



# DIFFERENTIAL $\gamma^*$ PRODUCTION WITH N3LO QCD CORRECTIONS RADCOR & LOOPFEST 2021

# ??? IN THE DETAILS

- Precision phenomenology examples
- g-2 exam with  $4.2\sigma$  significance

$$a_\mu = \frac{g - 2}{2} \quad a_\mu^{\text{EXP}} = 116592061(41) \times 10^{-11}$$

*Phys.Rev.Lett. 126, 141801 (2021)*

$$a_\mu^{\text{SM}} = 116591810(43) \times 10^{-11}$$

More in Massimo's talk [Phy. Reports 887 \(2020\) 1-116](#)

- Beauty-quark decays with  $3.1\sigma$  difference

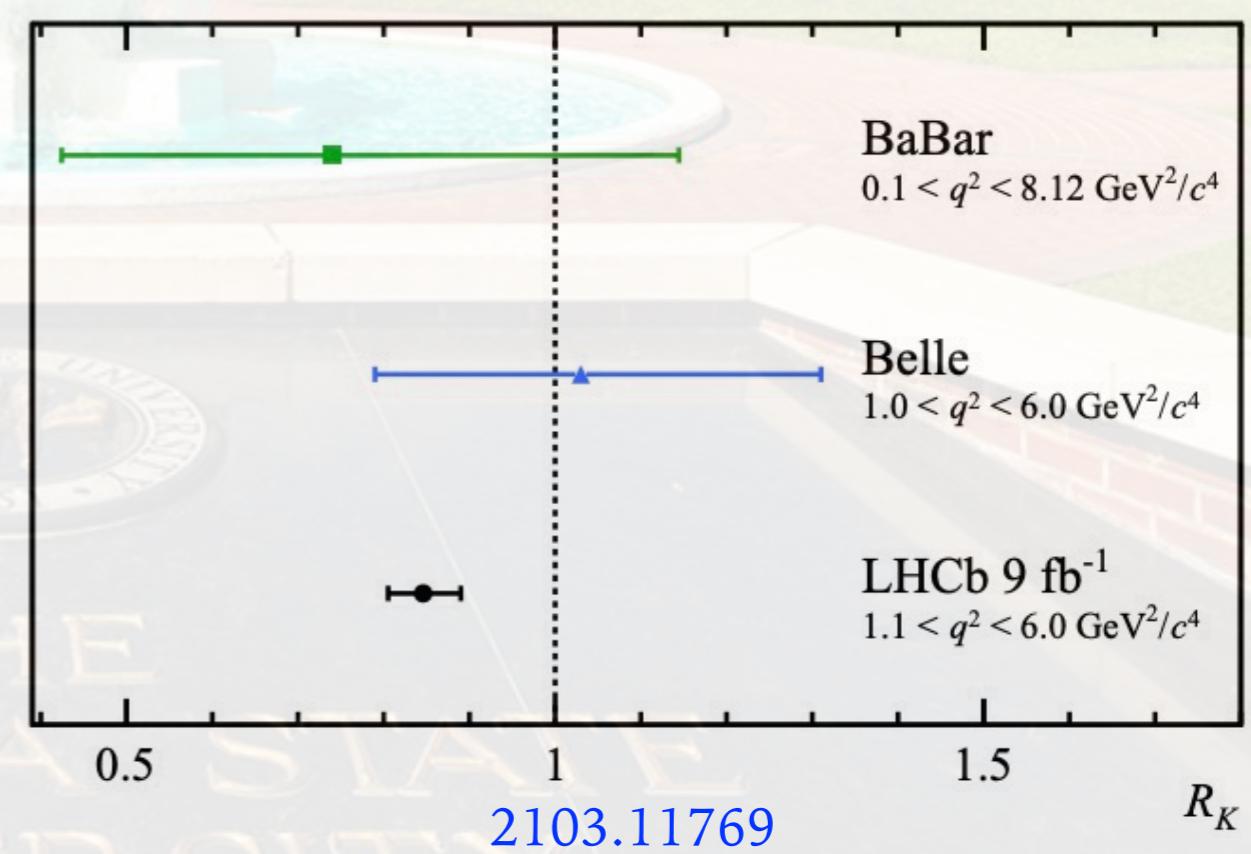
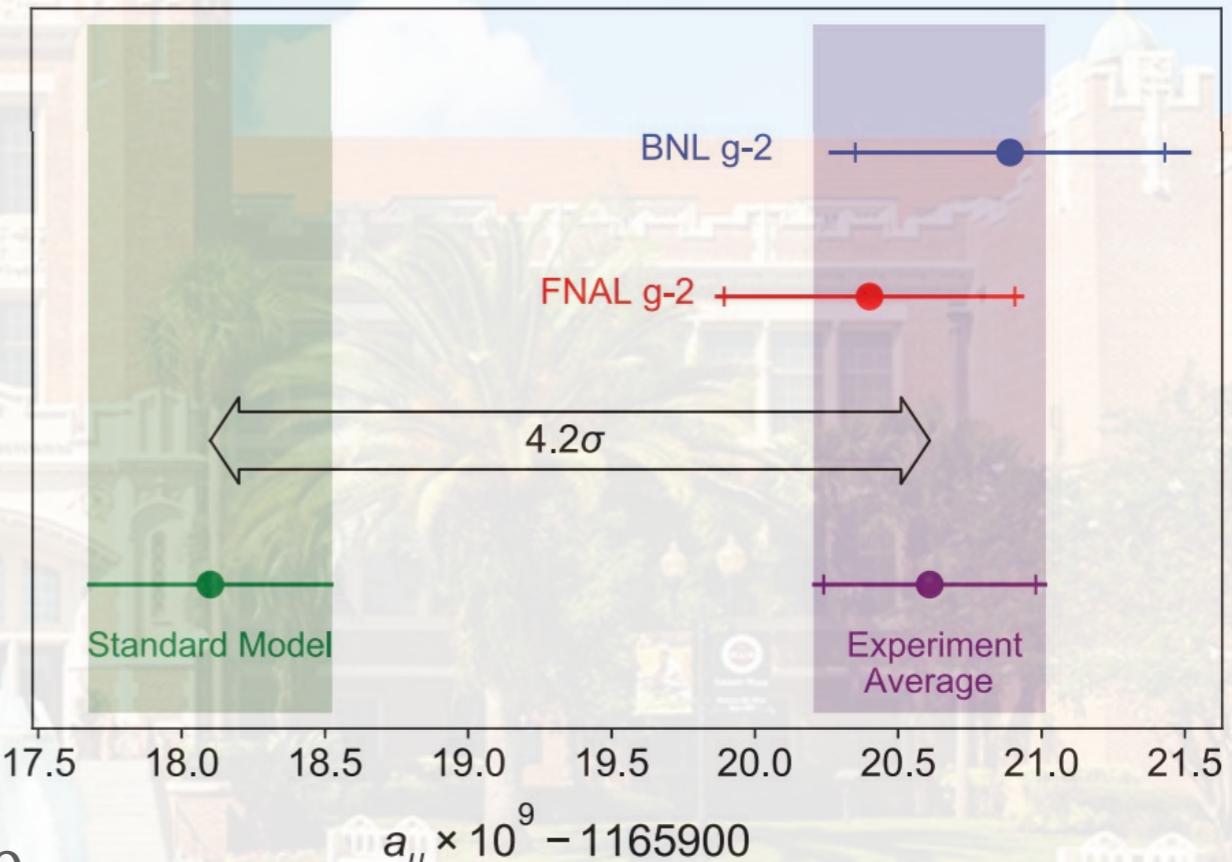
$$R_H \equiv \frac{\int_{q_{\min}^2}^{q_{\max}^2} \frac{d\mathcal{B}(B \rightarrow H \mu^+ \mu^-)}{dq^2} dq^2}{\int_{q_{\min}^2}^{q_{\max}^2} \frac{d\mathcal{B}(B \rightarrow H e^+ e^-)}{dq^2} dq^2}$$

$$R_K = 0.846^{+0.044}_{-0.041}$$

[2103.11769](#)

- Anomaly in fundamental parameters

- Discovery needs **more precision**
- Accurate interpretation of SM
- Manifest in other observables

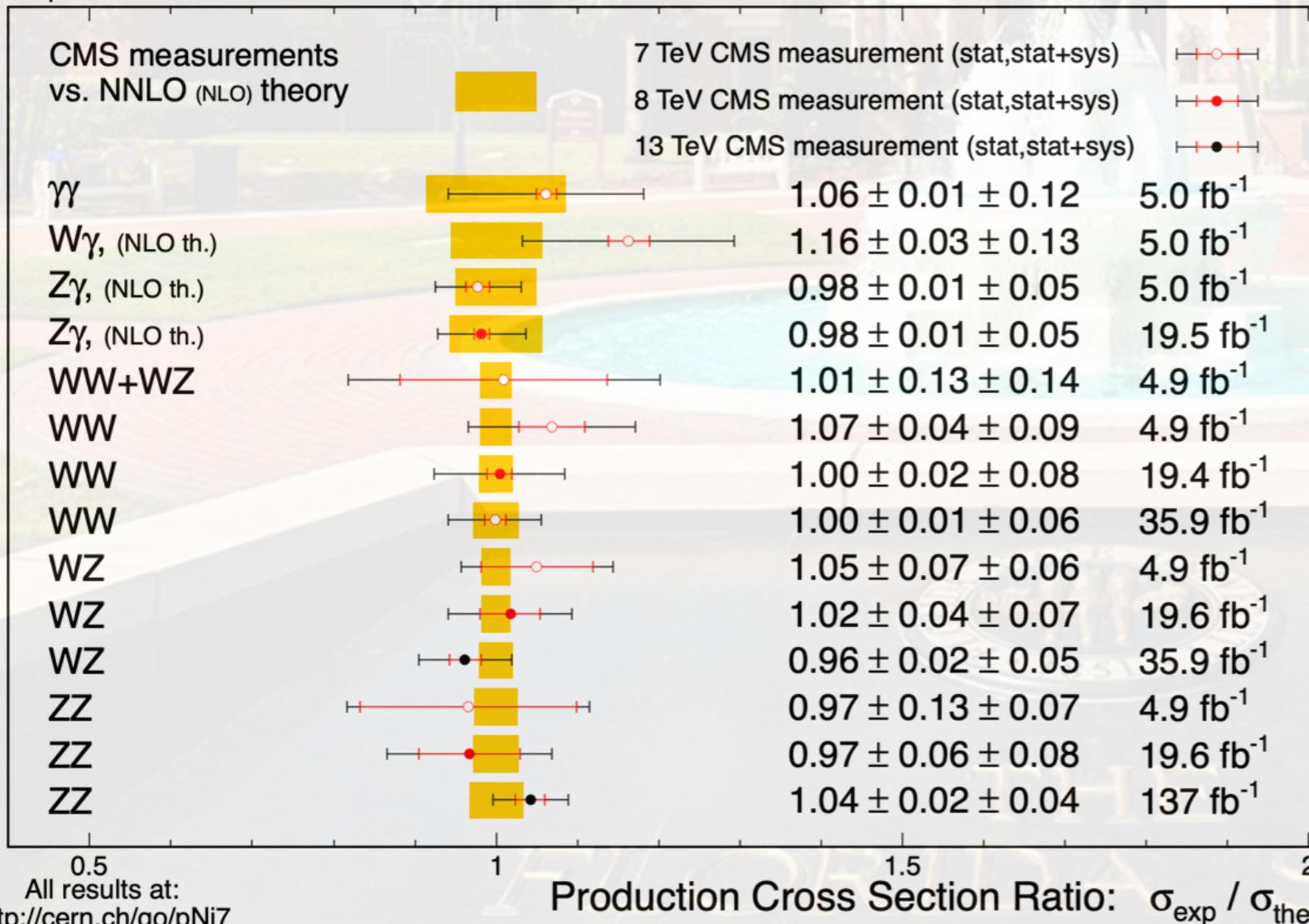


# PRECISION MEASUREMENT AT THE LHC

- Flip every stone searching for new physics: (More in Harrison's talk)
- Measured total cross section at 10% accuracy for di-boson productions
- Gauge boson production in association with jets (from 1 to 7 jets)
- HL-LHC projected Higgs cross section measurements at 2-4% precision

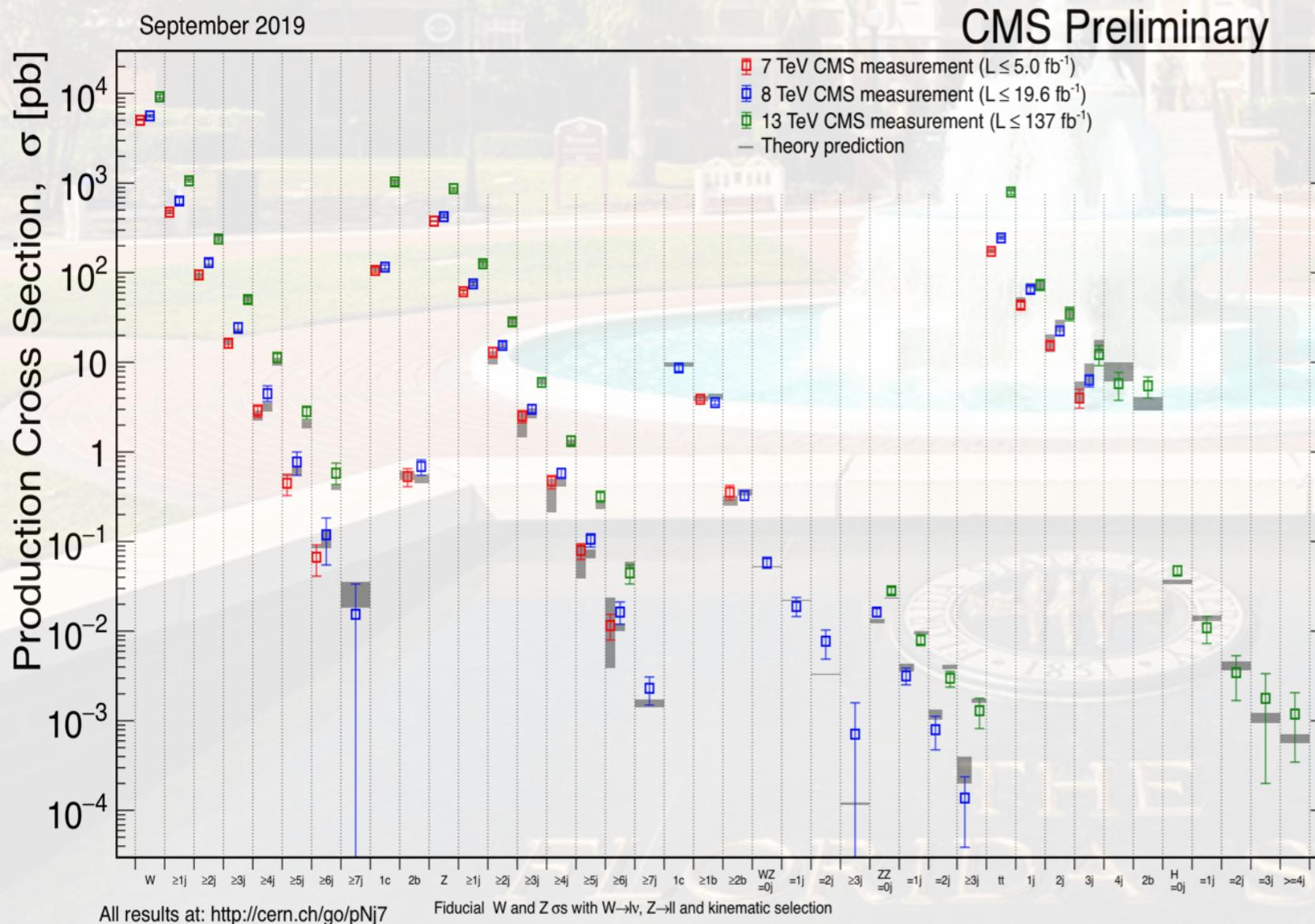
September 2020

CMS Preliminary



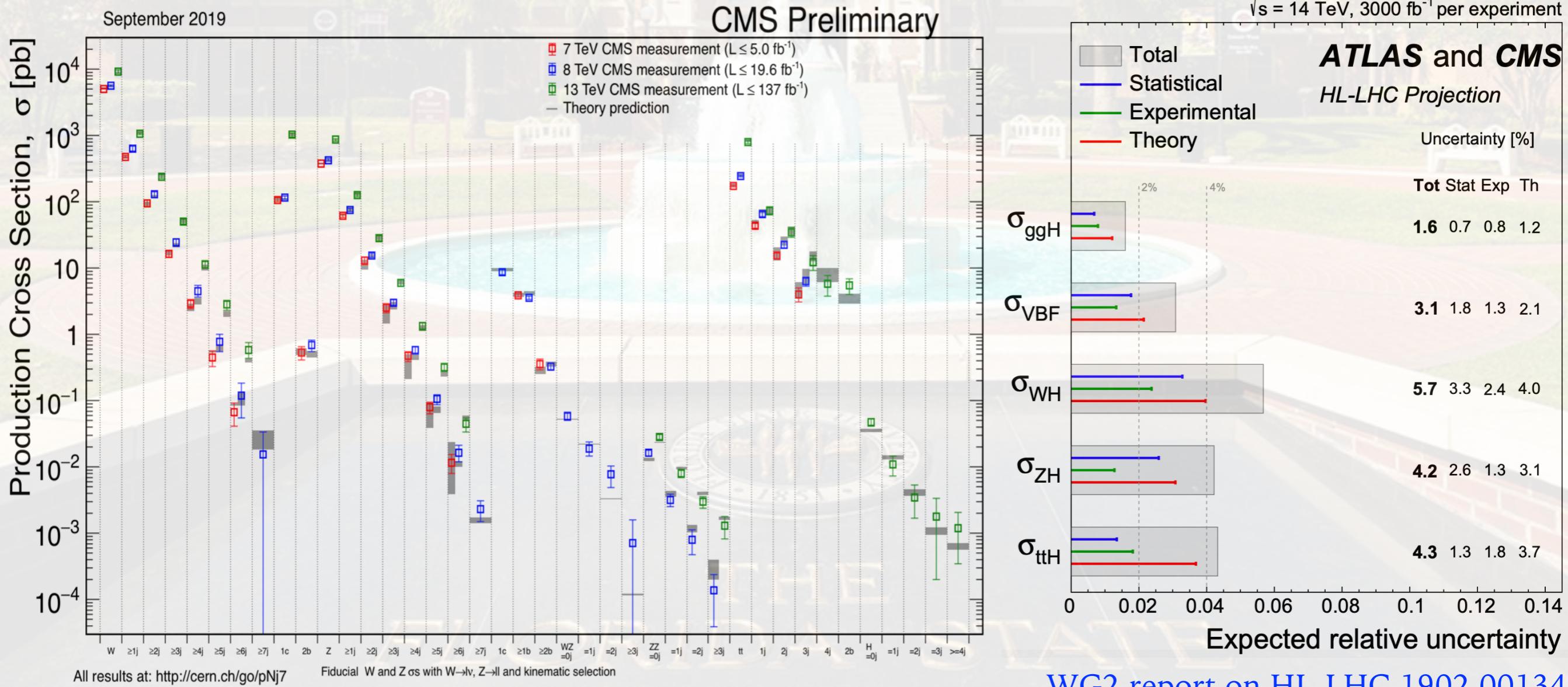
# PRECISION MEASUREMENT AT THE LHC

- Flip every stone searching for new physics: (More in Harrison's talk)
- Measured total cross section at 10% accuracy for di-boson productions
- Gauge boson production in association with jets (from 1 to 7 jets)
- HL-LHC projected Higgs cross section measurements at 2-4% precision



# PRECISION MEASUREMENT AT THE LHC

- Flip every stone searching for new physics: (More in Harrison's talk)
- Measured total cross section at 10% accuracy for di-boson productions
- Gauge boson production in association with jets (from 1 to 7 jets)
- HL-LHC projected Higgs cross section measurements at 2-4% precision



# THE PRECISION THEORY FRONTIER AT THE LHC

- Parton model

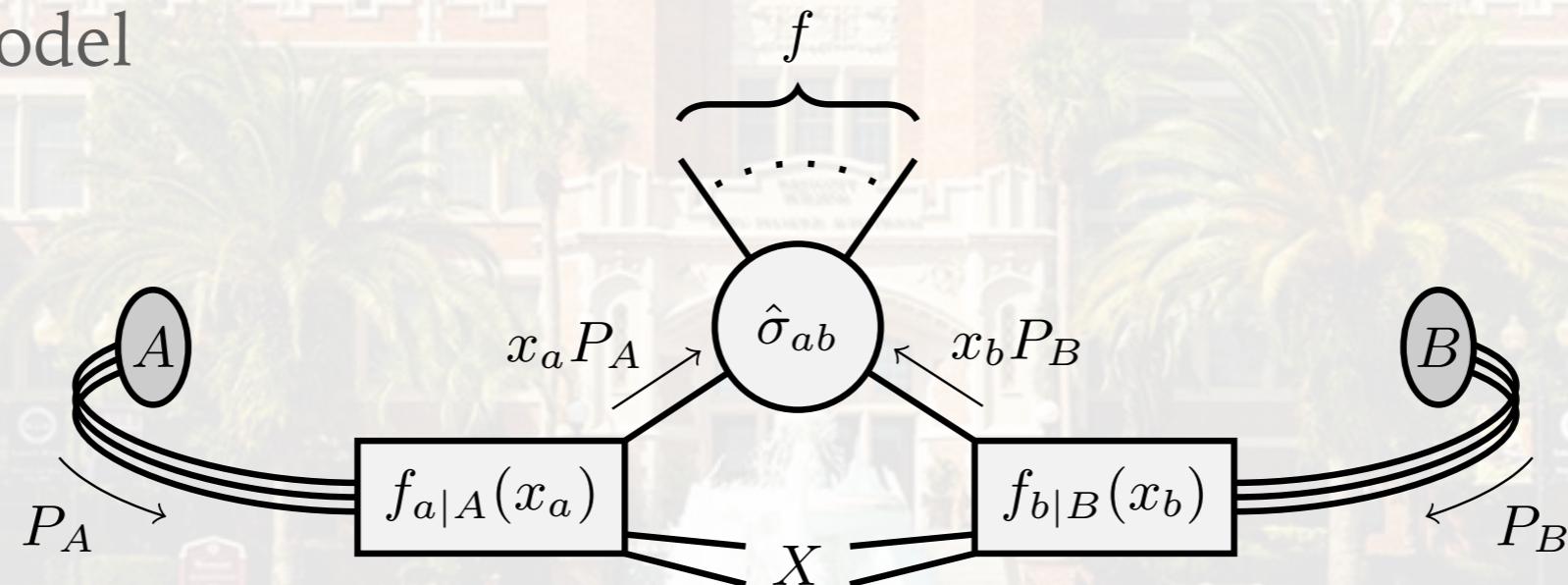


Figure by A. Huss

$$\sigma_{AB} = \sum_{ab} \int_0^1 dx_a \int_0^1 dx_b f_{a|A}(x_a) f_{b|B}(x_b) \hat{\sigma}_{ab}(x_a, x_b) (1 + \mathcal{O}(\Lambda_{\text{QCD}}/Q))$$

