ALGORITHMICALLY SOLVING THE TADPOLE PROBLEM

[2010.10519] ,[2012.XXXXX]

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string_data 2020

CERN, December 14

MOTIVATION

➤ String theory landscape:
vast number of flux vacua on CYs with large Hodge numbers

➤ Constraints on fluxes:

- Integer quantization
- Tadpole cancellation

in F-theory:

$$\frac{1}{2}\int G_4 \wedge G_4 = \frac{\chi(CY_4)}{24} \sim \frac{1}{4} h^{3,1}$$

➤ Tadpole conjecture: [Bena, Blåbäck, Graña, SL '20]

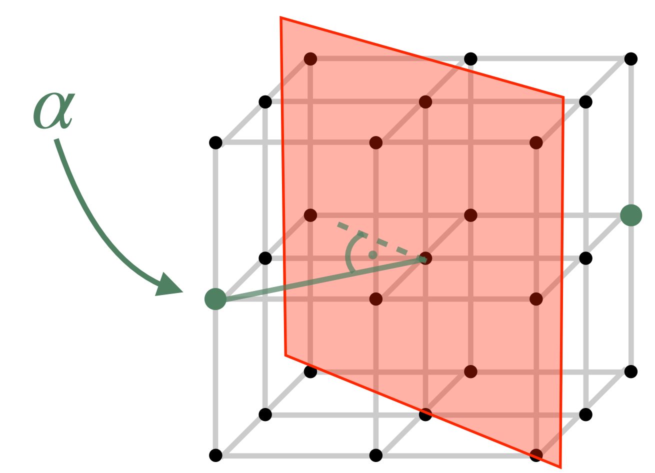
$$\frac{1}{2}\int G_4 \wedge G_4 \gtrsim \alpha \times h^{3,1}$$

(α: $\mathcal{O}(1)$-constant)

➡ Test conjecture for K3 × K3 with evolutionary algorithms

THE PROBLEM

- Input data: even lattice Λ with inner product $d \in \Lambda^\star \otimes \Lambda^\star$ (of indefinite signature)
- Search space: all matrices $G \in \Lambda \otimes \Lambda$ such that
  - GdG^Td and G^TdGd diagonalizable w/ non-negative eigenvalues
  - d has definite signature on all eigenspaces
  - no root $\alpha \in \Lambda$ orthogonal to positive norm eigenvectors



- Target: $Q_{\min}(\Lambda) = \frac{1}{2} \min_G \operatorname{tr}(GdG^Td) = ?$

Relation to Physics:

M-theory on $K3 \times K3$: [Braun, Hebecker, Ludeling, Valandro '08]

- $\Lambda = H^2(K3, \mathbb{Z}) = (-E_8) \oplus (-E_8) \oplus U \oplus U \oplus U$
- $Q(G) = \frac{1}{2} \int G_4 \wedge G_4$ (tadpole charge)

METHOD

- Global optimization problem on integer matrices
- Approach: Differential Evolution
  (in **Julia** using **BlackBoxOptim.jl** [Feldt et al.] and **bbsearch.jl** [Blåbäck])
- Challenges:
  - large search space $\sim e^{\dim(\Lambda)^2}$
  - each step: NP-hard problem (lattice reduction)
- Parameters:
  - population: 1000 - 10000,
  - runtime: few hours - multiple weeks
- Post-processing: local search (brute force) around minima

RESULTS

- Analyzed lattices of various dimensions
- Universal result:

$$Q_{\min}(\Lambda) \geq \dim(\Lambda) - 1$$

(not always saturated)

lattice Λ	$D = \dim(\Lambda)$	$Q_{\min}(\Lambda)$ (found)
$U \oplus U \oplus U$	6	5
$A_4 \oplus U \oplus U$	8	7
E_8	8	8
$E_8 \oplus U$	10	10
$E_8 \oplus U \oplus U$	12	12
$E_8 \oplus U \oplus U \oplus U$	14	13
$E_8 \oplus E_8$	16	16
$E_8 \oplus E_8 \oplus U$	18	20
$E_8 \oplus E_8 \oplus U \oplus U$	20	21
$E_8 \oplus E_8 \oplus U \oplus U \oplus U$	22	25

- K3 × K3: $Q_{\min}(2E_8 \oplus 3U) = 25$
- Tadpole cancellation (on smooth K3 × K3):

$$\frac{1}{2}\int G_4 \wedge G_4 = \frac{\chi}{24} = 24$$

$\lightning \geq 25$