

Towards Enumeration of Vacua

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with Mehmet Demirtas, Manki Kim, Jakob Moritz, and Andres Rios-Tascon

string_data 2020



de Sitter space

Simplest explanation for observed universe.

Should try to construct de Sitter solutions of string theory.

de Sitter solutions are not common.

So a search must survey many candidate geometries.

Not feasible with pen and paper.







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So a search must survey many candidate geometries.

Not feasible with pen and paper.

With partial computerization, human steps are rate-limiting.

Need efficient implementation of every step
from topological data to a stabilized vacuum.

SN 2011fe

Plan

I will describe progress we have made in this direction:

CYTools: software for computing string data

Demirtas, L.M., Rios-Tascon, 20

Demirtas, L.M., Rios-Tascon, to appear

Demirtas, Kim, L.M., Moritz, Rios-Tascon, work in progress

Some first fruits of this approach:

Vacua with small flux superpotential

Demirtas, Kim, L.M., Moritz 19

Demirtas, Kim, L.M., Moritz 20

This talk does not involve ML, but our data is a natural target.

Reality is hard

We seek a de Sitter solution of string theory.

Exact nonsupersymmetric solutions of superstring theories with 4d gravity seem out of reach.

This is not a cosmological issue.

e.g., no derivation of **Standard Model + Einstein gravity** (with no light moduli) to standard demanded for de Sitter.

Non-supersymmetric compactifications are hard, but this does not imply they do not exist.

We need **systematic approximations**.

Fundamental expansions are in α' and g_s .

Strategy for constructing vacua

Conceptually easy, impractical at present:

Find solution preserving $\mathcal{N} = 1$ SUSY, e.g. type II on $CY_3 - \mathcal{O}$.

Directly compute 4d EFT to N^k LO, in α' and g_s .

Exhibit de Sitter solutions in EFT at N^{k-1} LO,
show that N^k LO negligible.

Practical: apply further approximations.

e.g. find parameter regimes where sectors decouple
into **modules** that interact weakly.

Analyze modules in isolation, then weakly couple them.

Final **assembly** is a key challenge.

Our aim

Work under bright lamppost: Calabi-Yau compactifications.

Systematically compute topological and geometric data that enter 4d $\mathcal{N} = 1$ EFT, for a **vast ensemble** of CY_3 hypersurfaces in toric varieties.

Automate the construction of vacua.

In time this may reveal controlled de Sitter vacua, or not.

Long, L.M., McGuirk 14

Long, L.M., Stout 16

Braun, Long, L.M., Stillman, Sung 17

Demirtas, Long, L.M., Stillman 18

Demirtas, Kim, L.M., Moritz 19

Demirtas, L.M., Rios-Tascon, 20

Demirtas, Kim, L.M., Moritz 20

Demirtas, L.M., Rios-Tascon, to appear

Blumenhagen, Jurke, Rahn, Roschy 10

Braun, Walliser 11

Gao, Shulka 13

Altman, Gray, He, Jejjala, Nelson 14

Cicoli, Muia, Shukla 16

Braun, Lukas, Sun 17

Taylor, Wang 17

Huang, Taylor 18

Halverson, Long 20

Which string data?

Needs vary depending on model, so try to get ‘everything’.

At least, everything that follows directly from topological data.

Start: 4d reflexive polytope $\Delta^\circ \in$ Kreuzer-Skarke list

Fast
Automatic
Complete

Fine regular star triangulation \Rightarrow toric variety V
Intersection numbers κ_{ijk} of CY_3 $X \subset V$
Kähler cone of V , $\mathcal{K}(V) \subset \mathcal{K}(X)$

Fast
Automatic
Incomplete

True/extended Kähler cone of X
Gopakumar-Vafa invariants of curves in X
Cone of effective divisors on X

Limited

Orientifolds of X
F-theory uplifts Y of orientifolds of X
Divisors supporting $EM5 \subset Y$ or $ED3 \subset X$

Limited

Calabi-Yau metric on X

talks by Douglas and Krippendorf

Perturbative corrections in α' , g_s

CYTools

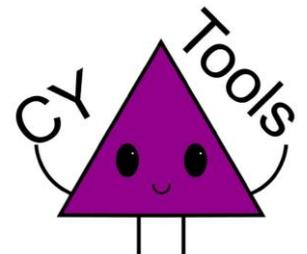
A Software Package for Analyzing Calabi-Yau Hypersurfaces

