Power Corrections to event shapes using eikonal dressed gluon exponentiation

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RADCOR-LoopFest 2021

May 18, 2021

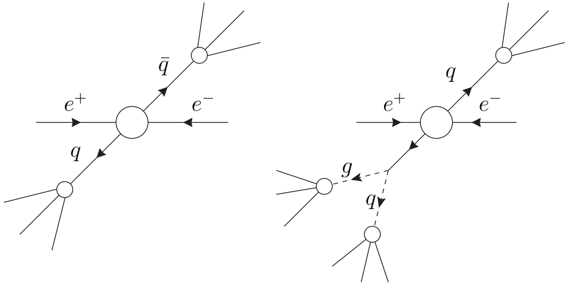
Outline

- Event Shape Variables
- Power Corrections
- Eikonal Dressed gluon exponentiation
- Summary

Event Shapes

What are Event Shapes

- Most basic final state observable in e^+e^- colliders.



- two-jet event $T = 1$
- three-jet event $2/3 \leq T \leq 1$

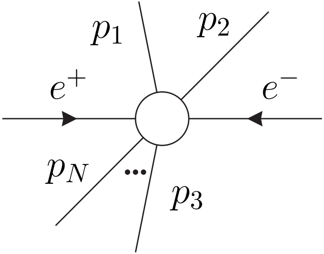
Event shapes

- Thrust
- C parameter
- Angularity
- Jet Mass
- Jet boardening

Definition of event shapes

- Thrust [Farhi (1977)]

$$T = \underset{\mathrm{n}}{\mathrm{Max}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i E_i},$$



- C-parameter [Parisi (1978)]

$$C = 3 - \frac{3}{2} \sum_{i,j} \frac{\left(p^{(i)} \cdot p^{(j)}\right)^2}{(p^{(i)} \cdot q)(p^{(j)} \cdot q)}$$

$$Q = \sum_i E_i,$$
$$q = p_1 + p_2 + p_3 + \dots p_N$$

- Angularity [Berger,Sterman (2003)]

$$\tau_a = \frac{1}{Q} \sum_i E_i (\sin\theta_i)^a (1 - |\cos\theta_i|)^{1-a}$$

Power Corrections

Power corrections

- A physical quantity has the form

$$\sigma \;=\; \sigma_{\rm pert} + \sum_n \sigma_n \left(\frac{\Lambda}{Q}\right)^n$$

- Calculation of Power corrections from perturbative corrections.

Power corrections

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$$\sigma = \sigma_{\rm pert} + \sum_n \sigma_n \left(\frac{\Lambda}{Q}\right)^n$$

- Calculation of Power corrections from perturbative corrections.
- $\sigma_{\rm Pert}$ is not well defined: factorial growth.
- σ_n is ambiguous
- Ambiguity in σ_n is compensated by the factorial growth of $\sigma_{\rm Pert}$.

Borel summation

- A physical quantity

$$R \approx \sum_{n=0}^{\infty} r_n \, \alpha^{n+1}$$

- Borel transform

$$B[R](u) = \sum_{n=0}^{\infty} r_n \, \frac{u^n}{n!}$$

- Borel sum

$$\tilde{R} = \int_0^{\infty} du \, e^{-\frac{u}{\alpha}} \, B[R](u)$$

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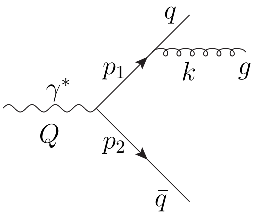
- pole for $B[R](u)$ is p/β_0, ambiguity $\delta\tilde{R} \propto (\frac{\Lambda}{Q})^{2p}$
  $\alpha_s \approx \frac{1}{\beta_0 log(Q^2/\Lambda^2)}$

Dressed gluon exponentiation



$\xi = k^2/Q^2$

- Single Dressed gluon [Beneke, Braun (1994))

$$\frac{1}{\sigma}\frac{d\sigma}{de}(e, Q^2) = -\frac{C_F}{2\beta_0}\int_0^1 d\xi \, \frac{d\mathcal{F}(e,\xi)}{d\xi} \, A(\xi Q^2)\,,$$

- $\mathcal{F}$ is Characteristic function
- Running Coupling

$$A(\xi Q^2) = \int_0^\infty du (Q^2/\Lambda^2)^{-u} \frac{\sin \pi u}{\pi u} \mathrm{e}^{\frac{5}{3}u} \xi^{-u}$$

Contd. . .

