Recent progress on two-loop massless pentabox integrals with one off-shell leg

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in collaboration with
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INPP, NCSR “Demokritos”, 15310 Athens, Greece

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Space-Time Approach to Quantum Electrodynamics

R. P. Feynman
Department of Physics, Cornell University, Ithaca, New York
(Received May 9, 1949)

In this paper two things are done. (1) It is shown that a considerable simplification can be attained in writing down matrix elements for complex processes in electrodynamics. Further, a physical point of view is available which permits them to be written down directly for any specific problem. Being simply a

and presumably consistent, method is therefore available for the calculation of all processes involving electrons and photons.

The simplification in writing the expressions results from an emphasis on the overall space-time view resulting from a study of the solution of the equations of electrodynamics. The relation

D. More Complex Problems

Matrix elements for complex problems can be set up in a manner analogous to that used for the simpler cases. We give three illustrations; higher order corrections to the Møller scatter-

Fig. 1. The fundamental interaction Eq. (4). Exchange of one quantum between two electrons.
Towards higher precision: NNLO and beyond
explosion of calculations in past 18 months

Missing generic approach

C.G.Papadopoulos (INPP)  Radcor-Loopfest 2021  FSU  4 / 27
NNLO QCD: $pp \rightarrow \gamma\gamma\gamma + X$

**Figure 1.** Predictions for the fiducial cross-section in LO (green), NLO (blue) and NNLO (red) QCD versus ATLAS data (black). Shown are predictions for six scale choices. The error bars on the theory predictions reflect scale variation only. For two of the scales only the central predictions are shown.

**Figure 2.** $p_T$ distribution of the hardest photon $\gamma_1$ (left), $\gamma_2$ (center) and the softest one $\gamma_3$ (right). Top plot shows the absolute distribution at NNLO (red), NLO (blue) and LO (green) versus ATLAS data (black). Middle plot shows same distributions but normalized to the NLO. Bottom plot shows central NNLO predictions for 6 different scale choices (only the central scale is shown) with respect to the default choice $\mu_0 = H_T/4$. The bands represent the $\tau$-point scale variations about the corresponding central scales.

NNLO QCD: $pp \rightarrow \gamma\gamma\gamma + X$

A fiducial setup for $pp \rightarrow \gamma\gamma\gamma + X$; used in the ATLAS 8 TeV analysis of Ref. [37]

- $p_{T,\gamma_1} \geq 27$ GeV,
- $p_{T,\gamma_2} \geq 22$ GeV,
- $p_{T,\gamma_3} \geq 15$ GeV,
- $0 \leq |\eta_\gamma| \leq 1.37$ or $1.56 \leq |\eta_\gamma| \leq 2.37$,
- $\Delta R_{\gamma\gamma} \geq 0.45$,
- $m_{\gamma\gamma} \geq 50$ GeV,
- Frizone isolation with $n = 1$, $\delta_0 = 0.4$, and $E_T^{iso} = 10$ GeV.

Table 1: Definition of phase space cuts.

![Graphs showing the dependency of cross sections on center-of-mass energy](image)

Figure 4: Fiducial cross sections for $pp \rightarrow \gamma\gamma\gamma + X$ as a function of the centre-of-mass energy at LO (black dotted), at NLO (red dashed), and at NNLO (blue, solid). The green data point at 8 TeV corresponds to the cross section measured by ATLAS in Ref. [37].


- B. Agarwal, F. Buccioni, A. von Manteuffel and L. Tancredi, “Two-loop leading colour QCD corrections to $q\bar{q} \rightarrow \gamma\gamma g$ and $qg \rightarrow \gamma\gamma q,”$ JHEP 2104 (2021) 201


+ rational terms, IBP reduction, computation of Master Integrals, etc.
TWO-LOOP GRAPH
The three planar pentaboxes of the families $P_1$ (left), $P_2$ (middle) and $P_3$ (right) with one external massive leg.