Demirtas, L.M., Rios-Tascon

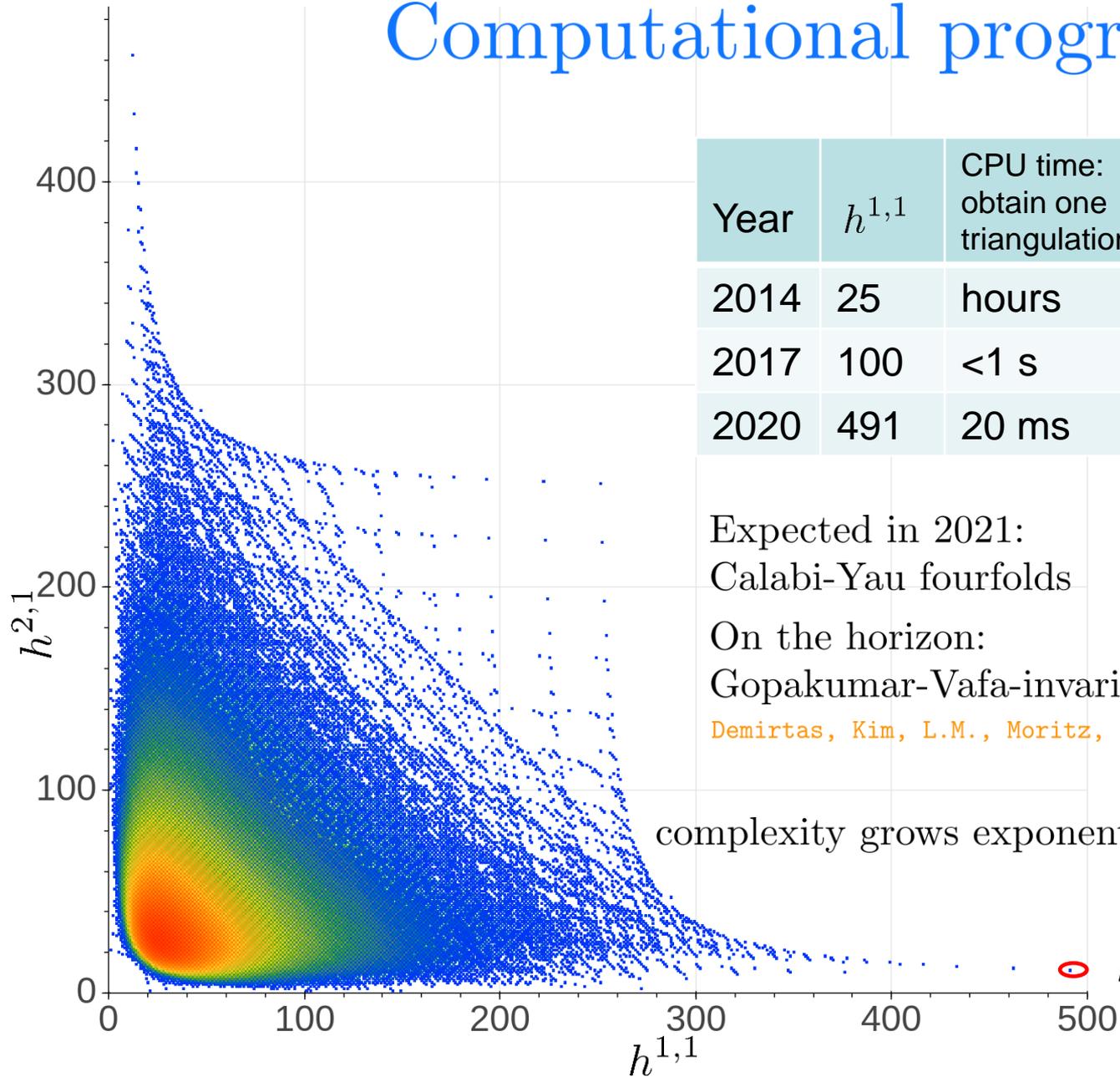
To appear in January 2021. Beta testers welcome.

Purpose-built to construct and analyze triangulations,
and associated Calabi-Yau hypersurfaces in toric varieties.

Orders of magnitude faster than available codes like Sage.



