Output Dim. Effects in Untrained NN

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Based on: arXiv:202x.xxxxx (to appear) with J. Halverson and K. Stoner

String_data 2020

N-pt functions at general d_{out}

NN-QFT Correspondence: Function space distribution treatment of NN outputs

$$f(x) = W_1(\sigma(W_0 x + b_0)) + b_1$$

[Halverson, AM, Stoner]

2-pt correlator invariant under $SO(d_{\text{out}})$ symmetry

$$G^{(2)}_{ij}(x_1, x_2) = \mathbb{E}[f_i(x_1) f_j(x_2)] = \mathbb{E}[f(x_1) f(x_2)]\delta_{ij} \qquad i, j = 1, 2 \cdots d_{\text{out}}$$

Mean-free GP: all correlation functions defined in terms of Kernel $K_{12} = \mathbb{E}[f(x_1) f(x_2)]$

➔ All infinite width NN correlation functions invariant under $SO(d_{\text{out}})$. Eg.

$$\begin{aligned} G^{(4)}_{ijkl}(x_1, x_2, x_3, x_4) &= \mathbb{E}[f_i(x_1) f_j(x_2) f_k(x_3) f_l(x_4)] \\ &= K_{12}K_{34}\delta_{ij}\delta_{kl} + K_{13}K_{24}\delta_{ik}\delta_{jl} + K_{14}K_{23}\delta_{il}\delta_{jk} \end{aligned}$$

Q. Is this a quantum symmetry?

Ward identity for NN Correlators

$$\int D\phi e^{-S[\phi]}\phi(x_1)\phi(x_2)\cdots\phi(x_n) = \int D\phi e^{-S[\phi]}\phi'(x_1)\phi'(x_2)\cdots\phi'(x_n)$$

