Background Field Method and Generalized Field Redefinitions in Effective Field Theories

Andrea Quadri
INFN, Sez. di Milano, Italy

RADCOR-LoopFest 2021

Outline

Background Field Method for Gauge EFTs

Dim.4 Lagrangian plus higher dim. ops. arranged in powers of a large inverse energy scale Λ

$$\mathscr{L}_{(SM)EFTs} = \mathscr{L}_4 + \frac{1}{\Lambda} \sum_i c_i^5 \mathscr{O}_i^5 + \frac{1}{\Lambda^2} \sum_i c_i^6 \mathscr{O}_i^6 + \dots$$

compatible with the low-energy symmetry pattern

Renormalizability in the modern sense

Gomis and Weinberg, Nucl.Phys. B469 (1996) 473-487

- Power-counting (p.c.) renormalizabilty is lost (more and more UV divergences at each loop order)
- Locality of the counter-terms (in the sense of formal power series in the fields, the external sources and the momenta) still holds provided that:
  1. generalized (i.e. non-linear) field redefinitons are first appropriately taken into account
  2. the renormalization of the gauge-invariant operators is carried out order by order in the perturbative loop expansion

Derivative interactions

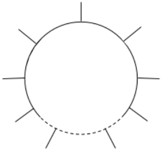
Prototype dim.6 operator

$$\phi^\dagger\phi(D^\mu\phi)^\dagger D_\mu\phi\,, \qquad \phi = \frac{1}{\sqrt{2}}(\sigma + v + i\chi)\,,$$

- Power-counting maximally violated

$$\phi^\dagger\phi(D^\mu\phi)^\dagger D_\mu\phi \supset \sigma^2\partial^\mu\sigma\partial_\mu\sigma$$

Infinite number of UV divergent amplitudes
already at one loop order



$$\delta = 4 + (2-2) + (2-2) + \cdots = 4$$

Field redefinitions vs. Gauge invariant ops.

Which UV divergences are reabsorbed by
(generalized) field redefinitions (GFRs) and which represent
genuine physical renormalizations of gauge inv. ops?

This is a difficult problem. Can the BFM be helpful?

Tree-level background-quantum splitting

$$\Phi = \widehat{\Phi} + Q_\Phi$$

- With an appropriate background gauge-fixing choice a background Ward identity holds true for the 1-PI vertex functional.
- Unlike the ST identity, the background Ward identity is linear in the background and the quantum fields.
- This property yields in turn significant simplifications in the counter-terms structure.

The p.c.-renormalizable case

- $\widehat{\Phi}$ renormalizes multiplicatively.
- This condition plus background gauge invariance ties together the background gauge field and coupling constant renormalizations, so that e.g. the charge renormalization factor can be obtained by evaluating just the gauge amplitude.

Summary of the results

A.Q., e-Print: 2102.10656 [hep-th]

- The tree-level background-quantum splitting is non-linearly deformed;
- A noticeable exception is the Landau gauge, where no such deformation of the tree-level background-quantum splitting happens, to all orders in the loop expansion;
- Background fields also undergo a non-linear redefinition;
- Background fields are (non-trivially) redefined in a background gauge invariant way.

Abelian Higgs-Kibble model supplemented by dim.6 operators:

$$-\frac{z}{2v^2}\Big(\phi^\dagger\phi-\frac{v^2}{2}\Big)\Box\Big(\phi^\dagger\phi-\frac{v^2}{2}\Big)\,,\quad \frac{g_1}{\Lambda^2}\Big(\phi^\dagger\phi-\frac{v^2}{2}\Big)(D^\mu\phi)^\dagger D_\mu\phi\,,$$
$$\frac{g_2}{\Lambda^2}\Big(\phi^\dagger\phi-\frac{v^2}{2}\Big)F^2_{\mu\nu}\,,\quad \frac{g_3}{6\Lambda^2}\Big(\phi^\dagger\phi-\frac{v^2}{2}\Big)^3$$

The background fields renormalize non-linearly

$$\begin{pmatrix}\widehat{\sigma}_R\\ \widehat{\chi}_R\end{pmatrix}=\Big[\alpha_0+\alpha_1\Big(\widehat{\phi}^\dagger\widehat{\phi}-\frac{v^2}{2}\Big)+\alpha_2\Big(\widehat{\phi}^\dagger\widehat{\phi}-\frac{v^2}{2}\Big)^2+\dots\Big]\begin{pmatrix}\widehat{\sigma}+v\\ \widehat{\chi}\end{pmatrix}$$

The coefficients α's depend on the gauge.

