IR-Improved Amplitude-Based Resummation in Quantum Field Theory: New Results and New Issues

B.F.L. Ward

Baylor University, Waco, TX, USA

5/21/21

partly in collaboration with

S. Jadach, W. Placzek, M. Skrzypek, Z. Was, S. A. Yost
OUTLINE

- Introduction
- Review of Exact Amplitude-Based Resummation Theory
- Precision LHC Physics: New Results and New Issues
- Precision FCC Physics: New Results and New Issues
- Quantum Gravity: New Results and New Issues
- Summary
Introduction

WHAT IS RESUMMATION (IR, UV, CL)?

- FAMILIAR SUMMATION: \( \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \)
- RESUMMATION:
  \[
  \sum_{n=0}^{\infty} C_n \alpha_s^n \begin{cases} 
  = F_{\text{RES}}(\alpha_s) \sum_{n=0}^{\infty} B_n \alpha_s^n, \text{EXACT, YFS} \\
  \approx G_{\text{RES}}(\alpha_s) \sum_{n=0}^{N} B'_n \alpha_s^n, \text{APPROX, J-S}
  \end{cases}
  \]

APPROX: THE ALGORITHM FOR B'\(_n\)(G_{\text{RES}}) TO ALL ORDERS IS UNKNOWN.

Would It Limit or Enhance Exactness: LO, NLO, NNLO, .... ?
As we indicated, the "two" classes of realizations are:

- Jackson-Scharre (JS) (APPROX) vs YFS (EXACT)
- JS → 'limit to precision', determined by N
- YFS → 'no limit to precision', algorithmically

1989 CERN Yellow Book article: Some were almost convinced, but not completely!

Today, new paradigms, analogous discussions: precision LHC/FCC physics and quantum gravity
54 YEARS of $SU_{2L} \times U_1$, S. Weinberg, PRL 19 (1967)


(SM@50, B. Lynn *et al.*, Case Western, June, 2018) ⇒

**Introduction**

54 YEARS of $SU_{2L} \times U_1$, S. Weinberg, PRL 19 (1967)


(SM@50, B. Lynn *et al.*, Case Western, June, 2018) ⇒

**The Standard Model at 50 Years:**

*The Standard Model at 50 Years:*

a celebratory symposium will take place in the

**Physics Department**

**Case Western Reserve University**

Cleveland, Ohio, June 1–4, 2018.

For more information, email SMat50@case.edu

see also PASCO52018, taking place immediately following SM@50

**Speakers**

- Steven Adler
- James “BJ” Bjorken
- Alain Blondel
- Norman Christ
- Savas Dimopoulos
- Henriette Elvang
- Jerome Friedman
- Mary K. Gaillard
- David Gross
- Gerard ’t Hooft
- Takeaki Kajita
- Bryan W. Lynn
- Pavel Fileviez Perez
- Michael Peskin
- Hellen Quinn
- Carlo Rubbia
- Jurgen Schukraft
- George Smoot
- Glenn Starkman
- Cyrus Taylor
- Samuel Ting
- Bennie F. L. Ward
- Steven Weinberg
- Mark Wise
- Sau Lan Wu

**Must Keep Historical Perspective**

Must keep historical perspective.
Future colliders

2021-2025: 3 studies: CLIC, FCC and muon colliders (new)
≥ 2026: single "High-energy frontier" line as "placeholder" for project selected by next ESPP

FCC
Budget ~ 20 M/y for feasibility study of infrastructure and colliders (as recommended by ESPP).
High-priority: tunnel, including high-risk zones, surface areas, administrative processes, environment;
R&D (superconducting RF for FCCee; magnets for FCCuh; see "Accelerator technology and R&D" line)
machine design → Goal is CDR++ with results of feasibility studies by ~ 2026

CLIC
Budget 4.5-6.5 M/y to continue R&D on key technology (X-band structure, beam dynamic, etc.) to maintain CLIC as option for a future collider. Klystrons and CLEAR moved to "Accelerator technology and R&D" line. Net budget reduction over 2024-2025: ~ 6.5 M.

Muon colliders
Budget 2 M/y to start efforts at CERN and to support European community.
Mainly personnel to work on accelerator and collider ring, design of interaction region, muon cooling, muon source, fastramping magnets and power converters, neutrino radiation and civil engineering.

Must keep historical perspective.
Exact Amplitude-Based Resummation -- Review

\[ d\bar{\sigma}_{\text{res}} = e^{\text{SUM}_{\text{IR}}(\text{QCED})} \sum_{n,m=0}^{\infty} \frac{1}{n! m!} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}} \]

\[ \prod_{j_2=1}^{m} \frac{d^3 k'_{j_2}}{k'_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy \cdot (p_1 + q_1 - p_2 - q_2 - \sum k_{j_1} - \sum k'_{j_2})} D_{\text{QCED}} \]

\[ \beta_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0}, \quad (1) \]

where new (YFS-style) non-Abelian residuals \( \beta_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m) \) have \( n \) hard gluons and \( m \) hard photons.
Here,

\[ \text{SUM}_{\text{IR}}(\text{QCED}) = 2\alpha_s \Re B_{\text{QCED}}^{\text{nls}} + 2\alpha_s \tilde{B}_{\text{QCED}}^{\text{nls}} \]

\[ D_{\text{QCED}} = \int \frac{d^3 k}{k^0} \left( e^{-iky} - \theta(K_{\text{max}} - k^0) \right) \tilde{S}_{\text{QCED}}^{\text{nls}} \]  

(2)

where \( K_{\text{max}} \) is “dummy” and

\[ B_{\text{QCED}}^{\text{nls}} \equiv B_{\text{QCD}}^{\text{nls}} + \frac{\alpha}{\alpha_s} B_{\text{QED}}^{\text{nls}}, \]

\[ \tilde{B}_{\text{QCED}}^{\text{nls}} \equiv \tilde{B}_{\text{QCD}}^{\text{nls}} + \frac{\alpha}{\alpha_s} \tilde{B}_{\text{QED}}^{\text{nls}}, \]

\[ \tilde{S}_{\text{QCED}}^{\text{nls}} \equiv \tilde{S}_{\text{QCD}}^{\text{nls}} + \tilde{S}_{\text{QED}}^{\text{nls}}. \]  

(3)

“nls” \( \equiv \) DGLAP-CS synthesization.