# THE PRECISION THEORY FRONTIER AT THE LHC

- Parton model

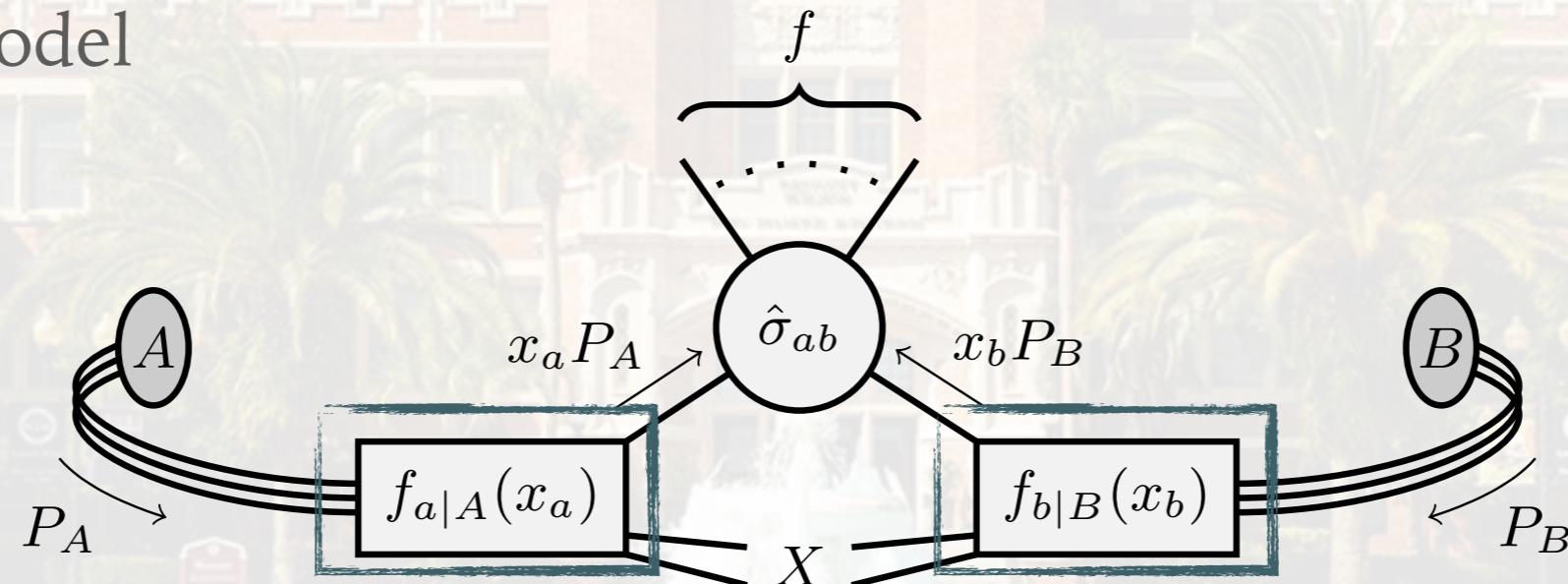


Figure by A. Huss

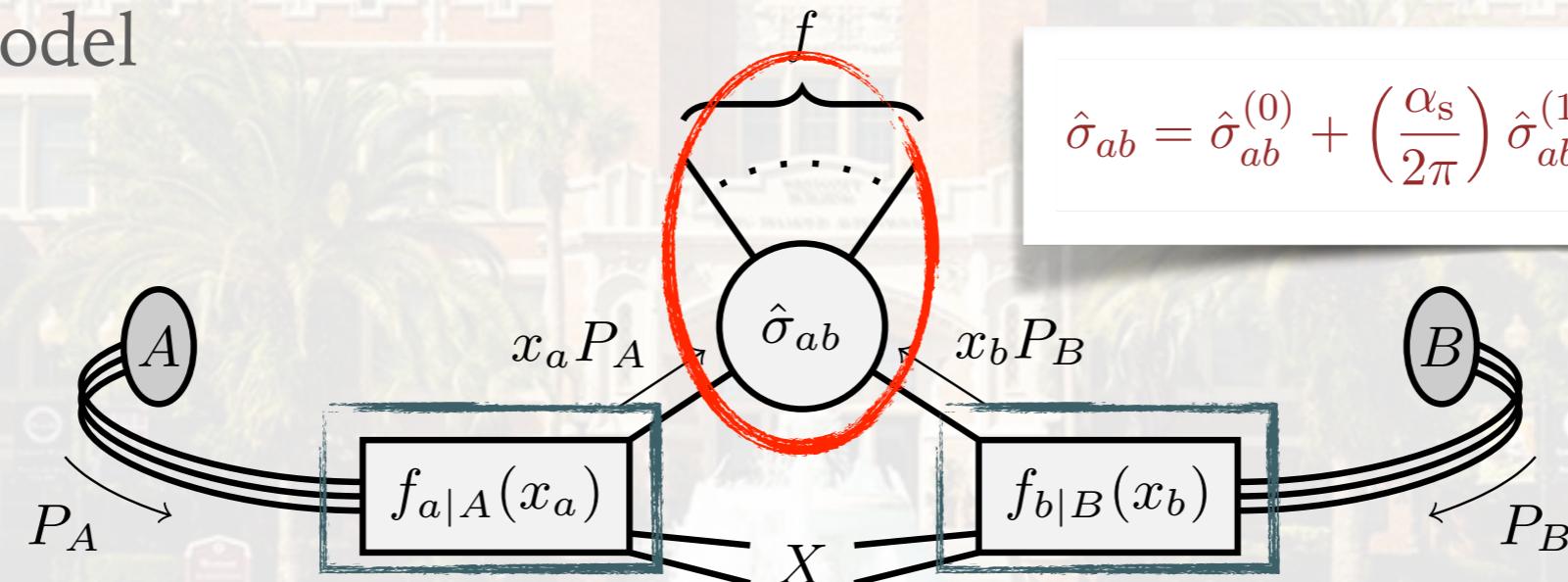
$$\sigma_{AB} = \sum_{ab} \int_0^1 dx_a \int_0^1 dx_b f_{a|A}(x_a) f_{b|B}(x_b) \hat{\sigma}_{ab}(x_a, x_b) (1 + \mathcal{O}(\Lambda_{\text{QCD}}/Q))$$

parton distribution functions  
(systematically, improvable)  
few % at LHC

THE  
FLORIDA STATE  
UNIVERSITY

# THE PRECISION THEORY FRONTIER AT THE LHC

- Parton model



$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}_{ab}^{(2)} + \dots$$

Figure by A. Huss

parton distribution functions  
(systematically, improvable)  
few % at LHC

hard scattering  
(systematically improvable)  
aim for few % level!

# THE PRECISION THEORY FRONTIER AT THE LHC

## ► Parton model

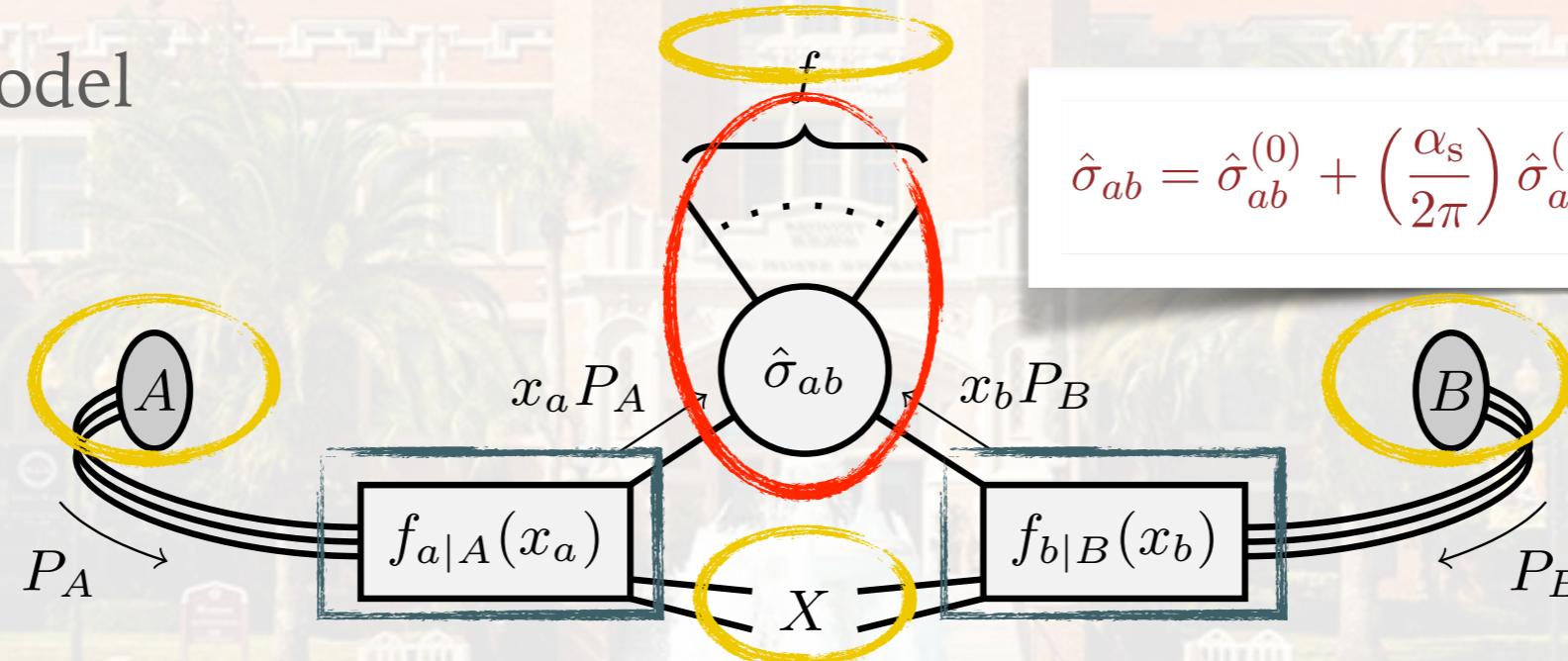


Figure by A. Huss

$$\sigma_{AB} = \sum_{ab} \int_0^1 dx_a \int_0^1 dx_b f_{a|A}(x_a) f_{b|B}(x_b) \hat{\sigma}_{ab}(x_a, x_b) (1 + \mathcal{O}(\Lambda_{\text{QCD}}/Q))$$

parton distribution functions  
(systematically, improvable)  
few % at LHC

hard scattering  
(systematically improvable)  
aim for few % level!

non-perturbative effects  
(no good understanding)  
~ few %?

# THE PRECISION THEORY FRONTIER AT THE LHC

- Remarkable progress at differential NNLO



# THE PRECISION THEORY FRONTIER AT THE LHC

- Remarkable progress at differential NNLO

- Complexity at 2 to 3 scattering

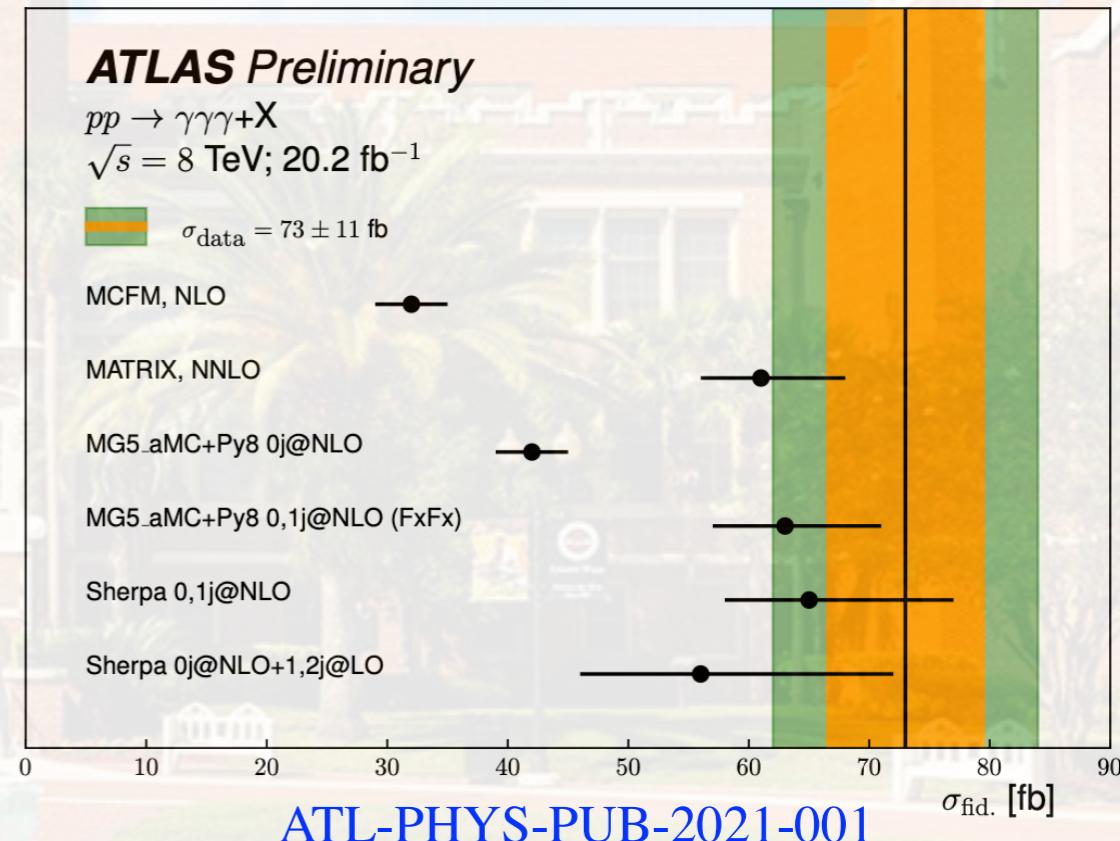
Tri-photon: [JHEP 02 \(2020\) 057](#)

[Phys.Lett.B 812 \(2021\) 136013](#)

Di-photon+Jet: [2105.06940](#)

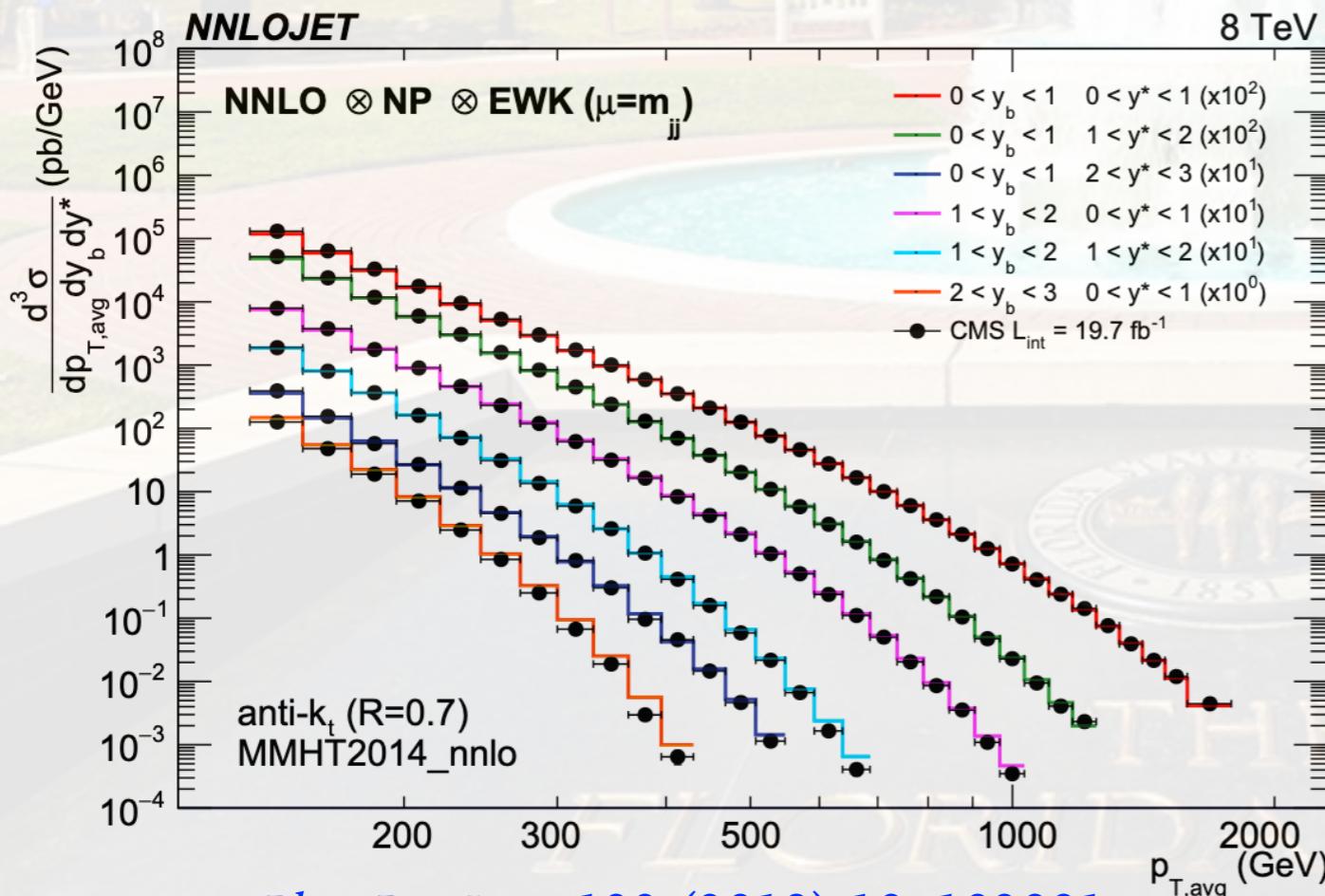
WH+Jet: [Phys.Lett.B 817 \(2021\) 136335](#)

- Di-jet triple differential predictions



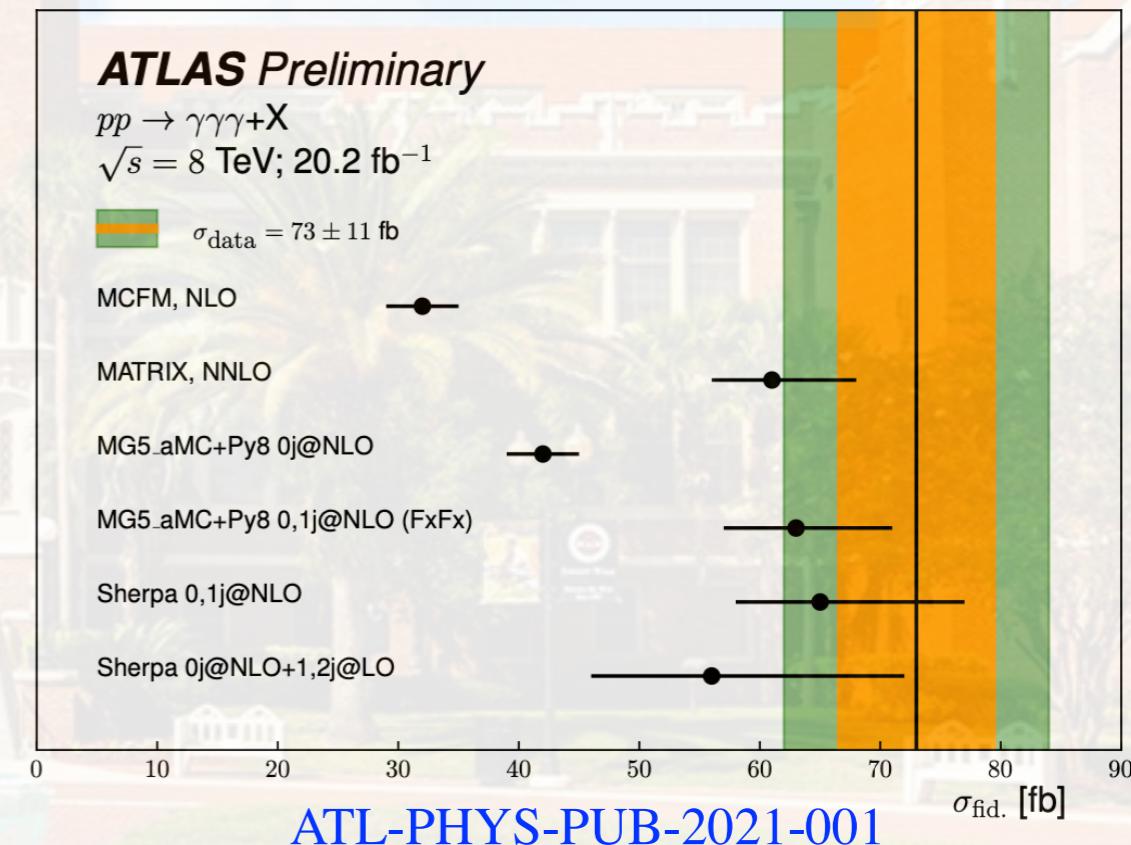
# THE PRECISION THEORY FRONTIER AT THE LHC

- Remarkable progress at differential NNLO
- Complexity at 2 to 3 scattering
  - Tri-photon: [JHEP 02 \(2020\) 057](#)  
[Phys.Lett.B 812 \(2021\) 136013](#)
  - Di-photon+Jet: [2105.06940](#)
  - WH+Jet: [Phys.Lett.B 817 \(2021\) 136335](#)
- Di-jet triple differential predictions