Computational progress



Year	$h^{1,1}$	CPU time: obtain one triangulation	CPU time: intersection numbers
2014	25	hours	hours
2017	100	<1 s	30 min
2020	491	20 ms	3 s

Expected in 2021:

Calabi-Yau fourfolds

On the horizon:

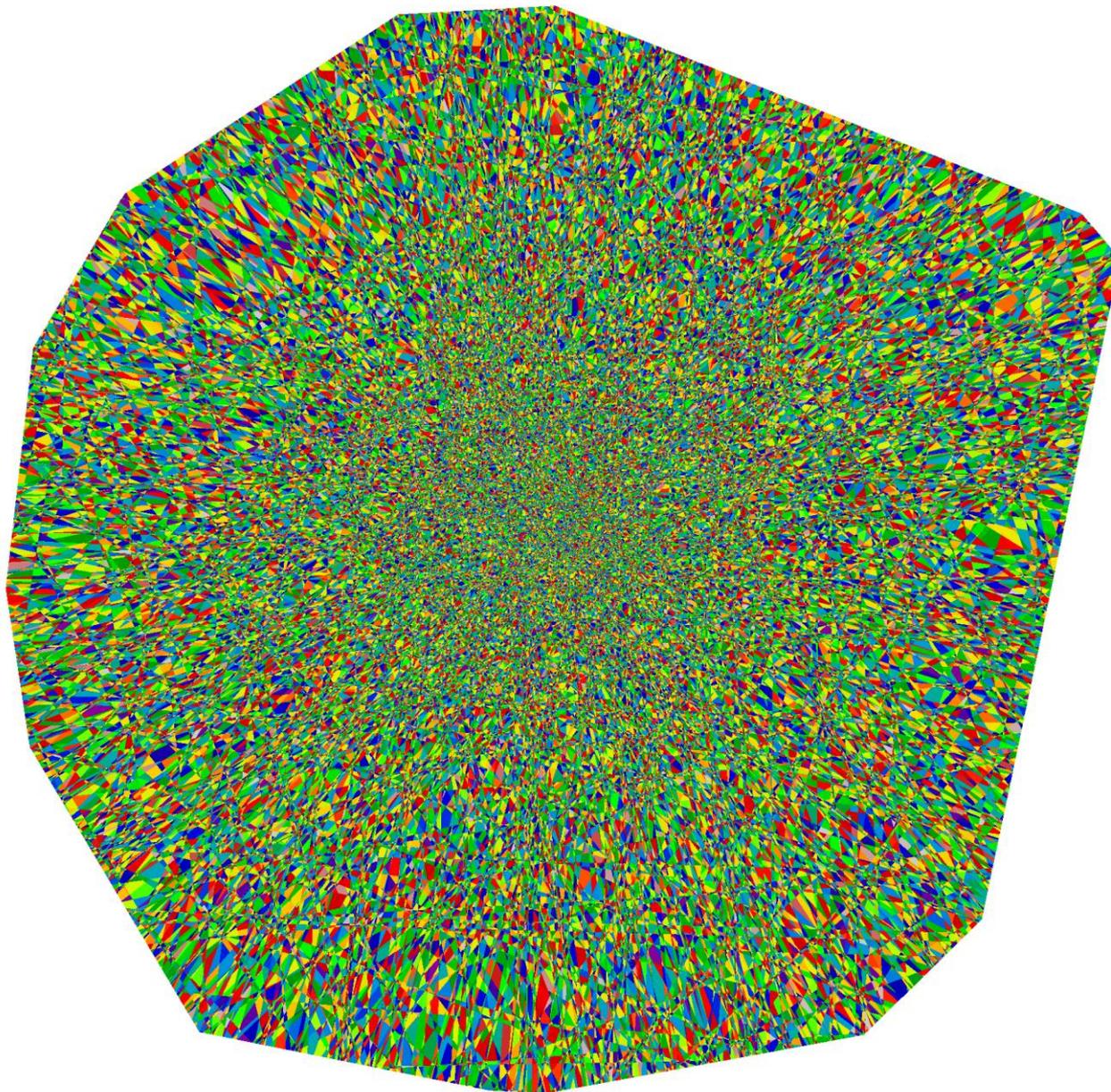
Gopakumar-Vafa-invariants

Demirtas, Kim, L.M., Moritz, Rios-Tascon, work in progress

complexity grows exponentially \rightarrow

$h^{1,1} = 491$

2d cross-section of Kähler cone in $h^{1,1} = 491$ threefold



Application:

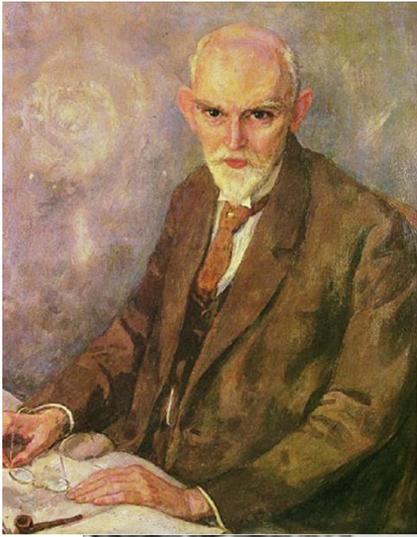
Vacua with Small Flux Superpotential

Demirtas, Kim, L.M., Moritz 19

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Computer-aided discovery of analytic results.

