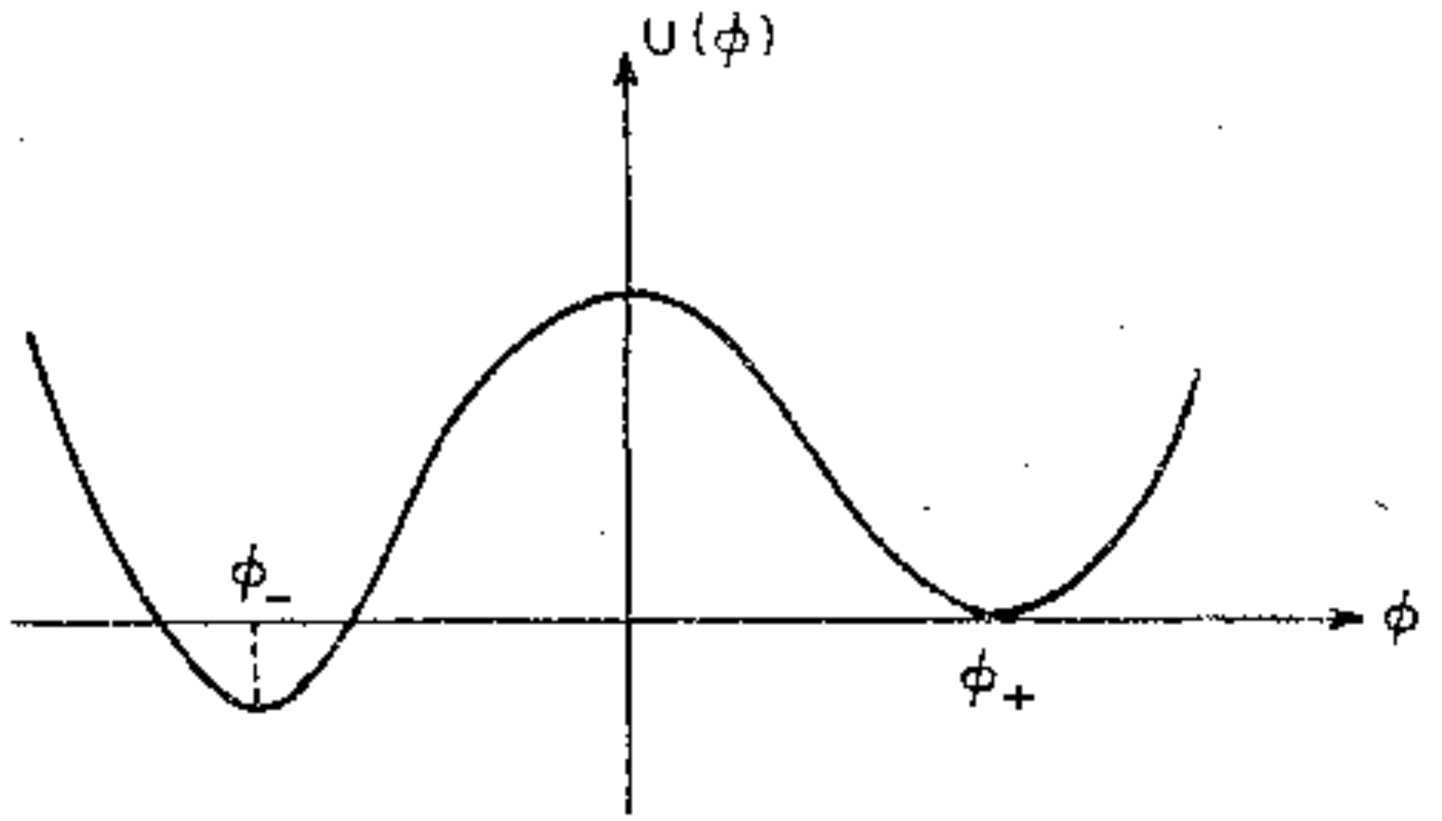
- However, the number of solutions consistent with our observed universe may still be enormous. How do we make predictions?
- We need a way of assigning importance to vacua, or assigning probability. This is known as prescribing a **measure**.
- Simple example: flat measure. Probability is given by simply counting. This measure would predict that we see features that show up the most often in the string landscape.

- However, the flat measure lacks physical motivation, in the sense that it is ignorant to how universes are produced (dynamics).
- Physically instead we should consider dynamical measures.