Shower/ME Matching: \( \hat{\beta}_{n,m} \rightarrow \beta_{n,m} \)
Precision LHC Physics: New Results and New Issues

- IR-Improved DGLAP-CS Theory: Herwiri1.031
  Interfaced to MC@NLO and MG5_aMC@NLO:
  Z and W+jets Production, ...
  KKMC-hh: Exact $O(\alpha^2L)$ CEEX EW Corrections Interfaced to Herwig6.5 and Herwiri1.031--new, interfaced to MG5_aMC@NLO

- In Z and W+ jets Production, IR-Improvement gives a comparable or better data fit without ad hoc parameters

- In KKMC-hh, IR-improvement allows to quantify role of ISR in precision predictions for Z production observables, as we now illustrate.
### Standard Model Input Parameters

DIZET uses a modified $G_\mu$ Scheme with an over-complete set of inputs to take advantage of precision measurements to the extent possible. The following input parameters are used, taken from the 2014 EW Benchmark study, S. Alioli et al., CERN-TH-2016-137 / arXiv:1606.02330

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/\alpha(0)$</td>
<td>137.03599991</td>
</tr>
<tr>
<td>$G_F$</td>
<td>$1.1663787 \times 10^{-5}$ GeV$^{-2}$</td>
</tr>
<tr>
<td>$\sin^2(\theta_W)$</td>
<td>0.2232290158</td>
</tr>
<tr>
<td>$\Gamma_Z$</td>
<td>2.4952 GeV</td>
</tr>
<tr>
<td>$\Gamma_W$</td>
<td>2.085 GeV</td>
</tr>
<tr>
<td>$m_d$</td>
<td>4.7 MeV</td>
</tr>
<tr>
<td>$m_s$</td>
<td>0.15 GeV</td>
</tr>
<tr>
<td>$m_b$</td>
<td>4.6 GeV</td>
</tr>
<tr>
<td>$m_e$</td>
<td>510.999 keV</td>
</tr>
<tr>
<td>$m_\tau$</td>
<td>1.777 GeV</td>
</tr>
<tr>
<td>$1/\alpha(M_Z)$</td>
<td>128.952</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>0.12018</td>
</tr>
<tr>
<td>$M_Z$</td>
<td>91.1876 GeV</td>
</tr>
<tr>
<td>$M_W$</td>
<td>80.385 GeV</td>
</tr>
<tr>
<td>$M_H$</td>
<td>125 GeV</td>
</tr>
<tr>
<td>$m_u$</td>
<td>2.2 MeV</td>
</tr>
<tr>
<td>$m_c$</td>
<td>1.2 GeV</td>
</tr>
<tr>
<td>$m_t$</td>
<td>173.5 GeV</td>
</tr>
<tr>
<td>$m_\mu$</td>
<td>105.6583 MeV</td>
</tr>
</tbody>
</table>
Angular Variables for $pp \rightarrow Z/\gamma^* \rightarrow \ell\bar{\ell}$

We will consider distributions of the angle $\theta_{CS}$ of the negative $\ell$ defined in the Collins-Soper frame: the CM frame of $\ell^\pm$, relative to a $\hat{z}$ axis oriented as shown relative to the proton beams.

If $P = p_\ell + p_{\ell}^-$ and $p^\pm = p^0 \pm p^z$ in the lab,

$$\cos(\theta_{CS}) = \text{sgn}(P^z) \frac{p_\ell^- p_{\ell}^- - p_{\ell}^- p_{\ell}^+}{\sqrt{P^2 P^+ P^-}}$$

$A_{FB}, A_4 \rightarrow \sin^2 \theta_W$
Consider recent ATLAS measurement of the angular coefficients in Z-boson events at 8 TeV, arXiv: 1606.00689

$Z/\gamma^*$ data with electron and muon pairs used: EW treated as 'small'
ISR: QED PDFs vs KKMC-hh

QED ISR enters the angular distributions at the order of several per-mil, and cannot be neglected.

There are two options at present:

1. Use a calculation that factorizes collinear effects and absorbs them into PDFs with a PDF that includes the collinear QED. Several are available. Current studies have focused on NNPDF3.1 NLO with LuxQED.

2. Use a complete ab-initio QED calculation, including collinear contributions, with a PDF that does not contain QED effects. The result will depend parametrically on quark masses. KKMC-hh follows this approach.

The two approaches should agree for variables which are not strongly sensitive to photon $P_T$.

The connection between these approaches should be studied in detail. KKMC-hh can be useful in such studies. Comparisons of quark momentum distributions could help determine the most appropriate values of the light quark masses.
• Results from KKMC-hh: arXiv:2002.11692
• We see clear evidence that the transverse degrees of freedom in the photon radiation for ISR do impact observables in single $Z/\gamma^*$ production: $\cos(\theta_{CS}), M_{ll}, A_4, A_{FB}, Y_{ll}, w/wo$ shower.

Illustrations: cuts - $60 \text{ GeV} < M_{ll} < 116 \text{ GeV}, P_T^{\ell\ell} < 30 \text{ GeV}, P_T^\ell > 25 \text{ GeV}, |\eta_\ell| < 2.5$.

### Showered Numerical Results: $\sigma, A_{FB}, A_4$

<table>
<thead>
<tr>
<th></th>
<th>Without Shower</th>
<th>With Shower</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncut $\sigma$(pb)</td>
<td>944.91(2)</td>
<td>938.44(4)</td>
<td>-0.684(7)%</td>
</tr>
<tr>
<td>Cut $\sigma$(pb)</td>
<td>442.33(1)</td>
<td>412.54(3)</td>
<td>-6.7307%</td>
</tr>
<tr>
<td>$A_{FB}$</td>
<td>0.01132(2)</td>
<td>0.01211(5)</td>
<td>0.00109(5)</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.06102(8)</td>
<td>0.06052(8)</td>
<td>-0.00050(8)</td>
</tr>
</tbody>
</table>
No cuts:
No cuts:
Precision LHC Physics: New Results and New Issues

No cuts:
• New Issues:
  Role of photon transverse degrees of freedom
  Role of quark masses:
    1. Observable parameters? YES
    2. Just unphysical (IR and CL regulators)? NO
Input data for non-QED PDFs at $Q_0 \sim 1$ GeV:
  1. Is this double counting if CL singular quark mass effects are not removed?
     QFT: processes at different space-time regimes cannot double count! (Shower!)
  2. Can we get a PDF with them removed
     -- probably YES.
Precision FCC Physics: New Results and New Issues

- FCC <-> FCC-ee + FCC-hh
- IR-Improvement of even the FCC-hh discovery spectra is needed--see arXiv:1801.03303
- For FCC-ee, a key issue is the theoretical precision of the Luminosity.
- Today, for illustration, we address the latter concern.
- We review what is the current state-of-the-art.
- We show the path forward to 0.01%
**General context: QED uncertainties in EW observables**