Finding de Sitter



KKLT de Sitter vacua

Compactification of type IIB on an orientifold X of a CY_3 , including:

three-form flux $G_3 = F_3 - \tau H_3 \in H_3(X, \mathbb{Z})$

an $\mathcal{N} = 1$ pure SYM sector on $N_c > 1$ D7-branes

a warped deformed conifold region Klebanov, Strassler 00

containing one or more anti-D3-branes

Claim [KKLT]: in a suitable parameter regime,
these sources can yield metastable dS_4 ,
and corrections to approximations are small.

Kachru, Kallosh, Linde, Trivedi 03

Setup

Type IIB string theory compactified on O3/O7 orientifold, X , of a CY_3 .

$$ds^2 = G_{AB}dX^A dX^B = e^{-6u(x)+2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{2u(x)-2A(y)} g_{ab} dy^a dy^b$$

Take $h_+^{1,1} = 1$, $\Sigma \in H_4(X, \mathbb{Z})$. Choose $G_3 = F_3 - \tau H_3 \in H_3(X, \mathbb{Z})$.

Moduli: axiodilaton $\tau := C_0 + ie^{-\phi}$

complex structure U_a , $a = 1, \dots, h^{2,1}$

Kähler: $T = e^{4u} + i \int_{\Sigma} C_4$.

4d $\mathcal{N} = 1$ supergravity:

$$W_{\text{flux}} = \int_X G_3 \wedge \Omega \quad \text{Gukov, Vafa, Witten 99}$$

$$\mathcal{K} = -3 \log(T + \bar{T}) - \log(-i(\tau - \bar{\tau})) - \log\left(-i \int_X \Omega \wedge \Omega\right)$$

For generic G_3 , solutions of $D_\tau W = D_{U_a} W = 0$ are isolated $\Rightarrow \tau, U_a$ fixed.

Setup

Below the scale $m_{U_a} \sim \frac{\alpha'}{\sqrt{\text{Vol}(X)}}$ of the complex structure moduli masses,

$$\mathcal{K} = -3 \log(T + \bar{T}) \quad W_{\text{flux}} \rightarrow \left\langle \int_X G_3 \wedge \Omega \right\rangle =: W_0$$

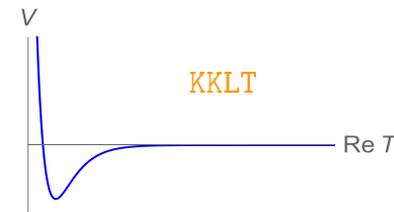
Consider a stack of D7-branes on Σ that support pure $SU(N_c)$ SYM.

$$W_{\text{np}} = -\frac{N_c}{32\pi^2} \langle \lambda\lambda \rangle = \mathcal{A} e^{-\frac{2\pi}{N_c} T} \quad W = W_0 + \mathcal{A} e^{-\frac{2\pi}{N_c} T}$$

[or, ED3 on Σ]
Witten 96

This supergravity theory has a **SUSY AdS_4 minimum**:

$$D_T W = 0 \Leftrightarrow W_0 = -\mathcal{A} e^{-\frac{2\pi}{N_c} T} \left(1 + \frac{2\pi}{3N_c} (T + \bar{T}) \right)$$



If $W_0 \ll 1$, the minimum is at large volume, $(T + \bar{T})_{\text{min}} \approx -\frac{N_c}{\pi} \ln(|W_0|)$

Statistical argument: $W_0 \ll 1$ is not generic, but should occur for some of the many choices of G_3 .

One should ask:

MODULI STABILIZATION

Do there exist consistent global models with:

- i. Quantized fluxes giving small classical superpotential **yes**
- ii. *and* a warped conifold region **yes**
- iii. *and* D7-brane gaugino condensates

Demirtas, Kim, L.M., Moritz 19

Demirtas, Kim, L.M., Moritz 20

Alvarez-Garcia, Blumenhagen, Brinkmann, Schlechter 20

Can one exhibit an explicit and fully-controlled compactification that unifies all necessary components? **TBD**

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Small Flux Superpotentials

$$(T + \bar{T})_{\min} \approx -\frac{N_c}{\pi} \ln(|W_0|) \Rightarrow \text{control requires } |W_0| \ll 1$$

Statistical arguments suggest $|W_0| \ll 1$ should occur,
but exponentially rarely,
and preferentially when moduli space dimension is large.