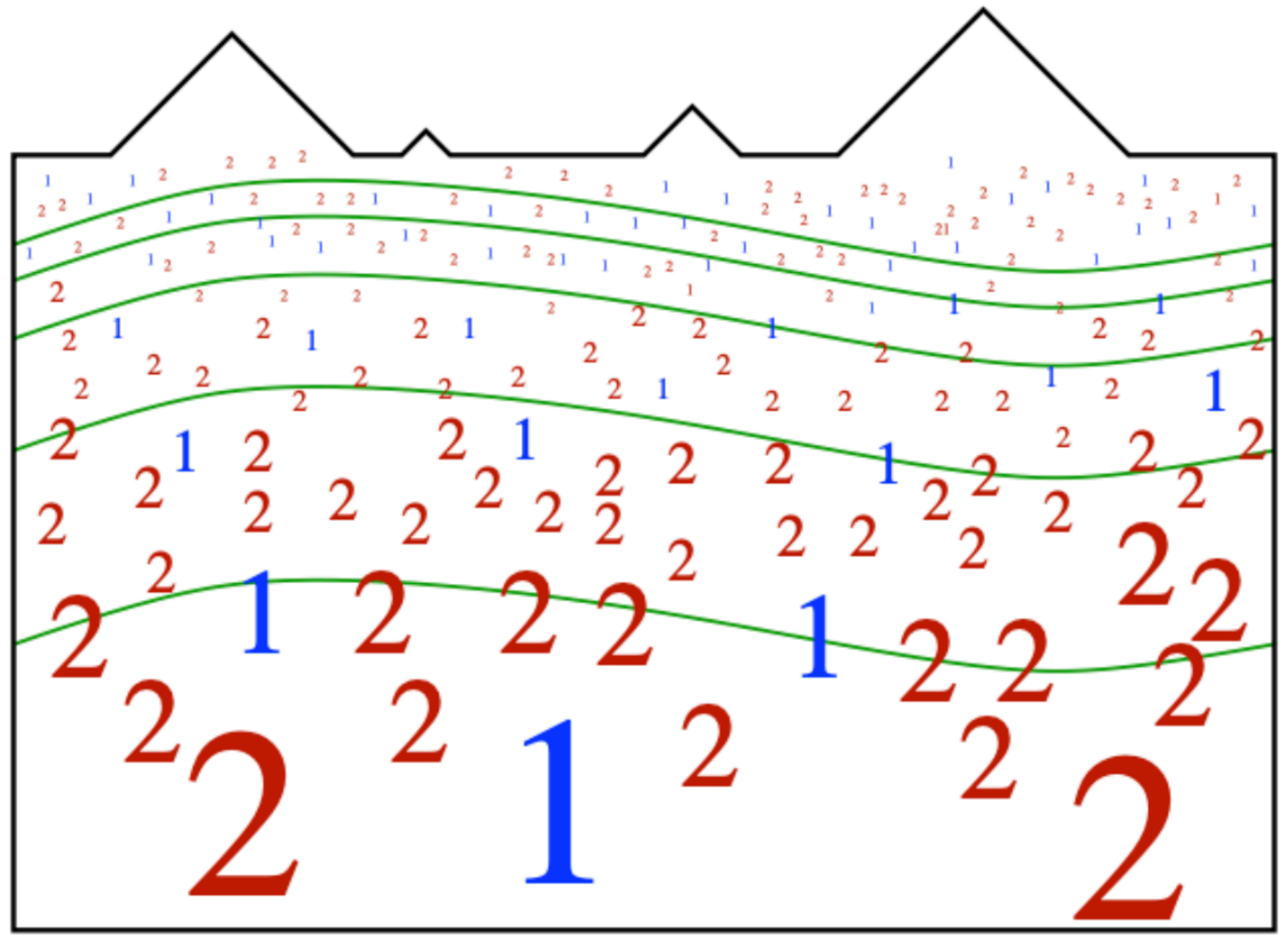
Dynamics

- In a landscape with multiple inflating/dS vacua, one starts in a universe corresponding to a single vacuum, but then bubbles of other vacua will form via tunneling (baby universes).

Coleman, de Luccia



- Bubbles with positive Λ will expand, and then nucleate other bubbles within. In the case that the expansion rates are faster than the nucleation rates (eternal inflation), this process will continue forever, producing an infinite number of bubbles.