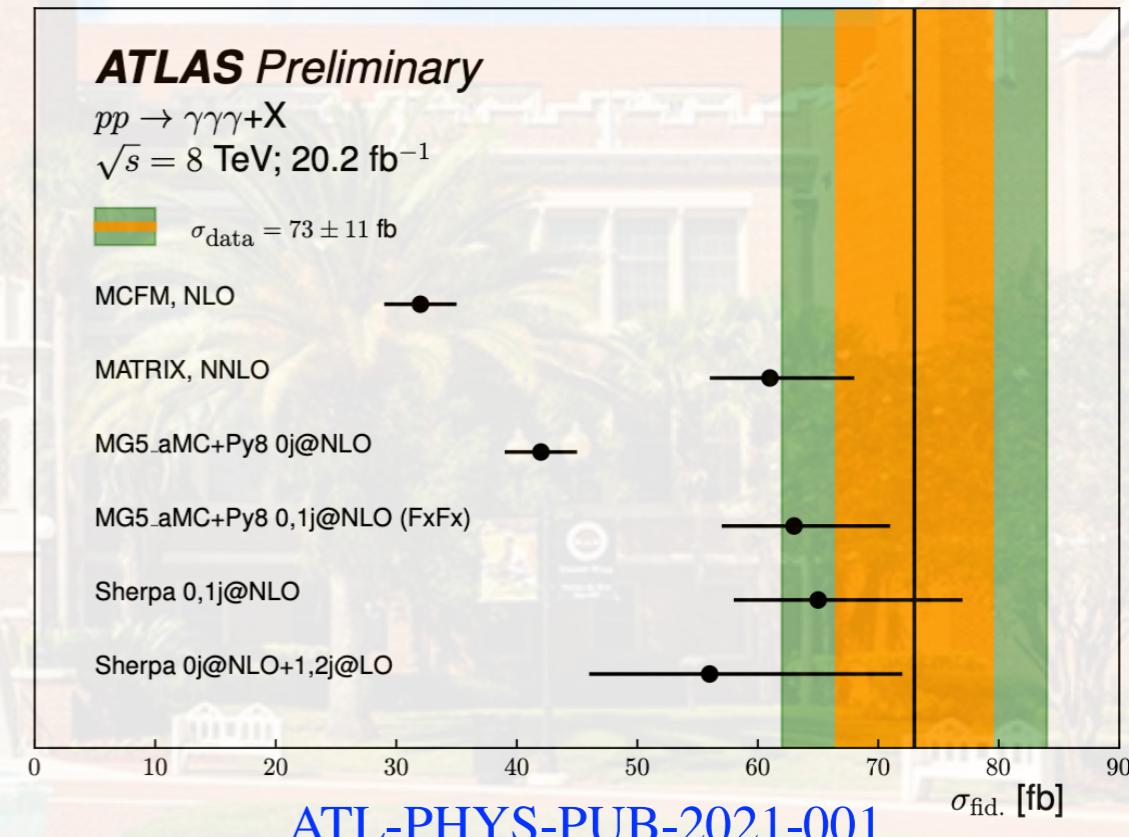
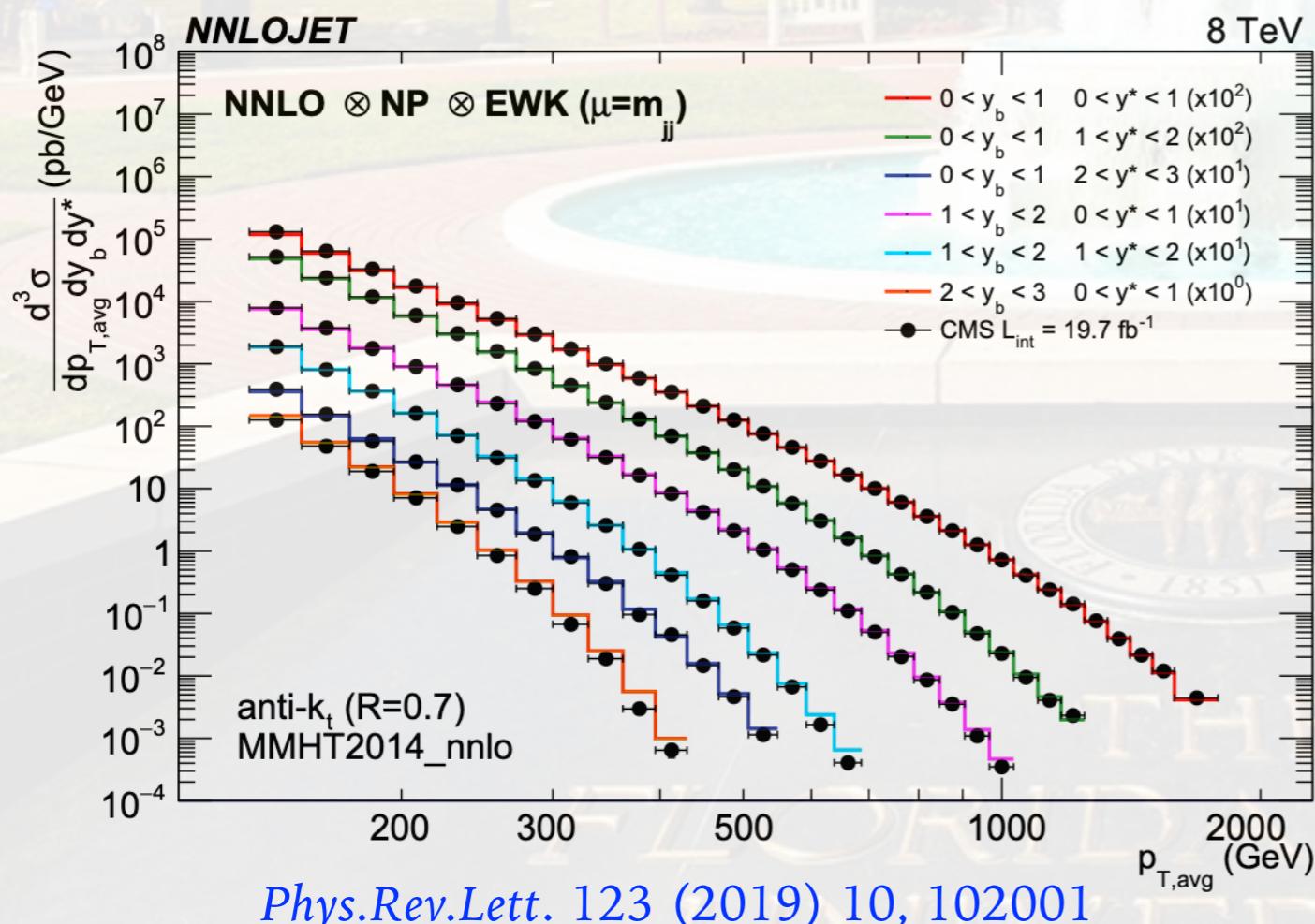
[Phys.Rev.Lett. 123 \(2019\) 10, 102001](#)

Fully differential  $\gamma^*$  production at LHC with N3LO QCD



# THE PRECISION THEORY FRONTIER AT THE LHC

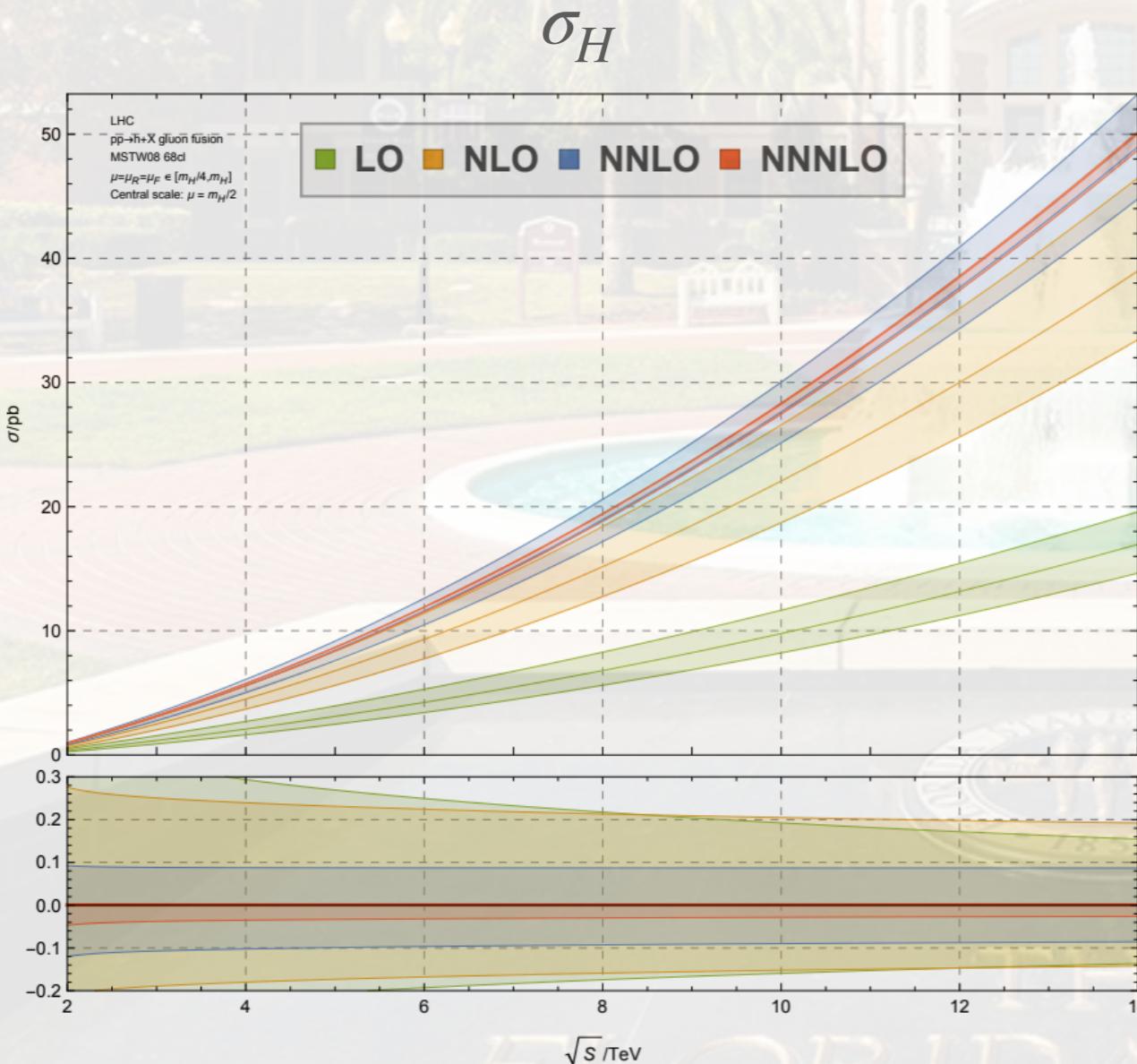
- Remarkable progress at differential NNLO
    - Complexity at 2 to 3 scattering  
Tri-photon: [JHEP 02 \(2020\) 057](#)  
[Phys.Lett.B 812 \(2021\) 136013](#)
    - Di-photon+Jet: [2105.06940](#)
    - WH+Jet: [Phys.Lett.B 817 \(2021\) 136335](#)
  - Di-jet triple differential predictions



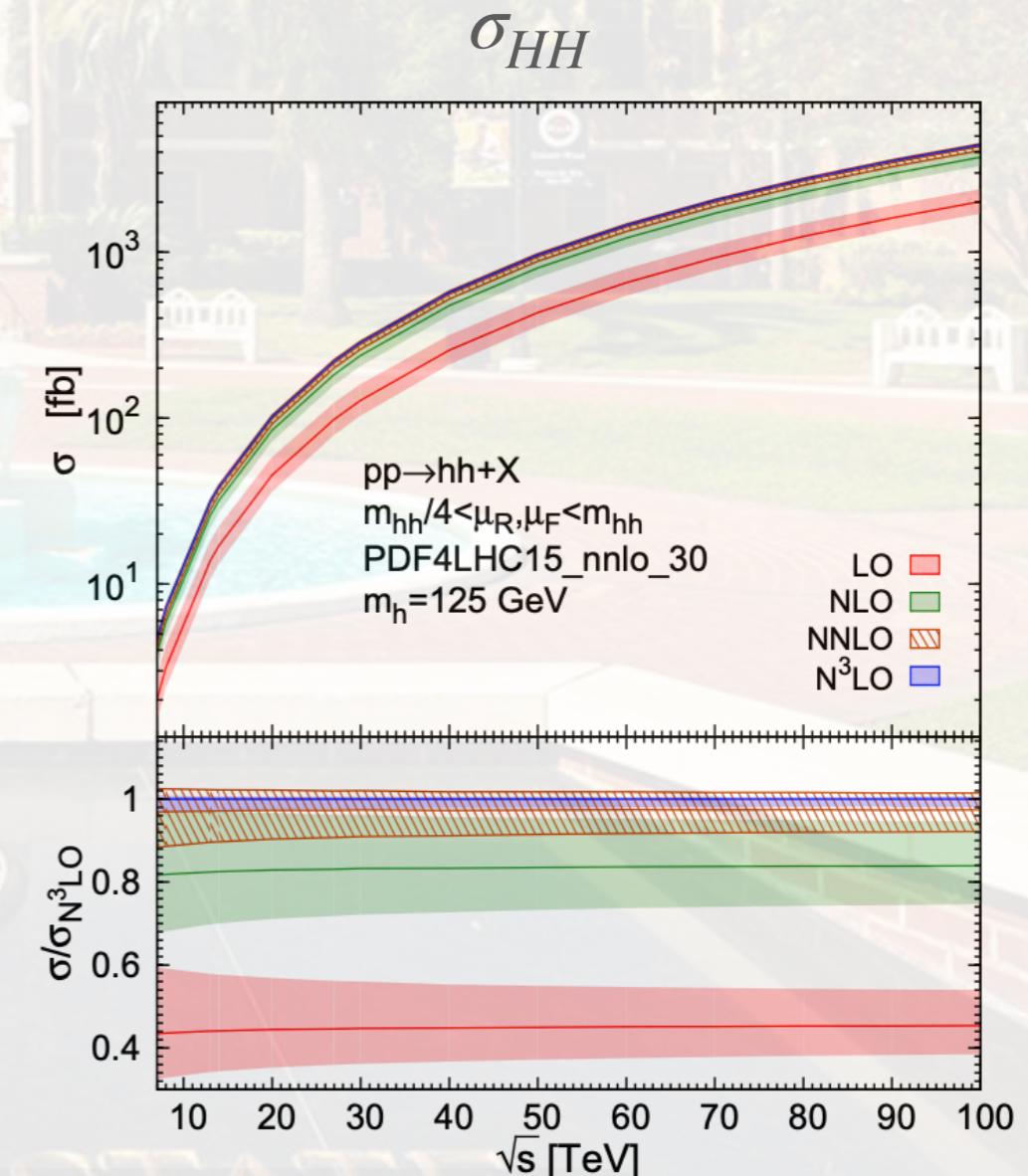
- RADCOR 2019 talks
    - 6 on NNLO pheno
    - 8 on 2-loop amplitudes
    - 2 on N3LO pheno+amplitude
  - RADCOR+LoopFest 2021 talks
    - 12 on NNLO pheno
    - 14 on 2-loop amplitudes
    - 5 on N3LO pheno+amplitude

# THE PRECISION THEORY FRONTIER AT THE LHC

- N3LO QCD corrections available for LHC observables
  - Total and differential predictions of Higgs production and decay
  - Gluon fusion channel (ggF) in heavy top quark limit (HTL): H and HH



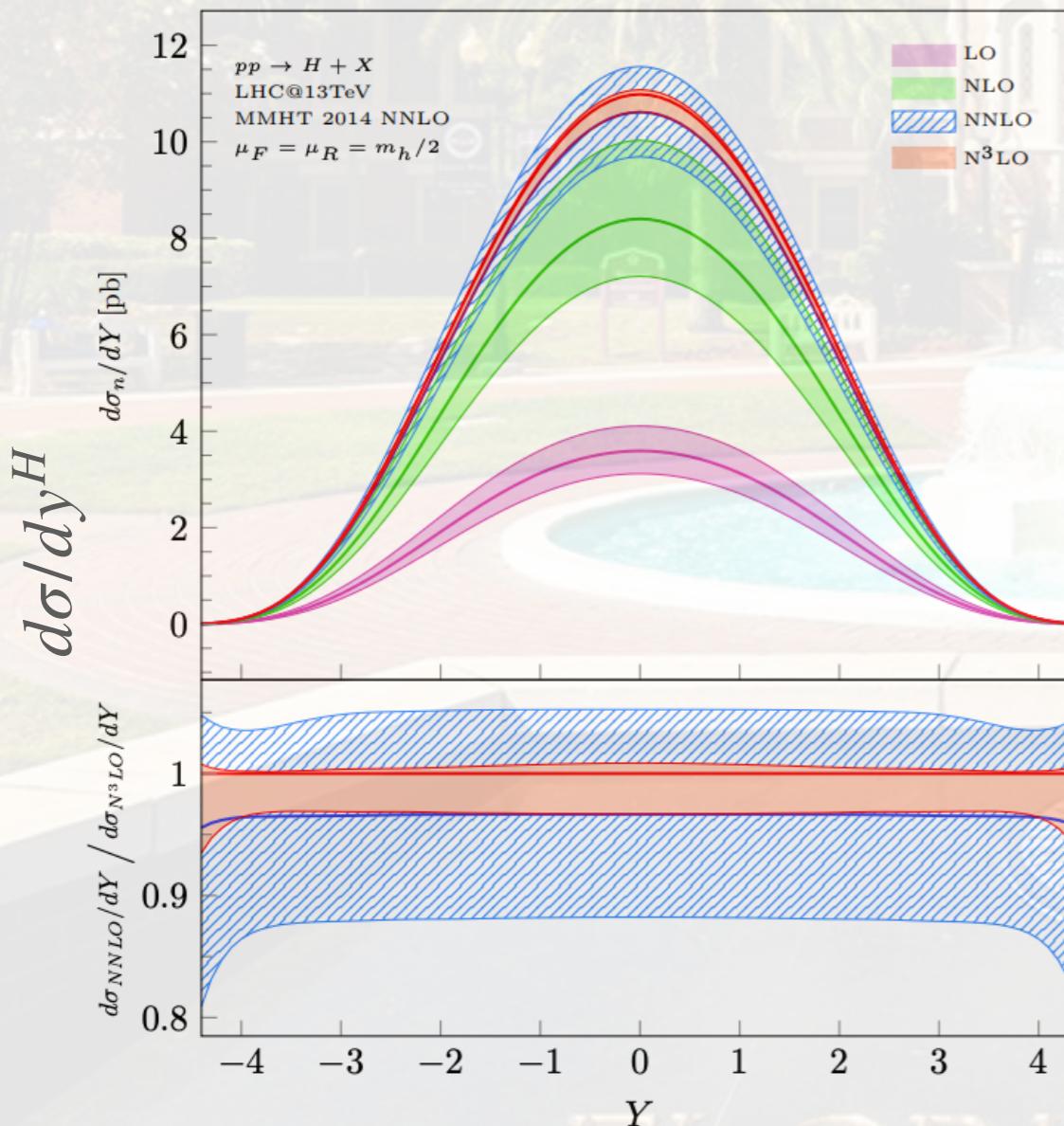
Phys.Rev.Lett. 114 (2015) 212001



Phys.Lett.B 803 (2020) 135292

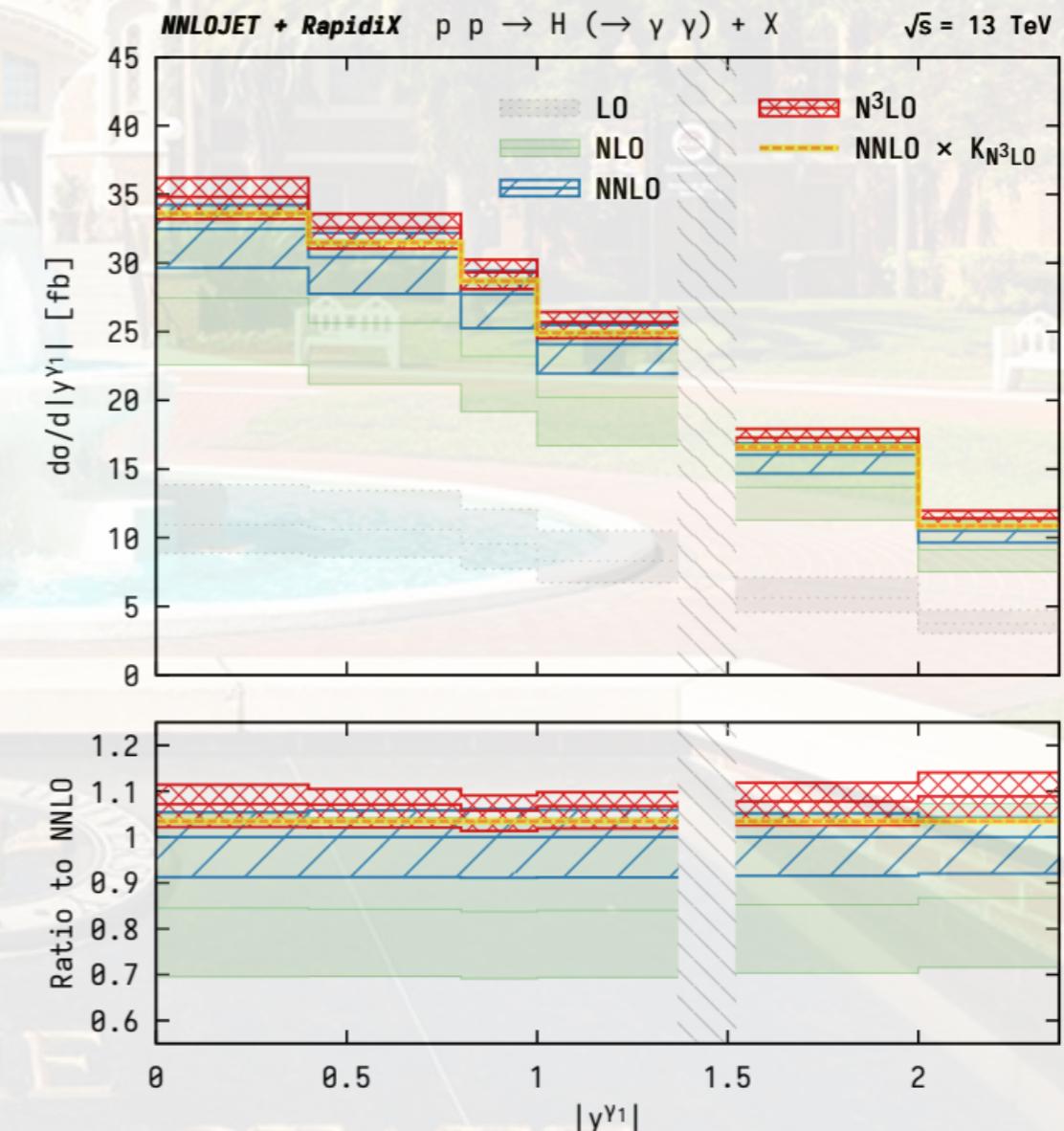
# THE PRECISION THEORY FRONTIER AT THE LHC

- N3LO QCD corrections available for LHC observables
  - Total and differential predictions of Higgs production and decay
  - Differential distributions from ggF channel in HTL



[Phys.Rev.D 99 \(2019\) 3, 034004](#)

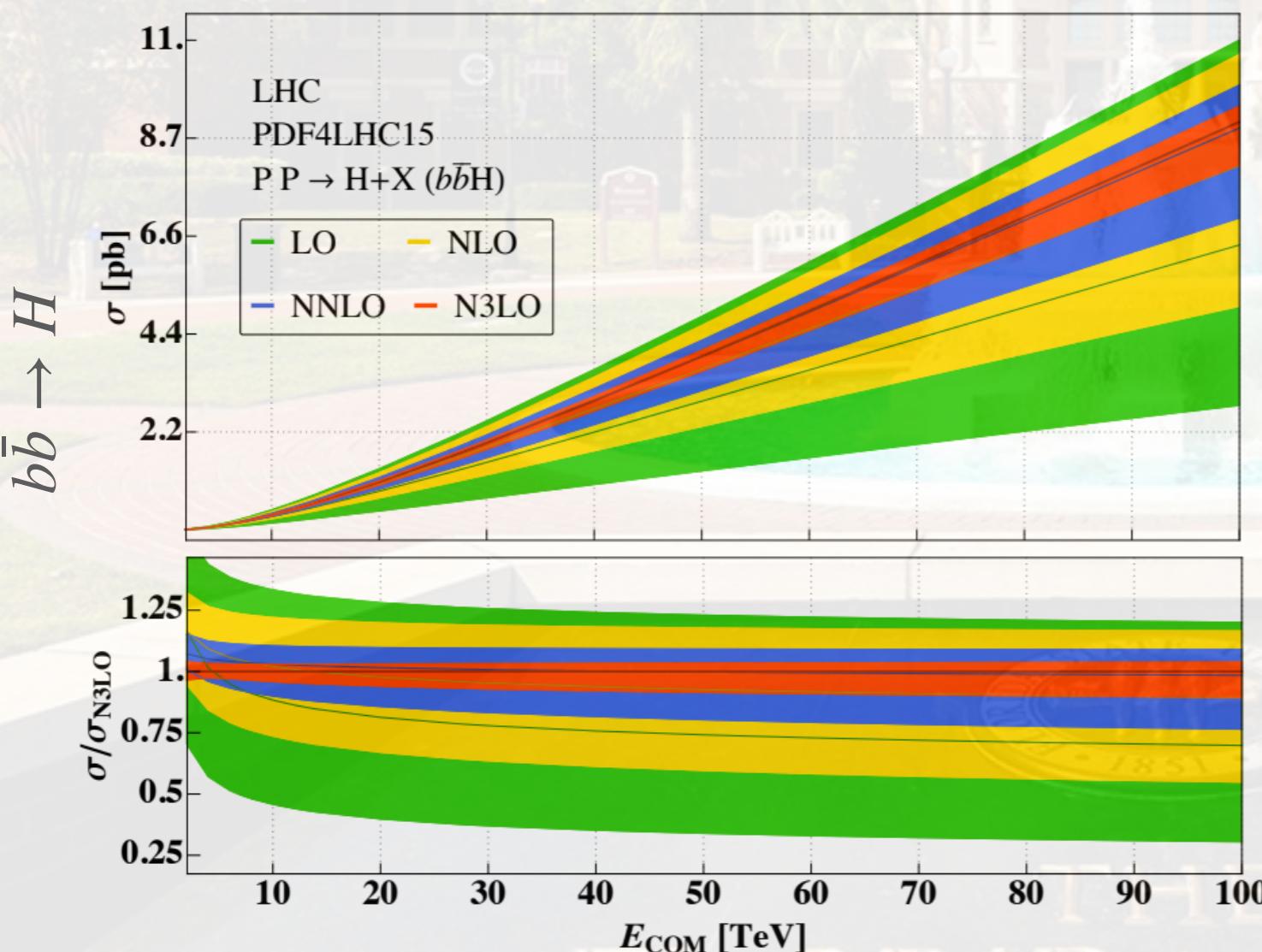
See also [JHEP 02 \(2019\) 096](#)