Effects of background field renormalizaion

c_j coefficients of gauge invariant operators $\mathcal{I}_j$

- Feynman gauge:

$$\overline{\Gamma}^{(1)\prime}_{\xi=1}\Big|_{Q_\Phi=0} = \sum_j c_j \mathcal{I}_j$$
$$-\int d^4x \frac{M_A^2}{8\pi^2 v^4}\frac{1}{\epsilon}\Big\{\frac{z}{1+z}\Big(\widehat{\phi}^\dagger\widehat{\phi}-\frac{v^2}{2}\Big)-\frac{1}{2v^2}\frac{z(3z-1)}{(1+z)^2}\Big(\widehat{\phi}^\dagger\widehat{\phi}-\frac{v^2}{2}\Big)^2\Big\}\,\mathcal{Z}_2|_{Q_\Phi=0}$$

- Landau gauge:

$$\overline{\Gamma}^{(1)\prime}_{\xi=0}\Big|_{Q_\Phi=0} = \sum_j c_j \mathcal{I}_j$$
$$+\int d^4x \frac{M_A^2}{32\pi^2 v^2}\frac{1}{\epsilon}\Big[2-\frac{4}{v^2}\Big(\frac{z}{1+z}+\frac{g_1 v^2}{\Lambda^2}\Big)\Big(\phi^\dagger\phi-\frac{v^2}{2}\Big)+$$
$$\Big[\frac{2}{v^4}\frac{z(3z-1)}{(1+z)^2}+\frac{g_1^2}{\Lambda^4}\Big]\Big(\phi^\dagger\phi-\frac{v^2}{2}\Big)^2\Big]\,\mathcal{Z}_2|_{Q_\Phi=0}$$

$$\mathcal{Z}_2|_{Q_\Phi=0} = \mathcal{S}_0\Big(\int d^4x\,(\phi^*\phi+h.c.)\Big)\Big|_{Q_\Phi=0} = \int d^4x\,\Big(\widehat{\phi}\frac{\delta\Gamma^{(0)}}{\delta\widehat{\phi}}+\widehat{\phi}^\dagger\frac{\delta\Gamma^{(0)}}{\delta\widehat{\phi}^\dagger}\Big)$$

$\mathcal{Z}_2$ background gauge invariant; differences between gauges vanish on-shell

Off-shell (all-order) Renormalization

- The full off-shell renormalization, as it happens for p.c. renormalizable theories, is very useful for several reasons:
  - Treatment of overlapping divergences by the Bogolubov R-operation (by now several 2-loop computations in the SMEFT are available)
  - One can go beyond the linearized approximation in the BSM couplings usually adopted in EFT computations
  - GFRs' contributions hard (impossible?) to identify by looking only at on-shell amplitudes
  - Symmetries hold off-shell (consistency checks of higher order computations)

Deformation of the Classical Background-Quantum Splitting

The Slavnov-Taylor (ST) identities control the background dependence:

$$\mathcal{S}(\Gamma) = \sum_i \int \mathrm{d}^4x \left(\frac{\delta\Gamma}{\delta\Phi_i^*}\frac{\delta\Gamma}{\delta\Phi_i} + \Omega_{\widehat{\Phi}} \frac{\delta\Gamma}{\delta\widehat{\Phi}} \right) = 0$$

They encode at the quantum level the BRST invariance of the gauge-fixed action, ensure the fulfillment of physical unitarity and uniquely fix the background dependence (at zero ghost number) of the vertex functional.

FDEs for the background

The ST identity uniquely fixes the dependence on the background in the physical sector at zero ghost fields.

Take a derivative w.r.t $\Omega_\mu, \Omega_{\widehat{\sigma}}, \Omega_{\widehat{\chi}}$ and then set all sources and quantum fields with ghost number one equal to zero (denoted by a prime):

$$\Gamma'_{\widehat{\sigma}} = -\int \mathrm{d}^4x \left[\Gamma'_{\Omega_{\widehat{\sigma}}\sigma^*}\Gamma'_{\sigma} + \Gamma'_{\Omega_{\widehat{\sigma}}\chi^*}\Gamma'_{\chi} \right],$$

$$\Gamma'_{\widehat{\chi}} = -\int \mathrm{d}^4x \left[\Gamma'_{\Omega_{\widehat{\chi}}\sigma^*}\Gamma'_{\sigma} + \Gamma'_{\Omega_{\widehat{\chi}}\chi^*}\Gamma'_{\chi} \right]$$