To be discussed in the following

<table>
<thead>
<tr>
<th>Observable</th>
<th>From</th>
<th>Present {QED}</th>
<th>FCC stat.</th>
<th>FCC syst.</th>
<th>Now{QED} FCC(exp.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$ [MeV]</td>
<td>$Z$ linesh. 2</td>
<td>91187.5 ± 2.1{0.3}</td>
<td>0.005</td>
<td><strong>0.1</strong></td>
<td>3</td>
</tr>
<tr>
<td>$\Gamma_Z$ [MeV]</td>
<td>$Z$ linesh. 2</td>
<td>2495.2 ± 2.1{0.2}</td>
<td>0.008</td>
<td><strong>0.1</strong></td>
<td>2</td>
</tr>
<tr>
<td>$\Gamma_h/\Gamma_l$</td>
<td>$\sigma(M_Z)$ 3</td>
<td>20.767 ± 0.025{0.012}</td>
<td>$1 \cdot 10^{-4}$</td>
<td>$\sim 10^{-3}$</td>
<td>12</td>
</tr>
<tr>
<td>$N_\nu$</td>
<td>$\sigma(M_Z)$ 2</td>
<td>2.984 ± 0.008{0.006}</td>
<td>$0.8 \cdot 10^{-4}$</td>
<td>4 · 10^{-4}</td>
<td><strong>15</strong></td>
</tr>
<tr>
<td>$N_\nu$</td>
<td>$Z+\gamma$ 4</td>
<td>2.69 ± 0.15{0.06}</td>
<td>1 · 10^{-3}</td>
<td>&lt; 10^{-3}</td>
<td>60</td>
</tr>
<tr>
<td>$\sin^2 \theta_W^e$</td>
<td>$A_{lept.}^{FB}$ 3</td>
<td>0.23099 ± 0.00053{0.06}</td>
<td>0.6 · 10^{-5}</td>
<td>&lt; 10^{-5}</td>
<td>10</td>
</tr>
<tr>
<td>$\sin^2 \theta_W^f$</td>
<td>$A_{pol.}^{FB}$ 2, 3</td>
<td>0.23159 ± 0.00041{0.03}</td>
<td>0.6 · 10^{-5}</td>
<td>&lt; 10^{-5}</td>
<td>20?</td>
</tr>
<tr>
<td>$M_W$ [MeV]</td>
<td>ADLO 5</td>
<td>80376 ± 33{7}</td>
<td>0.3</td>
<td><strong>0.3</strong></td>
<td>14</td>
</tr>
<tr>
<td>$A_{FB,\mu}$</td>
<td>$M_Z \pm 3.5$ GeV</td>
<td>$\pm 0.020${0.001}</td>
<td>$1.0 \cdot 10^{-5}$</td>
<td>0.3 · 10^{-5}</td>
<td>100</td>
</tr>
<tr>
<td>$\alpha_{QED}^{-1}(M_Z)$</td>
<td>≤ 10 GeV 6</td>
<td>128.952 ± 0.014</td>
<td>0.004</td>
<td>0.001</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: Experimental precision of electroweak observables, which are most sensitive to QED effects. In the braces \{...\} in 3-rd column are estimates of the systematic error due to QED calculation uncertainty. The necessary improvement factors of QED calculations for FCCee experiments are shown in the last column. FCCee systematic is without QED component. Uncertain numbers are marked with the question mark.

---


QED challenges at FCCee are of 2-fold type:

A. More higher (fixed) orders, better resummation, more sophisticated Monte Carlo programs

B. Possibly completely new methodology of the QED “deconvolution” and related new definition of the EW pseudo-observables (EWPO’s)

--S. Jadach, private communication

An illustrative example:
Low angle Bhabha for luminosity measurement which enters into many observables, notably neutrino counting.
• Motivation: better measurement of invisible Z width from Z peak x-section

• LEP legacy:

\[ R_{\text{inv}}^0 = \frac{\Gamma_{\text{inv}}}{\Gamma_{\ell\ell}} = \sqrt{\frac{12\pi R_{\ell}^0}{\sigma_{\text{had}}^0 m_Z^2}} - R_{\ell}^0 - (3 + \delta_\ell) \]

- assuming lepton universality

\[ (R_{\text{inv}}^0)_{\text{exp}} = N_\nu \left( \frac{\Gamma_{\nu\nu}}{\Gamma_{\ell\ell}} \right)_{\text{SM}} \]

- from LEP Z-peak measurements

\[ N_\nu = 2.9840 \pm 0.0082 \]
\[ \delta N_\nu \approx 10.5 \frac{\delta n_{\text{had}}}{n_{\text{had}}} \oplus 3.0 \frac{\delta n_{\text{lep}}}{n_{\text{lep}}} \oplus 7.5 \frac{\delta L}{L} \]
\[ \frac{\delta L}{L} = 0.061\% \implies \delta N_\nu = 0.0046 \]


• Recently, Janot and Jadach, arXiv:1912.02067, current status for LEP lumi theory error

\[ \leftrightarrow 0.037\% \cup \text{improved measurement error analysis (see also Voutsinas et al., arXiv:1908.01704)} \]
\[ \implies N_\nu = 2.9975 \pm 0.0074 \text{ with } \delta N_\nu = 0.0028 \text{ from } \frac{\delta L}{L} \]

• 7.5x0.061\%=0.0046. Shall we do better at FCCee?? YES!

• In 1999 lumi TH error 0.061\% was dominated by VP \( \implies \) No motivation tp improve QED components

Now, 0.037\% (JJ) dominated by photonic correction \( \implies \) motivation already to improve QED error. At FCCee VP error will be reduced by another factor 2 compared to today! New reality!

• Low angle Bhabha luminometer already defined, Mogens Dam, FCC Week 2018, 2019 wkshp
LEP legacy, lumi TH error budget

Example of low angle Bhabha (luminosity) at FCCee


- By the time of FCCee VP contribution will be merely 0.006% (F. Jegerlehner)

- QED corrections and Z contrib. come back to front!

- Z contr. easy to master, even if rises at FCCee, because (28-58)->(64-86) mrad.

- Our FCCee forecast is 0.001%, provided QED is improved.

Bibliography in last slides

Table 1: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for a generic angular range within $1^\circ$-$3^\circ$ (18-52 mrad), and for LEP2 energies up to 176 GeV and an angular range within $3^\circ$-$6^\circ$. Total uncertainty is taken in quadrature. Technical precision included in (a).