- Bubbles with non-positive Λ are called terminal, and are thought to not produce any new bubbles themselves. Act as probability sinks from inflating vacua.

Measures

- Two clear ways to assign importance:

    1. At late times probability of a given vacuum is proportional to number of the bubbles in that vacuum.

    $$p_i \sim N_i \,.$$

    Garriga, Schwartz-Perlov, Vilenkin, Winitzki

    2. At early times, probability of a given vacuum is determined by how easy it is to produce it, or how accessible it is.

    Denef, Douglas, Greene, Zukowski

    Khoury

- Today's focus is on the first type of measure, but very interesting to consider the second type, which is work in progress.

Selection at late times

Garriga, Schwartz-Perlov, Vilenkin, Winitzki

Nucleation rate from i to j

- f^j: fraction of comoving volume in vacuum j.
- Dynamics governed by $\frac{\partial f^j}{\partial t} = M^j{}_i f^i$, where $M^j{}_i = \kappa_{j,i} - \sum_k \kappa_{k,i} \delta^j{}_i$
- Late time solution given by $\mathbf{f}(t) = \mathbf{f}^{(0)} + \mathbf{s} e^{-qt} + \ldots$,
- $p_j \propto N_j = \sum_\alpha H^q_\alpha \kappa_{j\alpha} s_\alpha$, independent of initial conditions.
- Terminal vacua act as probability sinks.

Downwards

Olum, Schwartz-Perlov

- We will consider the downward approximation, where one only has transitions from higher Λ to lower Λ. Probably not completely accurate in a landscape as complex as the string landscape, but a useful first step.
- Define $D_\alpha = \sum_{\Lambda_i < \Lambda_\alpha} \kappa_{i\alpha}$, which is the total downward tunneling rate out of vacuum α.
- **Dominant vacuum** ★ given by inflating vacuum with smallest D_α . ★ has highest probability.
- Probability of other inflating vacua given by relative strength of upward transitions from ★ :

$$p_\alpha = \sum \frac{\kappa_{\alpha a}\kappa_{ab}\cdots\kappa_{z*}}{(D_a - D_*)(D_b - D_*)\cdots(D_z - D_*)}$$

- Goal: find ★.

Vacua in the F-theory/IIB Landscape

- The landscape is vast and we've likely only explored a tiny fraction of it. Still, can use the already incredibly complex landscape that's been constructed to understand how vacuum selection works.
- I will focus on flux vacua, as studied by Ashok, Denef, and Douglas (ADD), since most known vacua lie in this regime.
- Goal: incorporate what we know about flux vacua to understand selection, including transition rates $\kappa_{i\alpha}$. Consider types of vacua, density of vacua, and number of vacua.

The ADD Flux Landscape

- F-theory on elliptically fibered CY4 $X \to B$.
- Tadpole constraint: $$\frac{\chi(X)}{24} = n_{D3} + \frac{1}{2}\int_X G \wedge G,$$

  SUSY: $n_{D3} \geq 0$, $G = \star\, G$, write $G = N^I \Sigma_I$ and so can write

  $$\frac{1}{2} Q_{IJ} N^I N^J + n_{D3} = \frac{\chi(X)}{24} \equiv Q_c\ ,$$
- $$\frac{1}{2} Q_{IJ} N^I N^J \leq Q_c$$
- N^I are integral, so the counting of flux configurations corresponds to counting appropriate lattice points that satisfy the inequality.

ADD SUSY Vacua: Number and Density

- Given a flux configuration, one wishes to estimate the number of vacua. Let's count SUSY vacua.
- $$I_{vac} = \frac{(2\pi Q_c)^{\frac{b}{2}}}{(\frac{b}{2})!} \int_{\mathscr{M}} \frac{1}{\pi^n} \det(\mathscr{R} + \omega \cdot \mathbf{1}) \,,$$ Ashok, Denef, Douglas

  $\mathscr{R}, \omega$ are curvature, Kähler form on moduli space. b are the number of four-form fluxes with a single leg along the elliptic fiber, and $n = 2h^{3,1}$.
- We have $\int_{\mathscr{M}} \det(\omega \cdot \mathbf{1}) = (n+1)!\mathrm{vol}(\mathscr{M})$, and so ignoring the curvature gives average

  density of vacua of the form
  $$\boxed{\rho_{vac} = \frac{I_{vac}}{\mathrm{vol}(\mathscr{M})} = \frac{(2\pi Q_c)^{\frac{b}{2}}}{(\frac{b}{2})!\pi^n}(n+1)!}$$