Our tools permit exploring such moduli spaces.

But brute-force search still very challenging.

Halverson and Ruelle 18
Cole, Schachner, Shiu 19

Previous record: $\mathcal{O}(0.01)$

Denef, Douglas, Florea 04
Denef, Douglas, Florea, Grassi, Kachru 05

Better to invent and apply a **mechanism**.

Giryavets, Kachru, Tripathy, Trivedi 03
Denef, Douglas, Florea 04
Demirtas, Kim, L.M., Moritz 19



Racetrack Superpotential

$$W = W_{\text{pert}}(U) + W_{\text{inst}}(U)$$

$$W_{\text{inst}}(U) = \mathcal{A}_1 e^{2\pi i p_1 U} + \mathcal{A}_2 e^{2\pi i p_2 U} + \dots$$

$$\rightarrow 0 \text{ for } U \rightarrow i\infty$$

Goal: minimize potential at U_{min} s.t. $|W(U_{\text{min}})| \ll 1$

Suppose that

$$D_U W_{\text{pert}} = W_{\text{pert}} = 0 \text{ along } U = \{U_{\text{flat}}\}$$

$$\Rightarrow W_{\text{eff}}(U \in U_{\text{flat}}) = W_{\text{inst}}(U)$$

$$U_{\text{min}} := U \in U_{\text{flat}} \text{ s.t. } D_U W_{\text{inst}} \Big|_{U_{\text{min}}} = 0$$

Racetrack Superpotential

$$W_{\text{inst}}(U) = \mathcal{A}_1 e^{2\pi i p_1 U} + \mathcal{A}_2 e^{2\pi i p_2 U} + \dots$$

$$2\pi i U_{\text{min}} = -\frac{\log\left(-\frac{\mathcal{A}_1 p_1}{\mathcal{A}_2 p_2}\right)}{(p_1 - p_2)}$$

$$W_{\text{inst}}(U_{\text{min}}) = \frac{\mathcal{A}_2 (p_1 - p_2)}{p_1} \left(-\frac{\mathcal{A}_2 p_2}{\mathcal{A}_1 p_1}\right)^{\frac{p_2}{p_1 - p_2}}$$

$$\ll 1 \text{ if } |p_1 - p_2| \ll p_2, \quad \mathcal{A}_2 \ll \mathcal{A}_1$$

Incarnation as Flux Superpotential

Need to show that, in a bona fide solution of string theory:

$$W_{\text{flux}} = W_{\text{pert}}(U) + W_{\text{inst}}(U)$$

(1) with $W_{\text{inst}}(U) = \mathcal{A}_1 e^{2\pi i p_1 U} + \mathcal{A}_2 e^{2\pi i p_2 U} + \dots$

(2) s.t. $D_U W_{\text{pert}} = W_{\text{pert}} = 0$ along $U = \{U_{\text{flat}}\}$

(3) and $|p_1 - p_2| \ll p_2$ and $\mathcal{A}_2 \ll \mathcal{A}_1$

Then we'll have succeeded:

$$W_0 := \langle W_{\text{flux}} \rangle = W_{\text{inst}}(U_{\text{min}}) = \frac{\mathcal{A}_2(p_1 - p_2)}{p_1} \left(-\frac{\mathcal{A}_2 p_2}{\mathcal{A}_1 p_1} \right)^{\frac{p_2}{p_1 - p_2}} \ll 1$$

(1) is immediate: IIA worldsheet instantons

(2) we established a sufficient condition on topological data

(3) we found explicit examples

Flux Superpotential in a Calabi-Yau

Let X be a Calabi-Yau threefold, $\{A_a, B^b\}$ a symplectic basis for $H_3(X, \mathbb{Z})$

$$A_a \cap A_b = 0, A_a \cap B^b = \delta_a^b, B^a \cap B^b = 0.$$

$$\Pi = \begin{pmatrix} \int_{B^a} \Omega \\ \int_{A_a} \Omega \end{pmatrix} = \begin{pmatrix} \mathcal{F}_a \\ U^a \end{pmatrix}.$$

$\mathcal{F}(U) = \mathcal{F}_{\text{pert}}(U) + \mathcal{F}_{\text{inst}}(U)$ — work around large complex structure.