[2102.07607](#)

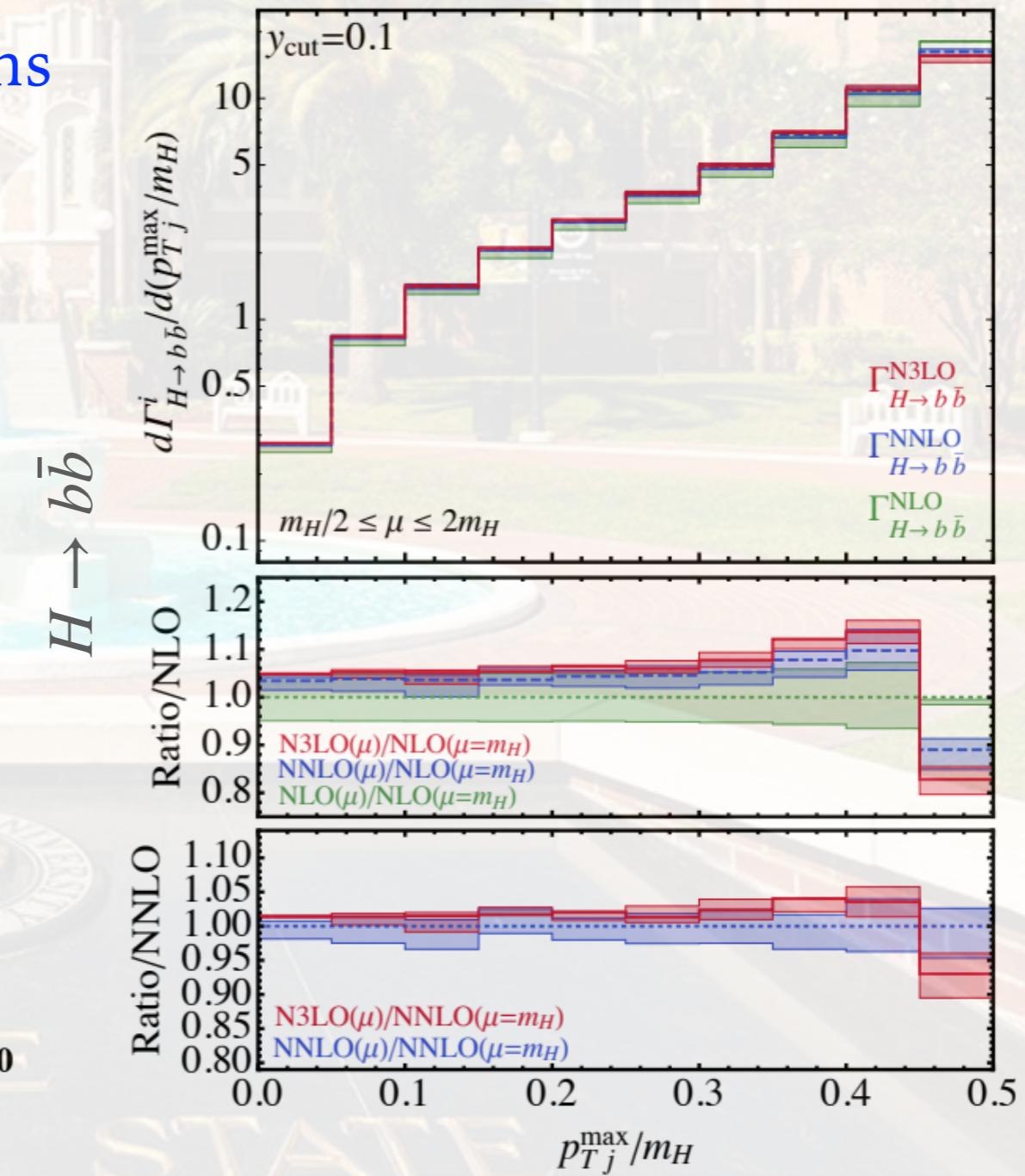
# THE PRECISION THEORY FRONTIER AT THE LHC

- N3LO QCD corrections available for LHC observables
  - Total and differential predictions of Higgs production and decay
  - $b\bar{b}H$  production and decay predictions



*Phys.Rev.Lett. 125 (2020) 5, 051804*

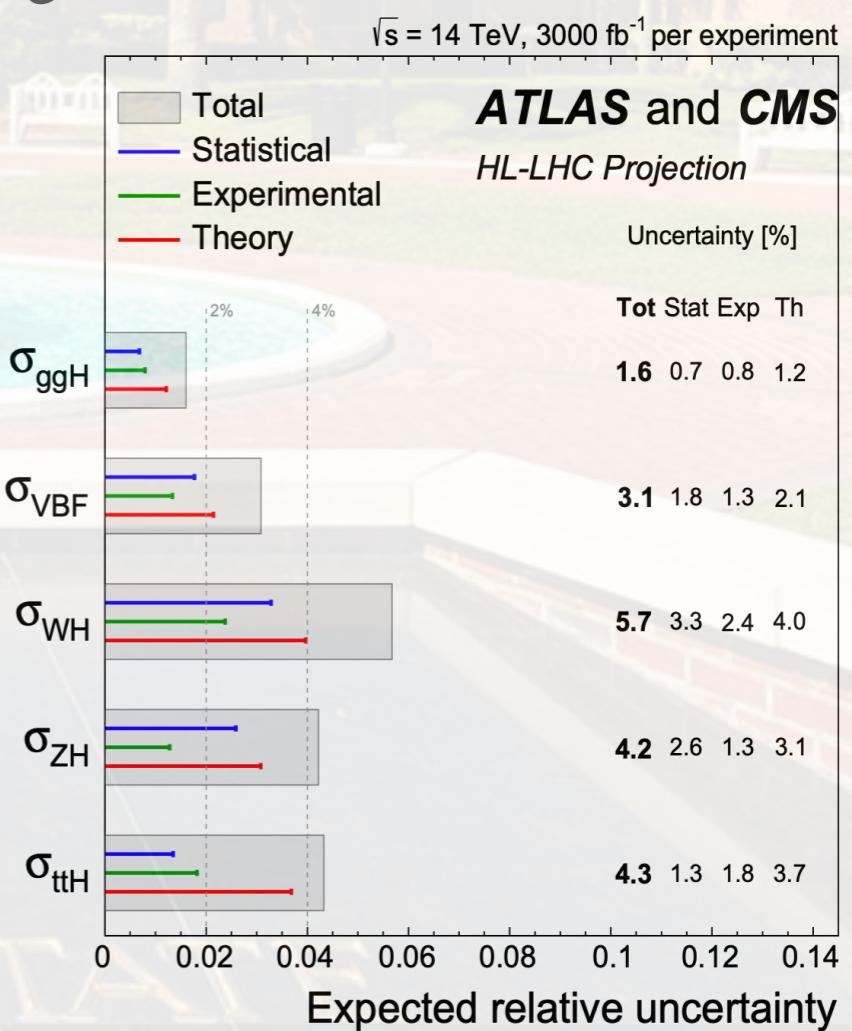
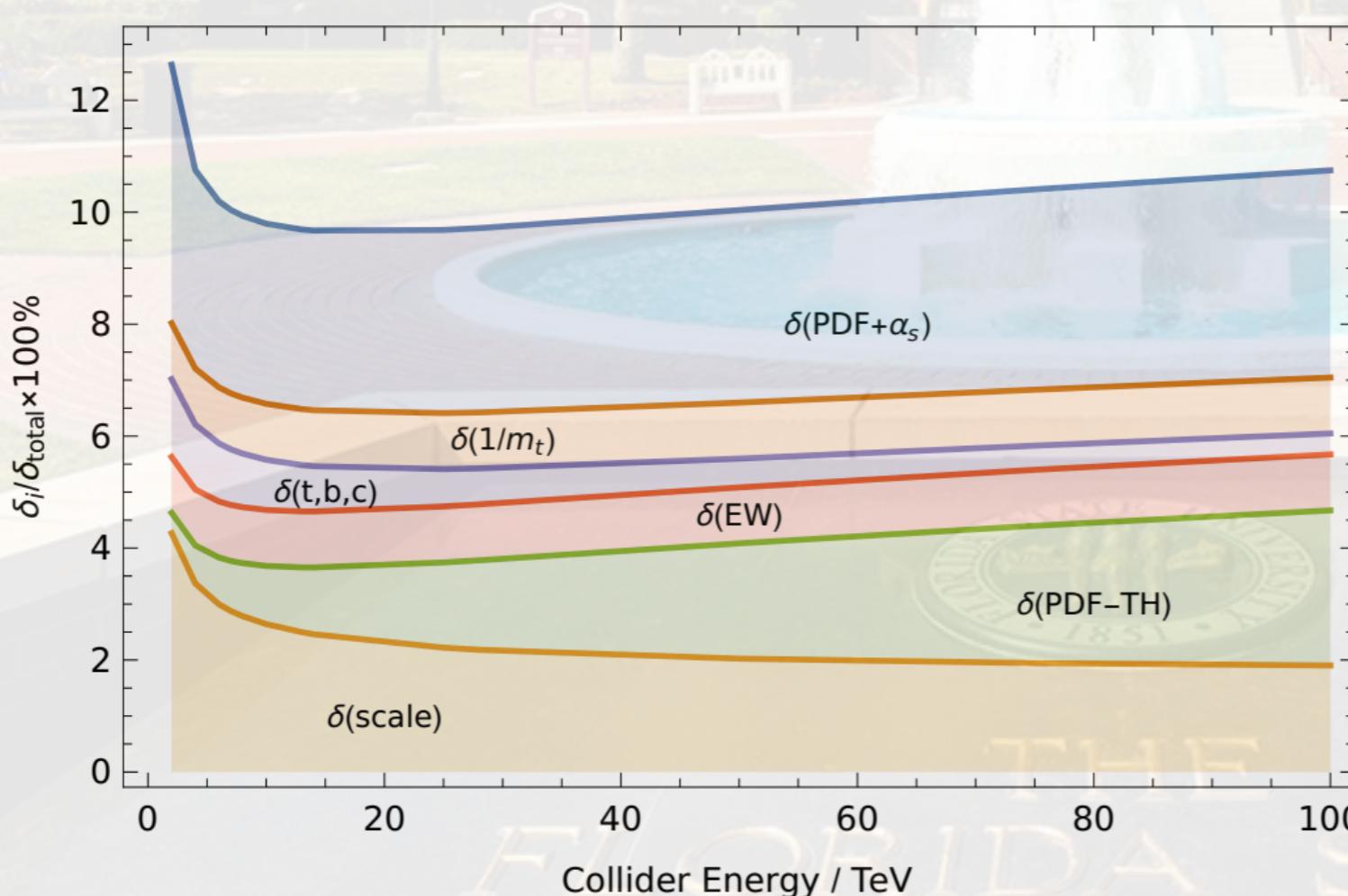
Fully differential  $\gamma^*$  production at LHC with N3LO QCD



*JHEP 06 (2019) 079*

# THE PRECISION THEORY FRONTIER AT THE LHC

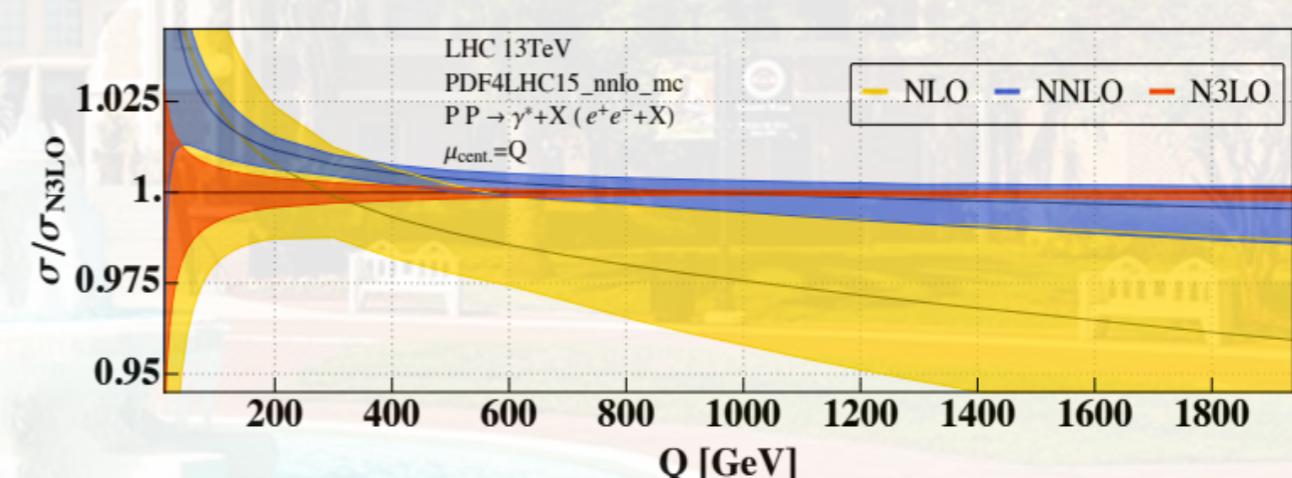
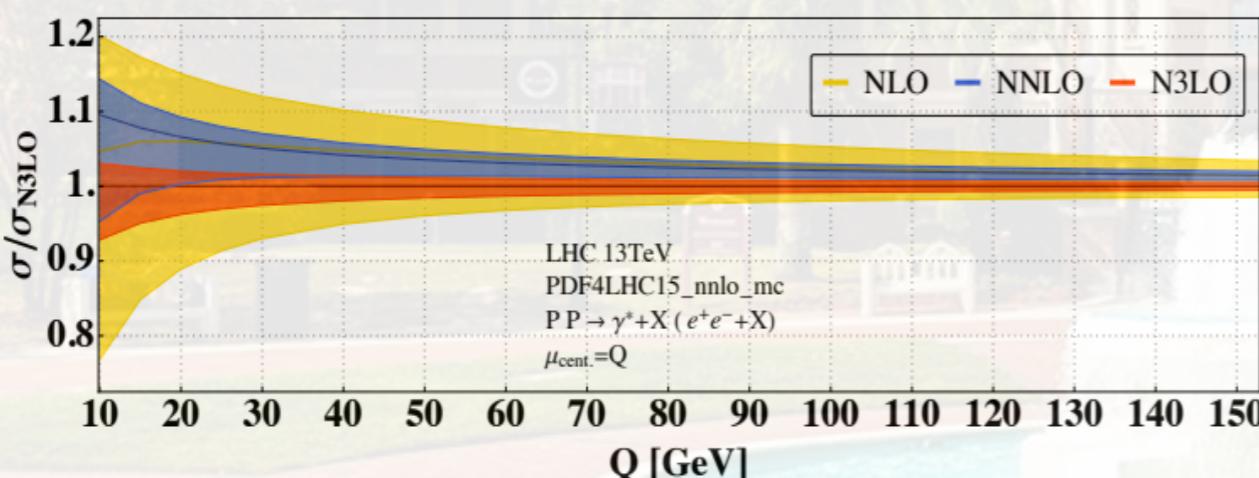
- N3LO QCD corrections available for LHC observables
  - Excellent convergence of perturbative series for Higgs related obs.
  - To answer the precision challenge from HL-LHC
  - Critical thinking needed for the new “shortest board of a bucket”
    - Uncertainties from PDF,  $\alpha_s$ , fixed couplings, mass effect etc.



# THE PRECISION THEORY FRONTIER AT THE LHC

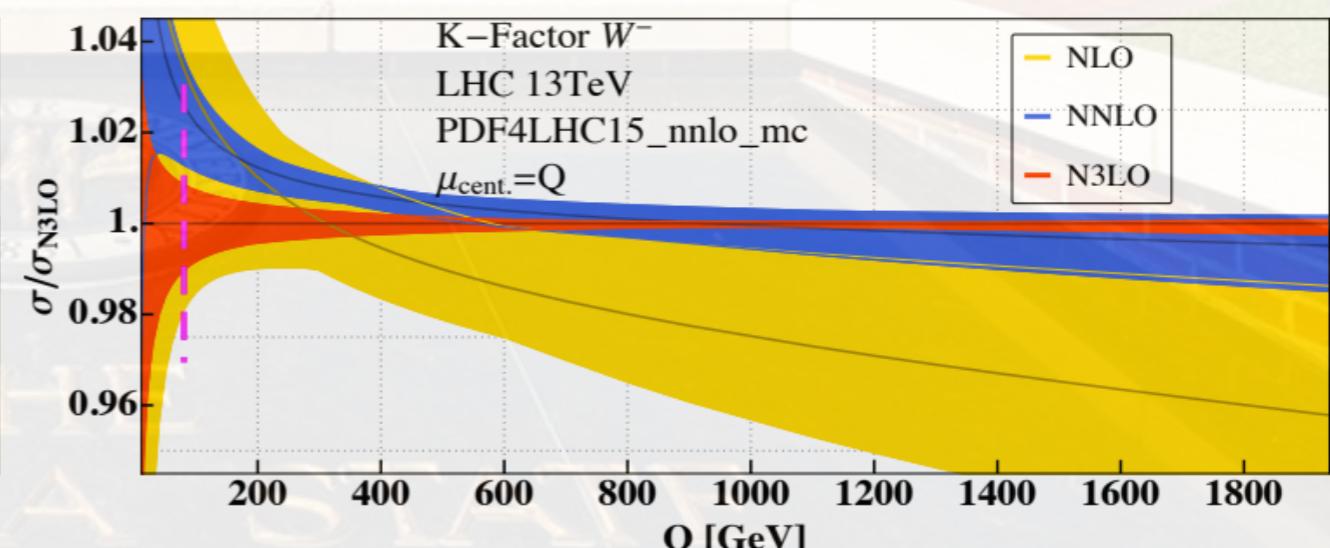
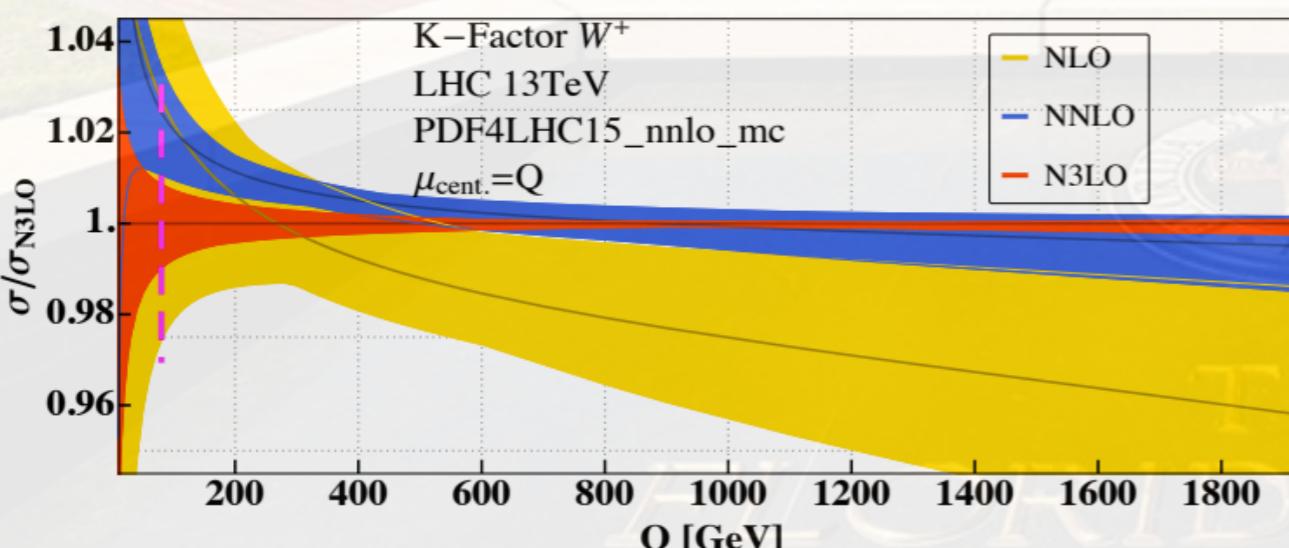
- N3LO QCD corrections available for LHC observables
  - Neutral and charged current total cross section (**CFB collaboration**)
  - Tension for perturbative convergence of fixed order corrections