The five non-planar families with one external massive leg.
5-Box - One Leg Off-shell: P1

\[ q_1 \rightarrow p_{123} - xp_{12}, \quad q_2 \rightarrow p_4, \quad q_3 \rightarrow -p_{1234}, \quad q_4 \rightarrow xp_1 \]

SDE parametrisation: \( n \) off-shell legs $\rightarrow$ \( n - 1 \) off-shell legs + the \( x \) variable.

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[talk by N. Syrrakos]

- \( p_i, \ i = 1 \ldots 5 \), satisfy \( \sum_{1}^{5} p_i = 0 \), with \( p_i^2 = 0, \ i = 1 \ldots 5, \ p_{i \ldots j} := p_i + \ldots + p_j \).

The set of independent invariants: \{ \( S_{12}, S_{23}, S_{34}, S_{45}, S_{51}, x \) \}, with \( S_{ij} := (p_i + p_j)^2 \).

\[ q_1^2 = (1 - x)(S_{45} - S_{12}x), \quad s_{12} = (S_{34} - S_{12}(1 - x))x, \quad s_{23} = S_{45}, \quad s_{34} = S_{51}x, \]
\[ s_{45} = S_{12}x^2, \quad s_{15} = S_{45} + (S_{23} - S_{45})x \]
5Box - One Leg off-shell: P1
4-POINT UP TO TWO LEGS OFF-SHELL

J. M. Henn, K. Melnikov and V. A. Smirnov, JHEP 1405 (2014) 090
F. Caola, J. M. Henn, K. Melnikov and V. A. Smirnov, JHEP 1409 (2014) 043

Figure 3. The parametrization of external momenta for the three planar double boxes of the families $P_{12}$ (left), $P_{13}$ (middle) and $P_{23}$ (right) contributing to pair production at the LHC. All external momenta are incoming.

Figure 4. The parametrization of external momenta for the three non-planar double boxes of the families $N_{12}$ (left), $N_{13}$ (middle) and $N_{34}$ (right) contributing to pair production at the LHC. All external momenta are incoming.

As well as planar and nonplanar double box with one off-shell leg expressed in UT basis.
where \( q_{i\ldots j} := q_i + \ldots + q_j \).
\[ d \vec{g} = \epsilon \sum_a d \log (W_a) \tilde{M}_a \vec{g} \]

- Also from direct differentiation of MI wrt to \( x \). Just \( g \) in terms of FI.

\[ \frac{d \vec{g}}{dx} = \epsilon \sum_b \frac{1}{x - \ell_b} M_b \vec{g} \]

- \( \ell_b \) are independent of \( x \), some depending only on the reduced invariants, \( \{ S_{12}, S_{23}, S_{34}, S_{45}, S_{51} \} \). \( M_b \) are independent of the invariants.
- Number of letters smaller than in AIMPTZ representation.
- Main contribution for us from AIMPTZ: the canonical basis (+ numerics).
$d\vec{g} = \epsilon \sum_a d\log(W_a) \tilde{M}_a \vec{g}$

$\frac{d \log(W_a)}{dx}$

- Also from direct differentiation of MI wrt to $x$. Just $g$ in terms of FI.

$\frac{d\vec{g}}{dx} = \epsilon \sum_b \frac{1}{x - \ell_b} M_b \vec{g}$

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number of letters smaller than in AIMPTZ representation

Main contribution for us from AIMPTZ: the canonical basis (+ numerics)
\[
\frac{dg}{dx} = \sum_a \frac{1}{x - \ell_a} M_a g
\]

\[
g = \epsilon^0 b_0^{(0)} + \epsilon \left( \sum G_a M_a b_0^{(0)} + b_0^{(1)} \right) \\
+ \epsilon^2 \left( \sum G_{ab} M_a M_b b_0^{(0)} + \sum G_a M_a b_0^{(1)} + b_0^{(2)} \right) \\
+ \epsilon^3 \left( \sum G_{abc} M_a M_b M_c b_0^{(0)} + \sum G_{ab} M_a M_b b_0^{(1)} + \sum G_a M_a b_0^{(2)} + b_0^{(3)} \right) \\
+ \epsilon^4 \left( \sum G_{abcd} M_a M_b M_c M_d b_0^{(0)} + \sum G_{abc} M_a M_b M_c b_0^{(1)} \\
+ \sum G_{ab} M_a M_b b_0^{(2)} + \sum G_a M_a b_0^{(3)} + b_0^{(4)} \right) + \ldots
\]

\[
G_{ab\ldots} := G(\ell_a, \ell_b, \ldots; x)
\]