Density and Number

- In the large n limit we have $Q_c \sim n/8$, $b \sim n$, and so we find

$$\rho_{\text{vac}} \sim \left(\frac{n}{\sqrt{2\pi \mathrm{e}}}\right)^n \to \text{average distance between vacua} \quad \boxed{d \sim \frac{1}{n}}$$

$$N \sim \left(\frac{n}{\sqrt{2\pi \mathrm{e}}}\right)^n \text{vol}\,\mathscr{M}$$

- This was all SUSY vacua. However our main interest is in dS vacua.
- Assume that number of dS vacua is on order or (much) smaller than the number of SUSY AdS vacua.
- Typical distance between a dS vacuum and any other vacuum is $1/n$. Always expect a nearby AdS vacuum to tunnel towards.

Estimating decay rates and Perturbativity

- To bound the downward decay rate, we want to compute the most important decay rate for a given vacuum α to produce another vacuum β.
- However, to compute this, we need to be able to compute it: need to be in a regime where semi-classical and perturbative expansions are valid.
- Dine: when there are many fields n, they can enhance loop diagrams by factors of $n^p, p > 0$, and so this imposes a scaling on the couplings with n.
- Updated question: If $\star$ is in the regime of control, where is $\star$?
- In particular, at what n is $\star$ located?

Estimating decay rates and Perturbativity

- Consider Lagrangian of the form

$$\mathscr{L} = -\frac{1}{2}\sum_i \left(\partial\phi_i\right)^2 - \frac{1}{2}\sum_{i,j}\mu_{ij}^2\phi_i\phi_j + \sum_{m>2}\sum_{i_1,\ldots,i_m} g^{(m)}\phi_{i_1}\cdots\phi_{i_m}$$

- Assumption for perturbative control:

$$g^{(m)} \sim n^{\frac{1-m}{2}}$$

Dine

- In particular, the cubic/quartic coupling satisfies

$$\gamma \equiv g^{(3)} \sim n^{-1}, \quad g^{(4)} \sim n^{-3/2}$$

Estimating decay rates

- Transition rate for tunneling is $\sim e^{-B}$. Need to compute leading bounce action B.
- Following Dine and Paban, we will model the bounce by a straight line path in field space, with toy single field potential

$$V(\phi) = \mu^2\phi^2 - \gamma\phi^3 + \ldots$$

See also Greene, Kagan, Masoumi, Mehta, Weinberg, Xiao

- Field distance through the barrier is then $\Delta\phi = \dfrac{\mu^2}{\gamma}$.
- Bounce action derived numerically by Sarid, given by

$$B = B_0 y\,; \qquad y \equiv \frac{\mu^2}{\gamma^2}\,, \qquad B_0 \equiv 2.376 \times 2\pi^2$$

- Given distributions for μ^2, γ^2 we can derive a distribution for bounce action.

Distributions for μ^2, γ

- Via Dine's perturbativity argument, we take the probability distribution function (PDF) for γ to be uniform distribution between $[0, 1/n]$.

- Assume μ^2 uniformly distributed between $[0, C]$, determine C by matching with average ADD distance between vacua $\sim 1/n$.

- Field distance governed by ratio PDF:

$$f_{\Delta\Phi}(\Delta\phi) = \int_0^\infty \mathrm{d}\gamma\, \gamma f_\Gamma(\gamma) f_{M^2}(\gamma\Delta\phi) = \begin{cases} \frac{1}{2Cn} & 0 < \Delta\phi < Cn\,; \\ \frac{nC}{2(\Delta\phi)^2} & \Delta\phi > Cn\,, \end{cases}$$

- Matching the median Cn to $1/n$ gives $\boxed{C = 1/n^2.}$

Bounce distribution

- Define $y = B = \dfrac{\mu^2}{\gamma^2}$, PDF again defined by a ratio PDF.