$pp \rightarrow \gamma^*$



C. Duhr, F. Dulat and B. Mistlberger *Phys.Rev.Lett.* 125 (2020) 17, 172001

$pp \rightarrow W^\pm$



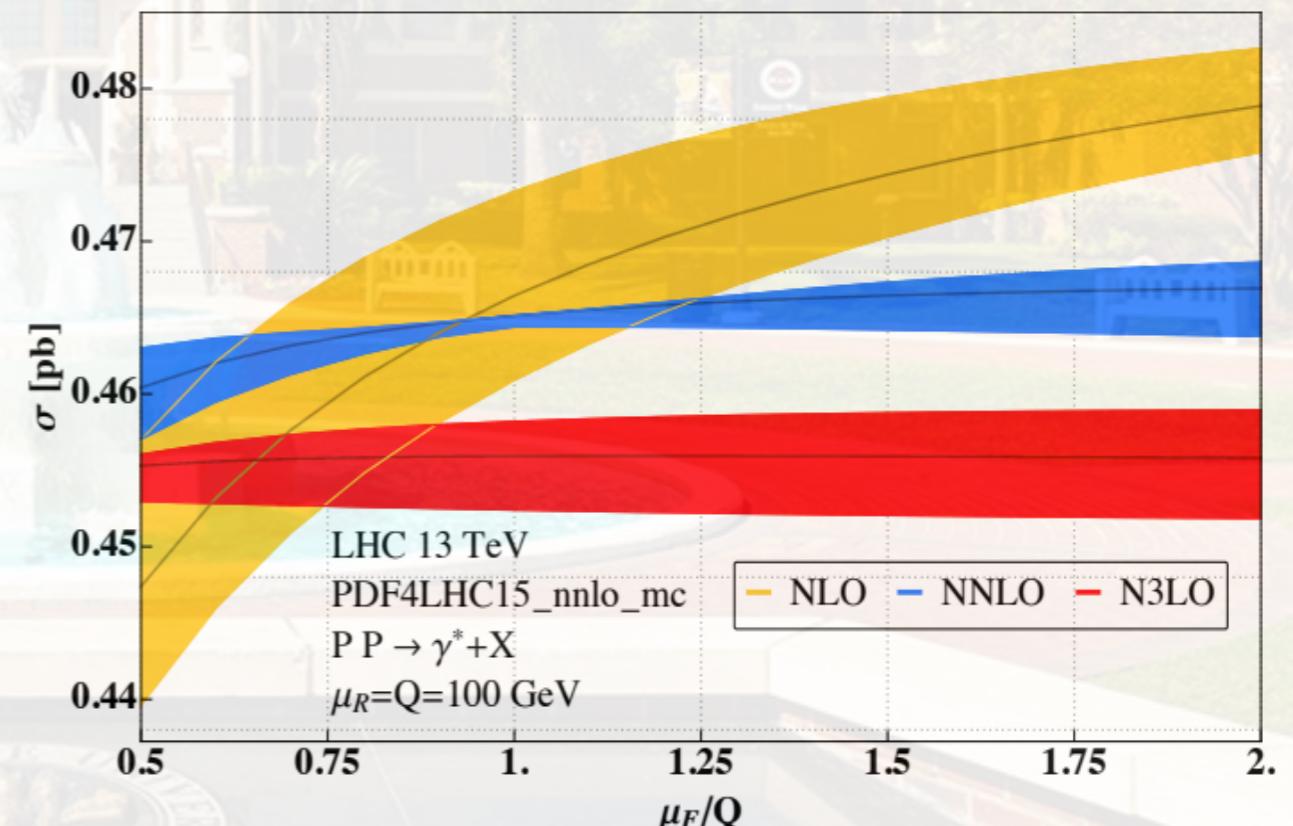
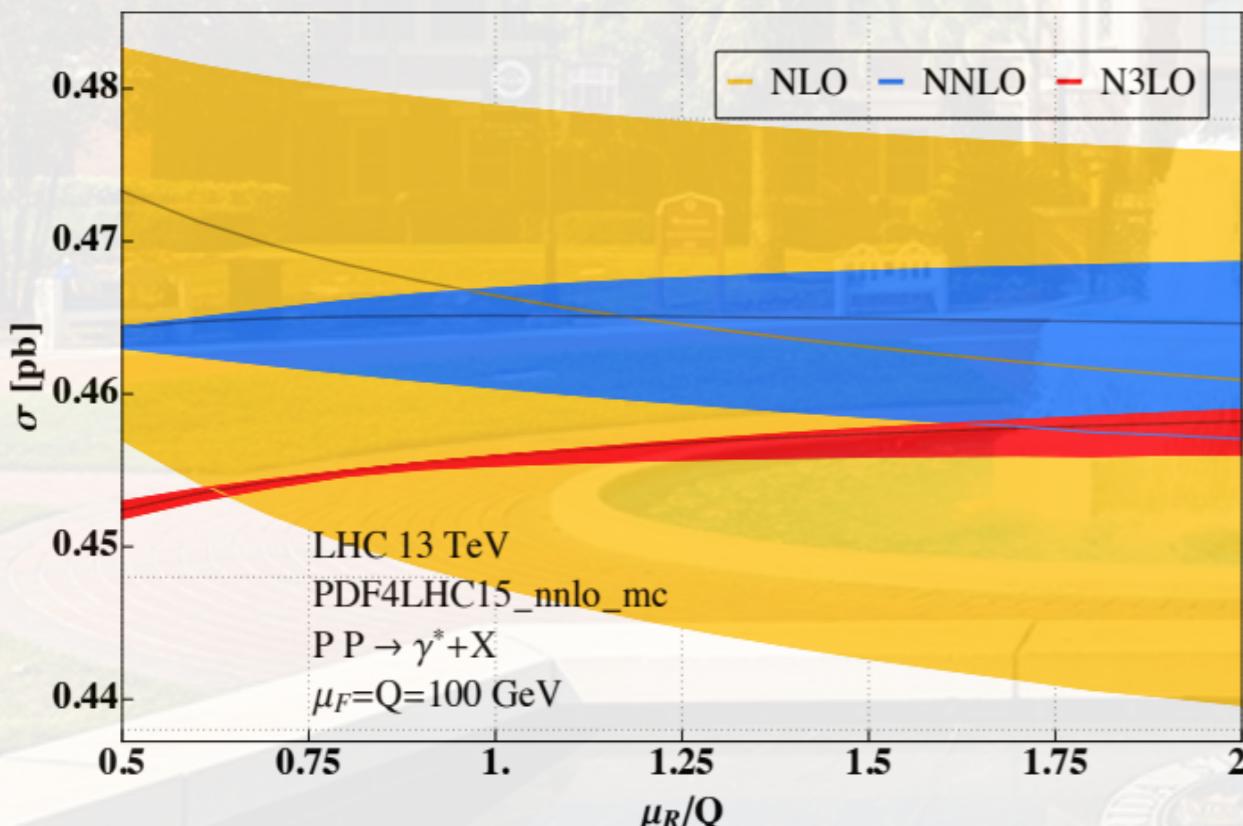
C. Duhr, F. Dulat and B. Mistlberger *JHEP* 11 (2020) 143

Fully differential  $\gamma^*$  production at LHC with N3LO QCD

# THE PRECISION THEORY FRONTIER AT THE LHC

- N3LO QCD corrections available for LHC observables
  - Neutral and charged current total cross section (CFB collaboration)
    - Tension for perturbative convergence of fixed order corrections

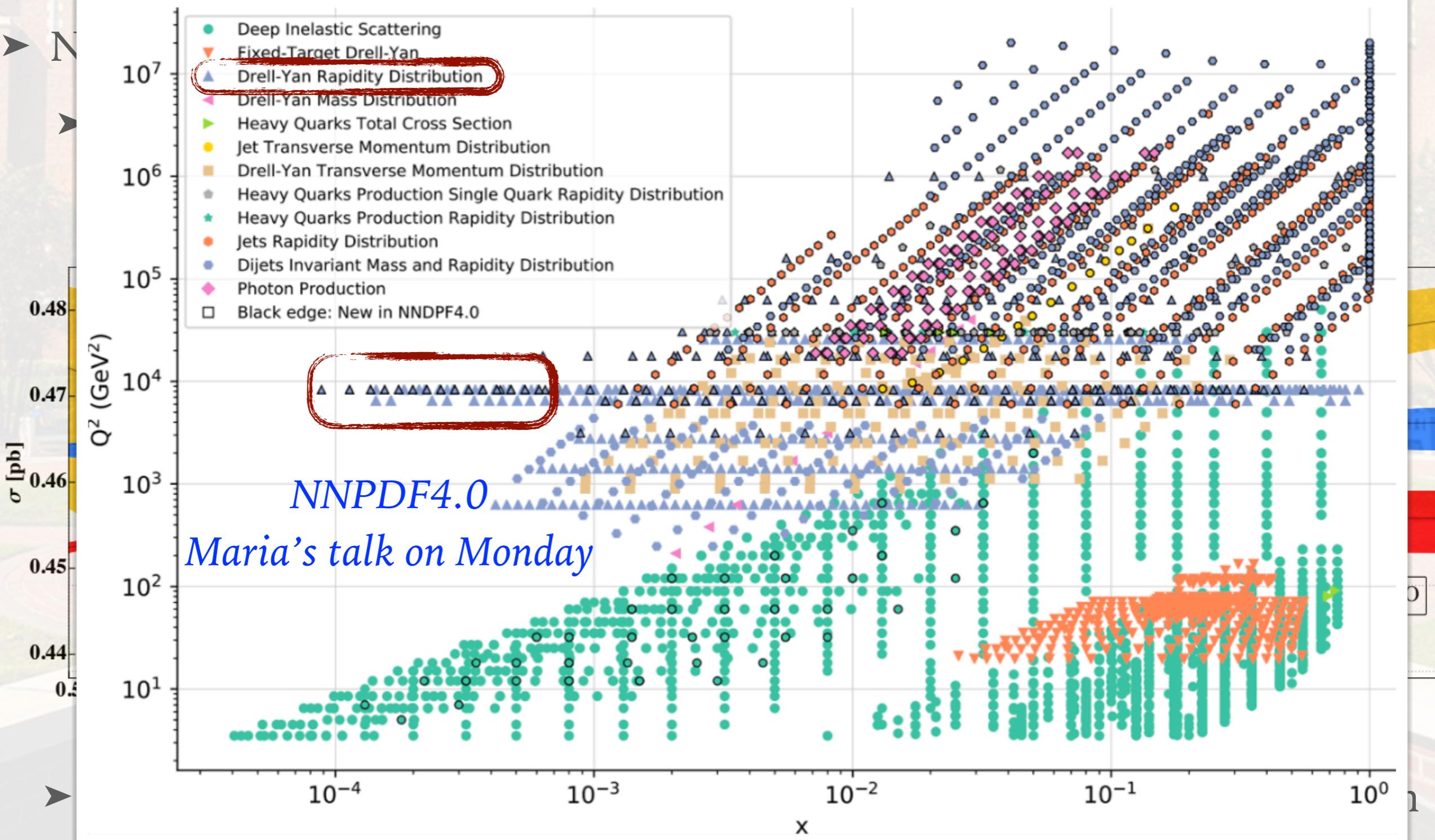
$pp \rightarrow \gamma^*$



C. Duhr, F. Dulat and B. Mistlberger *Phys.Rev.Lett.* 125 (2020) 17, 172001

- Require 2nd calculation to check the -2% correction & scale variation
- Need to study differential observables with wide dynamic range
  - To feed coherent predictions for PDF and  $\alpha_s$  fittings

# THE PRECISION THEORY FRONTIER AT THE LHC



- Need to study differential observables with wide dynamic range
  - To feed coherent predictions for PDF and  $\alpha_s$  fittings

# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

XC, T. Gehrmann, N. Glover, A. Huss, T. Z. Yang, H. X. Zhu

- Apply qt-subtraction at N3LO with **SCET factorisation** and expand to N3LO:

$$\frac{d^3\sigma}{dQ^2 d^2\vec{q}_T dy} = \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} \sum_q \sigma_{\text{LO}}^{\gamma^*} H_{q\bar{q}} \left[ \sum_k \int_{x_1}^1 \frac{dz_1}{z_1} \mathcal{I}_{qk}(z_1, b_T^2, \mu) f_{k/h_1}(x_1/z_1, \mu) \right. \\ \times \sum_j \int_{x_2}^1 \frac{dz_2}{x_2} \mathcal{I}_{\bar{q}j}(z_2, b_T^2, \mu) f_{j/h_2}(x_2/z_2, \mu) \mathcal{S}(b_\perp, \mu) + (q \leftrightarrow \bar{q}) \left. \right] + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

- All factorised functions are recently known up to N3LO:
  - 1) 3-loop hard function  $H_{q\bar{q}}^{(3)}$  from *JHEP* 06 (2010) 094
  - 2) TMD soft function  $S(b_\perp, \mu)$  at  $\alpha_s^3$  from *Phys.Rev.Lett.* 118 (2017) 022004
  - 3) Matching kernel of TMD beam function  $I_{qk}$  at  $\alpha_s^3$  from:  
*Phys.Rev.Lett.* 124 (2020) 092001 and *JHEP* 09 (2020) 146

- Apply qt cut to factorise N3LO contribution into two parts:

$$d\sigma_{N^3LO}^{\gamma^*} = [\mathcal{H}^{\gamma^*} \otimes d\sigma^{\gamma^*}]_{N^3LO} \Big|_{\delta(p_{T,\gamma^*})} + [d\sigma_{NNLO}^{\gamma^*+jet} - d\sigma_{N^3LO}^{\gamma^* CT}]_{p_{T,\gamma^*} > qt}$$

- $d\sigma_{NNLO}^{\gamma^*+jet}$  from NNLOJET with excellent numerical stability at small  $p_{T,\gamma^*}$

# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

XC, T. Gehrmann, N. Glover, A. Huss, T. Z. Yang, H. X. Zhu

- Apply qt-subtraction at N3LO with **SCET factorisation** and expand to N3LO:

$$\frac{d^3\sigma}{dQ^2 d^2\vec{q}_T dy} = \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} \sum_q \sigma_{\text{LO}}^{\gamma^*} H_{q\bar{q}} \left[ \sum_k \int_{x_1}^1 \frac{dz_1}{z_1} \mathcal{I}_{qk}(z_1, b_T^2, \mu) f_{k/h_1}(x_1/z_1, \mu) \right. \\ \times \sum_j \int_{x_2}^1 \frac{dz_2}{x_2} \mathcal{I}_{\bar{q}j}(z_2, b_T^2, \mu) f_{j/h_2}(x_2/z_2, \mu) \mathcal{S}(b_\perp, \mu) + (q \leftrightarrow \bar{q}) \left. \right] + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

- All factorised functions are recently known up to N3LO:

- 1) 3-loop hard function  $H_{q\bar{q}}^{(3)}$  from [JHEP 06 \(2010\) 094](#)
- 2) TMD soft function  $S(b_\perp, \mu)$  at  $\alpha_s^3$  from [Phys.Rev.Lett. 118 \(2017\) 022004](#)
- 3) Matching kernel of TMD beam function  $I_{qk}$  at  $\alpha_s^3$  from:  
[Phys.Rev.Lett. 124 \(2020\) 092001](#) and [JHEP 09 \(2020\) 146](#)

- Apply qt cut to factorise N3LO contribution into two parts:

$$d\sigma_{N^3LO}^{\gamma^*} = [\mathcal{H}^{\gamma^*} \otimes d\sigma^{\gamma^*}]_{N^3LO} \Big|_{\delta(p_{T,\gamma^*})} + [d\sigma_{NNLO}^{\gamma^*+jet} - d\sigma_{N^3LO}^{\gamma^* CT}]_{p_{T,\gamma^*} > qt}$$

- $d\sigma_{NNLO}^{\gamma^*+jet}$  from NNLOJET with excellent numerical stability at small  $p_{T,\gamma^*}$

# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

XC, T. Gehrmann, N. Glover, A. Huss, T. Z. Yang, H. X. Zhu

- Apply qt-subtraction at N3LO with **SCET factorisation** and expand to N3LO:

$$\frac{d^3\sigma}{dQ^2 d^2\vec{q}_T dy} = \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} \sum_q \sigma_{\text{LO}}^{\gamma^*} H_{q\bar{q}} \left[ \sum_k \int_{x_1}^1 \frac{dz_1}{z_1} \mathcal{I}_{qk}(z_1, b_T^2, \mu) f_{k/h_1}(x_1/z_1, \mu) \right. \\ \times \sum_j \int_{x_2}^1 \frac{dz_2}{x_2} \mathcal{I}_{\bar{q}j}(z_2, b_T^2, \mu) f_{j/h_2}(x_2/z_2, \mu) \mathcal{S}(b_\perp, \mu) + (q \leftrightarrow \bar{q}) \left. \right] + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

- All factorised functions are recently known up to N3LO:

- 1) 3-loop hard function  $H_{q\bar{q}}^{(3)}$  from [JHEP 06 \(2010\) 094](#)
- 2) TMD soft function  $S(b_\perp, \mu)$  at  $\alpha_s^3$  from [Phys.Rev.Lett. 118 \(2017\) 022004](#)
- 3) Matching kernel of TMD beam function  $I_{qk}$  at  $\alpha_s^3$  from:  
[Phys.Rev.Lett. 124 \(2020\) 092001](#) and [JHEP 09 \(2020\) 146](#)

- Apply qt cut to factorise N3LO contribution into two parts:

$$d\sigma_{N^3LO}^{\gamma^*} = [\mathcal{H}^{\gamma^*} \otimes d\sigma^{\gamma^*}]_{N^3LO} \Big|_{\delta(p_{T,\gamma^*})} + [d\sigma_{NNLO}^{\gamma^*+jet} - d\sigma_{N^3LO}^{\gamma^* CT}]_{p_{T,\gamma^*} > \textcolor{red}{qt}}$$

- $d\sigma_{NNLO}^{\gamma^*+jet}$  from NNLOJET with excellent numerical stability at small  $p_{T,\gamma^*}$

# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

XC, T. Gehrmann, N. Glover, A. Huss, T. Z. Yang, H. X. Zhu

- Apply qt-subtraction at N3LO with **SCET factorisation** and expand to N3LO:

$$\frac{d^3\sigma}{dQ^2 d^2\vec{q}_T dy} = \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} \sum_q \sigma_{\text{LO}}^{\gamma^*} H_{q\bar{q}} \left[ \sum_k \int_{x_1}^1 \frac{dz_1}{z_1} \mathcal{I}_{qk}(z_1, b_T^2, \mu) f_{k/h_1}(x_1/z_1, \mu) \right. \\ \times \sum_j \int_{x_2}^1 \frac{dz_2}{x_2} \mathcal{I}_{\bar{q}j}(z_2, b_T^2, \mu) f_{j/h_2}(x_2/z_2, \mu) \mathcal{S}(b_\perp, \mu) + (q \leftrightarrow \bar{q}) \left. \right] + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

- All factorised functions are recently known up to N3LO:
  - 1) 3-loop hard function  $H_{q\bar{q}}^{(3)}$  from [JHEP 06 \(2010\) 094](#)
  - 2) TMD soft function  $S(b_\perp, \mu)$  at  $\alpha_s^3$  from [Phys.Rev.Lett. 118 \(2017\) 022004](#)
  - 3) Matching kernel of TMD beam function  $I_{qk}$  at  $\alpha_s^3$  from:  
[Phys.Rev.Lett. 124 \(2020\) 092001](#) and [JHEP 09 \(2020\) 146](#)

- Apply qt cut to factorise N3LO contribution into two parts:

$$d\sigma_{N^3LO}^{\gamma^*} = [\mathcal{H}^{\gamma^*} \otimes d\sigma^{\gamma^*}]_{N^3LO} \Big|_{\delta(p_{T,\gamma^*})} + [d\sigma_{NNLO}^{\gamma^*+jet} - d\sigma_{N^3LO}^{\gamma^* CT}]_{p_{T,\gamma^*} > \textcolor{red}{qt}}$$

- $d\sigma_{NNLO}^{\gamma^*+jet}$  from **NNLOJET** with excellent numerical stability at small  $p_{T,\gamma^*}$   
[JHEP 12 \(2018\) 132](#) and [Eur.Phys.J.C 79 \(2019\) 10](#)

# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

- Computational setup for  $pp \rightarrow \gamma^*$   
(identical to CFB paper *Phys.Rev.Lett.* 125 (2020) 17, 172001)
  - Fix Q value for  $\gamma^*$  at 100 GeV (NNLO and N3LO scale variations deviate)
  - Use central value of PDF4LHC15\_nnlo\_mc as benchmark input
  - $\alpha_s(m_Z) = 0.118$  with scale variation values calculated from LHAPDF
  - $G_\mu$  EW-scheme with fixed  $\alpha$  value
  - $\mu_R = \mu_F = 100$  GeV for central QCD scale and use 7-point variations for uncertainty estimation
- Apply  $p_{T,\gamma^*} > 0.25$  GeV constrain for NNLO  $\gamma^* + Jet$  without jet definition
- Use SCET factorisation to integrate contributions below qt cut.
- Dedicated MC adaption to four sub-channels:  
 $gg, qqb = (qqb+qbq), qg = (qg+gq+qbg+gqb), qq = (qq+qbqb)$

# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

- Computational setup for  $pp \rightarrow \gamma^*$   
(identical to CFB paper *Phys.Rev.Lett.* 125 (2020) 17, 172001)
  - Fix Q value for  $\gamma^*$  at 100 GeV (NNLO and N3LO scale variations deviate)
  - Use central value of PDF4LHC15\_nnlo\_mc as benchmark input
  - $\alpha_s(m_Z) = 0.118$  with scale variation values calculated from LHAPDF
  - $G_\mu$  EW-scheme with fixed  $\alpha$  value
  - $\mu_R = \mu_F = 100$  GeV for central QCD scale and use 7-point variations for uncertainty estimation
- Apply  $p_{T,\gamma^*} > 0.25$  GeV constrain for NNLO  $\gamma^* + Jet$  without jet definition
- Use SCET factorisation to integrate contributions below qt cut.
- Dedicated MC adaption to four sub-channels:  
 $gg$ ,  $qq\bar{b}$  =  $(q\bar{q}b + q\bar{b}q)$ ,  $qg$  =  $(qg + gq + q\bar{b}g + g\bar{q}b)$ ,  $qq$  =  $(q\bar{q} + q\bar{b}q\bar{b})$

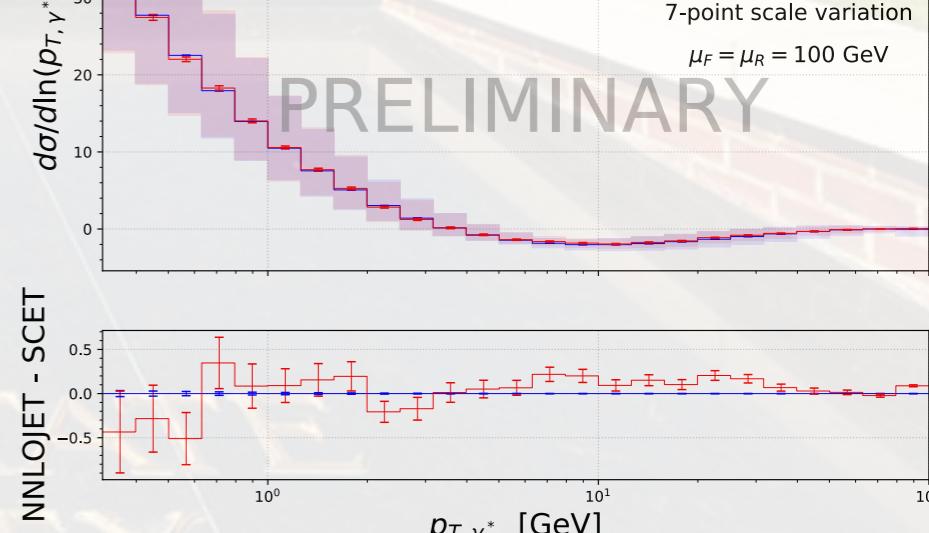
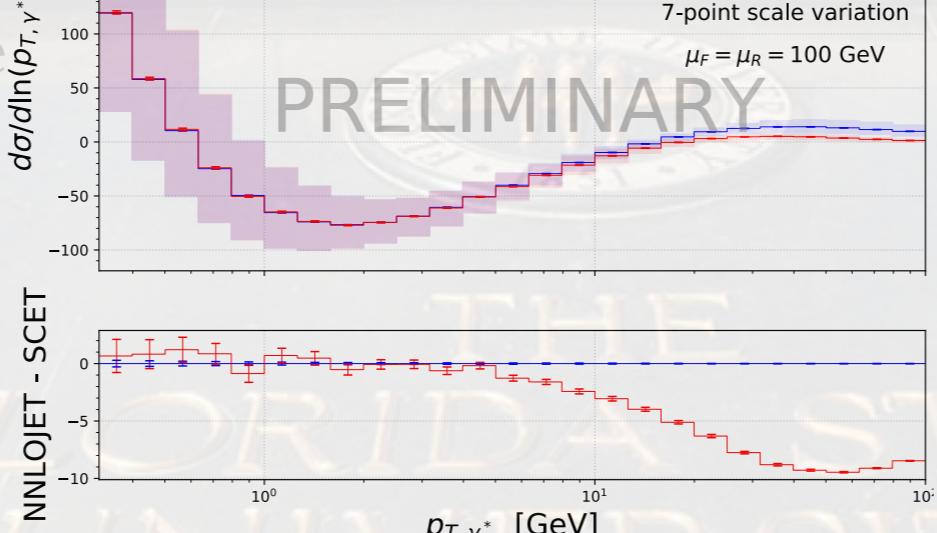
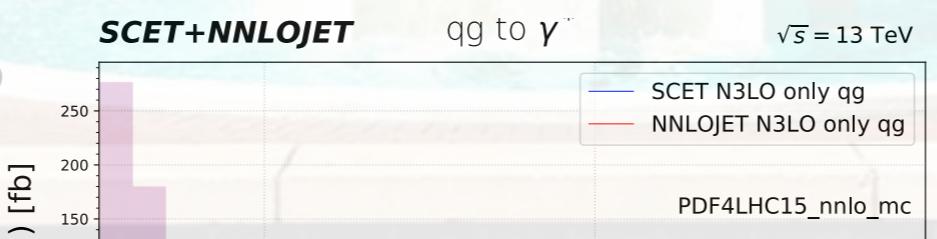
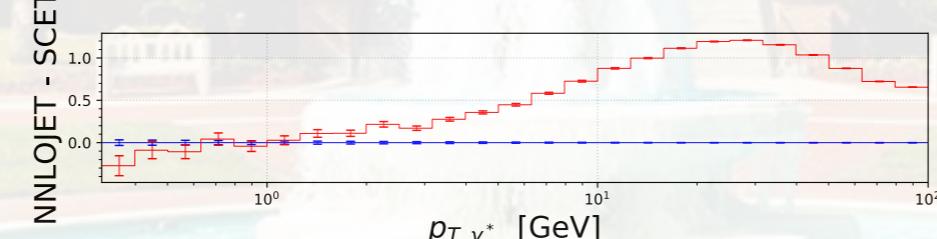
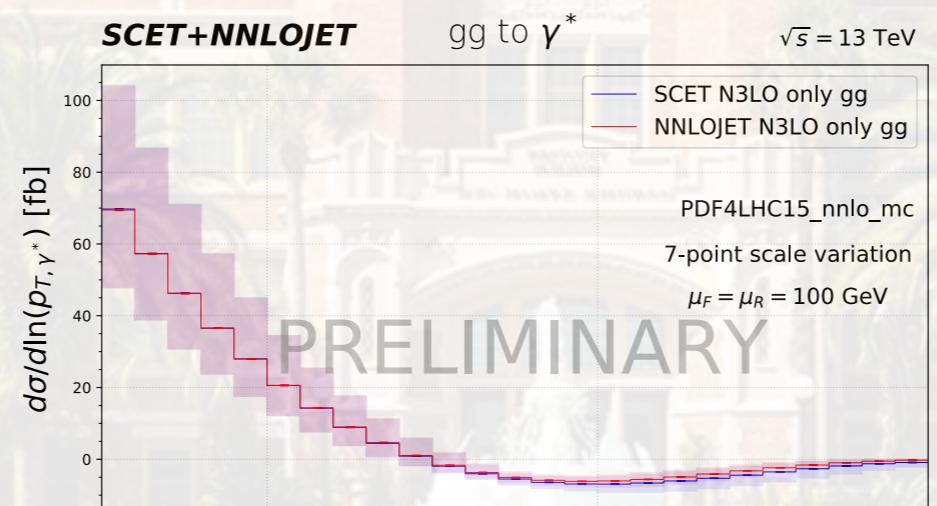
# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