5box - one leg off-shell: Boundary conditions

- starting from the full equation

\[ \frac{d\vec{g}}{dx} = \frac{1}{x} M_0 \vec{g} + \mathcal{O}(x^0) \]

- using all letters \( W_a \), with the solution \( (b := \sum_{i=0}^{4} \epsilon^i b_0^{(i)}) \)

\[ g_0 = S e^{\epsilon \log(x) D} S^{-1} b \]

- \( S \) and \( D \) are obtained through Jordan decomposition of the \( M_0 \)

- Resummed: \( R_0 = S e^{\epsilon \log(x) D} S^{-1} \)

- What we know about:

\[ R_0 = \sum_i x^{n_i \epsilon} R_{0i} + \sum_j \epsilon x^{n_j \epsilon} \log(x) R_{0j0} \]
IBP reduction in terms of Master Integrals

\[ g = \mathbf{T}G. \]


Expansion by regions. [no logarithmic terms]

\[ G_i = \sum_j x^{b_j + a_j \varepsilon} G_i^{(j)} \]

Linear equations:

\[ g_0 := \mathbf{R}_0 \mathbf{b} = \lim_{x \to 0} \mathbf{T}G \bigg|_{O(x^{0+a_j \varepsilon})} \]

Matrix \( \mathbf{T} \) is horrible-looking depending on \( x, \varepsilon \) and \( S_{ij} \). But

\[ \mathbf{R}_0 \mathbf{b} \to \varepsilon, x, \text{Rationals} \otimes \text{polyLogs} \quad G_i^{(j)} \to \text{Simple} [S_{ij}] \otimes \text{polyLogs} \]

so we can afford IBP reduction with only \( x, \varepsilon \) symbolic: i.e. FIRE6 or Kira2.
No regions in the top-sector are needed.

To obtain expressions for regions, $G_i^{(j)}$, in Feynman parameter space, we use FIESTA \texttt{asyexpand}, for $x \to 0$ limit (SDE).

In most cases integration is straightforward and the resulting $2F_1$ hypergeometric functions are expanded with HypExp.

In few cases we use Mellin-Barnes techniques using the MB, MBSums and XSummer along with the in-house (A. Kardos) package Gsuite.

Boundary terms only depends on 12 Goncharov

$$G \left[0, 1, -\frac{S_{12} - S_{34}}{S_{51}}\right], G \left[1, -\frac{S_{12} - S_{34}}{S_{51}}\right], G \left[0, 0, 1, -\frac{S_{12} - S_{34}}{S_{51}}\right], G \left[0, 1, 1, -\frac{S_{12} - S_{34}}{S_{51}}\right],$$

$$G \left[1, 0, 1, -\frac{S_{12} - S_{34}}{S_{51}}\right], G \left[0, 0, 1, 1, -\frac{S_{12} - S_{34}}{S_{51}}\right], G \left[0, 1, 1, 1, -\frac{S_{12} - S_{34}}{S_{51}}\right],$$

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and 4 Logarithms $\{\log[-S_{12}], \log[-S_{45}], \log[S_{12} - S_{34}], \log[-S_{51}]\}$.

