Light Quark Mediated Higgs Boson Threshold Production in NLL

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RADCOR-LoopFest

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Topics discussed

- Motivation
- Anatomy of mass suppressed logarithms
- Bottom quark mediated Higgs production at the LHC

Based on

C. Anastasiou, A.A. Penin, JHEP **2007** *195 (2020)*

T. Liu, A.A. Penin, Phys.Rev.Lett. **119** *(2017) 262001*

Higgs production at the LHC

- Total cross section at 13 GeV $\boxed{\sigma_{pp\to H+X} = 48.68\ pb}$
  - N^{3}LO with $m_t = \infty$ and $m_b = 0$

    Anastasiou et al. Phys. Rev. Lett. **114**, 212001 (2015)
  - NLO with finite $m_{t,b}$

    Graudenz, Spira, Zerwas, Phys. Rev. Lett. **70**, 1372 (1993)
- Dominant theory uncertainties

  Anastasiou et al. JHEP **1605**, 058 (2016)
  - scale choice $^{+0.10}_{-1.15}\ pb$
  - PDF N^{3}LO $\pm 0.56\ pb$
  - $m_t < \infty$ NNLO $\pm 0.49\ pb$ removed *Czakon et al. arXiv:2105.04436*
  - $m_b > 0$ NNLO+ $\pm 0.40\ pb$

Bottom quark mass effect

- Fixed order NNLO (partial)
  - *Higgs plus jet cross section (small-mass expansion)*

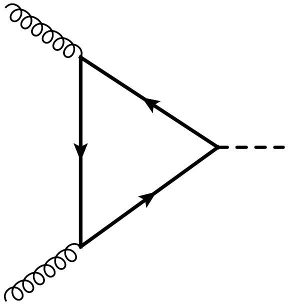
    Lindert, Melnikov, Tancredi, Wever, Phys. Rev. Lett. **118**, 252002 (2017)
  - *Higgs plus jet master integrals (full mass dependence)*

    Frellesvig, Hidding, Maestri, Moriello, Salvatori, JHEP **06**, 093 (2020)
  - *3-loop $gg \to H$ amplitude (full mass dependence)*

    Czakon, Niggetiedt, JHEP **2005**, 149 (2020); Niggetiedt, JHEP **2104**, 196 (2021)
- *The problem is beyond fixed order calculation!*

Bottom quark mass effect

- Leading contribution



$$\propto \alpha_s \ln^2(m_H^2/m_b^2)\frac{m_b^2}{m_H^2}$$

- *effective expansion parameter* $\boxed{\alpha_s \ln^2(m_H^2/m_b^2) \sim 40\alpha_s}$

➥ *resummation is mandatory*

- Large logs at subleading power
  - *one of a few big challenges for EFT RG approach*

Large logs beyond the leading power

- $e^-\mu^-$ backward scattering

  Gorshkov, Gribov, Lipatov, Frolov, Yad. Fiz. **6**, 129 (1967)

- QED form factors — Penin, Phys. Lett. B **745**, 69 (2015)

- soft radiation

  Bonocore, Laenen, Magnea, Vernazza, White, JHEP **1612**, 121 (2016)

- jettiness — Boughezal, Liu, Petriello, JHEP **1703**, 160 (2017)

- event shapes — Moult, Stewart, Vita, Zhu, JHEP **1808**, 013 (2018)

- threshold production

  Beneke, Garny, Jaskiewicz, Szafron, Vernazza, Wang, JHEP **2001**, 094 (2020)

- *many other recent studies ...*

Higgs boson production and decays

- $H \to \gamma\gamma$ LL and NLL $\ln(m_H/m_b)$

  Kotsky, Yakovlev, Phys. Lett. B **418**, 335 (1998)

  Liu, Mecaj, Neubert, Wang, JHEP **2101**, 077 (2021)

- $gg \to Hg$ abelian LL $\ln(m_H/m_b)$, $\ln(p_\perp/m_b)$

  Melnikov, Penin, JHEP **1605**, 172 (2016)

- $gg \to H$ full LL $\ln(m_H/m_b)$

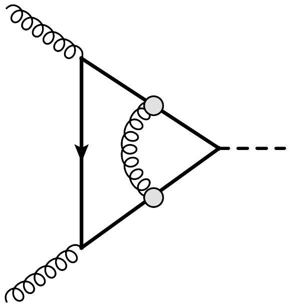
  Liu, Penin, Phys. Rev. Lett. **119**, 262001 (2017); JHEP **1811**, 158 (2018)