$$\mathcal{F}_{\text{pert}}(U) = -\frac{1}{3!} \mathcal{K}_{abc} U^a U^b U^c + \frac{1}{2} a_{ab} U^a U^b + b_a U^a - \frac{\zeta(3)\chi}{2(2\pi i)^3},$$

$$\mathcal{F}_{\text{inst}}(U) = \frac{1}{(2\pi i)^3} \sum_{\vec{q}} A_{\vec{q}} e^{2\pi i \vec{q} \cdot \vec{U}}.$$

$$W_{\text{flux}} = \sqrt{\frac{2}{\pi}} \left(F - \tau H \right)^T \cdot \Sigma \cdot \Pi, \quad \Sigma = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix},$$

$$W_{\text{flux}} = W_{\text{pert}} + W_{\text{inst}}$$

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$$\mathcal{F}_{\text{inst}}(U) = \frac{1}{(2\pi i)^3} \sum_{\vec{q}} A_{\vec{q}} e^{2\pi i \vec{q} \cdot \vec{U}}. \quad \text{computable, rational data}$$

$$W_{\text{flux}} = \sqrt{\frac{2}{\pi}} \left(F - \tau H \right)^T \cdot \Sigma \cdot \Pi, \quad \Sigma = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix},$$

$$W_{\text{flux}} = W_{\text{pert}} + W_{\text{inst}}$$

Flux Superpotential in a Calabi-Yau

Let \tilde{X} be the Calabi-Yau threefold hypersurface in $\mathbb{CP}_{[1,1,1,6,9]}$

$$h^{1,1}(\tilde{X}) = 2, \quad h^{2,1}(\tilde{X}) = 272$$

$X =$ mirror of $\tilde{X} =$ resolution of $\tilde{X} / (\mathbb{Z}_6 \times \mathbb{Z}_{18})$

$$h^{2,1}(X) = 2, \quad h^{1,1}(X) = 272$$

$$\mathcal{F}_{\text{pert}}(U) = -\frac{1}{3!} \mathcal{K}_{abc} U^a U^b U^c + \frac{1}{2} a_{ab} U^a U^b + b_a U^a - \frac{\zeta(3)\chi}{2(2\pi i)^3},$$

$$\mathcal{K}_{111} = 9, \quad \mathcal{K}_{112} = 3, \quad \mathcal{K}_{122} = 1,$$

$$a_{ab} = \frac{1}{2} \begin{pmatrix} 9 & 3 \\ 3 & 0 \end{pmatrix}, \quad b_a = \frac{1}{4} \begin{pmatrix} 17 \\ 6 \end{pmatrix}.$$

$$\mathcal{F}_{\text{inst}}(U) = \frac{1}{(2\pi i)^3} \sum_{\vec{q}} A_{\vec{q}} e^{2\pi i \vec{q} \cdot \vec{U}}.$$

$$(2\pi i)^3 \mathcal{F}_{\text{inst}}(U_1, U_2) = -540q_1 - 3q_2 - \frac{1215}{2}q_1^2 + 1080q_1q_2 + \frac{45}{8}q_2^2 + \dots$$

$$q_a := \exp(2\pi i U^a)$$

Flux Superpotential in a Calabi-Yau

We find an O3/O7 orientifold of X, and quantized fluxes F, H

$$F = (\vec{M} \cdot \mathbf{b}, \vec{M}^T \cdot \mathbf{a}, 0, \vec{M}^T), \quad H = (0, \vec{K}^T, 0, 0),$$

$$\text{with } \vec{M} = \begin{pmatrix} -16 \\ 50 \end{pmatrix}, \quad \vec{K} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}, \quad Q_{D3}^{\text{flux}} = 124$$

s.t. $\vec{U} = \tau \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \frac{\tau}{10} \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ is a flat direction, and

$$W_{\text{inst}} = \mathcal{A}_1 e^{2\pi i p_1 \tau} + \mathcal{A}_2 e^{2\pi i p_2 \tau} + \dots$$

$$\text{with } p_1 = \frac{2}{5}, \quad p_2 = \frac{3}{10} \quad \frac{\mathcal{A}_2}{\mathcal{A}_1} = -\frac{5}{288}$$

Moduli are stabilized at

$$\langle \tau \rangle = 6.856i, \quad \langle U_1 \rangle = 2.742i, \quad \langle U_2 \rangle = 2.057i,$$

$$\text{with } |W_0| = 2.037 \times 10^{-8}.$$

Single-instantons dominate

Totally explicit, no unknown factors in leading-order data.

Just a finite set of computable rational numbers.

Higher orders strongly suppressed:

$$(2\pi i)^3 \mathcal{F}_{\text{inst}}(U_1, U_2) = -540q_1 - 3q_2 - \frac{1215}{2}q_1^2 + 1080q_1q_2 + \frac{45}{8}q_2^2 + \dots$$

$\mathcal{O}(10^{-5})$ smaller $\mathcal{O}(10^{-10})$ smaller

Remark: mass^2 along previously-flat direction is $\mathcal{O}(|W_0|)$

Comments

We easily find many more examples.