Euclidean region:

\[ \left\{ S_{12} \rightarrow -2, S_{23} \rightarrow -3, S_{34} \rightarrow -5, S_{45} \rightarrow -7, S_{51} \rightarrow -11, x \rightarrow \frac{1}{4} \right\} \]

no letter / in the region \([0, x]\), all boundary terms real. [very fast GiNaC]

<table>
<thead>
<tr>
<th>Family</th>
<th>W=1</th>
<th>W=2</th>
<th>W=3</th>
<th>W=4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_1) ((g_{72}))</td>
<td>17 (14)</td>
<td>116 (95)</td>
<td>690 (551)</td>
<td>2740 (2066)</td>
</tr>
<tr>
<td>(P_2) ((g_{73}))</td>
<td>25 (14)</td>
<td>170 (140)</td>
<td>1330 (1061)</td>
<td>4950 (3734)</td>
</tr>
<tr>
<td>(P_3) ((g_{84}))</td>
<td>22 (12)</td>
<td>132 (90)</td>
<td>1196 (692)</td>
<td>4566 (2488)</td>
</tr>
</tbody>
</table>

**Table:** Number of GP entering in the solution, as explained in the text.

- with timings, running the GiNaC Interactive Shell `ginsh`, given by 1.9, 3.3, and 2 seconds for \(P_1\), \(P_2\) and \(P_3\) respectively and for a precision of 32 significant digits
- A very different canonical basis, several elements start at \(\epsilon^4\).
One-scale integrals - closed form

\[ (-s_{34})^{-\epsilon} = (-S_{51})^{-\epsilon} x^{-\epsilon} \]
\[ (-s_{45})^{-\epsilon} = (-S_{12})^{-\epsilon} x^{-2\epsilon} \]
\[ (-s_{15})^{-\epsilon} = (-S_{45})^{-\epsilon} \left( 1 - \frac{S_{45} - S_{23}}{S_{45}} x \right)^{-\epsilon} \]
\[ (-p_{1s})^{-\epsilon} = (1 - x)^{-\epsilon} (-S_{45})^{-\epsilon} \left( 1 - \frac{S_{12}}{S_{45}} x \right)^{-\epsilon} \]
\[ (-s_{12})^{-\epsilon} = x^{-\epsilon} (S_{12} - S_{34})^{-\epsilon} \left( 1 - \frac{S_{12}}{S_{12} - S_{34}} x \right)^{-\epsilon} \]

One-scale integrals - expanded form

\[ \text{Log}[-p_{1s} - i\delta] \rightarrow G[1, x] + G \left[ \frac{S_{45}}{S_{12}}, x \right] + \text{Log}[-S_{45}], \]
\[ \text{Log}[-s_{34} - i\delta] \rightarrow \text{Log}[-S_{51}] + \text{Log}[x], \]
\[ \text{Log}[-s_{12} - i\delta] \rightarrow G \left[ \frac{S_{12} - S_{34}}{S_{12}}, x \right] + \text{Log}[S_{12} - S_{34}] + \text{Log}[x], \]
\[ \text{Log}[-s_{45} - i\delta] \rightarrow \text{Log}[-S_{12}] + 2 \text{Log}[x], \]
\[ \text{Log}[-s_{15} - i\delta] \rightarrow G \left[ \frac{S_{45}}{-S_{23} + S_{45}}, x \right] + \text{Log}[-S_{45}] \]
In general many letters will be now in \([0, x]\). This has two consequences:

1. Need to fix infinitesimal imaginary part of \( \frac{l_i}{x} \)
2. Increasing CPU time in GiNaC.

Since the \( \mathcal{F} \) polynomial maintains the sign of the \( i0 \) prescription of Feynman propagators with all original invariants assuming \( s_{ij}(p_{1s}) \rightarrow s_{ij}(p_{1s}) + i\delta \), we determine the corresponding infinitesimal imaginary part of \( \frac{l_i}{x} \) from

\[
p_{1s} + i\delta = (1 - x)(S_{45} - S_{12}x), \quad s_{12} + i\delta = (S_{34} - S_{12}(1 - x))x,
\]
\[
s_{23} + i\delta = S_{45}, \quad s_{34} + i\delta = S_{51}x,
\]
\[
s_{45} + i\delta = S_{12}x^2, \quad s_{15} + i\delta = S_{45} + (S_{23} - S_{45})x
\]

with \( S_{ij} \rightarrow S_{ij} + i\delta \eta_{ij}, \ x \rightarrow x + i\delta \eta_x, \)

- Building a Fibration Basis using for instance PolyLogTools.
All regions of AIMPTZ checked @precision

One-loop pentagon at order $\mathcal{O}(\varepsilon^4)$ [any order, analytic]


[talk by N. Syrrakos]

Taken the limit $x = 1$ in all families to obtain the result for on-shell planar 5box

SDE is not only capable to produce analytic results for off-shell MI but it can also give, almost for free, the on-shell MI.