$$\boxed{f_Y(y) = \int_0^\infty \mathrm{d}\gamma^2\,\gamma^2 f_{\Gamma^2}\left(\gamma^2\right) f_{M^2}\left(\gamma^2 y\right) = \begin{cases} \frac{1}{3} & 0 < y < 1\,; \\ \frac{1}{3y^{3/2}} & y > 1\,. \end{cases}}$$

- Cumulative distribution:

$$F_Y(y) \equiv \Pr(Y \leq y) = \int_0^y \mathrm{d}t\, f_Y(t) = \begin{cases} \frac{y}{3} & 0 < y < 1\,; \\ 1 - \frac{2}{3\sqrt{y}} & y > 1\,, \end{cases}$$

Lifetime probability distribution

- In n dimensions, decay rate set by smallest bounce action among its n allowed transitions. Denote minimal bounce action by $B_{\min} \equiv B_0 W$, where $W \equiv \min(Y_1, \ldots, Y_n)$.

- Would like to compute the median of W. Since W is the min of Y_i we have the complementary cumulative distribution function (CCDF):

$$\begin{aligned}
\overline{F}_W(w) &= \Pr(Y_1, \ldots, Y_n > w) \\
&= \Pr(Y_1 > w) \cdots \Pr(Y_n > w) \\
&= \overline{F}_{Y_1}(w) \cdots \overline{F}_{Y_n}(w) \\
&= \left(\overline{F}_Y(w)\right)^n ,
\end{aligned}$$

Lifetime probability distribution

- We then have

$$\overline{F}_W(w) = \begin{cases} \left(1 - \frac{w}{3}\right)^n & 0 < w < 1\,; \\ \left(\frac{2}{3\sqrt{w}}\right)^n & w > 1\,. \end{cases}$$

- Median smallest bounce $\bar{w}$ given by $\overline{F}_W(\bar{w}) = 1/2$. Anticipating at large n that $\bar{w} \ll 1$, we find

$$\left(1 - \frac{\bar{w}}{3}\right)^n = \frac{1}{2} \quad \Longrightarrow \quad \boxed{\bar{w} \simeq \frac{3\log 2}{n}}$$

Consistent with scaling found by Dine and Paban.

- Thus vacua in large-n geometries have shorter lifetimes on average.

Pressure from above

- Our calculation, Dine-Paban, Greene et. al: average bounce action decreases with n.

- In addition, probability of a generic critical point to be classically stable also decreases with n.

- Studied in random matrix models of Hessians (mass matrices), probability of classical stability: **Aazami, Easther**

  $P(n) \sim e^{-cn^{3/2}}$ for generic iid entries of the Hessian (wasteland). **Marsh, McAllister, Wrase**

  $P(n) \sim e^{-cn}$ for nearly-SUSY vacua, also for perturbativity. **Bachlechner, Marsh, McAllister, Wrase** **Dine**

- Expect probability of classical and quantum stability to decrease with n.

Longest Lived Vacuum

- Let $W_1, \ldots, W_N$ denote the random variables determining the bounce actions of the inflating vacua in an n-dimensional moduli space.

- The longest-lived vacuum among these corresponds to $W_{\max} = \max\left(W_1, \ldots, W_N\right)$.

- Consider its CDF, $F_{W_{\max}}(w_{\max}) = \Pr(W_{\max} \leq w_{\max})$. Since $W_{\max}$ is the largest of all W's

$$
\begin{aligned}
F_{W_{\max}}(w_{\max}) &= \Pr(W_1, \ldots, W_N \leq w_{\max}) \\
&= \Pr(W_1 \leq w_{\max}) \cdots \Pr(W_N \leq w_{\max}) \\
&= F_{W_1}(w_{\max}) \cdots F_{W_N}(w_{\max}) \\
&= \left(F_W(w_{\max})\right)^N \, .
\end{aligned}
$$

Longest Lived Vacuum

$$F_{W_{\max}}(w_{\max}) = \begin{cases} \left[1 - \left(1 - \frac{w_{\max}}{3}\right)^n\right]^N & 0 < w_{\max} < 1\,; \\ \left[1 - \left(\frac{2}{3\sqrt{w_{\max}}}\right)^n\right]^N & w_{\max} > 1\,. \end{cases}$$