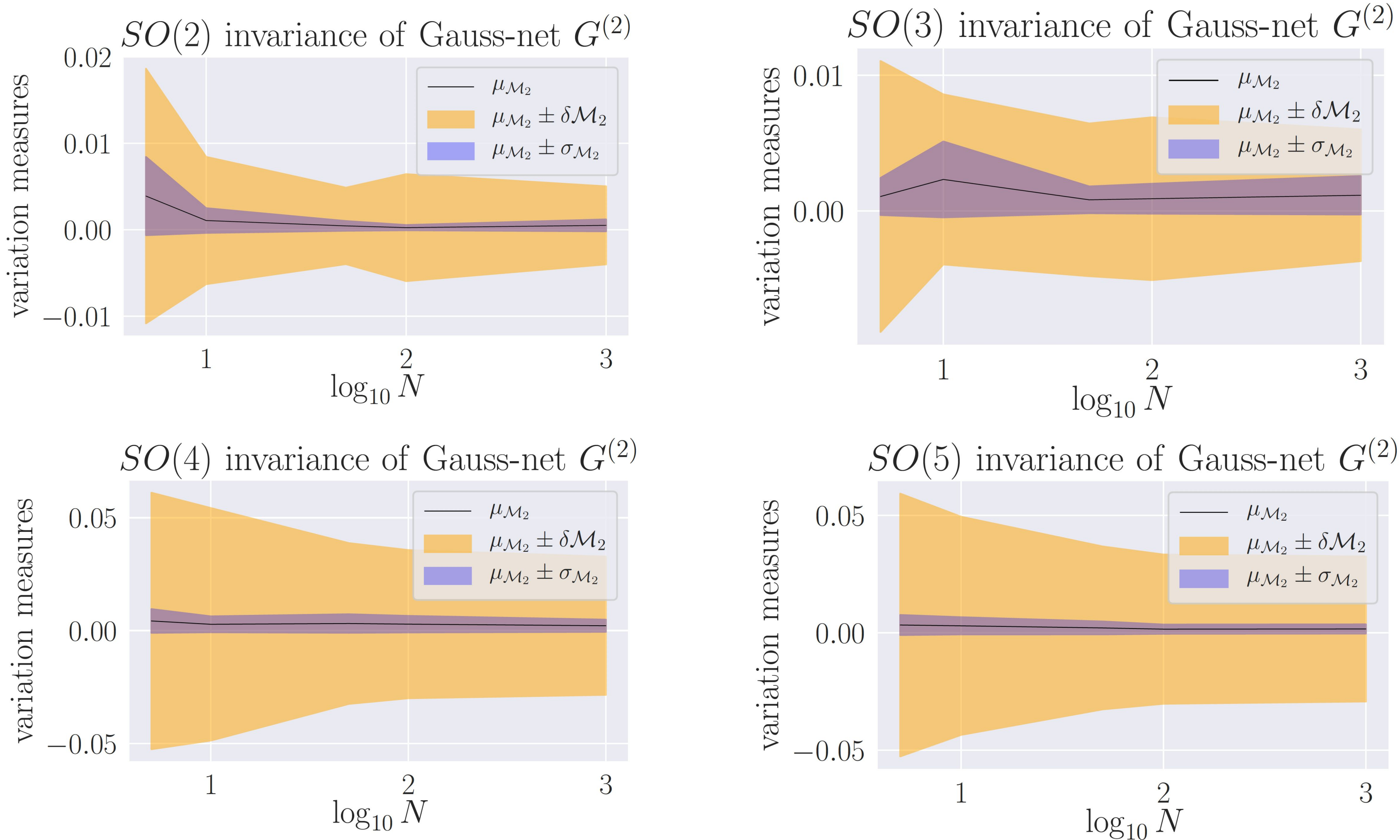
QFT correlators in Euclidean space

NN-QFT Correspondence: $\phi(x_k) \sim f(x_k)$

- Is Ward identity true for infinite width NN correlation functions?
- What about finite width NN correlation functions?

Test Ward identity experimentally

- Generate 1000 random states of $SO(d_{\text{out}})$ group at d_{out} = 2, 3, 4, 5.
- Act upon 2-pt and 4-pt functions of Gaussnet architecture at widths N = 5, 10, 50, 100, 1000 using 10^5 neural nets.
- Measure variation due to $SO(d_{\text{out}})$ transformation as function of width.



$SO(2)$ invariance of Gauss-net $G^{(2)}$
$SO(3)$ invariance of Gauss-net $G^{(2)}$
$SO(4)$ invariance of Gauss-net $G^{(2)}$
$SO(5)$ invariance of Gauss-net $G^{(2)}$
variation measures
$\log_{10} N$
$\mu_{\mathcal{M}_2}$
$\mu_{\mathcal{M}_2} \pm \delta\mathcal{M}_2$
$\mu_{\mathcal{M}_2} \pm \sigma_{\mathcal{M}_2}$
0.02
0.01
0.00
−0.01
0.05
−0.05
1
2
3

2-pt function shows Ward identity

$\mu_{\mathcal{M}_2}$: Mean of variations due to $SO(d_{\text{out}})$ transformation.