➥ *problem of scale dependence and convergence* ⇨ *NLL analysis*

Leading “Double” Logarithms

$gg \to H$ amplitude

- Non-Sudakov logs   T. Liu, A.A. Penin, Phys.Rev.Lett. **119** (2017) 262001



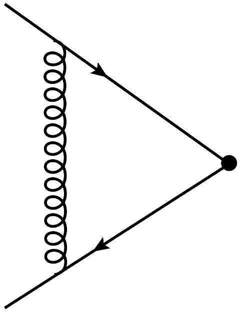
- Factorization formula $$\boxed{\mathcal{M}^b_{gg\to H} = Z_g^{2\,LL}\, g(x)\, \mathcal{M}^{b\,(0)}_{gg\to H}}$$
  - *gluon Sudakov factor* $Z_g^{2\,LL} = \exp\left[-\frac{C_A}{\varepsilon^2}\frac{\alpha_s}{2\pi}\frac{\mu^{2\varepsilon}}{Q^{2\varepsilon}}\right]$
  - *Non-Sudakov double logarithms* $g(x) = {}_2F_2\,(1,1;3/2,2;x/2)$
  - *Double-log variable* $x = (C_A - C_F)\frac{\alpha_s}{4\pi}L^2,\ L = \ln(Q^2/m_q^2)$

➥ *eikonal color nonconservation, exponential enhancement*

Next-to-Leading Logarithms

Sudakov form factor

- Origin of next-to-leading logs $\alpha_s^n \ln^{2n-1}(Q^2)$



- *RG logs*
- *collinear logs*

- NLL form factor ($m_q^2 \ll |p_i^2| \ll Q^2$)

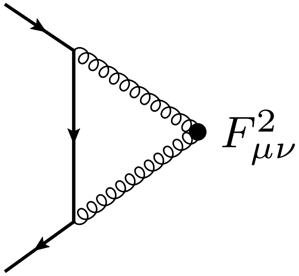
$$F_1^{NLL} = \exp\left\{-\frac{C_F\alpha_s}{2\pi}\ln\left(\frac{Q^2}{-p_1^2}\right)\ln\left(\frac{Q^2}{-p_2^2}\right)\left[1-\beta_0\frac{\alpha_s}{8\pi}\left(\ln\left(\frac{-p_1^2}{\mu^2}\right)+\ln\left(\frac{-p_2^2}{\mu^2}\right)\right)\right]\right.$$

$$\left.+\gamma_q^{(1)}\frac{\alpha_s}{4\pi}\left[\ln\left(\frac{Q^2}{-p_1^2}\right)+\ln\left(\frac{Q^2}{-p_2^2}\right)\right]\right\},$$

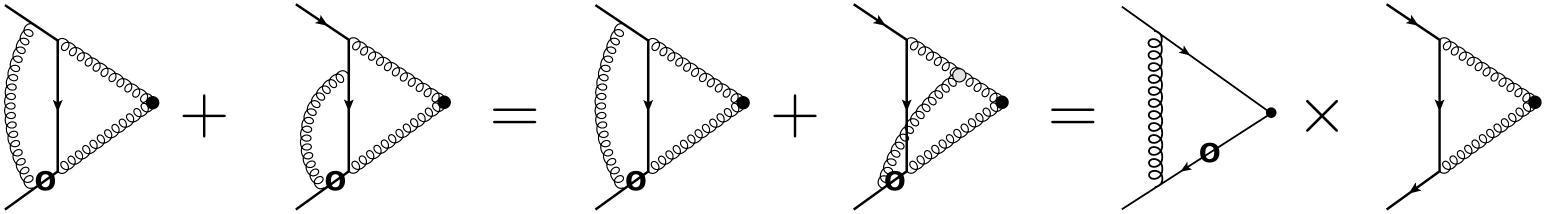
collinear anomalous dimension $\gamma_q^{(1)} = 3C_F/2$

Mass-suppressed amplitude

- Quark scattering in QED
    - *no one-loop single logs*



- Two loops
    - p_i *collinear factorization*



➥ *no subtraction necessary*

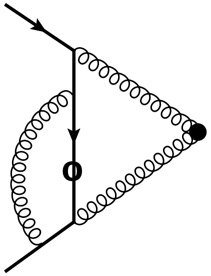
Mass-suppressed amplitude

- *New variables*
  - *Sudakov parameters* $l = up_1 + vp_2 + l_\perp$
  - *hypercube coordinates*