- Interchanging the order of integration

$$\frac{1}{\sigma}\frac{d\sigma}{de}(e, Q^2) = \frac{C_F}{2\beta_0}\int_0^\infty du(Q^2/\Lambda^2)^{-u}B(e, u)$$

- Borel function

$$B(e, u) = -\frac{\sin \pi u}{\pi u}e^{\frac{5}{3}u}\int_0^\infty d\xi\ \xi^{-u}\frac{d\mathcal{F}(e, \xi)}{d\xi}$$

- $B(e, u)$ is free from any $u = 0$ poles.

Sudakov Logs

- Additive property of the event shape variables with respect to multiple gluon emissions
- Exponentiation in Laplace space

$$\frac{1}{\sigma}\frac{d\sigma(e,Q^2)}{de} = \int \frac{d\nu}{2\pi i}\, e^{\nu e} \exp[S(\nu,Q^2)]$$

- Sudakov Logs [Gardi (2001)]

$$S(\nu,Q^2) = \frac{C_F}{2\beta_0}\int_0^\infty du \left(\frac{Q^2}{\Lambda^2}\right)^{-u} B_\nu^e(u)$$

- Borel function is Laplace space

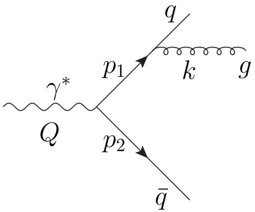
$$B_\nu^e(u) = \int_0^1 de\, B(e,u)\,(e^{-\nu e}-1).$$

Eikonal dressed gluon exponentiation

Steps for EDGE

- Soft approximated Matrix element.
- Factorization of phase space
- Soft approximated version of the relevant event shapes
- Produces the Leading terms without the painful calculations

Matrix element



$$\xi = k^2/Q^2$$

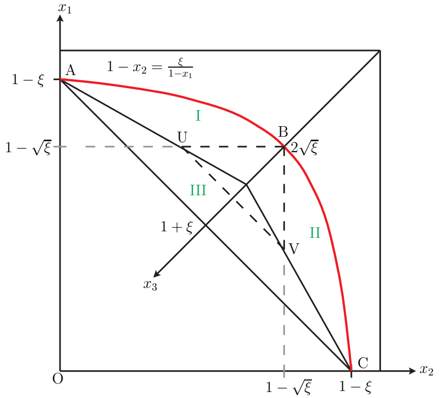
- Energy fractions

$$x_1 = \frac{2p_1 \cdot Q}{Q^2}, \quad x_2 = \frac{2p_2 \cdot Q}{Q^2}, \quad x_3 = \frac{2k \cdot Q}{Q^2}.$$

- Soft approximated matrix element

$$\mathcal{M}_{\mathrm{soft}}(x_1, x_2, \xi) = \frac{2}{(1-x_1)(1-x_2)}.$$

Phase Space



- Characteristic function [Webber et.al. (1995)]

$$\mathcal{F}(e,\xi) = \int dx_1 dx_2\, \mathcal{M}_{\text{soft}}(x_1,x_2,\xi)\, \delta\left(e - \bar{e}_{\text{eik}}(x_1,x_2,\xi)\right)$$

Eikonal definitions of event shapes

T	$\text{Max}\left\{x_1, x_2, \sqrt{x_3^2 - 4\xi}\right\}$
$c_{eik}(x_1, x_2)$	$\frac{(1-x_1)(1-x_2)}{(1-x_1)+(1-x_2)}$
$\tau_a^{eik}(x_1, x_2, \xi)$	$(1 - x_1)^{1-a/2}(1 - x_2)^{a/2}$

Borel function under EDGE

$B(t,u)$	$4\frac{\sin\pi u}{\pi u}e^{\frac{5u}{3}}\frac{1}{u}\frac{1}{t}\left(\frac{1}{t^{2u}}-\frac{1}{t^{u}}\right)$
$B(c,u)$	$4\frac{\sin\pi u}{\pi u}e^{\frac{5u}{3}}\frac{1}{c}\left[\frac{1}{(2c)^{2u}}\frac{\sqrt{\pi}\Gamma(u)}{\Gamma(u+\frac{1}{2})}-\frac{1}{uc^{u}}\right]$
$B(\tau_a,u)$	$4\frac{\sin\pi u}{\pi u}e^{\frac{5}{3}u}\frac{1}{1-a}\frac{1}{\tau_a}\left[\frac{1}{\tau_a^{2u}}-\frac{1}{\tau_a^{\frac{2u}{2-a}}}\right]$

[Agarwal, Mukhopadhyay, SP, Tripathi (2021)]

- $B(e,u)$ are free from any $u=0$ poles.
- $B(t,u)$, $B(c,u)$ and $B(\tau_a,u)$ produces the leading order terms correctly as compared to the full results.