- $d\sigma_{NNLO}^{\gamma^*+jet}$  vs.  $d\sigma_{N^3LO}^{\gamma^* CT}$  at small  $p_{T,\gamma^*}$

- Check each channel for N3LO only

- Convergence** between SCET and NNLOJET:  
gg below  $\sim 1$  GeV  
qg below  $\sim 4$  GeV  
qq below  $\sim 6$  GeV

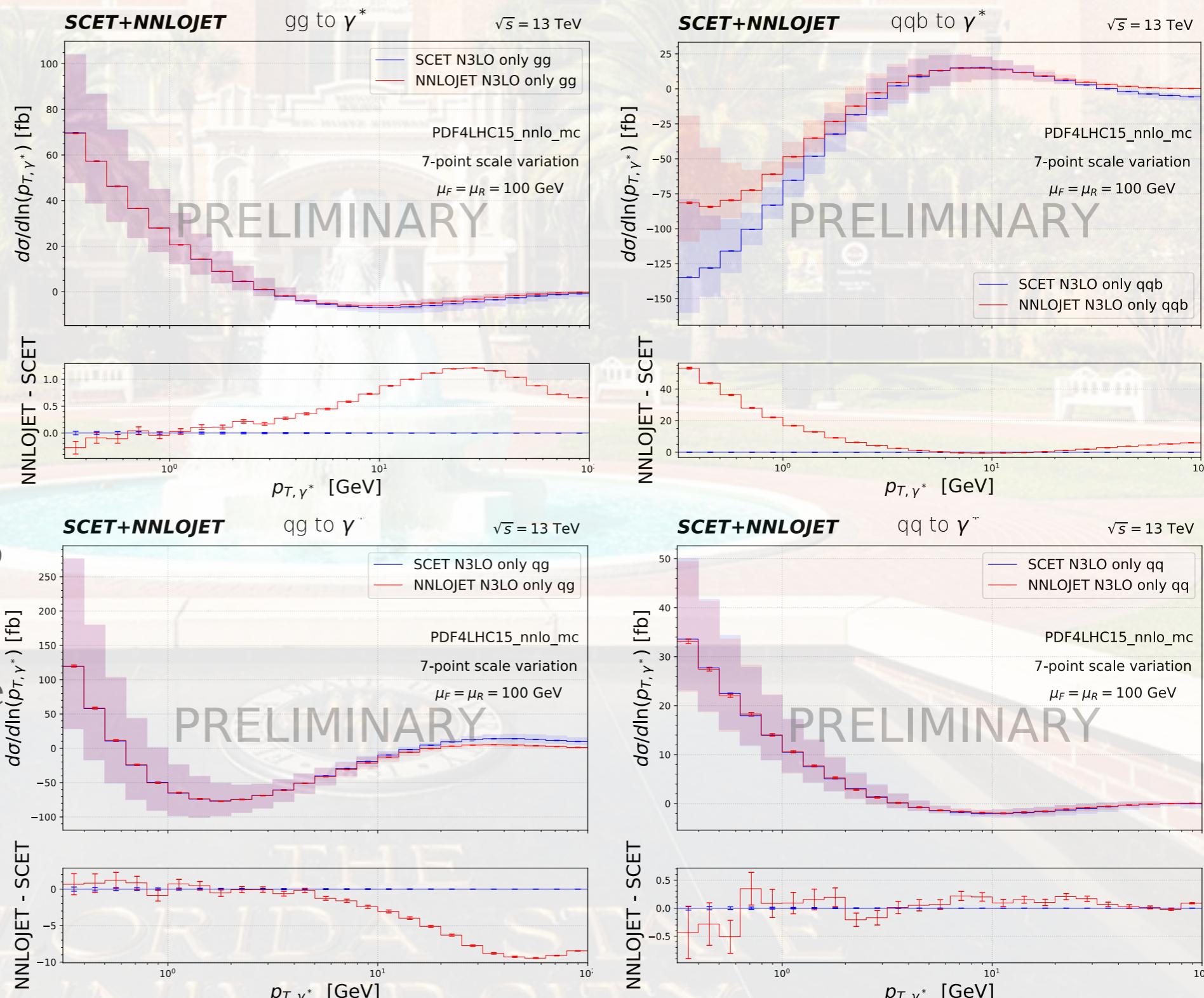
- Deviations seen in qqb obvious below 4 GeV
- Surface only with large statistics at small  $p_{T,\gamma^*}$
- Large cancellation between qg and qqb



# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

- $d\sigma_{NNLO}^{\gamma^*+jet}$  vs.  $d\sigma_{N^3LO}^{\gamma^* CT}$  at small  $p_{T,\gamma^*}$

- Check each channel for N3LO only
- Convergence** between SCET and NNLOJET:  
 gg below  $\sim 1$  GeV  
 qg below  $\sim 4$  GeV  
 qq below  $\sim 6$  GeV
- Deviations seen** in qqb obvious below 4 GeV
- Surface only with large statistics at small  $p_{T,\gamma^*}$
- Large cancellation between qg and qqb



# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

- $d\sigma_{NNLO}^{\gamma^*+jet}$  vs.  $d\sigma_{N^3LO}^{\gamma^* CT}$  at small  $p_{T,\gamma^*}$
- Dive into each colour level of qqb channel:  $\sigma_{q\bar{q}}^{\gamma^*}|_{N3LO \text{ only}} = (N^2 - 1) \times [N^0 + N_f N^{-1} + N^{-1} + N_f N^{-2} + N^{-2} + N_f N^{-3} + N^{-3} + N^{-4}]$

# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

- $d\sigma_{NNLO}^{\gamma^*+jet}$  vs.  $d\sigma_{N^3LO}^{\gamma^* CT}$  at small  $p_{T,\gamma^*}$
- Dive into each colour level of qqb channel:  $\sigma_{q\bar{q}}^{\gamma^*}|_{N^3LO \text{ only}} = (N^2 - 1) \times [N^0 + N_f N^{-1} + N^{-1} + N_f N^{-2} + N^{-2} + N_f N^{-3} + N^{-3} + N^{-4}]$ 
$$\sigma_{q\bar{q}}^{\gamma^*}|_{N^3LO \text{ only}} = (N^2 - 1) \times [N^0 + N_f N^{-1} + N^{-1} + N_f N^{-2} + N^{-2} + N_f N^{-3} + N^{-3} + N^{-4}]$$
$$Nc0$$
$$Ncm1$$
$$Ncm2$$
$$Ncm3$$
$$Ncm4$$

# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

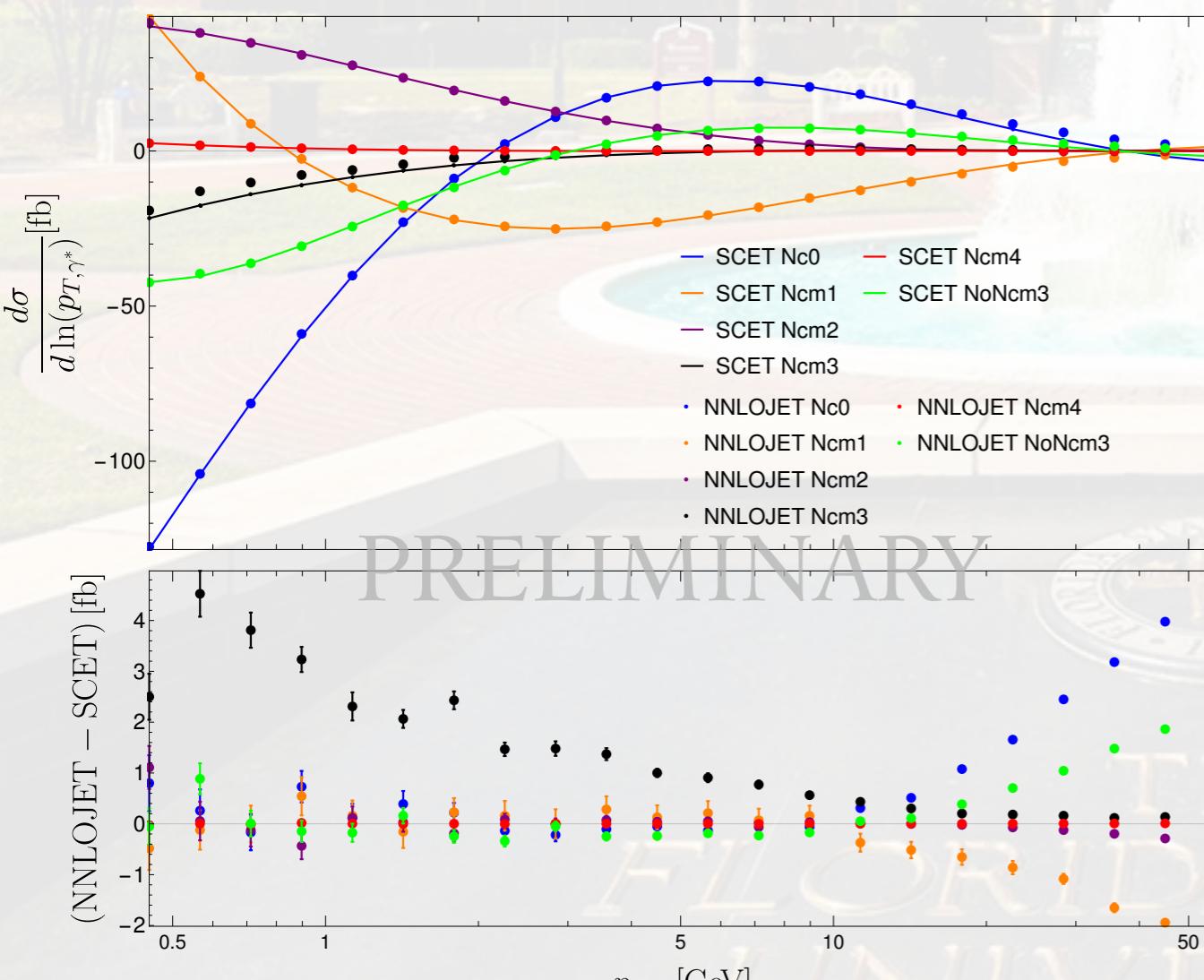
- $d\sigma_{NNLO}^{\gamma^*+jet}$  vs.  $d\sigma_{N^3LO}^{\gamma^* CT}$  at small  $p_{T,\gamma^*}$

- Dive into each colour level of qqb channel:  $\sigma_{q\bar{q}}^{\gamma^*}|_{N^3LO \text{ only}} = (N^2 - 1) \times$

$$[N^0 + N_f N^{-1} + N^{-1} + N_f N^{-2} + N^{-2} + N_f N^{-3} + N^{-3} + N^{-4}]$$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$   
 $Nc0 \quad Ncm1 \quad Ncm2 \quad Ncm3 \quad Ncm4$

$q\bar{q} (\mu_R = \mu_F = Q = 100 \text{ GeV})$



- Excellent agreement in leading colour
- Problematic part in Ncm3
- More specifically in  $N_f N^{-3}$  SLC
- Dangerous for recent application in N3LO+N3LL' matching:  
[2102.08039](https://arxiv.org/abs/2102.08039), [2103.04974](https://arxiv.org/abs/2103.04974)
- Still under investigation
- Proceed without Ncm3 (temporary)

# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

- $d\sigma_{NNLO}^{\gamma^* + jet}$  vs.  $d\sigma_{N^3LO}^{\gamma^* CT}$  at small  $p_{T,\gamma^*}$

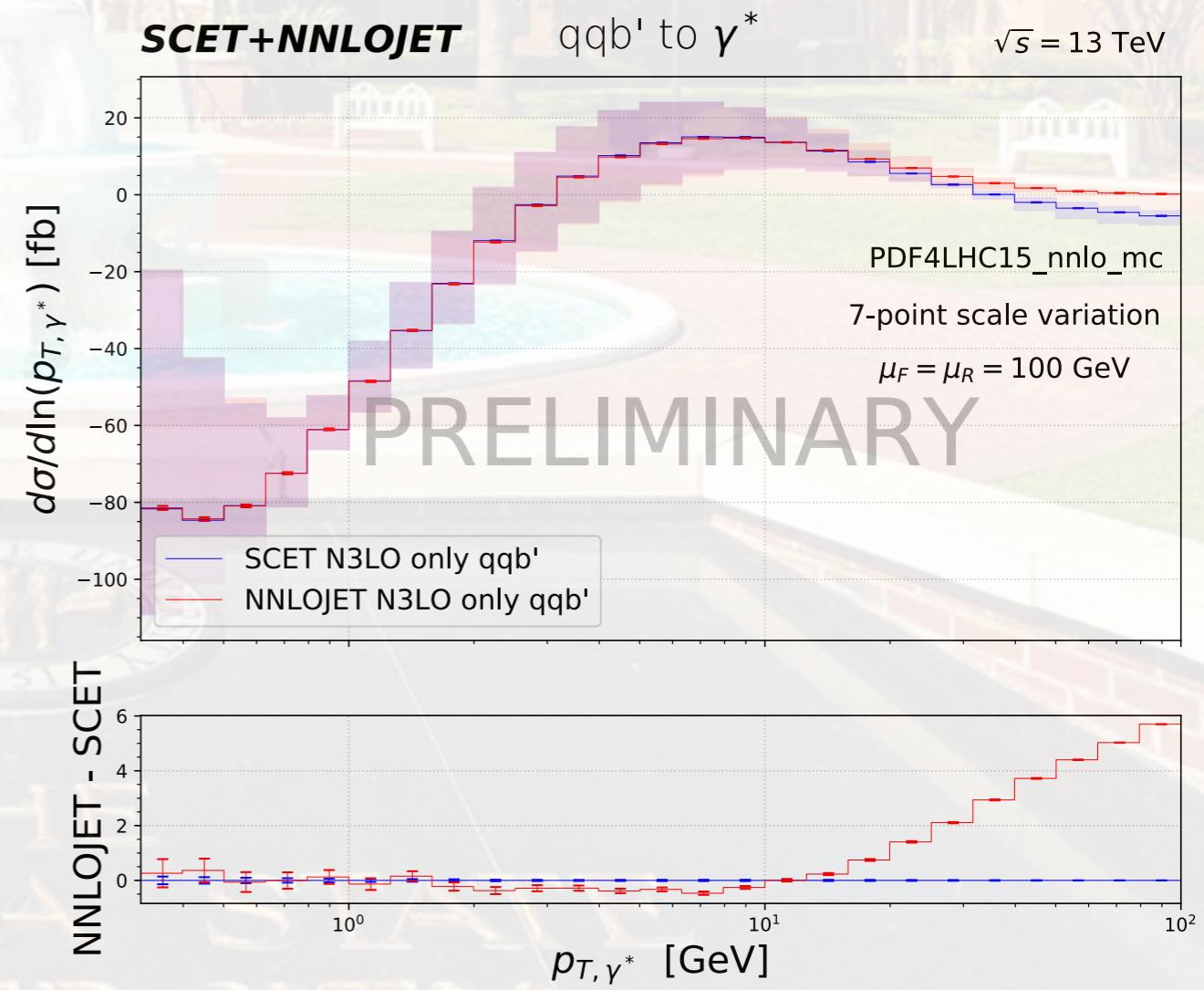
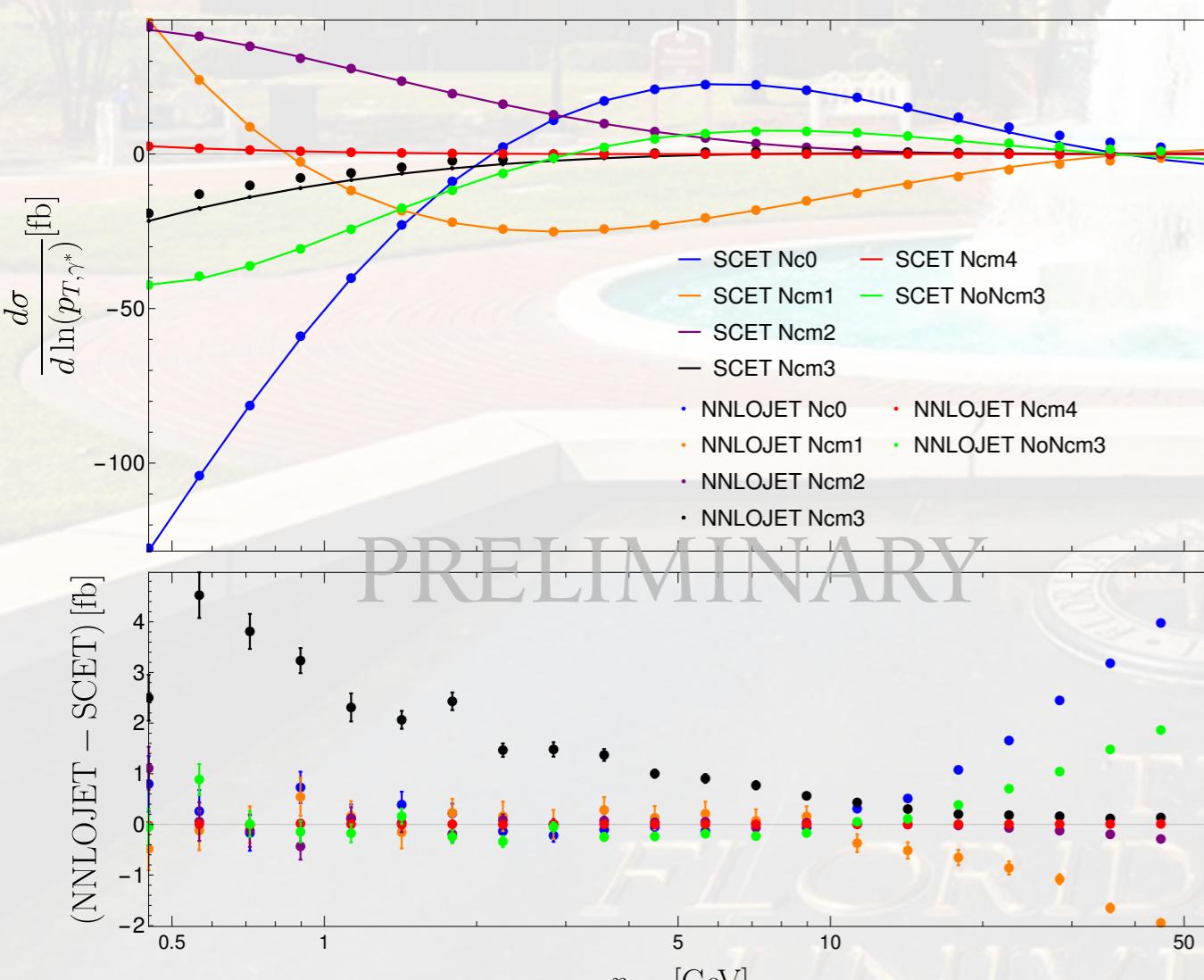
- Dive into each colour level of qqb channel:  $\sigma_{a\bar{a}}^{\gamma^*} \Big|_{N^3LO \text{ only}} = (N^2 - 1) \times$

$$\left[ N^0 + N_f N^{-1} + N^{-1} + N_f N^{-2} + N^{-2} + N_f N^{-3} + N^{-3} + N^{-4} \right] \rightarrow \sigma_{q\bar{q}'}^{\gamma^*}$$

$\downarrow$        $\downarrow$        $\downarrow$        $\downarrow$        $\downarrow$