    $\eta = \ln v / \ln(m_q^2/Q^2),\ \xi = \ln u / \ln(m_q^2/Q^2),\ 0 < \eta,\ \xi < 1$
  - *strict ordering* $\eta_i < \eta_j,\ \ldots$, *onshell condition* $\eta_i + \xi_i < 1$

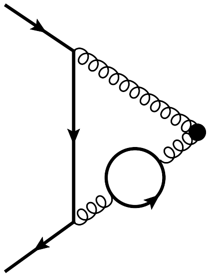
Mass-suppressed amplitude

- Two loops
  - *nonfactorizable collinear log*



↪ $e^{\gamma_q^{(1)}\frac{\alpha_q L}{4\pi}(2-\eta-\xi)}$

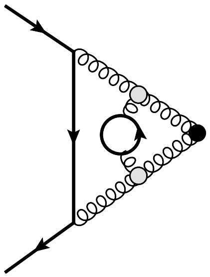
  - *RG running of LO coupling*



↪ $$\frac{1}{\left[1+\beta_0\frac{\alpha_q L}{4\pi}(1-\eta)\right]\left[1+\beta_0\frac{\alpha_q L}{4\pi}(1-\xi)\right]}$$

Mass-suppressed amplitude

- Three loops
  - *RG running of LL terms*



$$e^{2x\eta\xi\beta_0\frac{\alpha_q L}{4\pi}\left(\frac{L_\mu}{L}-\frac{\eta+\xi}{2}\right)}, \qquad L_\mu=\ln(Q^2/\mu^2)$$

- Final NLL result

$$\mathcal{G}^{NLL}=2\int_0^1 \mathrm{d}\xi\int_0^{1-\xi}\mathrm{d}\eta\,\frac{e^{\left\{-2x\eta\xi\left[1-\beta_0\frac{\alpha_q L}{4\pi}\left(\frac{L_\mu}{L}-\frac{\eta+\xi}{2}\right)\right]+\gamma_q^{(1)}\frac{\alpha_q L}{4\pi}(2-\eta-\xi)\right\}}}{\left[1+\beta_0\frac{\alpha_q L}{4\pi}(1-\eta)\right]\left[1+\beta_0\frac{\alpha_q L}{4\pi}(1-\xi)\right]}\,Z_q^{2\,NLL}\mathcal{G}^{(0)}$$

Mass-suppressed amplitude

$$\boxed{\mathcal{G}^{NLL} = C\, {Z_q^2}^{NLL}\, \mathcal{G}^{(0)}}$$

expanding in $\alpha_q L$

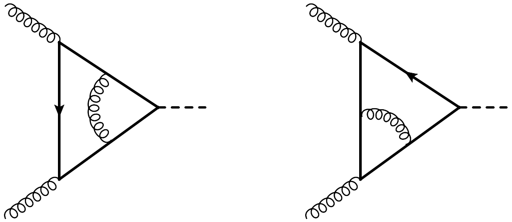
$$C = \left[g(-x) + \frac{\alpha_s L}{4\pi}(2\left(\gamma_q^{(1)} - \beta_0\right)(g(-x) - g_\gamma(-x)) - \beta_0 g_\beta(-x))\right]$$

$$g_\gamma(x) = \frac{1}{x}\left[\left(\frac{\pi e^x}{2x}\right)^{1/2} \mathrm{erf}(\sqrt{x/2}) - 1\right]$$

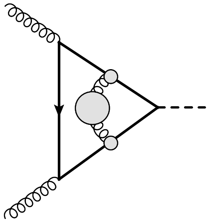
$$g_\beta(x) = \left[\left(\frac{\pi e^x}{2x}\right)^{1/2} \mathrm{erf}(\sqrt{x/2}) - g(x)\right]\frac{L_\mu}{L} + \frac{3}{2x}\left[\left(1 - \frac{x}{3}\right)\left(\frac{\pi e^x}{2x}\right)^{1/2} \mathrm{erf}(\sqrt{x/2}) - 1\right]$$

$gg \to H$ amplitude

- Two loops
  - *nonfactorizable collinear log*
  - *RG running of Yukawa coupling*