(in the variables $\Phi, \widehat{\Phi}$. Afterwards one sets $\Phi = Q_\Phi + \widehat{\Phi}$)

- Order one in the loop expansion:

$$\Gamma^{(1)\prime}_{\widehat{\sigma}} = -\int \mathrm{d}^4x \left[\Gamma^{(1)\prime}_{\Omega_{\widehat{\sigma}}\sigma^*} \Gamma^{(0)\prime}_{\sigma} + \Gamma^{(1)\prime}_{\Omega_{\widehat{\sigma}}\chi^*} \Gamma^{(0)\prime}_{\chi} \right],$$
$$\Gamma^{(1)\prime}_{\widehat{\chi}} = -\int \mathrm{d}^4x \left[\Gamma^{(1)\prime}_{\Omega_{\widehat{\chi}}\sigma^*} \Gamma^{(0)\prime}_{\sigma} + \Gamma^{(1)\prime}_{\Omega_{\widehat{\chi}}\chi^*} \Gamma^{(0)\prime}_{\chi} \right],$$

  since there are no tree-level mixing of Ω's with the antifields σ^*, χ^* (sources of the BRST transformations of σ, χ)
- The kernels in blue are gauge-dependent. They vanish in the Landau gauge, i.e. the classical background-quantum splitting does not receive any radiative correction.

In this gauge no dependence of the kernels on the background fields, so we obtain a linear dependence on the background fields themselves (a bar denotes the UV divergent part):

$$\overline{\Gamma}^{(1)'} = -\int \mathrm{d}^4x \left[\left(\widehat{\sigma}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\sigma}}\sigma^*} + \widehat{\chi}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\chi}}\sigma^*} \right) \Gamma^{(0)'}_{\sigma} + \left(\widehat{\sigma}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\sigma}}\chi^*} + \widehat{\chi}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\chi}}\chi^*} \right) \Gamma^{(0)'}_{\chi} \right] + \overline{\Gamma}^{(1)'} \Big|_{\widehat{\sigma}=\widehat{\chi}=0} .$$

The background-quantum splitting is non-trivially modified at the quantum level according to (the classical terms are in red):

$$\sigma \to \widehat{\sigma} + Q_\sigma - \widehat{\sigma}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\sigma}}\sigma^*}[\Phi, \Phi^*] - \widehat{\chi}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\chi}}\sigma^*}[\Phi, \Phi^*] \,,$$

$$\chi \to \widehat{\chi} + Q_\chi - \widehat{\sigma}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\sigma}}\chi^*}[\Phi, \Phi^*] - \widehat{\chi}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\chi}}\chi^*}[\Phi, \Phi^*] \,.$$

In the limit of zero BSM couplings we obtain

$$\overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\sigma}}\sigma^*} = \gamma_{\Omega_{\widehat{\sigma}}\sigma^*} = const\,, \quad \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\chi}}\sigma^*} = 0\,,$$
$$\overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\sigma}}\chi^*} = 0\,, \quad \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\chi}}\chi^*} = \gamma_{\Omega_{\widehat{\chi}}\chi^*}\Big|_{z=0} = 0\,,$$

so the background-quantum splitting is modified linearly, as expected by power-counting renormalizability.

Eventually one has to replace in $\left.\overline{\Gamma}^{(1)\prime}\right|_{\widehat{\sigma}=\widehat{\chi}=0}$ the fields $\Phi \rightarrow \widehat{\Phi} + Q_\Phi$ and then set $Q_\Phi = 0$.

This induces an additional contribution to the deformation of the background fields, that can be studied as follows.

There are two contributions in $\left.\overline{\Gamma}^{(1)\prime}\right|_{\widehat{\sigma}=\widehat{\chi}=0}$: the sum over gauge invariants plus the (cohomologically trivial) part proportional to the classical equation of motion, denoted by $\overline{Y}^{(1)}$.

Cohomologically trivial terms

$$
\begin{aligned}
\overline{Y}^{(1)}\Big|_{\xi=1} = \int \mathrm{d}^4x\, \frac{M_A^2}{8\pi^2 v^2}\frac{1}{\epsilon}\Big\{ & \frac{zv}{2(1+z)}\frac{\chi^2}{v^2}\Gamma_\sigma^{(0)} \\
& + \Big[\frac{1}{1+z} - \frac{2z}{(1+z)^2}\frac{\sigma}{v} + \frac{z(3z-1)M_A^2}{(1+z)^3 v^2}\frac{\sigma^2}{v^2} \\
& + \frac{z(z-1)}{2(1+z)^2}\frac{\chi^2}{v^2}\Big]\left(\sigma\Gamma_\sigma^{(0)} + \chi\Gamma_\chi^{(0)}\right)\Big\} + \dots
\end{aligned}
$$