$Nc0$        $Ncm1$        $Ncm2$        ~~$Ncm3$~~        $Ncm4$

$q\bar{q} (\mu_R = \mu_F = Q = 100 \text{ GeV})$

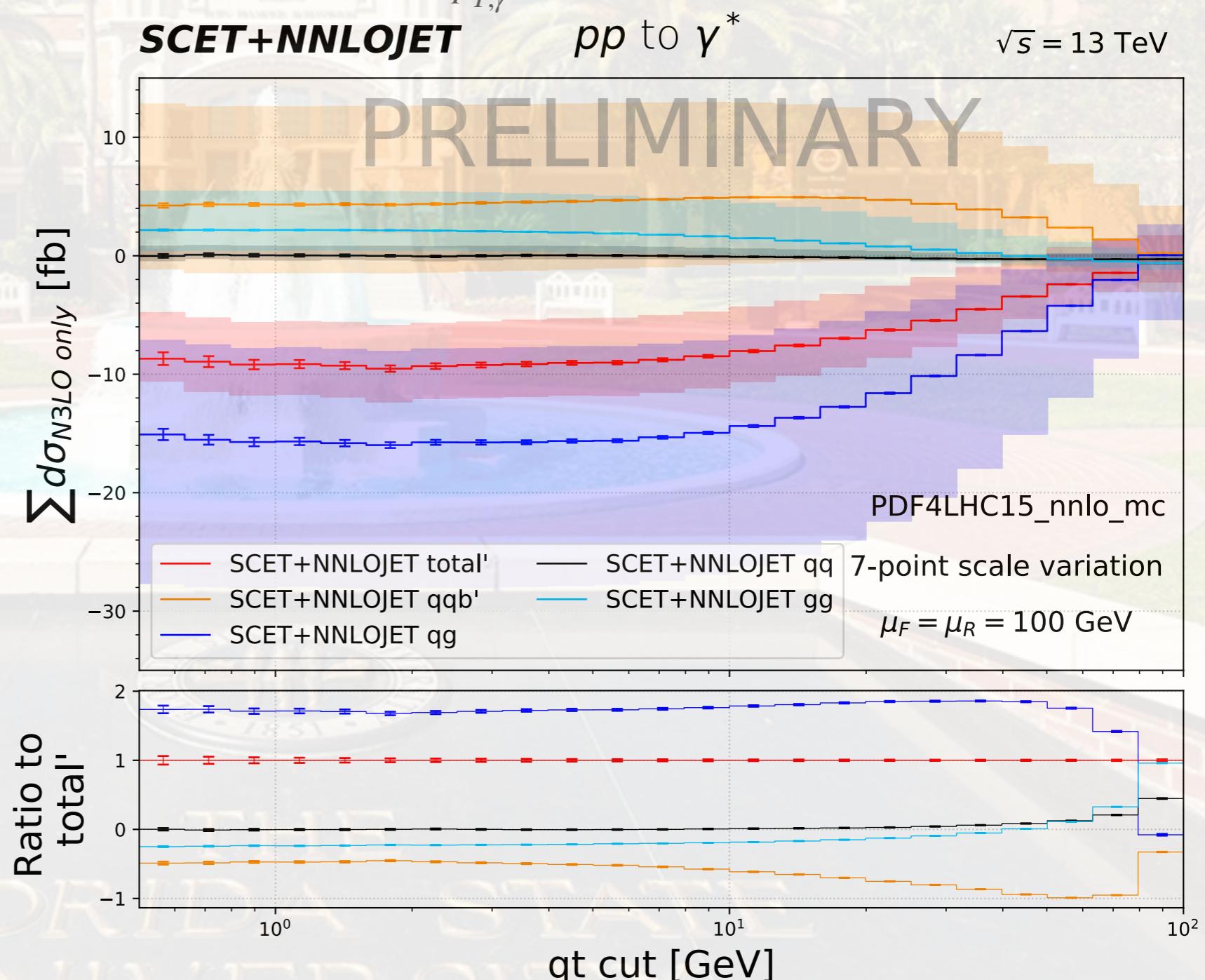


# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

- N3LO accumulated cross section from  $p_{T,\gamma^*}$  (N3LO' coefficient)

$$\sum \frac{d\sigma_{N3LO}^{\gamma^*}}{dp_{T,\gamma^*}} \equiv \sum \frac{d\sigma_{NNLO}^{\gamma^*+jet}}{dp_{T,\gamma^*}} \Big|_{p_{T,\gamma^*} > qt} + \sum \frac{d\sigma_{N^3LO}^{\gamma^* SCET}}{dp_{T,\gamma^*}} \Big|_{p_{T,\gamma^*} \in [0, qt]}$$

- More challenging than  $p_{T,\gamma^*}$
- Large scale cancellation among channels

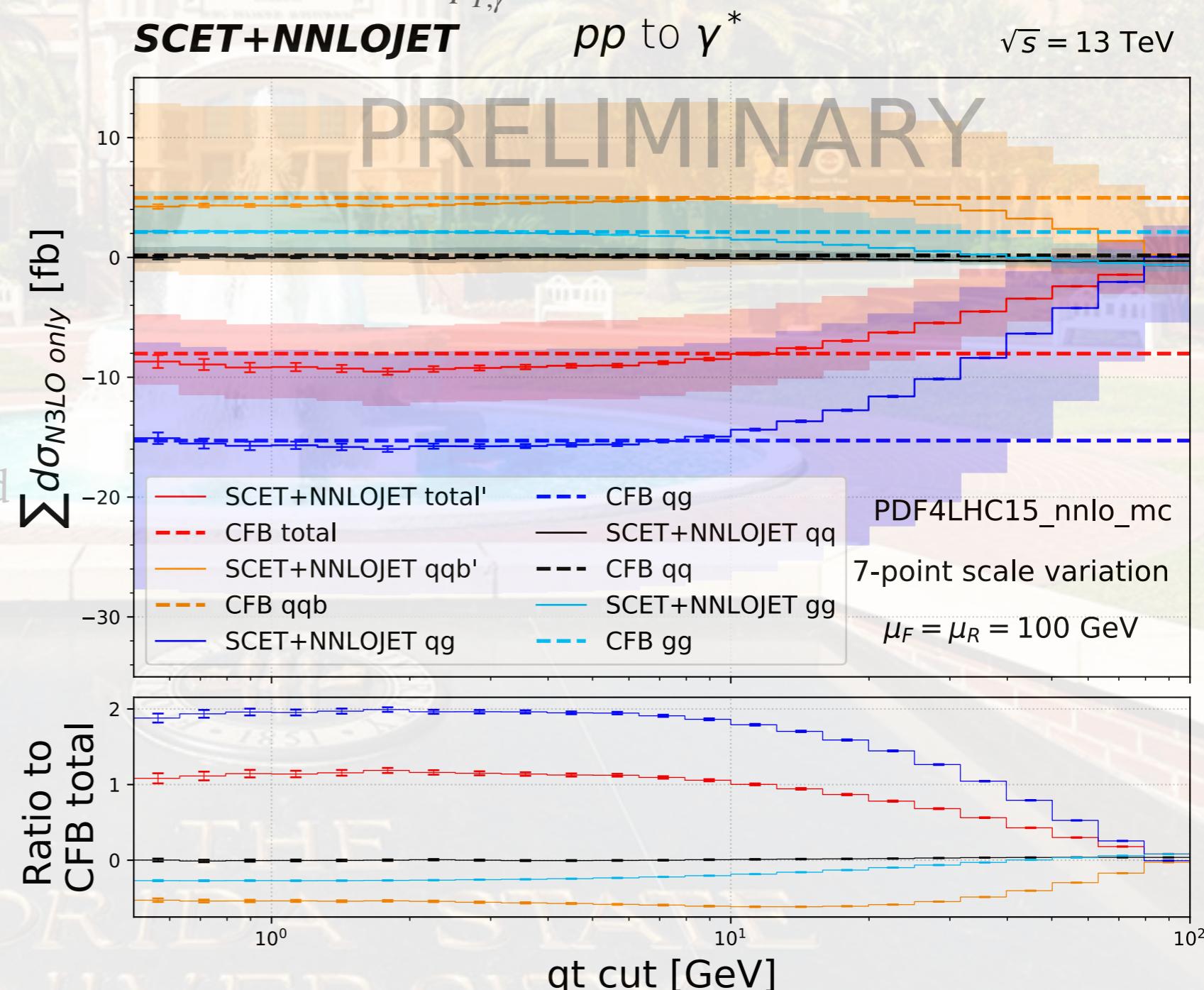


# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

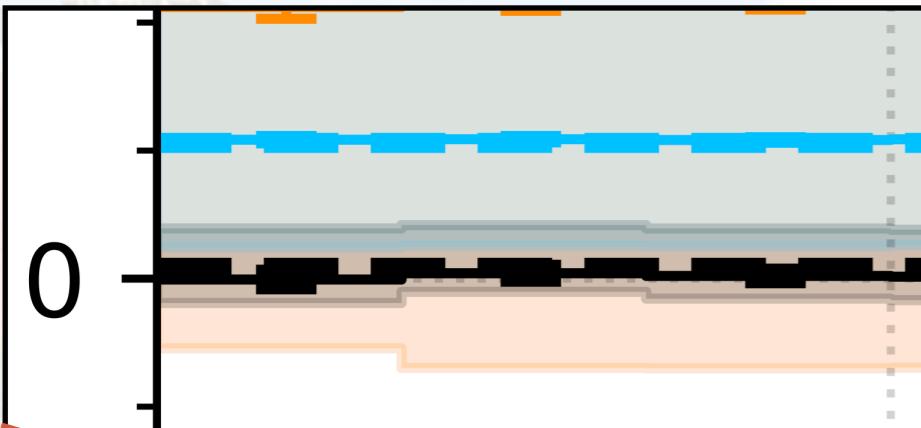
- N3LO accumulated cross section from  $p_{T,\gamma^*}$  (N3LO' coefficient)

$$\sum \frac{d\sigma_{N3LO}^{\gamma^*}}{dp_{T,\gamma^*}} \equiv \sum \frac{d\sigma_{NNLO}^{\gamma^*+jet}}{dp_{T,\gamma^*}} \Big|_{p_{T,\gamma^*} > qt} + \sum \frac{d\sigma_{N^3LO}^{\gamma^* SCET}}{dp_{T,\gamma^*}} \Big|_{p_{T,\gamma^*} \in [0, qt]}$$

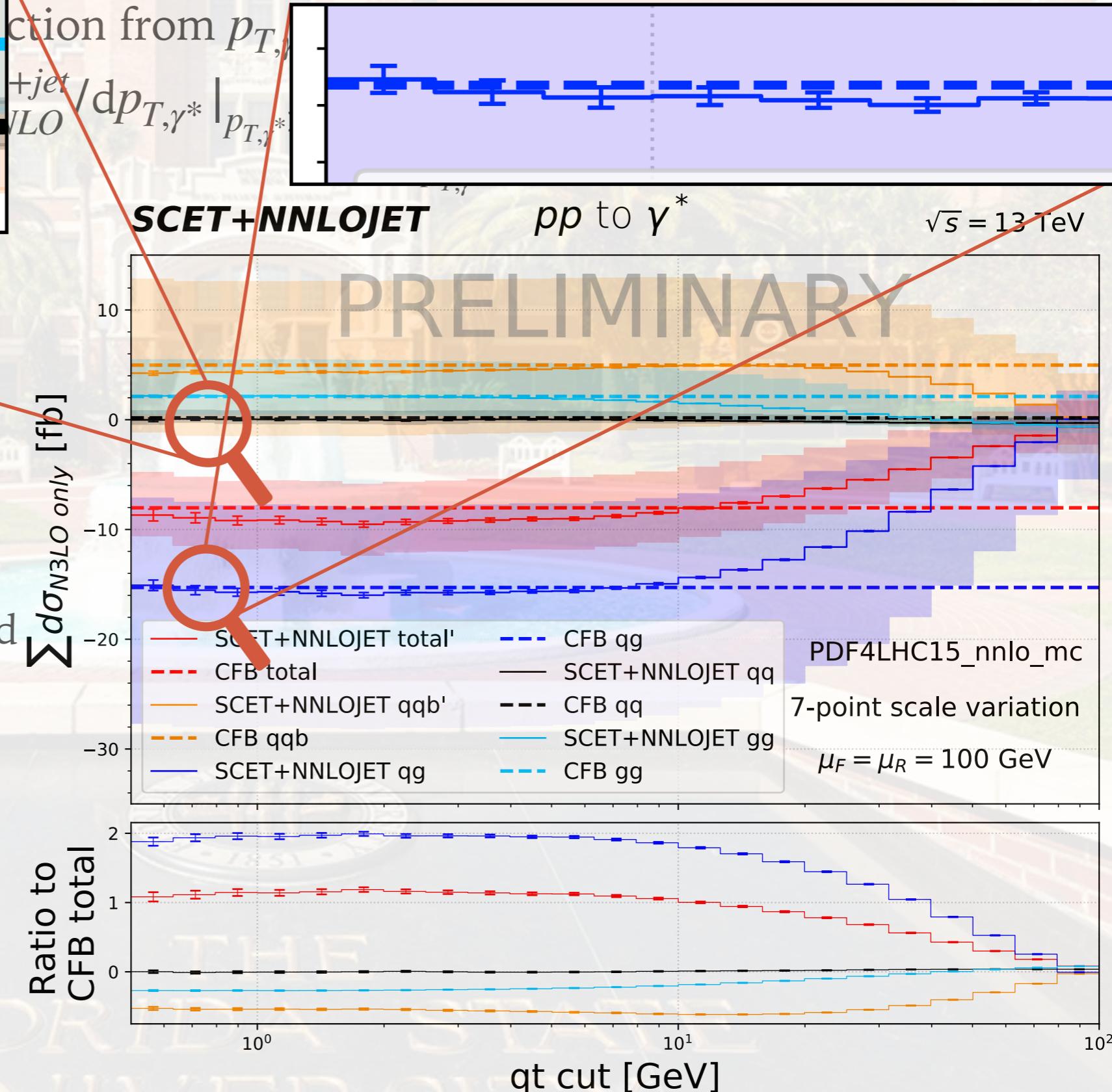
- More challenging than  $p_{T,\gamma^*}$
- Large scale cancellation among channels
- Compare to CFB total cross section results
- First validation for gg, qq and qg channels (below 1 GeV)
- Missing Ncm3 contribution in qqb channel at  $\sim 13\%$  (8% with respect to N3LO)
- Within MC error of full channel N3LO coefficient



# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

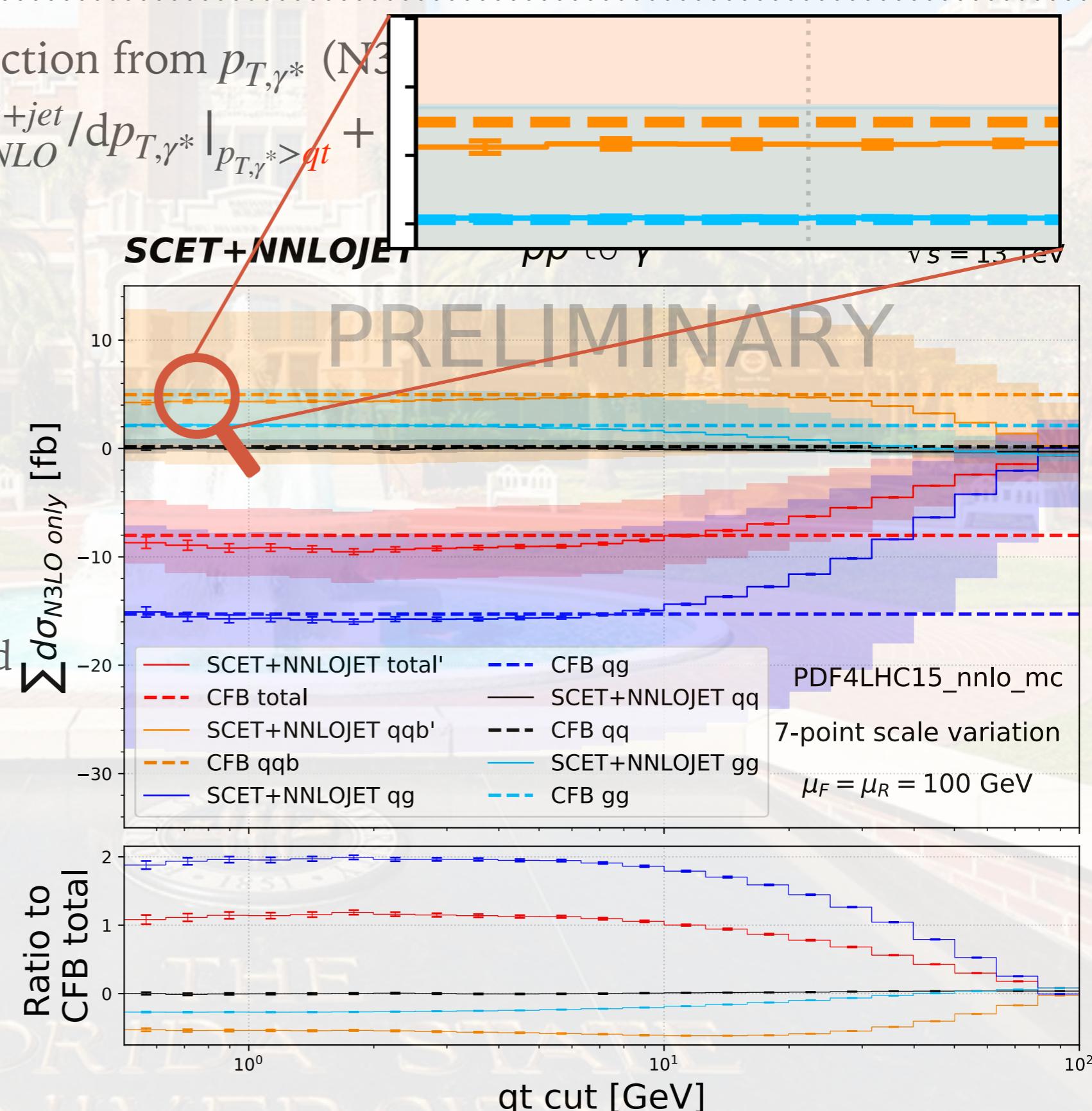


- More challenging than  $p_{T,\gamma^*}$
- Large scale cancellation among channels
- Compare to CFB total cross section results
- First validation for gg, qq and qg channels (below 1 GeV)
- Missing Ncm3 contribution in qqb channel at  $\sim 13\%$  (8% with respect to N3LO)
- Within MC error of full channel N3LO coefficient



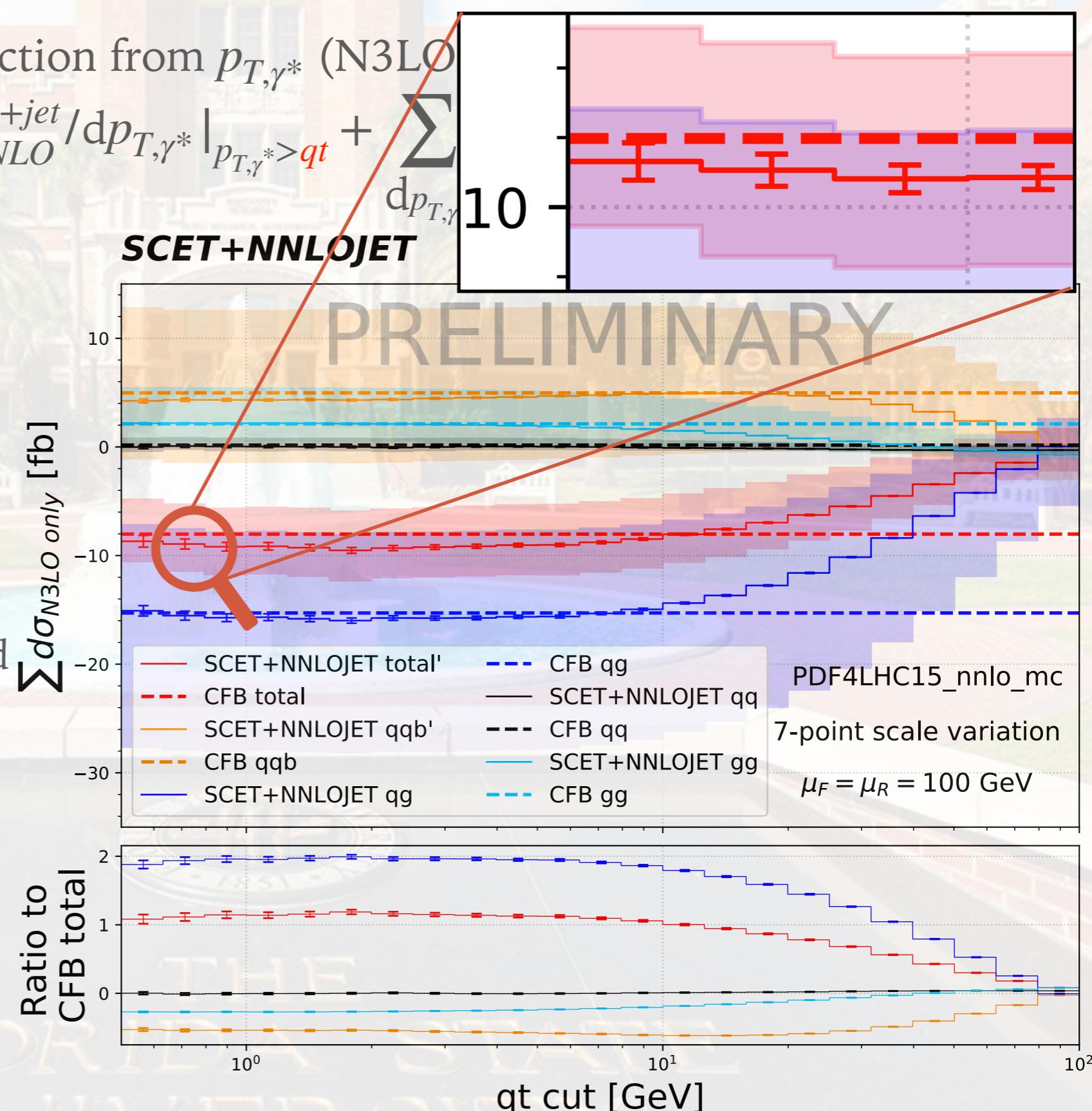
# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

- N3LO accumulated cross section from  $p_{T,\gamma^*}$  (N3LO)
 
$$\sum d\sigma_{N3LO}^{\gamma^*} \equiv \sum \frac{d\sigma_{NNLO}^{\gamma^*+jet}}{dp_{T,\gamma^*}} \Big|_{p_{T,\gamma^*} > qt} +$$
- More challenging than  $p_{T,\gamma^*}$
- Large scale cancellation among channels
- Compare to CFB total cross section results
- First validation for gg, qq and qg channels (below 1 GeV)
- Missing Ncm3 contribution in qqb channel at  $\sim 13\%$  (8% with respect to N3LO)
- Within MC error of full channel N3LO coefficient