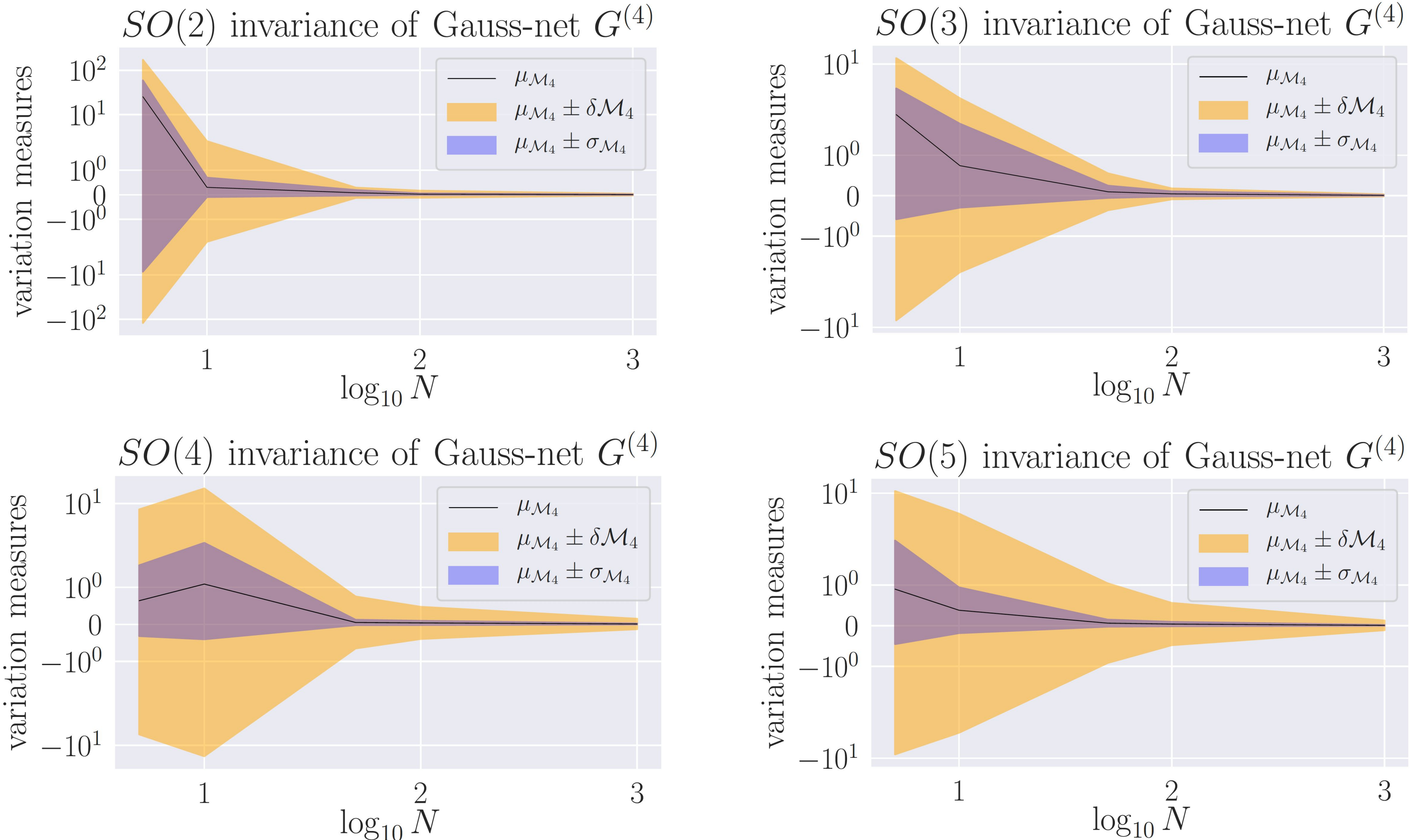
$\mu_{\mathcal{M}_2} \pm \delta\mathcal{M}_2$: Predicted error bounds, using experimental errors.

$\mu_{\mathcal{M}_2} \pm \sigma_{\mathcal{M}_2}$: Actual statistical fluctuations in variation due to $SO(d_{\text{out}})$ transformation.

Gaussnet: Kernel = exact 2-pt function at all widths.
But,

➔ Experimentally, at very low widths, actual means of weights and biases deviate away from 0.

➔ Causing mean of experimental $SO(d_{\text{out}})$ variation measures to be non-zero at low widths.