- Consider, the median maximal bounce $\bar{w}_{\max}$, given by $F_{W_{\max}}(\bar{w}_{\max}) = 1/2$. Anticipating that $\bar{w}_{\max} \gg 1$ N is enormous, we obtain

$$\left[1 - \left(\frac{2}{3\sqrt{\bar{w}_{\max}}}\right)^n\right]^N = \frac{1}{2} \quad \Longrightarrow \quad \bar{w}_{\max} \simeq \frac{4}{9}\left(\frac{N}{\log 2}\right)^{2/n}$$

Longest Lived Vacuum

- $$\left[1-\left(\frac{2}{3\sqrt{\bar{w}_{\max}}}\right)^{n}\right]^{N}=\frac{1}{2} \quad\Longrightarrow\quad \bar{w}_{\max}\simeq\frac{4}{9}\left(\frac{N}{\log 2}\right)^{2/n}$$

- Model number of inflating vacua N as the number of SUSY vacua, multiplied by a wasteland factor (accounting for stability):

$$N\sim\left(\frac{n}{\sqrt{2\pi e}}\right)^{n}\text{vol}\,\mathscr{M}\times e^{-cn^{p}}=\exp\left(n\log\left(\frac{n}{\sqrt{2\pi e}}\right)-cn^{p}\right)\text{vol}\,\mathscr{M}$$

- For nearly-SUSY/perturbative case with $p=1$, the first term dominates at large n (for instance, largest known $n=2\times 303{,}148$), and we have exponentially many inflating vacua.

- However, for $p>1$ there is tension with the wasteland factor.

Longest Lived Vacuum

- Assuming $p = 1$, we find

$$\boxed{\bar{w}_{\max} \sim \frac{2}{9\pi \mathrm{e}} \left(\frac{\text{vol}\,\mathscr{M}}{\log 2} \right)^{2/n} n^2}$$

- Unless vol $\mathscr{M}$ decreases rapidly with n, we see that the lifetime of the longest-lived vacuum grows with n. In the known F-theory landscape this would select B_{max}.

Taylor, Wang

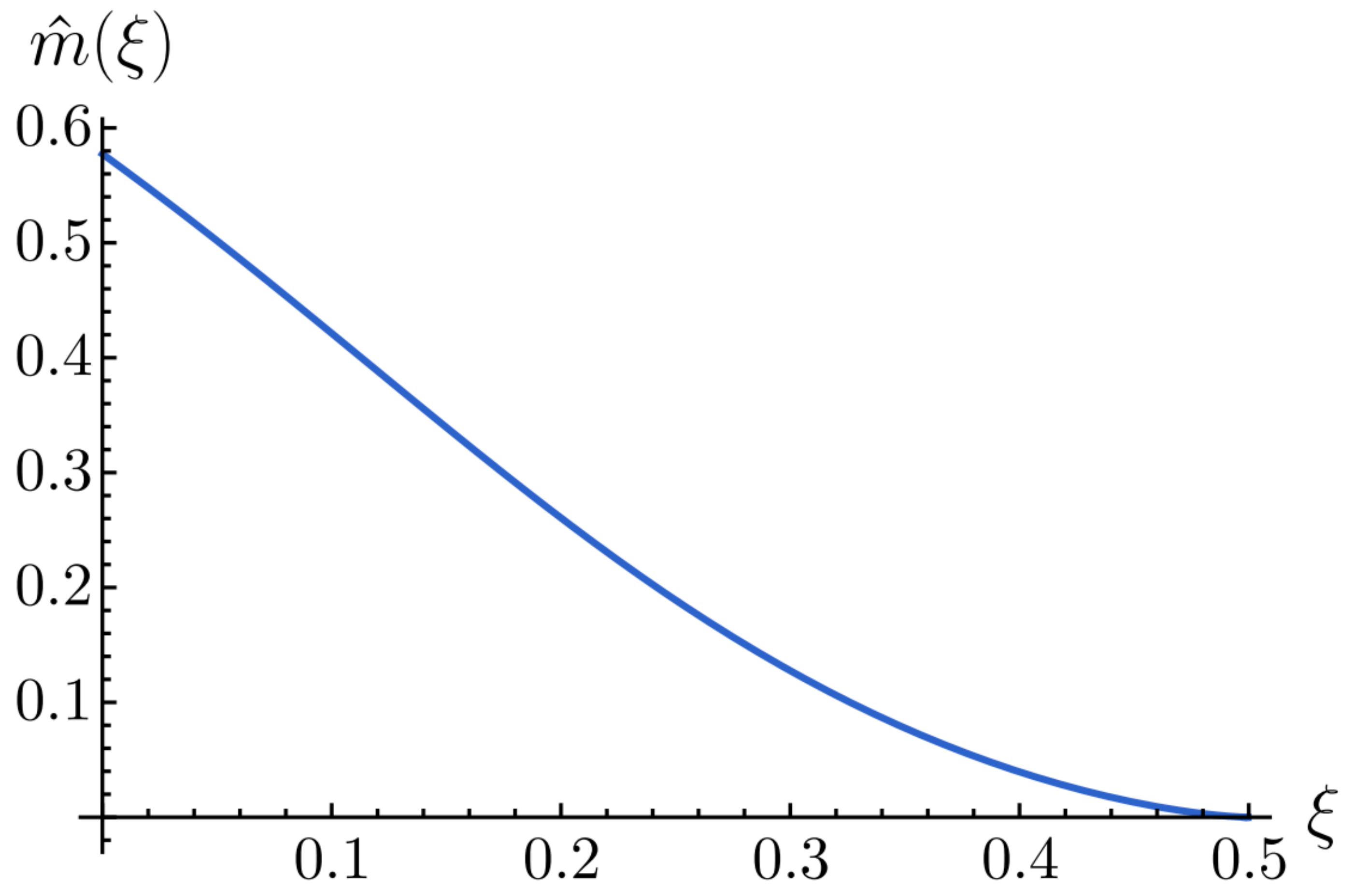
- This can be understood intuitively. Although vacua in large-n geometries tend to be shorter-lived, as we have seen, these geometries host an enormous number of vacua $N \sim n^n$. As a result, the longest-lived vacuum becomes increasingly stable with increasing n.