[talk by D. Canko]

Evaluating phase-space points for $pp \rightarrow W^+ j_1 j_2$ generated by HELAC-PHEGAS, i.e. arbitrary floating points.
Non-planar families
- We have completed the first hexa-box family, N1.
- Preliminary check against AIMPTZ group results @lowaccuracy: at least 13 digits.
- N2 and N3 families UT bases from AIMPTZ: analytic results expected soon.
- Complete all 5-point families: penta-pentagon N4 and N5.
- Obtain all 5-point families with on-shell legs, $x \to 1$. [new wrt pentagon functions?]
- MI: 4-point with up to 2 off shell legs and 5-point with up to one off-shell leg.

Speed-up numerical evaluation
- Improving GPLs analytic continuation.
- Study letters ordering in physical regions, use different mappings and/or fibrations.
- Combine analytics with numerics. [talk by Martijn Hidding]

Massive internal particles.

**HELAC2LOOP**: generic approach to amplitude reduction and evaluation
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5BOX - ONE LEG OFF-SHELL: OUTLOOK

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**HELAC2LOOP**: generic approach to amplitude reduction and evaluation
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Thank you for your attention!
Backup slides
5box - one leg off-shell: P2-3

The two-loop diagram representing the decoupling basis element.

- Basis element 46 for P2 (53 for P3) known from double box P23 family; starts at $\mathcal{O}(\epsilon^4)$. [decoupling]

\[ q_1 \to P_{123} - y P_{12}, \quad q_2 \to y P_1, \quad q_3 \to P_4, \quad q_4 \to -P_{1234}, \quad q_5 \to y P_2 \]

\[ q_1 \to p_{123} - x p_{12}, \quad q_2 \to p_4, \quad q_3 \to -p_{1234}, \quad q_4 \to x p_1, \quad q_5 \to x p_2 \]

\[ q_1^2 = (1 - y)(S_{45}' - S_{12}'), s_{12} = S_{45}' - (S_{12} + S_{23})y, \quad s_{23} = (S_{34}' - S_{12}(1 - y))y, \]

\[ s_{34} = S_{45}', \quad s_{45} = -(S_{12}' - S_{34}' + S_{51}')y, \quad s_{15} = S_{45}' + S_{23}y \]

\[ q_1^2 = (1 - x)(S_{45} - S_{12}x), \quad s_{12} = (S_{34} - S_{12}(1 - x))x, \quad s_{23} = S_{45}, \quad s_{34} = S_{51}x, \]

\[ s_{45} = S_{12}x^2, \quad s_{15} = S_{45} + (S_{23} - S_{45})x \]
\[ d\vec{g} = \left[ \sum_b d \log (x - \ell_b) \, M_b + \sum_c d \log (y - \ell_c) \, \tilde{M}_c + d \log (W_{58} (x, y)) \, \tilde{M}_{58} \right] \vec{g} \]

- All letters \( W_a \), except \( W_{58} \), are linear functions only of \( x \) or \( y \).
- \( M \) matrices have zeroes in the row and the column corresponding to the basis element 46 for P2 (53 for P3).
- \( \tilde{M} \) matrices have non-zero matrix elements only in the row and the column corresponding to the basis element 46 for P2 (53 for P3).
- \( \tilde{M} \) matrix have non-zero matrix elements only in the column corresponding to the basis element 46 for P2 (53 for P3).

\[ \frac{d\vec{g}'}{dx} = \sum_a \frac{1}{x - \ell_a} \, M_a \vec{g}' \]