- Three loops
  - *RG running of LL terms*



now $x = (C_A - C_F)\frac{\alpha_s(\mu)}{4\pi}L^2, \quad L = \ln(m_H^2/m_b^2), \quad L_\mu = \ln(m_H^2/\mu^2)$

$gg \to H$ amplitude

$$\mathcal{M}^{b\,NLL}_{gg\to H} = C_b \underbrace{\left(\frac{\alpha_s(m_H)}{\alpha_s(m_b)}\right)^{\gamma_m^{(1)}/\beta_0}}_{\text{Yukawa RG factor}} \underbrace{Z_g^{2\,NLL}}_{\text{gluon Sudakov form factor}} \underbrace{\left[-\frac{3}{2}\frac{m_b^2}{m_H^2}L^2\,\mathcal{M}^{t\,(0)}_{gg\to H}\right]}_{\text{LO amplitude}}$$

C_b *incorporates all the bottom mass logs*

Abelian part agrees with $H \to \gamma\gamma$ NLL result

Liu, Mecaj, Neubert, Wang,
JHEP **2101**, 077 (2021)

$gg \to H$ amplitude

$$\mathcal{M}^{b\,NLL}_{gg\to H} = C_b \underbrace{\left(\frac{\alpha_s(m_H)}{\alpha_s(m_b)}\right)^{\gamma_m^{(1)}/\beta_0}}_{\text{Yukawa RG factor}} \underbrace{Z_g^{2\,NLL}}_{\text{gluon Sudakov form factor}} \underbrace{\left[-\frac{3}{2}\frac{m_b^2}{m_H^2}L^2\,\mathcal{M}^{t\,(0)}_{gg\to H}\right]}_{\text{LO amplitude}}$$

$$C_b = \left[g(x) + \frac{\alpha_s L}{4\pi}\left(2\gamma_q^{(1)} g_\gamma(x) - \beta_0 g_\beta(x)\right)\right] = 1 + \sum_{n=1}^{\infty} c_n$$

$$\begin{aligned} c_1 &= \frac{x}{6} + C_F\frac{\alpha_s L}{4\pi}\,, \quad c_2 = \frac{x^2}{45} + \frac{x}{5}\frac{\alpha_s L}{4\pi}\left[\frac{3}{2}C_F - \beta_0\left(\frac{5}{6}\frac{L_\mu}{L} - \frac{1}{3}\right)\right], \\ c_3 &= \frac{x^3}{420} + \frac{x^2}{5}\frac{\alpha_s L}{4\pi}\left[\frac{5}{21}C_F - \beta_0\left(\frac{2}{9}\frac{L_\mu}{L} - \frac{2}{21}\right)\right], \quad \ldots \end{aligned}$$

3-loop coefficient c_2 *agrees with*

Harlander, Prausa, Usovitsch, JHEP **1910**, 148 (2019) *(analytic n_l part)*

Czakon, Niggetiedt, JHEP **2005**, 149 (2020) *(numerical, all ten digits)*

Niggetiedt, JHEP **2104**, 196 (2021)

Top-bottom interference at the threshold

- $Z_g^2\mathcal{M}^{t\,(0)}$ IR logs identical to heavy top EFT amplitude
- soft radiation identical to heavy top EFT