<table>
<thead>
<tr>
<th>Type of correction/error</th>
<th>LEP1</th>
<th>1996</th>
<th>0.10%</th>
<th>0.027%</th>
<th>0.20%</th>
<th>0.04%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1999</td>
<td>0.015%</td>
<td>0.015%</td>
<td>0.03%</td>
<td>0.03%</td>
</tr>
<tr>
<td>(a) Missing photonic $O(\alpha^2)$ [4,5]</td>
<td></td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.10%</td>
<td>0.10%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(c) Vacuum polarization [7,8]</td>
<td>0.03%</td>
<td>0.03%</td>
<td>0.05%</td>
<td>0.05%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(d) Light pairs [9,10]</td>
<td>0.015%</td>
<td>0.015%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(e) Z-exchange [11,12]</td>
<td>0.11% [12]</td>
<td>0.061% [13]</td>
<td>0.25% [12]</td>
<td>0.12% [13]</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type of correction / Error</th>
<th>1999</th>
<th>Update 2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Photonic $O(L_\gamma \alpha^2)$</td>
<td>0.027% [5]</td>
<td>0.027%</td>
</tr>
<tr>
<td>(b) Photonic $O(L_3 \alpha^3)$</td>
<td>0.015% [6]</td>
<td>0.015%</td>
</tr>
<tr>
<td>(c) Vacuum polariz.</td>
<td>0.040% [7,8]</td>
<td>0.013% (0.011%[JJ])</td>
</tr>
<tr>
<td>(d) Light pairs</td>
<td>0.030% [10]</td>
<td>0.010% [18,19]</td>
</tr>
<tr>
<td>(e) s-channel Z-exchange</td>
<td>0.015% [11,12]</td>
<td>0.015%</td>
</tr>
<tr>
<td>(f) Up-down interference</td>
<td>0.0014% [27]</td>
<td>0.0014%</td>
</tr>
<tr>
<td>(f) Technical Precision</td>
<td>-</td>
<td>(0.027%)</td>
</tr>
<tr>
<td>Total</td>
<td>0.061% [13]</td>
<td>0.038% (0.037%[JJ])</td>
</tr>
</tbody>
</table>
The Path to 0.01% Theoretical Luminosity Precision for the FCC-ee

S. Jadach\textsuperscript{a}, W. Płaczek\textsuperscript{b}, M. Skrzypek\textsuperscript{a}, B.F.L. Ward\textsuperscript{c,d} and S.A. Yost,\textsuperscript{e}

\textsuperscript{a}Institute of Nuclear Physics, Polish Academy of Sciences, ul. Radzikowskiego 152, 31-342 Kraków, Poland
\textsuperscript{b}Marian Smoluchowski Institute of Physics, Jagiellonian University, ul. Łojasiewicza 11, 30-348 Kraków, Poland
\textsuperscript{c}Baylor University, Waco, TX, USA
\textsuperscript{d}Max Planck Institute für Physik, München, Germany
\textsuperscript{e}The Citadel, Charleston, SC, USA

Abstract

The current status of the theoretical precision for the Bhabha luminometry is critically reviewed and pathways are outlined to the requirement targeted by the FCC-ee precision studies. Various components of the pertinent error budget are discussed in detail – starting from the context of the LEP experiments, through their current updates, up to prospects of their improvements for the sake of the FCC-ee. It is argued that with an appropriate upgrade of the Monte Carlo event generator BHLUMI and/or other similar MC programs calculating QED effects in the low angle Bhabha process, the total theoretical error of 0.01% for the FCC-ee luminometry can be reached. A new study of the Z and s-channel \( \gamma \) exchanges within the angular range of the FCC-ee luminometer using the BH-MIDE Monte Carlo was instrumental in obtaining the above result. Possible ways of BHLUMI upgrade are also discussed.
All of LEP/SLD luminosity QED error estimates represent corrections missing in BHLUMI v.4.04 Monte Carlo, used by all LEP and SLD collaborations.

BHLUMI features $O(\alpha^1)$ and $O(L_e^2\alpha^2)$ corrections with YFS resumation, neglecting photonics interferences between $e^+$ and $e^-$ lines, where $L_e = \ln(|t|/m_e^2)$.

Vacuum polarisation and pairs not dominant any more — QED photonic corrections and Z-exchange come back to front line!
1. Photonic corrections are large, but higher orders contrib. known, hence soft/collinear re-summation is mandatory!
2. M.E. in BHLUMI includes \( O(\alpha^1) \) and \( O(L_e^2 \alpha^2) \) corrections within YFS soft photon re-summation, neglecting photonics interferences between \( e^+ \) and \( e^- \) lines (suppressed by \(|t|/s\) factor).
3. Photonics 2nd order NLO \( O(L_e \alpha^2) \) and 3rd order LO \( O(\alpha^3 L_e^3 \alpha^2) \) corrections were calculated long ago [4], [6]. Presently they are not in BHLUMI v4.02 and accounted for in the error budget. Once included, error estimate is done for \( O(L_e^0 \alpha^2) \), \( O(\alpha^4 L_e^4) \) and \( O(\alpha^3 L_e^2) \) corrections.
4. Corrections \( O(L_e^0 \alpha^2) \sim 10^{-5} \) are not quoted in FCC error budget because are known.
5. Using scaling rules of thumb we estimate \( O(\alpha^4 L_e^4) \) as \( 0.015% \times \gamma = 0.6 \times 10^{-5} \) and \( O(\alpha^3 L_e^2) \sim \gamma^2 \alpha/\pi \approx 10^{-5} \). 
6. N.B. BHLUMI with \( O(L_e \alpha^2) \) has been already realised but not published because VP was dominant in 1998.

\[
\gamma = \frac{\alpha}{\pi} \ln \frac{\sqrt{t}}{m_e^2} = 0.042
\]

\[
|t|^{1/2} = (|t|)^{1/2} \approx 3.25 \text{ GeV}
\]


Z and s-channel gamma exchange for FCCee angular range 64-86mrad

<table>
<thead>
<tr>
<th>Type of correction / Error</th>
<th>Update 2018</th>
<th>FCCee forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Photonic $O(L^4_\alpha^4)$</td>
<td>0.027%</td>
<td>$6.0 \times 10^{-5}$</td>
</tr>
<tr>
<td>(b) Photonic $O(L^2_\alpha^3)$</td>
<td>0.015%</td>
<td>$1.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>(c) Vacuum polariz.</td>
<td>0.014% [25]</td>
<td>$6.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>(d) Light pairs</td>
<td>0.010% [18,19]</td>
<td>$5.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>(e) Z and s-channel $\gamma$ exchange</td>
<td>0.090% [11]</td>
<td>$1.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>(f) Up-down interference</td>
<td>0.009% [27]</td>
<td>$1.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>(f) Technical Precision</td>
<td>(0.027)%</td>
<td>$1.0 \times 10^{-4}$</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0.097%</td>
<td>$1.0 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

1. With respect to **dominant** t-channel gamma exchange $|\gamma_t|^2 = \gamma_t \otimes \gamma_t$, all other contributions are suppressed (near Z) by factor $<|t|>/s = 1.3 \cdot 10^{-3}$ (instead $0.4 \cdot 10^{-3}$ for LEP!)