Need: knowledge of prepotential \mathcal{F} around LCS
suitable orientifold

some luck with the numbers $p_1, p_2, \mathcal{A}_2/\mathcal{A}_1, Q_{D3}^{\text{flux}}$

Upshot: **small $|\mathbf{W}_0|$ is in the landscape.**

But for KKLT we want much more:

warped conifold region

Kähler moduli stabilization ($h^{1,1} = 272$ in example!)

anti-D3-brane supersymmetry breaking

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Conifold vacua

Thus far, have expanded around LCS: $\mathcal{F}(U) = \mathcal{F}_{\text{pert}}(U) + \mathcal{F}_{\text{inst}}(U)$

Want flux vacuum near a conifold point and with $|W_0| \ll 1$.

So, **analytically continue** from LCS to conifold.

$$\mathcal{F}_{\text{inst}}(U) = -\frac{1}{(2\pi i)^3} \sum_{[\mathcal{C}]} n_{\mathcal{C}}^0 \text{Li}_3(q^{\mathcal{C}}), \quad q^{\mathcal{C}} := \exp(2\pi i b_{\mathcal{C}}^a U_a) \quad \text{Li}_k(q) := \sum_{n=1}^{\infty} q^n / n^k$$

$\mathcal{C} = b_{\mathcal{C}}^a \Sigma_a$ effective, Σ_a basis of $H_2(\tilde{X}, \mathbb{Z})$
 $n_{\mathcal{C}}^0$: genus-0 Gopakumar-Vafa invariant

Mirror of developing conifold singularity in X can be:

collapsing \tilde{X}

collapsing divisor(s) in \tilde{X}

collapsing curve(s) in \tilde{X}

Often, infinite instanton sum contributes at conifold.

Conifold vacua

But **if** the conifold singularity is mirror to collapsing
a curve \mathcal{C}_1 s.t. $n_{k\mathcal{C}_1}^0 \neq 0$ only for **finitely many** k
then one need only continue finitely many polylogs!

$$-\frac{\text{Li}_2(e^{2\pi iz})}{(2\pi i)^2} = \frac{1}{24} + \frac{z}{2\pi i} \ln(1 - e^{2\pi iz}) + \frac{\text{Li}_2(1 - e^{2\pi iz})}{(2\pi i)^2}$$

Euler 1729

We obtained an analytic result for \mathcal{F} in such cases:

$$\mathcal{F}_1 = n_{\text{cf}} \frac{z_1}{2\pi i} \ln(1 - e^{2\pi iz_1}) + n_{\text{cf}} \left(\frac{1}{24} + \frac{\text{Li}_2(1 - e^{2\pi iz_1})}{(2\pi i)^2} \right) \\ - \frac{1}{2} \mathcal{K}_{1ab} z^a z^b + a_{1a} z^a + b_1 - \frac{1}{(2\pi i)^2} \sum_{[c] \neq [c_1]} n_c^0 b_1^c \text{Li}_2(q^c).$$

In an explicit orientifold with $h^{1,1} = 99$, $h^{2,1} = 3$,
we exhibited near-conifold vacua with $e^{2A} \approx |W_0| \ll 1$.

Conclusions

KKLT scenario for de Sitter vacua requires special structures in classical flux compactification:

exponentially small flux superpotential W_0
warped conifold region

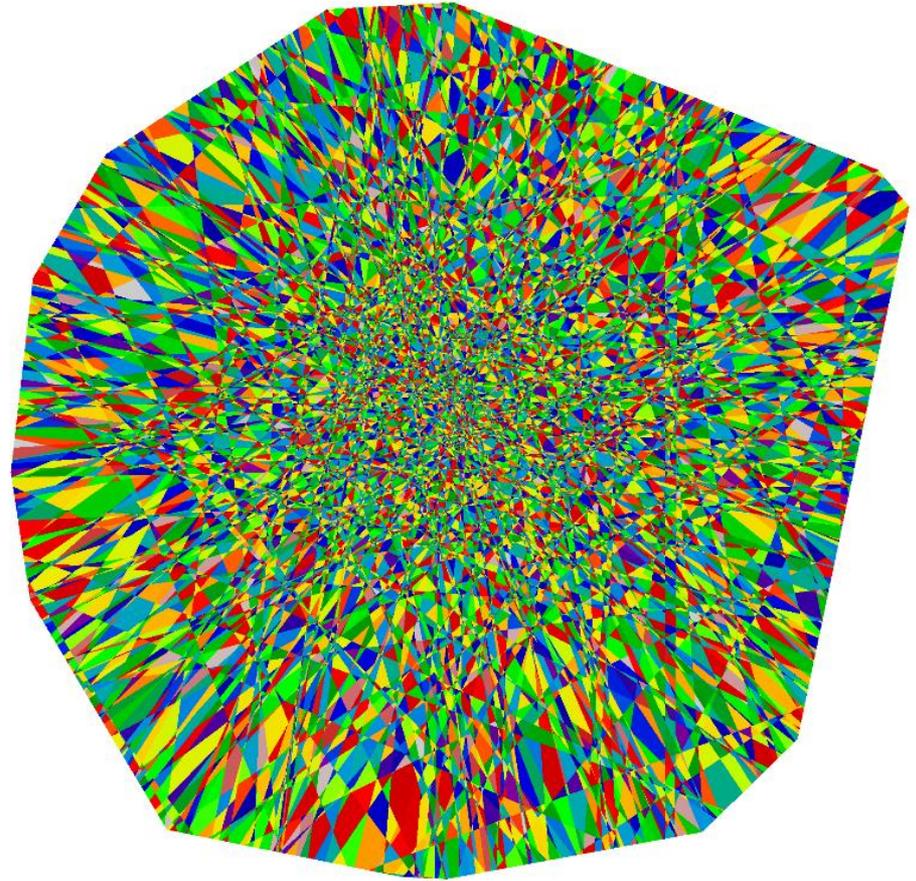
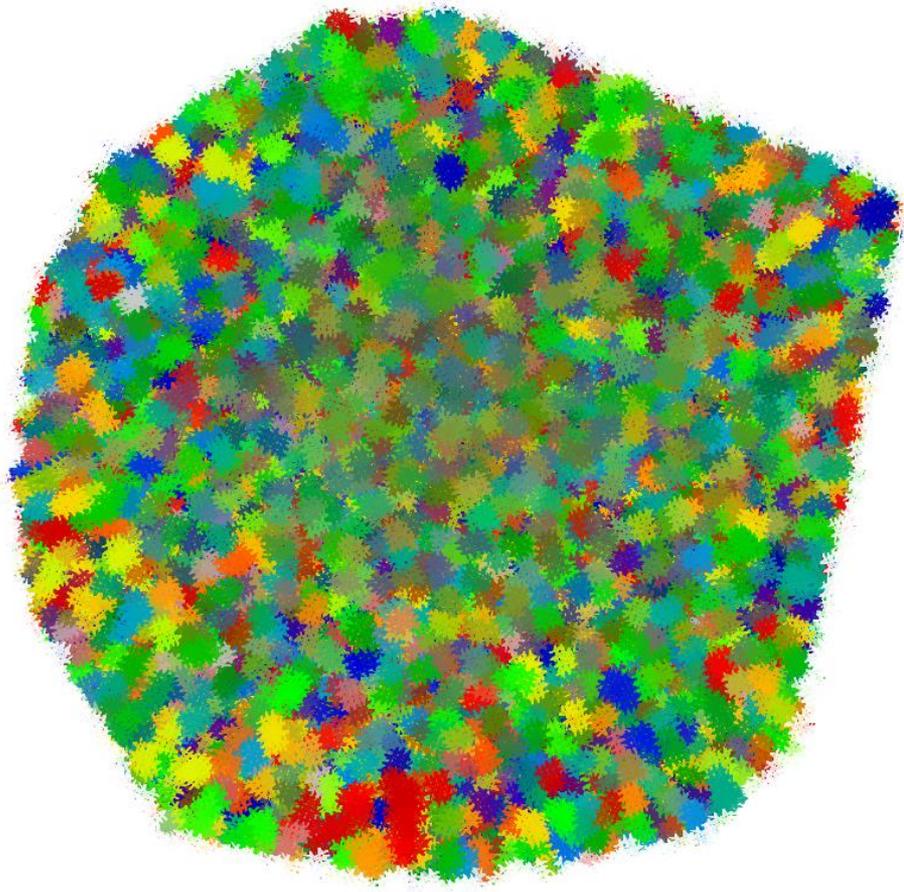
We presented a mechanism for constructing such solutions, via a racetrack of IIA worldsheet instantons.

We gave complete explicit examples.

Small $|W_0|$ is in the landscape.

Our search is automated; large-scale studies possible.

Quantum side of KKLT, and uplift, remain challenging.



Thanks!