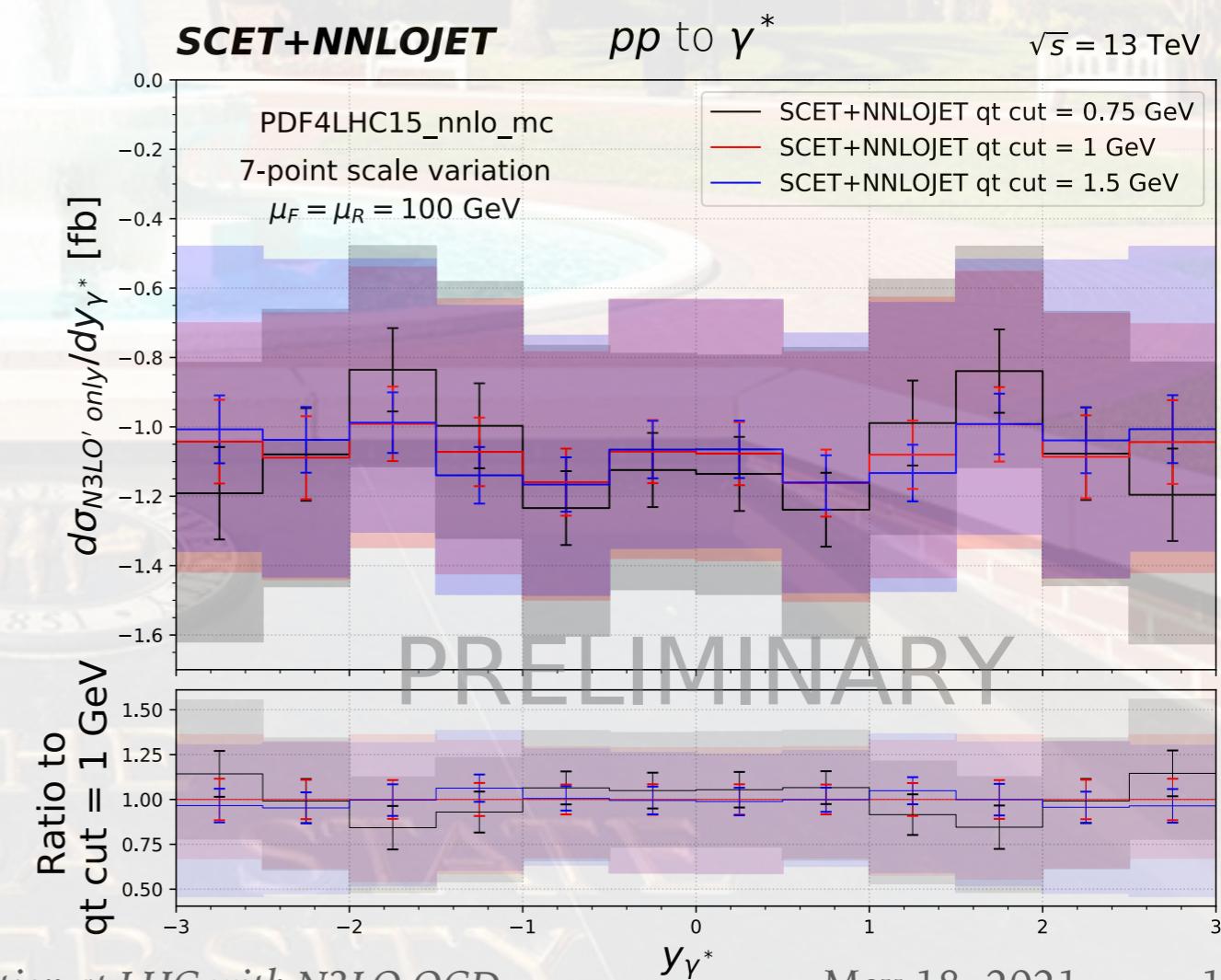
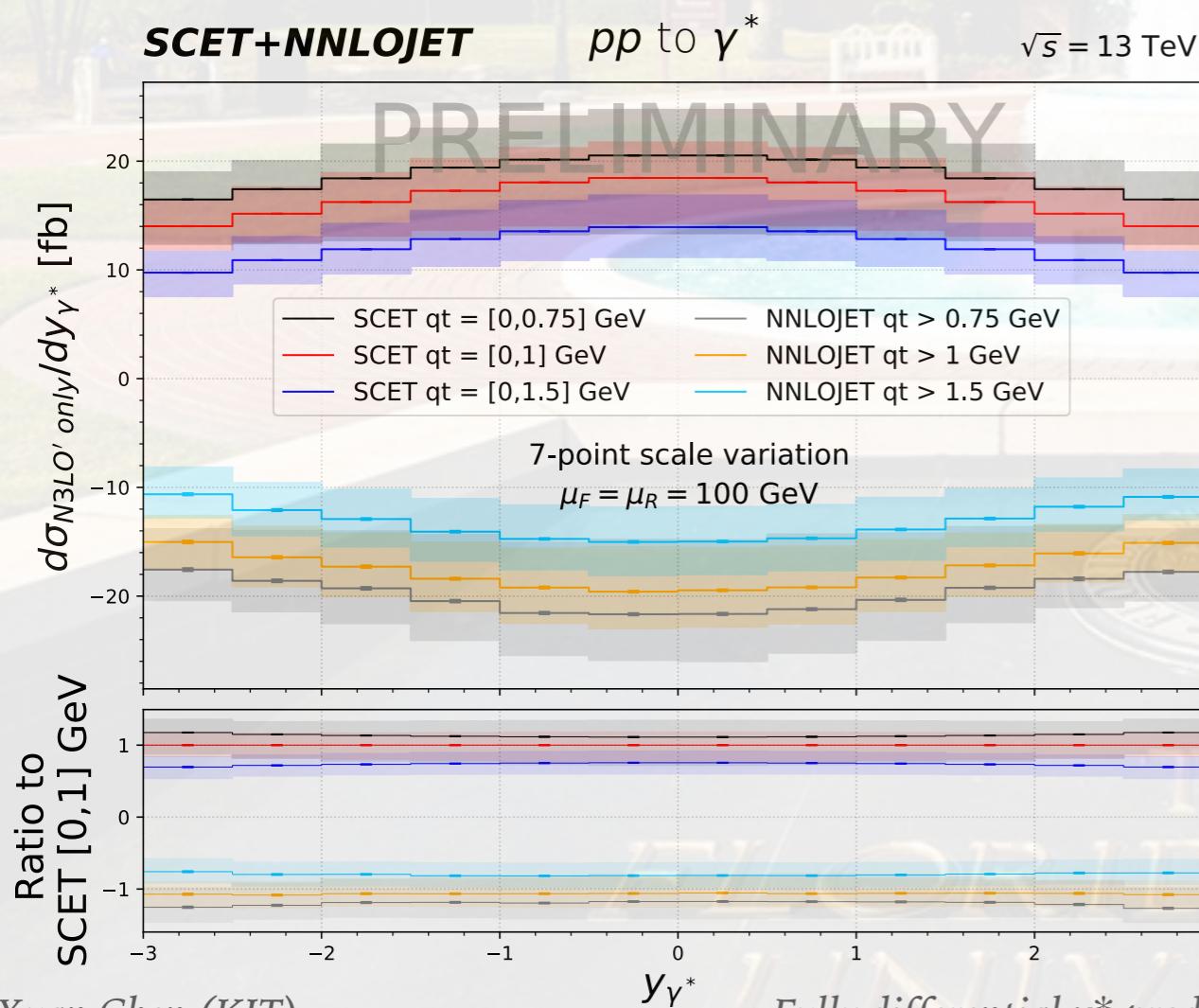
# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

- N3LO accumulated cross section from  $p_{T,\gamma^*}$  (N3LO)
 
$$\sum d\sigma_{N3LO}^{\gamma^*} \equiv \sum \frac{d\sigma_{NNLO}^{\gamma^*+jet}}{dp_{T,\gamma^*}} \Big|_{p_{T,\gamma^*} > qt} + \sum \frac{d\sigma_{N3LO}^{\gamma^*}}{dp_{T,\gamma^*}}$$
- More challenging than  $p_{T,\gamma^*}$
- Large scale cancellation among channels
- Compare to CFB total cross section results
- First validation for gg, qq and qg channels (below 1 GeV)
- Missing Ncm3 contribution in qqb channel at  $\sim 13\%$  (8% with respect to N3LO)
- Within MC error of full channel N3LO coefficient



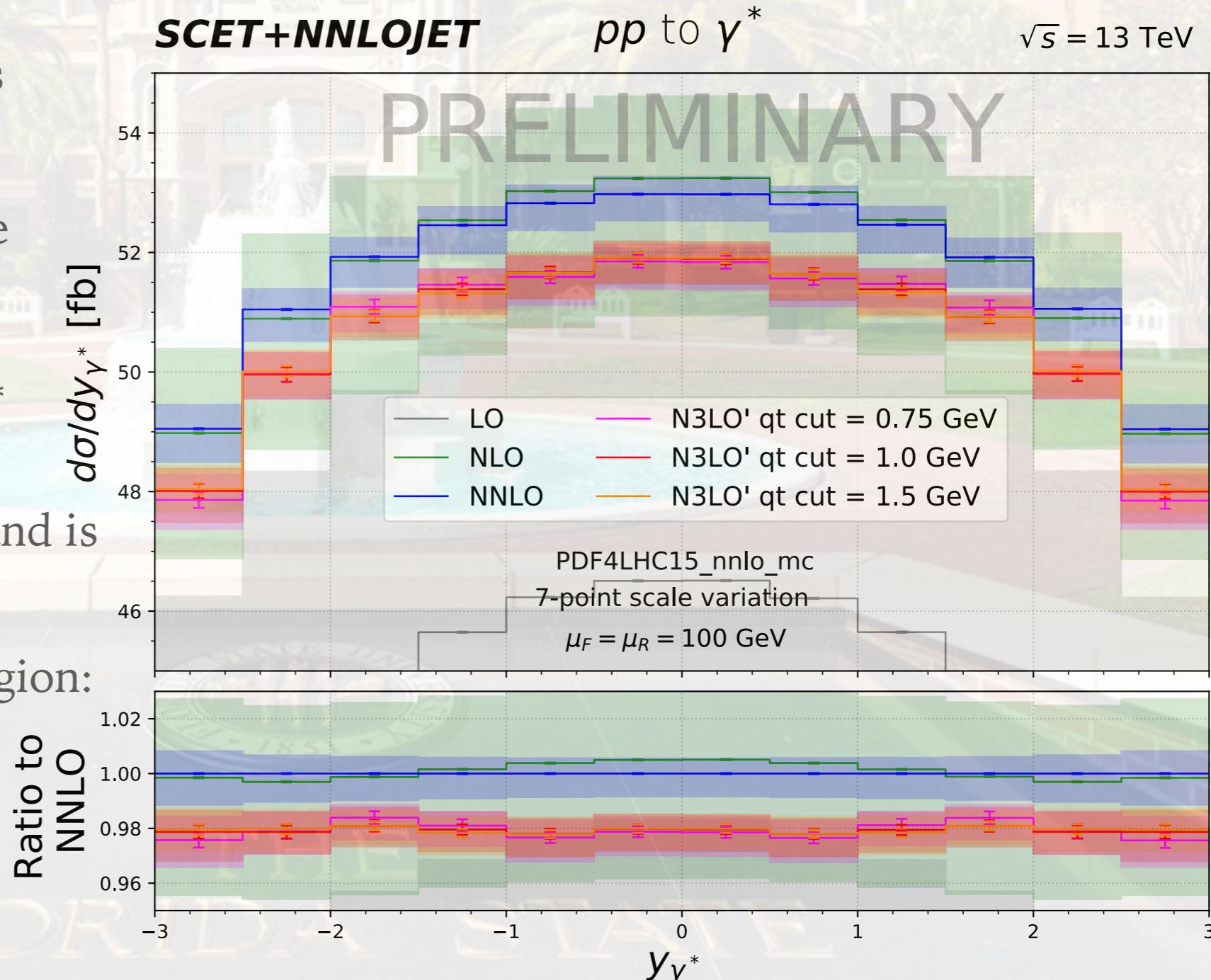
# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

- N3LO differential cross sections (N3LO' coefficient)
  - Apply SCET+NNLOJET calculations for  $y_{\gamma^*}$  distribution
  - Large cancellation in each histogram bin with  $\times 20$  multiplier  
 $\pm 0.5\%$  accuracy  $\rightarrow \pm 10\%$  fluctuation
  - Stable predictions with qt cut variation at 1 GeV



# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

- N3LO differential cross sections
  - Apply SCET+NNLOJET calculations for  $y_{\gamma^*}$  distribution
- First N3LO differential cross section for  $\gamma^*$  production
- N3LO corrections contribute -2% with respect to NNLO
- Uniformly distributed for  $y_{\gamma^*}$  cross central rapidity region
- The N3LO scale variation band is outside the NNLO ones
- Scale variations at central region:  
 $\text{NNLO: } +0.26\% \quad -0.4\%$     $\text{N3LO: } +0.26\% \quad -0.33\%$
- Scale variation range mildly increase towards large  $y_{\gamma^*}$



# SUMMARY

---

- Precision phenomenology could be the key to reveal new physics principles.
- Comprehensive understanding requires comparable precision between theory and experiment to achieve accurate interpretation/reveal disagreement.
- For theory predictions of LHC observables, there has been rapid progress in perturbative QCD calculations towards to NNLO and N3LO.
- Our standard methodology to estimate theoretical uncertainties via scale variation is challenged by N3LO calculations of charge and neutral current.
- We perform an independent calculation of the N3LO QCD correction to  $\gamma^*$  production and confirm the N3LO scale uncertainty disagree with NNLO for  $Q = 100 \text{ GeV}$ .
- We use  $q_t$  slicing method to further calculate N3LO differential  $y_{\gamma^*}$  distribution and find disagreement of scale uncertainties at differential level.

# SUMMARY

- During the N3LO calculation, we find mismatch between FO and SCET expansion for contributions from sub-leading colour qqb channel.
- The mismatch is very likely due to numerical instability issues from FO and more investigation is needed. (We have to fix this)
- Luckily, by omitting this sub-leading colour contribution, we only miss 0.16% of the N3LO total cross section of  $\gamma^*$  production.
- The performance of qt slicing is stable with qt cut around 1 GeV. This estimation will get worse when applying fiducial cuts on decay products of  $\gamma^*$ .
- With all factorisation functions at N3LO available, N3LO+N3LL' precision is expected but require careful validation of matching for inclusive production and could be very challenging with fiducial cuts upon decay products.
- Progress on qt cut power correction of LL at NNLO would help in the future.
- Differential N3LO DY and W production in the future, hopefully we could better estimate uncertainties due to missing higher order contributions.

# SUMMARY

---

- During the N3LO calculation, we find mismatch between FO and SCET expansion for contributions from sub-leading colour qqb channel.
- The mismatch is very likely due to numerical instability issues from FO and more investigation is needed. (We have to fix this)
- Luckily, by omitting this sub-leading colour contribution, we only miss 0.16% of the N3LO total cross section of  $\gamma^*$  production.
- The performance of qt slicing is stable with qt cut around 1 GeV. This estimation will get worse when applying fiducial cuts on decay products of  $\gamma^*$ .
- With all factorisation functions at N3LO available, N3LO+N3LL' precision is expected but require careful validation of matching for inclusive production and could be very challenging with fiducial cuts upon decay products.
- Progress on qt cut power correction of LL at NNLO would help in the future.
- Differential N3LO DY and W production in the future, hopefully we could better estimate uncertainties due to missing higher order contributions.

*Thank You for Your Attention*

# BACK SLIDES

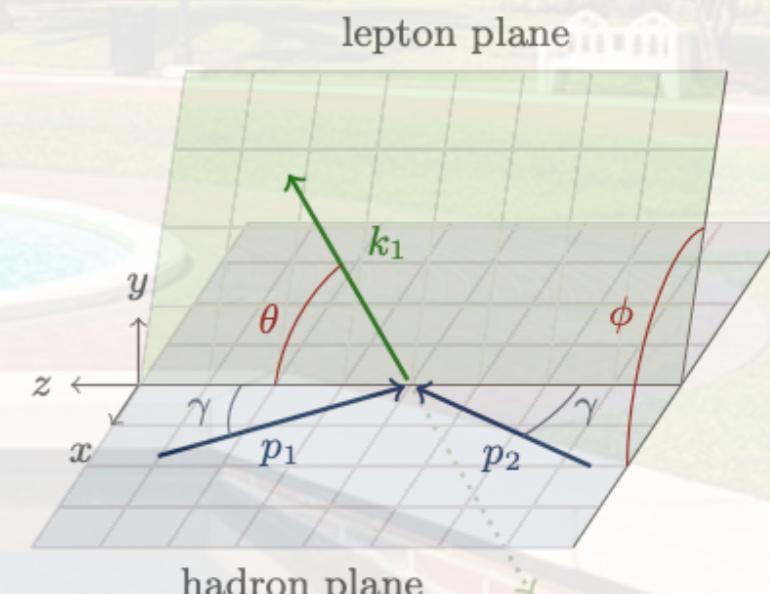
## Triple-differential Drell-Yan cross section

- Lepton pair production: EW precision observable

$$\frac{d^3\sigma}{dm_{ll}dy_{ll}d\cos\theta^*} = \frac{\pi\alpha^2}{3m_{ll}s} \sum_q P_q(\cos\theta^*) [f_q(x_1, Q^2)f_{\bar{q}}(x_2, Q^2) + (q \leftrightarrow \bar{q})]$$

- ATLAS 8 TeV measurement [1710.05167]

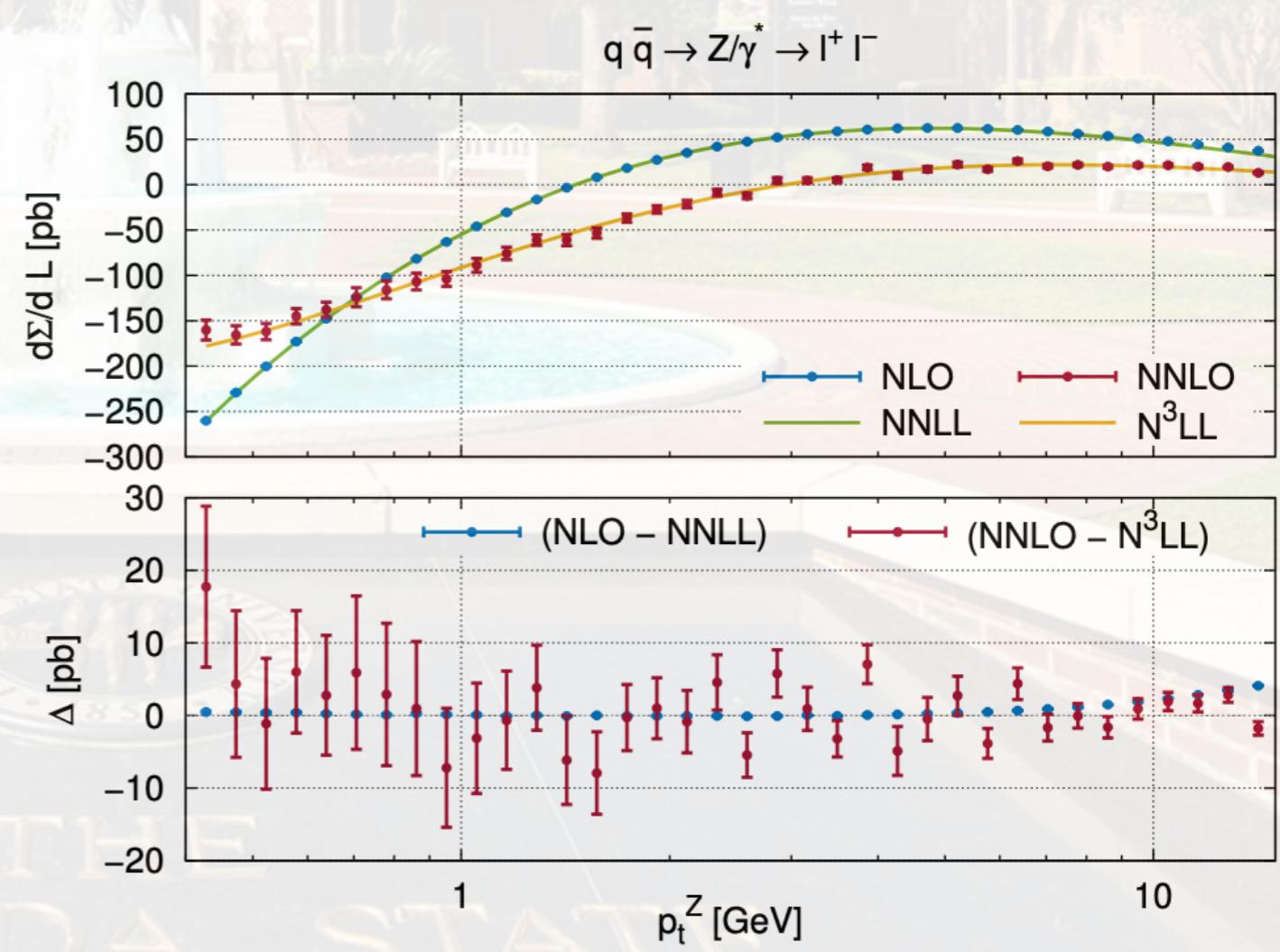
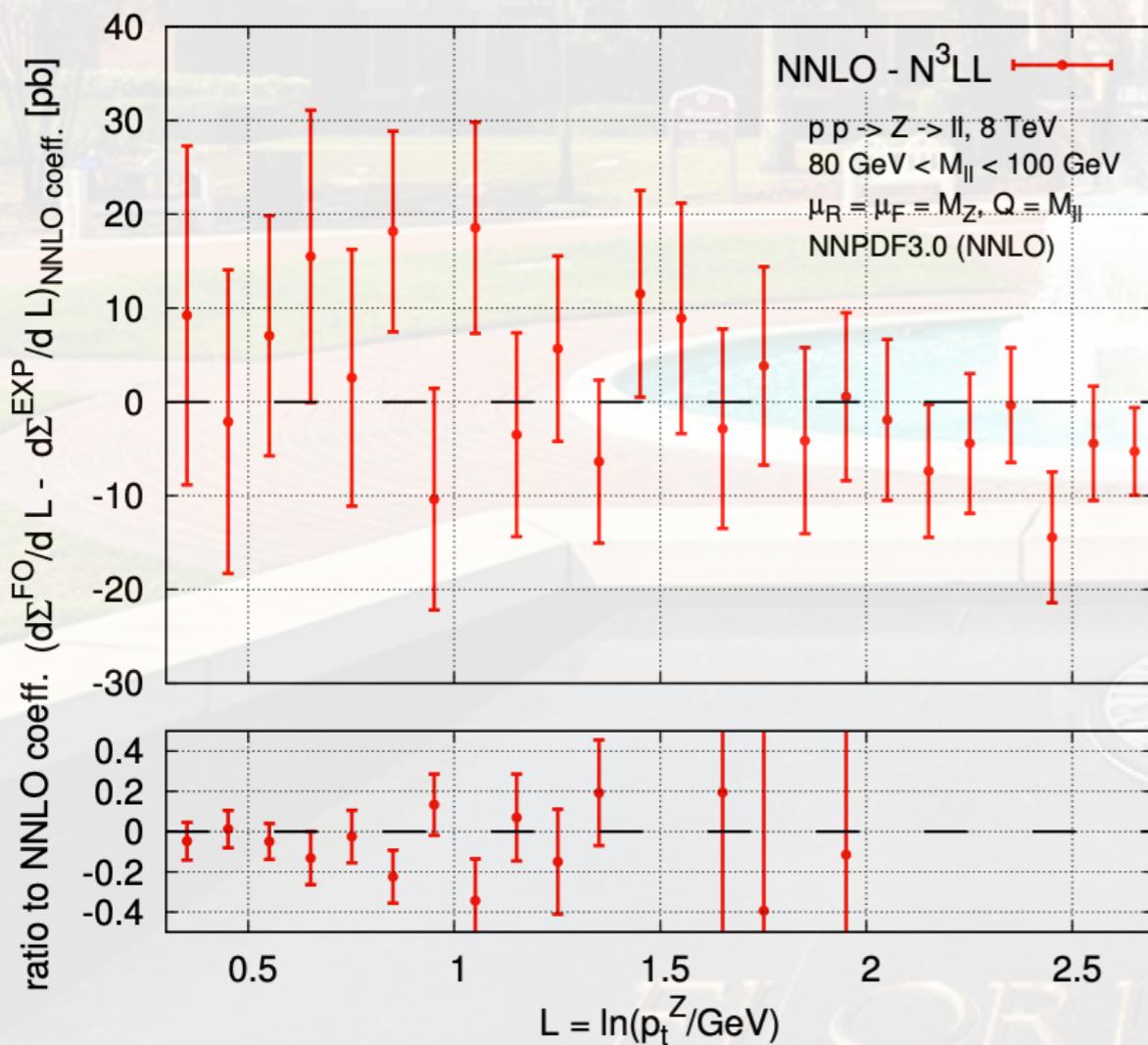
Observable	Central-Central	Central-Forward
$m_{ll}$ [GeV]	[46,66,80,91,102,116,150,200]	[66,80,91,102,116,150]
$ y_{ll} $	[0,0.2,0.4,0.6,0.8,1,1.2, 1.4,1.6,1.8,2,2.2,2.4]	[1.2,1.6,2,2.4,2.8,3.6]
$\cos\theta^*$	[-1,-0.7,-0.4,0,0.4,0.7,1]	[-1,-0.7,-0.4,0,0.4,0.7,1]
Total Bin Count:	504	150



Slide from T. Gehrmann at LHCP 2020

# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

- $p_{T,Z}$  validation with single MC adaption for full channel
- General good agreement between FO and RadISH for qqb channel
- Large numerical error with fluctuated plateau
- Matching  $p_{T,Z}$  distribution at NNLO+N3LL accuracy with transition region at medium  $p_{T,Z}$  is fine but may not be accurate



# DIFFERENTIAL $\gamma^*$ PRODUCTION AT LHC WITH N3LO CORRECTIONS

- N3LO differential cross sections
  - Apply SCET+NNLOJET calculations for  $y_{\gamma^*}$  distribution
  - Left: with mirror statistics. Right: without mirror statistics