2. However, **resonant** Zs exchange gets enhanced by $M_Z^2/\Gamma_Z$ and $\gamma_t \otimes Z_s$ term will be up to 1%. It is included in BHLUMI at the complete 1-st order level (with QED running couplings). Using results of ref. [11] its uncertainty due to QED corrections is **presently** estimate above as 0.090%.

3. **Non-resonant** $\gamma_t \otimes \gamma_s \sim 0.1\%$ is included in BHLUMI, gets small QED cor. with uncertainty 0.01%.

4. Other contribution not in BHLUMI are: $|Z_s|^2 \sim 0.01\%$, $\gamma_t \otimes Z_s \sim 3 \cdot 10^{-5}$, $|\gamma_s|^2 \sim 10^{-6}$ and $|Z_t|^2 \sim 10^{-6}$.

5. It will be straightforward to reduce the above uncertainties to $\sim 10^{-4}$ level by means of upgrade of the BHLUMI matrix element to the level of BHWIDE (EEX type).

6. With the implementation of the mat.el. of the CEEX type, as in KKMC, one could get for this group of contributions precision level of $\sim 10^{-5}$.

---

**Study of Z and s-channel $\gamma$ exchanges using BHWIDE**

<table>
<thead>
<tr>
<th>$E_{CM}$ [GeV]</th>
<th>$\Delta_{tot}$ [%]</th>
<th>$\delta_{QED}$ [%]</th>
<th>$\delta_{\alpha,\alpha}$ [%]</th>
<th>$\delta_{b,\alpha}$ [%]</th>
<th>$\delta_{gaug}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.1876</td>
<td>+0.642 (12)</td>
<td>-0.152 (59)</td>
<td>+0.034 (38)</td>
<td>-0.005 (12)</td>
<td></td>
</tr>
<tr>
<td>91.1876</td>
<td>+0.041 (11)</td>
<td>+0.148 (59)</td>
<td>-0.035 (38)</td>
<td>+0.009 (12)</td>
<td></td>
</tr>
<tr>
<td>92.1876</td>
<td>-0.719 (13)</td>
<td>+0.348 (59)</td>
<td>-0.081 (38)</td>
<td>+0.039 (13)</td>
<td></td>
</tr>
</tbody>
</table>

---

1. The error due to imprecise knowledge of the QED coupling constant for the t-channel exchange is \( \frac{\delta_{VP}\sigma}{\sigma} = \frac{2\delta \alpha_{eff}(t)}{\alpha_{eff}(t)} \).

2. With \( \Delta \alpha^{(5)}(-s_0) = (64.09 \pm 0.63) \times 10^{-4} \), of ref. [26], at \( s_0=2\text{GeV} \) we get \( (\delta_{VP}\sigma)/\sigma = 1.3 \times 10^{-4} \).

3. Anticipating improvement of hadronic \( e^+e^- \) cross section we expect by the FCCee time factor 2 improvement down to \( \delta_{VP}\sigma/\sigma = 0.65 \times 10^{-4} \).

4. N.B. The above is part of strategy of obtaining \( \alpha_{eff}(M_Z^2) \) in two steps:
   (a) obtaining \( \Delta \alpha^{(5)}(-s_0) \) from \( \sigma_{had}(s), s^{1/2} \lesssim 2.5\text{GeV} \), using dispersion relations,
   (b) calculating \( \Delta \alpha^{(5)}(M_Z^2) - \Delta \alpha^{(5)}(-s_0) \) using perturbative QCD.
   Getting \( \Delta \alpha^{(5)}(-s_0) \) for Bhabha luminometry from \( \alpha_{eff}(M_Z^2) \) could be an interesting crosscheck:)

---


1. Additional light fermion pair production in Bhabha process \( e^- e^+ \rightarrow e^- e^+ f \bar{f}, \ f = e, \mu, \tau, u, d, s \) together with the corresponding virtual correction (fermion loop on photon line) is a valid 2nd order correction.

2. Numerically most sizeable is electron pair production subprocess \( e^- e^+ \rightarrow e^- e^+ \gamma, \ \gamma^* \rightarrow e^- e^+ \) which very well known [9,10,18,19,53-60] and its precision is usually quoted to be \( \sim 0.5 \cdot 10^{-4} \).

3. Second pair production \( e^- e^+ \rightarrow e^- e^+ 2(e^- e^+) \) and addition photon production \( e^- e^+ \rightarrow e^- e^+ e^- e^+ \gamma \) are calculable [10,18,54] and quoted to be negligible.

4. Contributions from heavier leptons and light quarks \( f = \mu, \tau, u, d, s \) are typically \( \sim 0.8 \cdot 10^{-4} \) and in LEP context were entirely accounted as part of an error. They can be however calculated with the precision \( \ll 0.5 \cdot 10^{-4} \).

5. These corrections can be incorporated only partly in BHLUMI (electron pair exponentiation in [10]), most likely auxiliary MC programs will be needed to calculate them.
1. From ref. [27] this photonics (1st order correction) is known to be \( \frac{\delta \sigma}{\sigma} \simeq 0.07 \frac{|t|}{s} \) and for the luminometry it was negligible.

2. For FCCee it will come in a natural way in the upgrade M.E. of BHLUMI, to be done either as in BHWIDE or in KKMC.

3. We use conservatively factor \( 2 \gamma \simeq 0.1 \) in its precision estimate.

<table>
<thead>
<tr>
<th>Type of correction / Error</th>
<th>Update 2018</th>
<th>FCCee forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Photonic ( O(L_e^4 \alpha^4) )</td>
<td>0.027%</td>
<td>( 0.6 \times 10^{-5} )</td>
</tr>
<tr>
<td>(b) Photonic ( O(L_e^2 \alpha^3) )</td>
<td>0.015%</td>
<td>( 0.1 \times 10^{-4} )</td>
</tr>
<tr>
<td>(c) Vacuum polariz.</td>
<td>0.014% [25]</td>
<td>( 0.6 \times 10^{-4} )</td>
</tr>
<tr>
<td>(d) Light pairs</td>
<td>0.010% [18, 19]</td>
<td>( 0.5 \times 10^{-4} )</td>
</tr>
<tr>
<td>(e) Z and s-channel ( \gamma ) exchange</td>
<td>0.090% [11]</td>
<td>( 0.1 \times 10^{-4} )</td>
</tr>
<tr>
<td>(f) Up-down interference</td>
<td>0.009% [27]</td>
<td>( 0.1 \times 10^{-4} )</td>
</tr>
<tr>
<td>(f) Technical Precision</td>
<td>(0.027)%</td>
<td>( 0.1 \times 10^{-4} )</td>
</tr>
<tr>
<td>Total</td>
<td>0.097%</td>
<td>( 1.0 \times 10^{-4} )</td>
</tr>
</tbody>
</table>
1. Technical precision is the hardest problem!
2. In LEP workshop ref. [29] (1998) it was based on two pillars: comparison with semi-analytical calculation in ref. [45] and on comparison of BHLUMI with two hybrid MCs, LUMLOG+OLBBIS and SABSPV.
3. It was established to be 0.27%, together with missing photonics corrections.
4. Later on another BabaYaga MC was developed [20-24] based on the parton shower algorithm, and in principle could be used to evaluate technical precision independently.
5. However, once BHLUMI will be upgraded to include complete $O(L^4 \alpha^4)$ and $O(L^2 \alpha^3)$ the problem will come back, because it will be much harder to upgrade BabaYaga to the same NNLO level due to known peculiarities of the parton shower methodology.
6. Alternative solution could/should be worked out. See S. Frixione, 1909.03886, V. Bertone et al., 1911.12040.