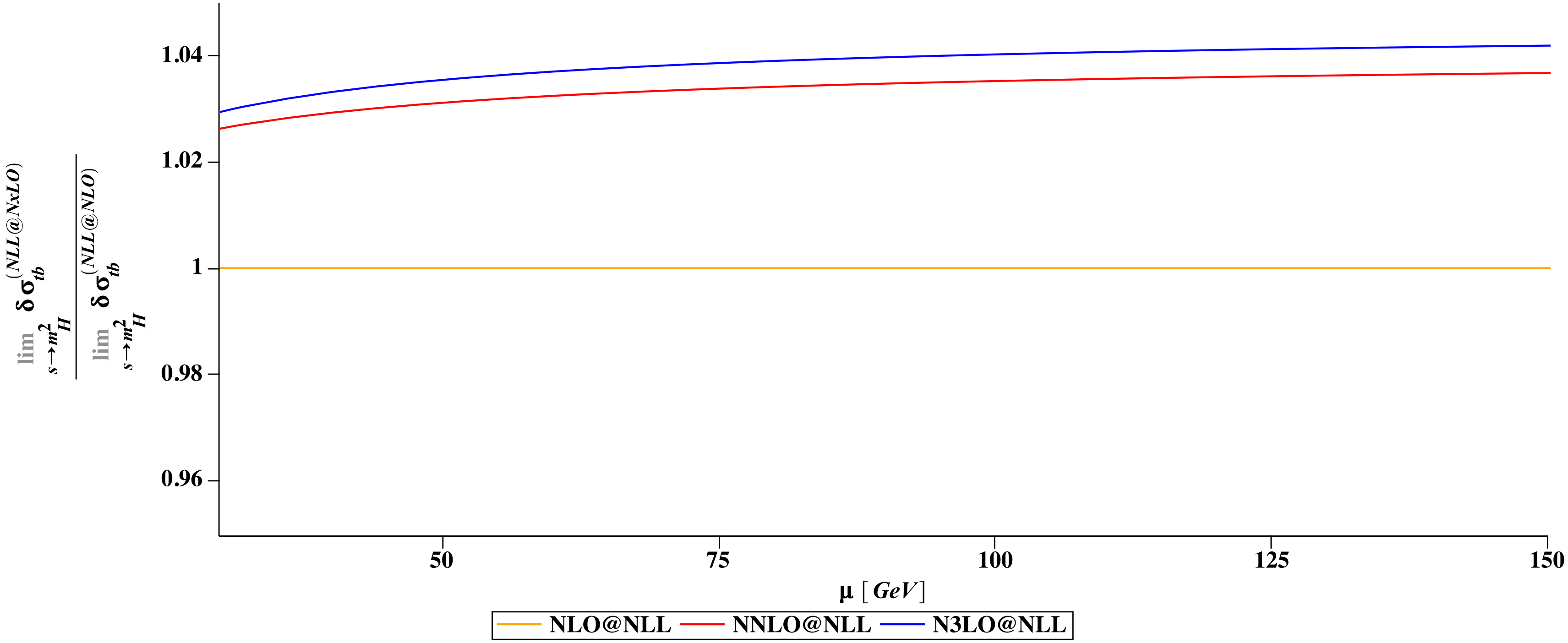
➥ *partonic threshold cross section (top-bottom interference)*

$$\delta\sigma_{gg\to H+X}(s) = C_b\left(\frac{\alpha_s(m_H)}{\alpha_s(m_b)}\right)^{\gamma_m^{(1)}/\beta_0}\left[-\frac{3}{2}\,\frac{m_b^2}{m_H^2}\,L^2\right] C_t\;\sigma^{\rm eff}_{gg\to H+X}$$

bottom mediated amplitude *heavy top EFT cross section*

heavy top EFT Wilson coefficient

NLL K-factors through N^{3}LO



computed with respect to NLO NLL result

Top-bottom interference in threshold cross section

	LO	NLO	NNLO	N^{3}LO
$\delta\sigma^{\rm LL}_{pp\to H+X}$	-1.420	-1.640	-1.667	-1.670
$\delta\sigma^{\rm NLL}_{pp\to H+X}$	-1.420	-2.048	-2.183	-2.204
$\delta\sigma_{pp\to H+X}$	-1.023	-2.000		

- NLL K-factors with full threshold $\delta\sigma^{NLO}_{pp\to H+X}$

$$\boxed{\delta\sigma^{\rm NNLO}_{gg\to H+X} \approx -0.13\ pb}$$

$$\boxed{\delta\sigma^{\rm N^3LO}_{gg\to H+X} \approx -0.02\ pb}$$

Bottom contribution to $\sigma_{pp\to H+X}$ beyond NLO

- Uncertainty
  - *Threshold (from N^{3}LO top mediated result)* $\pm 50\%$
  - *Higher orders in α_s (from N^{3}LO NLL result)* $\pm 20\%$
  - *Subleading logs* $\pm 100\%$
    - *LO result* -43%
    - *NLO result* -3%
    - *NNLO three-loop virtual* *(Czakon, Niggetiedt, 2020)* -40%
    - *NNLO electroweak logs* *(Kuhn, Moch, Penin, Smirnov, 2002)* $+100\%$

Total $\pm 170\%$

- Final estimate $\boxed{-0.34 \text{ to } 0.08 \text{ pb}}$ (factor two interval reduction)

Summary

- The first NLL subleading power result in QCD

- Bottom quark effect on $\sigma_{pp\to H+X}$
  - *PT series converges despite large logs*
  - *beyond NLO contribution* -0.34 *to* 0.08 *pb*