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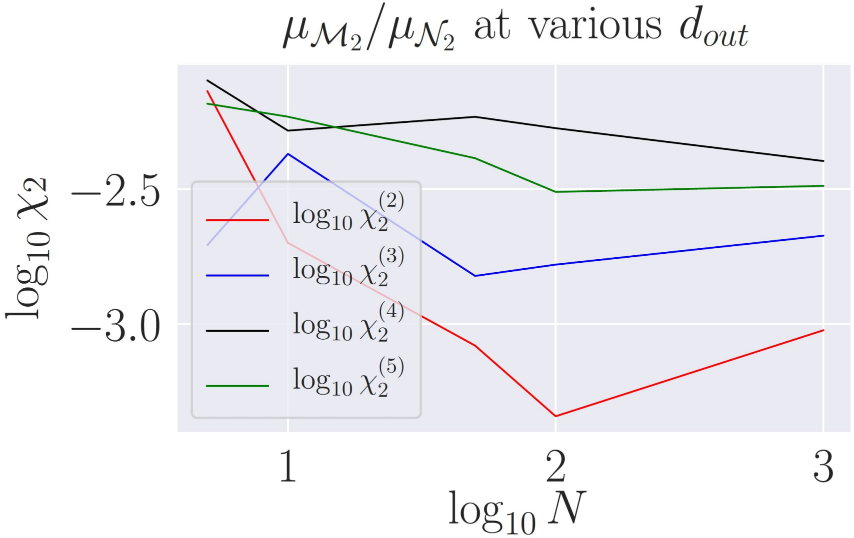
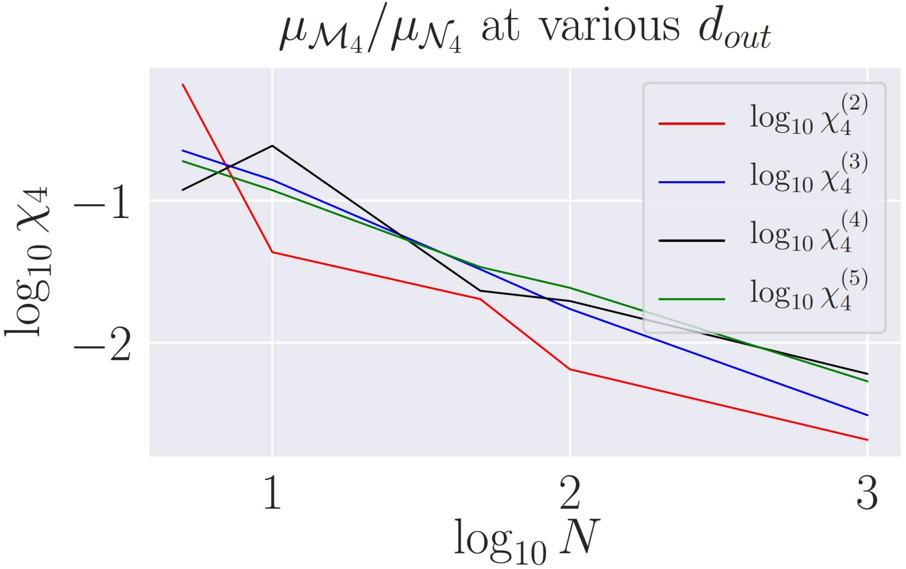
variation measures
$\log_{10} N$
$\mu_{\mathcal{M}_4}$
$\mu_{\mathcal{M}_4} \pm \delta\mathcal{M}_4$
$\mu_{\mathcal{M}_4} \pm \sigma_{\mathcal{M}_4}$
10^2
10^1
10^0
0
-10^0
-10^1
-10^2
1
2
3

➔ 4-pt function shows same artifact at low widths, and Ward identity at high widths.

Compare $SO(d_{\text{out}})$ variation results with random transformations





$\mu_{\mathcal{M}_n}$: Mean of variation of n-pt function due to random $SO(d_{\text{out}})$ transformation

$\mu_{\mathcal{N}_n}$: Mean of variation of n-pt function due to random transformation

Thank you!

- Output dimension induces quantum symmetry
- Away from GP, couplings in dual EFT depend on output dimension of NNs
- Training can change distributions that weights and biases are drawn from. Will break this symmetry.