<table>
<thead>
<tr>
<th>Type of correction / Error</th>
<th>Update 2018</th>
<th>FCCee forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Photonic $O(L^4 \alpha^4)$</td>
<td>0.027%</td>
<td>$0.6 \times 10^{-5}$</td>
</tr>
<tr>
<td>(b) Photonic $O(L^2 \alpha^3)$</td>
<td>0.015%</td>
<td>$0.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>(c) Vacuum polariz.</td>
<td>0.014% [25]</td>
<td>$0.6 \times 10^{-4}$</td>
</tr>
<tr>
<td>(d) Light pairs</td>
<td>0.010% [18,19]</td>
<td>$0.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>(e) Z and s-channel $\gamma$ exchange</td>
<td>0.090% [11]</td>
<td>$0.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>(f) Up-down interference</td>
<td>0.009% [27]</td>
<td>$0.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>(f) Technical Precision</td>
<td>(0.027)%</td>
<td>$1.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>Total</td>
<td>0.097%</td>
<td>$1.0 \times 10^{-4}$</td>
</tr>
</tbody>
</table>


Historically, our exact $O(\alpha^2 L_e)$ corrections were done for BHLUMI 4 precision $\Rightarrow$ Combined via crossing with CEEX $\Rightarrow$ KKMC for state-of-art 2f production $\Rightarrow$ KKMC-hh for Z production in pp

KKMC-hh $\Rightarrow$ MG5_aMC@NLO/KKMC-hh (to appear) $\Rightarrow$ exact QCD NLO $\otimes$ exact $O(\alpha^2 L_e)$ EW

When we add to BHLUMI QED matrix element corrections of $O(L_e \alpha^2)$ and $O(\alpha^3 L_e^3)$

$\Rightarrow$ Already reduce $\delta N_\nu$ from $\delta \mathcal{L}/\mathcal{L}$ to 0.0015.

We now need to take CEEX to BHLUMI (a technical precision solution)

$\Rightarrow$ For FCCee, take CEEX to all the EEX YFS realizations for LEP:

- YFSWW3 & KORALW (see Skrzypek)
- YFSZZ
- BHWIDE

We do need sufficient theory resources.
SYNERGIES

• For example, $A_{FB}$: Jadach & Yost, arXiv: 1801.08611 => use CEEX
  => KKMC for state-of-art 2f production => already have $\Delta [A_{FB}]_{IF} \sim 10^{-4}$

• $\Delta M_W$: (Skrzypek(FCCee Workshp, 2020)):

  • Threshold & Reconstruction: Need $\sim 0.3$ MeV for FCC-ee
  • CEEX extension of the LEP2 MC YFSWW3 & KORALW needed in both cases:
  • In progress: Jadach et al., arXiv: 1906.09071 -- CEEX formalism applied
  • to $e^+e^-\rightarrow WW+\gamma \rightarrow 4f+\gamma$
  • Note: Contact with the usual Kleiss-Stirling spinor product-based
    photon helicity infrared factors in CEEX via

$$e j_{X}^{\mu}(k_i) = e Q_X \theta_X \frac{p_{X}}{2 p_X k_i} \rightarrow s(k_i) = e Q_X \theta_X \frac{b_{\sigma_i}(k_i, p_X)}{2 p_X k_i},$$

with

$$b_{\sigma}(k, p) = \sqrt{2} \frac{\bar{u}_{\sigma}(k) \ p \ u_{\sigma}(\zeta)}{\bar{u}_{-\sigma}(k) u_{\sigma}(\zeta)}$$

• The way forward is open.
All of LEP/SLD luminosity QED error estimates represent corrections missing in BHLUMI v.4.04 Monte Carlo, used by all LEP and SLD collaborations.

BHLUMI features $O(\alpha^1)$ and $O(L_e^2 \alpha^2)$ corrections with YFS resumation, neglecting photonics interferences between $e^+$ and $e^-$ lines, where $L_e = \ln(|t|/m_e^2)$.

One has to add to BHLUMI QED matrix element corrections of $O(L_e \alpha^2)$ and $O(\alpha^3 L_e^3)$.

They were calculated by Cracow-Knoxville collaboration long time ago (1996-99), but there was no strong motivation to publish them in the MC form, because of large VP uncertainty.

Interferences between $e^+$ and $e^-$ lines should be added at 1-st order, with resummation.

This class of corrections are implemented in the KKMC and BHWIDE since 1999.

Corrections due to Z exchange and s-channel gamma are big but easy to master (ME upgrade).

There is (almost) enough auxiliary programs and calculations to control light pair corrections.

Summarising there is no hard obstacles on the way to 0.01% QED precision on the theory side.

The sticky issue is that of “technical precision”.--The New Issue!!

If BabaYaga Monte Carlo team makes sufficient progress this problem is solved (Piccinini).

Alternative solutions are available: comparing CEEX and EEX upgrades of BHLUMI, Frixione et al., Sherpa, ....

We do need sufficient theory resources.
Bibliography

References


Bibliography


Preliminary Remarks

Overview of Resummed Quantum Gravity

Planck Scale Cosmology

An Estimate of $\Lambda$

An Open Question?

Einstein-Heisenberg Consistency Condition

Constraints on SUSY GUTs
Preliminary Remarks

- **IS QUANTUM GRAVITY (Einstein-Hilbert Theory) CALCULABLE IN RELATIVISTIC QFT?**
- **STRING THEORY: NO.** You need superstrings, supersymmetric one-dimensional objects of Planck length size, $1.62 \times 10^{-33}$ cm.
- **LOOP QUANTUM GRAVITY: NO.** You need Planck length size loops that are the fundamental constructs for quantum gravity.
- **HORAVA-LIFSHITZ THEORY: NO.** You need anisotropic scaling at Planck length scales: Time and space differ by a factor of $z$ in scale dimension at Planck length distances with $z = 3$ in the original proposal–this violates local Lorentz invariance.
Preliminary Remarks

- New Approach: Exact Amplitude-Based Resummation of Feynman’s Formulation of Einstein’s Theory – Resummed Quantum Gravity (RQG)

RESULT (1): UV Finiteness!
RESULT (2): Constraints on SUSY GUT’s
RESULT (3): Prediction for the Cosmological Constant Λ with Relatively Small Theoretical Uncertainty.
RESULT (4): Consistent with Weinberg’s Asymptotic Safety Ansatz, as realized by Exact Field Space Renormalization Group Program of Reuter et al.
Preliminary Remarks

RESULT (5): Consistent with Kreimer’s Leg Renormalizability Results ...