- Interestingly, this is very sensitive to the growth of $N \sim n^n = e^{n \log n}$. If instead we had $N \sim e^n$ (via a e.g. a wasteland factor), $\bar{w}_{\max}$ would be constant with n.

A comment on LCS

- Recent work by Blanco-Pillado, Sousa, Urkiola, Wachter suggests that the complex structure moduli masses at large complex structure take a universal form

$$\frac{\mu^2_{\pm\lambda}}{m^2_{3/2}} = \begin{cases} \left(1 \pm \frac{\sqrt{(1-2\xi)}}{\sqrt{3}}\hat{m}(\xi)\right)^2 & \lambda = 0, \\ \left(1 \pm \frac{\sqrt{(1-2\xi)}}{\sqrt{3}\hat{m}(\xi)}\right)^2 & \lambda = 1, \\ \left(1 \pm \frac{1+\xi}{3}\right)^2 & \lambda = 2, \ldots, h^{2,1}_{-}. \end{cases}$$



$\xi \in \left(-1, \frac{1}{2}\right)$, $\xi = 0$ is the LCS point.

- Perturbativity in this case requires $m^2_{3/2} \sim \dfrac{1}{n^2}$, and so the perturbative vacua are expected to have SUSY breaking scale that decreases with n.

A consistency check

- The perturbative vacua that are selected are then large n vacua with very low SUSY-breaking scale.
- Mike Douglas, 2012: “Thus, a reasonable guess is that the master vacuum is some flux sector in a vacuum with the smallest Λ_{SUSY}…

  The question of how to get small Λ_{SUSY} deserves detailed study, but it is a very reasonable guess that this will be achieved by taking the topology of the extra dimensions to be as complicated as possible, and even more specifically by an extra dimensional manifold with the largest possible Euler number χ …

  Thus, we might look for the master vacuum as an F theory compactification on the fourfold with maximal χ, which (as far as I know) is the hypersurface in weighted projective space given in (reference) with $\chi = 24 \cdot 75852$.”

  (1204.6626)
- This is B_{max}, studied further by Taylor and Wang, which has the maximal n.

Summary and Musings

- Density of vacua grows exponentially with n, average bounce action decrease as $1/n$.

- However, the enormous number of vacua at large n allows the tails of the distribution of bounce actions to be probed, leading to an increasing lifetime of the most stable vacuum as a function of n. Therefore expect $\star$ to be at large n.

- Important considerations are wasteland factors, which could provide a mechanism for selection at smaller n.

- Work in progress includes considering other distributions for the couplings, Kähler moduli/axions, and similar analyses for early-time measures.

Thanks!