Event shapes in $k_\perp$ and y

- In the soft limit [Salam, Wicke (2001)]

$$\bar{e}(k, Q) = \sqrt{\frac{k_\perp^2 + k^2}{Q^2}}\, h_e(y)$$

- $h_e(y)$

1-Thrust (t)	$e^{-\|y\|}$
c-parameter	$\frac{1}{\cosh\, y}$
Angularity	$e^{-\|y\|(1-a)}$

- Characteristic function in these co-ordinates

$$\mathcal{F}(e, \xi) = \frac{8}{e}\int_{y_{\min}} dy$$

Borel function using these co-ordinates

- $y_{\min}$ for different event shapes

t	$\ln\left(\frac{1}{t}\sqrt{\xi}\right)$
c	$\cosh^{-1}\left(\sqrt{\xi}/(2c).\right)$
τ_a	$\frac{1}{1-a}\ \ln\left(\frac{1}{\tau_a}\sqrt{\xi}\right)$

- Reproduces the same $\mathcal{F}$.
- Reproduces the same Borel function $B(t,u)$, $B(c,u)$ and $B(\tau_a,u)$. compared to energy fractions.

Borel function in Laplace space

$$B_\nu^{t,\text{eik}}(u) = 2\, e^{\frac{5}{3}u} \frac{\sin \pi u}{\pi u} \times \left[\Gamma(-2u)\,(\nu^{2u}-1)\frac{2}{u} - \Gamma(-u)\,(\nu^{u}-1)\frac{2}{u}\right]$$

[Agarwal, Mukhopadhyay, SP, Tripathi (2021)]

$$B_\nu^{t}(u) = 2\mathrm{e}^{\frac{5}{3}u} \frac{\sin \pi u}{\pi u} \times \left[\Gamma(-2u)\,(\nu^{2u}-1)\,\frac{2}{u} - \Gamma(-u)\,(\nu^{u}-1)\left(\frac{2}{u}+\frac{1}{1-u}+\frac{1}{2-u}\right)\right]$$

[Gardi, Rathsman (2001)]

- No extra renormalon singularities as compared to full result.
- $B_t^{\nu,\text{eik}}$ has renormalon poles only at half-integer values of u.

Expansion

$$B_\nu^{t,\text{eik}}(u) = -\,2\,L^2 - 2.31\,L + \left(-2\,L^3 - 6.79\,L^2 - 15.71\,L\right)u + \left(-1.167\,L^4 - 6.02\,L^3 - 19.10\,L^2 - 44.56\,L\right)u^2 + \ldots$$

[Agarwal, Mukhopadhyay, SP, Tripathi (2021)]

$$B_\nu^{t}(u) = -2\,L^2 + 0.691\,L + \left(-2\,L^3 - 5.297\,L^2 - 6.485\,L\right)\,u + \left(-1.167\,L^4 - 5.527\,L^3 - 14.491\,L^2 - 31.655\,L\right)\,u^2 + \ldots,$$

[Gardi, Magnea (2003)]

$L = \log(\nu)$, the leading log terms are produced correctly.

Soft and collinear corrections

$$S(\nu, Q^2) \approx \int_0^\infty du \left(\frac{Q^2}{\Lambda^2}\right)^{-u} B_\nu^t(u)$$

order	soft correction	collinear correction
ν^1	$8\frac{\bar{\Lambda}}{Q}$	$-\frac{2}{\pi}(\frac{\bar{\Lambda}}{Q})^2$

In LEP the ratio of the size of the collinear correction to the soft correction for ν^1 is approximately -0.0017

Summary and Conclusions

- Perturbative series has a factorial growth which makes the series divergent.
- Power corrections can be estimated from the ambiguity in the perturbative series.
- Eikonal approximation of matrix element and event shapes produces the leading order terms correctly in few simple steps.
- EDGE works in both energy fraction and light cone co-ordinates.
- This process may be useful for complicated hadron event shapes in future.


Thank you