It is convenient to parameterize $\overline{Y}^{(1)}$ as

$$\overline{Y}^{(1)} = \int \mathrm{d}^4x \left(F_\sigma^{(1)} \Gamma_\sigma^{(0)} + F_\chi^{(1)} \Gamma_\chi^{(0)} \right) + \dots$$

with $F_\sigma^{(1)}, F_\chi^{(1)}$ are gauge-dependent functionals depending on the fields and the external sources.

$$\sigma_R = \widehat{\sigma} + q_\sigma - \widehat{\sigma}\, \overline{\Gamma}^{(1)\prime}_{\Omega_{\widehat{\sigma}} \sigma^*} - \widehat{\chi}\, \overline{\Gamma}^{(1)\prime}_{\Omega_{\widehat{\chi}} \sigma^*} + F_\sigma^{(1)} \,,$$

$$\chi_R = \widehat{\chi} + q_\chi - \widehat{\sigma}\, \overline{\Gamma}^{(1)\prime}_{\Omega_{\widehat{\sigma}} \chi^*} - \widehat{\chi}\, \overline{\Gamma}^{(1)\prime}_{\Omega_{\widehat{\chi}} \chi^*} + F_\chi^{(1)} \,.$$

The BGFRs are obtained from the r.h.s. of the above equation after setting $Q_\Phi = 0$ (or equivalently $\Phi = \widehat{\Phi}$).

The contributions from the background-quantum splitting deformation and from the $\overline{Y}^{(1)}$-sector conspire in giving back a background gauge-invariant field redefinition:

$$\begin{pmatrix} \widehat{\sigma}_R \\ \widehat{\chi}_R \end{pmatrix} = \left[\alpha_0 + \alpha_1 \Big(\widehat{\phi}^\dagger \widehat{\phi} - \frac{v^2}{2} \Big) + \alpha_2 \Big(\widehat{\phi}^\dagger \widehat{\phi} - \frac{v^2}{2} \Big)^2 + \dots \right] \begin{pmatrix} \widehat{\sigma} + v \\ \widehat{\chi} \end{pmatrix}$$

Describe the physical scalar mode (of mass M) with the variable X_2 :

$$X_2 \sim \frac{1}{v}\Big(\phi^\dagger\phi - \frac{v^2}{2}\Big)$$

For instance in the X-representation

$$\frac{1}{v}\phi^\dagger\phi(D^\mu\phi)^\dagger D_\mu\phi \sim X_2(D^\mu\phi)^\dagger D_\mu\phi$$

Amplitudes with external X_2-legs are fixed in terms of amplitudes with other fields and external sources insertions, enjoying either a better UV behaviour or resummation.

Background Generalized Field Redefinitions (BGFRs)

- Parameterizing in a gauge-invariant way the physical scalar displays a more regular UV behaviour of the amplitudes.
- With dim.6 ops. a weak p.c. emerges in the fields sector (only a finite number of UV divergent amplitudes at each loop order).
- Decorate with insertions of gauge-invariant external sources (resummation).
- Going on-shell with X_2 and the associated Lagrange multiplier for the X_2-constraint we recover the amplitudes of the ϕ-theory.

Outlook and Conclusions

- BFM in EFTs does not prevent the occurrence of GFRs. They are not even polynomial even for background fields. They matter also in the linearized approximation.
- Background Field Redefinitions are background gauge-invariant (subtle cancellations between the deformation of the classical background quantum-splitting and the cohomologically trivial sector of the fields counter-terms)
- Cohomological tools provide a way to handle this problem and identify the genuine physical renormalizations of gauge invariant operators and to control the BFM renormalization
- Systematic recursive approach in the loop expansion
- Further steps: $SU(2) \times U(1)$

References

D. Binosi, A. Q., JHEP 05 (2020) 141 - e-Print: 2001.07430 [hep-ph]

D. Binosi, A. Q., JHEP 09 (2019) 032 - e-Print: 1904.06692 [hep-ph]

D. Binosi, A. Q., Eur.Phys.J.C 80 (2020) 9, 807 - e-Print: 1904.06693 [hep-ph]

D. Binosi, A. Q., JHEP 04 (2018) 050 - e-Print: 1709.09937 [hep-th]

A. Q., Int.J.Mod.Phys.A 32 (2017) 16, 1750089 - e-Print: 1610.00150 [hep-th]