Today we give highlights on the status and outlook for this new RQG approach.
Overview of Resummed Quantum Gravity

SM ⇔ Many Massive Point Particles.
Feynman: spin is an inessential complication – checked. We replace $L_{SM}^{G}(x)$ with that a free physical Higgs field, $\varphi(x)$, with a rest mass 125 GeV (ATLAS, CMS) ⇒ the representative model


$$L(x) = \frac{1}{2\kappa^2} R \sqrt{-g} + \frac{1}{2} \left( g^{\mu\nu} \partial_{\mu} \varphi \partial_{\nu} \varphi - m_{0}^{2} \varphi^{2} \right) \sqrt{-g}$$

$$= \frac{1}{2} \left\{ h^{\mu\nu,\lambda} \bar{h}_{\mu\nu,\lambda} - 2 \eta^{\mu\mu'} \eta^{\lambda\lambda'} \bar{h}_{\mu,\lambda} \eta^{\sigma\sigma'} \bar{h}_{\mu',\sigma,\sigma'} \right\}$$

$$+ \frac{1}{2} \left\{ \varphi,_{\mu} \varphi,_{\mu} - m_{0}^{2} \varphi^{2} \right\} - \kappa h^{\mu\nu} \left[ \frac{\varphi,_{\mu} \varphi,_{\nu}}{\varphi,_{\mu} \varphi,_{\nu}} + \frac{1}{2} m_{0}^{2} \varphi^{2} \eta_{\mu\nu} \right]$$

$$- \kappa^2 \left[ \frac{1}{2} h_{\lambda\rho} \bar{h}^{\rho\lambda} \left( \varphi,_{\mu} \varphi,_{\mu} - m_{0}^{2} \varphi^{2} \right) - 2 \eta_{\rho\rho'} h^{\mu\rho} \bar{h}^{\rho'\nu} \varphi,_{\mu} \varphi,_{\nu} \right] + \cdots \tag{1}$$
Overview of Resummed Quantum Gravity

where $\varphi,\mu \equiv \partial_\mu \varphi$ and we have

- $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$,
  $\eta_{\mu\nu} = \text{diag}\{1, -1, -1, -1\}$

- $\bar{y}_{\mu\nu} \equiv \frac{1}{2} (y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu} y_{\rho\rho})$ for any tensor $y_{\mu\nu}$

- Feynman rules already worked-out by Feynman (op. cit.), where we use his gauge, $\partial^\mu \bar{h}_{\nu\mu} = 0$

$\Leftrightarrow$ Quantum Gravity is just another quantum field theory where the metric now has quantum fluctuations as well.
Overview of Resummed Quantum Gravity

YFS resum the propagators in the NON-ABELIAN gauge theory of QG:
⇒ from the YFS formula

\[ iS'_F(p) = \frac{ie^{-\alpha B''_{\gamma}}}{S^{-1}_F(p) - \Sigma'_F(p)}, \quad (2) \]

we find for Quantum Gravity, proceeding as above, the analogue of

\[ \alpha B''_{\gamma} = \int \frac{d^4 \ell}{(2\pi)^4} \frac{-i\eta^{\mu\nu}}{(\ell^2 - \chi^2 + i\epsilon) \left( \ell^2 - 2\ell k + \Delta + i\epsilon \right) \left( \ell^2 - 2\ell k' + \Delta' + i\epsilon \right)} \bigg|_{k = k'} \]

as \(-B''_g(k)\) with

\[ B''_g(k) = -2iK^2 k^4 \int \frac{d^4 \ell}{16\pi^4} \frac{1}{\ell^2 - \chi^2 + i\epsilon \left( \ell^2 + 2\ell k + \Delta + i\epsilon \right)^2} \]

(4)

for \(\Delta = k^2 - m^2\) ⇒ for a scalar field

\[ i\Delta'_F(k) |_{YFS-resummed} = \frac{ie^{B''_g(k)}}{(k^2 - m^2 - \Sigma'_s + i\epsilon)}. \]
Expand theory with the 'improved Born' propagators

\[ iP_{\alpha_1 \ldots ; \alpha'_1 \ldots} \Delta'_F(k) |_{YFS\text{-resummed}, \Sigma'_s=0} = \frac{iP_{\alpha_1 \ldots ; \alpha'_1 \ldots} e^{B''_g(k)}}{(k^2 - m^2 + i\epsilon)} \]  

(5)

where in the DEEP UV we get

\[ B''_g(k) = \frac{\kappa^2 |k^2|}{8\pi^2} \ln \left( \frac{m^2}{m^2 + |k^2|} \right), \]  

(6)

\[ \Rightarrow \text{ALL PROPAGATORS FALL FASTER THAN ANY POWER OF } |k^2| \Rightarrow \text{QG IS FINITE (SEE MPLA17 (2002) 2371; hep-ph/0607198)}! \]
CONTACT WITH ASYMPTOTIC SAFETY APPROACH

OUR RESULTS IMPLY

\[ G(k) = G_N / (1 + \frac{k^2}{a^2}) \]

⇒ FIXED POINT BEHAVIOR FOR \( k^2 \to \infty \),
IN AGREEMENT WITH THE PHENOMENOLOGICAL
ASYMPTOTIC SAFETY APPROACH OF BONANNO &

OUR RESULTS ⇒ AN ELEMENTARY PARTICLE HAS
NO HORIZON. THIS AGREES WITH BONANNO & REUTER
THAT A BLACK HOLE WITH A MASS LESS THAN
\( M_{cr} \sim M_{Pl} \)
HAS NO HORIZON.
BASIC PHYSICS:
\( G(k) \) VANISHES FOR \( k^2 \to \infty \).
Bonanno and Reuter see arXiv.org:0803.2546, and refs. therein – phenomenological approach to Planck scale cosmology: STARTING POINT IS THE EINSTEIN-HILBERT THEORY

\[ \mathcal{L}(x) = \frac{1}{2\kappa^2} \sqrt{-g} (R - 2\Lambda) \]  

PHENOMENOLOGICAL EXACT RENORMALIZATION GROUP FOR THE WILSONIAN COARSE GRAINED EFFECTIVE AVERAGE ACTION IN FIELD SPACE ⇒ RUNNING NEWTON CONSTANT \( G_N(k) \) AND COSMOLOGICAL CONSTANT \( \Lambda(k) \) APPROACH UV FIXED POINTS AS \( k \) GOES TO \( \infty \) IN THE DEEP EUCLIDEAN REGIME – \( k^2 G_N(k) \rightarrow g_* \), \( \Lambda(k) \rightarrow \lambda_* k^2 \).

Due to the thinning of the degrees of freedom in Wilsonian field space renormalization theory, the arguments of Foot et al. (PLB664(2008)199) are obviated. – See also MPLA 25(2010)607; SHAPIRO & SOLA, PLB682(2009)105
CONTACT WITH COSMOLOGY PROCEEDS AS FOLLOWS: PHENOMENOLOGICAL CONNECTION BETWEEN THE MOMENTUM SCALE $k$ CHARACTERIZING THE COARSENESS OF THE WILSONIAN GRAININESS OF THE AVERAGE EFFECTIVE ACTION AND THE COSMOLOGICAL TIME $t$, B-R SHOW STANDARD COSMOLOGICAL EQUATIONS ADMIT (see also Bonanno et al., 1006.0192) THE FOLLOWING EXTENSION:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{1}{3} \Lambda + \frac{8\pi}{3} G_N \rho$$

$$\dot{\rho} + 3(1 + \omega) \frac{\dot{a}}{a} \rho = 0$$

$$\dot{\Lambda} + 8\pi \rho \dot{G}_N = 0$$

$$G_N(t) = G_N(k(t))$$

$$\Lambda(t) = \Lambda(k(t)) \quad (8)$$

FOR DENSITY $\rho$ AND SCALE FACTOR $a(t)$
Planck Scale Cosmology

WITH ROBERTSON-WALKER METRIC REPRESENTATION

$$ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right)$$  \(9\)

\(K = 0, 1, -1 \Leftrightarrow \) RESPECTIVELY FLAT, SPHERICAL AND PSEUDO-SPHERICAL 3-SPACES FOR CONSTANT TIME \(t\)

FOR A LINEAR RELATION BETWEEN THE PRESSURE \(p\) and \(\rho\) \((\text{EQN. OF STATE})\)

$$p(t) = \omega \rho(t).$$  \(10\)
FUNCTIONAL RELATIONSHIP BETWEEN MOMENTUM SCALE $k$ AND COSMOLOGICAL TIME $t$ DETERMINED PHENOMENOLOGICALLY VIA (see also Shapiro and Sola, PLB475(2000)236)

\[ k(t) = \frac{\xi}{t} \quad (11) \]

WITH POSITIVE CONSTANT $\xi$.

Using the UV fixed points for $k^2 G_N(k) = g_*$ and $\Lambda(k)/k^2 = \lambda_*$ B-R SHOW THAT (8) ADMITS, FOR $K = 0$, A SOLUTION IN THE PLANCK REGIME ($0 \leq t \leq t_{\text{class}}$, with $t_{\text{class}}$ a few times the Planck time $t_{Pl}$), WHICH JOINS SMOOTHLY ONTO A SOLUTION IN THE CLASSICAL REGIME ($t > t_{\text{class}}$) which agrees with standard Friedmann-Robertson-Walker phenomenology but with the horizon, flatness, scale free Harrison-Zeldovich spectrum, and entropy problems solved by Planck scale quantum physics.
PHENOMENOLOGICAL NATURE OF THE ANALYSIS: THE fixed-point results $g_*, \lambda_*$ depend on the cut-offs used in the Wilsonian coarse-graining procedure.

KEY PROPERTIES OF $g_*, \lambda_*$ USED FOR THE B-R ANALYSES: they are both positive and the product $g_*\lambda_*$ is cut-off/threshold function independent.
In Phys. Dark Univ. 2 (2013) 97, using (5) and (6) we get rigorous cut-off independent values for the fixed points $g_*, \lambda_*$ and the following estimate of $\Lambda$:

$$\rho_\Lambda(t_0) \cong -M_{Pl}^4 (1 + c_{2,\text{eff}} k_{tr}^2 / (360\pi M_{Pl}^2))^2 \frac{64}{\sum_j (-1)^j n_j \rho_j^2}$$

$$\times \frac{t_{tr}^2}{t_{eq}^2} \times \left(\frac{t_{eq}^2}{t_0^2/3}\right)^3$$

$$\simeq -M_{Pl}^2 (1.0362)^2 (-9.194 \times 10^{-3}) (25)^2 \frac{64}{t_0^2}$$

$$\simeq (2.4 \times 10^{-3} \text{eV})^4,$$  

(12)

where the age of the universe is $t_0 \cong 13.7 \times 10^9$ yrs.

Compare: $\rho_\Lambda(t_0)|_{\text{expt}} \cong ((2.37 \pm 0.05) \times 10^{-3} \text{eV})^4$.  

B.F.L. Ward

RADCOR2021
A MAIN UNCERTAINTY: $t_{tr}$
B-R: NUMERICAL STUDIES $\Rightarrow t_{tr} \approx 25/M_{Pl}$
IN GENERAL, A FACTOR of $\mathcal{O}(100)$ IS ALLOWED
CAN WE DO BETTER - NEW ISSUE?
Einstein-Heisenberg Consistency Condition

- In MPLA\textbf{30} (2015)1550206, we use the de Sitter space solutions of Duerr et al. to get the Einstein-Heisenberg consistency condition

\[
k \geq \frac{\sqrt{5}}{2w_0} = \frac{\sqrt{5}}{2} \frac{1}{\sqrt{3/\Lambda(k)}}
\]

(13)

from the Heisenberg uncertainty relation \(\Delta p \Delta q \geq \frac{1}{2}\), with \(\Delta p = k\) and \((w_0 = \sqrt{3/\Lambda})\)

\[
(\Delta q)^2 \approx \frac{\int_0^{w_0} dww^2 w^2 <\cos^2 \theta>}{\int_0^{w_0} dww^2} = \frac{1}{5} w_0^2.
\]

(14)

Violation of (13) ends Planck scale inflation: solving for \(k_{\text{tr}}\)

\[\Rightarrow k_{\text{tr}} \approx M_{Pl}/25.3,\] in agreement with what Bonnano and Reuter suggested from numerical studies.

\[\Rightarrow\text{uncertainty on our estimate of } \rho_\Lambda\text{ is } \mathcal{O}(10)\text{-YES!}\]
Note

$$\langle 0 | H | 0 \rangle \sim \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2} \omega(k) = \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2}$$


Intermediate Stage:

$$SU_{2L} \times SU_{2R} \times SU(3)^c$$

SM Stage at $\sim 2\text{TeV} = M_R$:

$$SU_{2L} \times U_1 \times SU(3)^c$$

SUSY Breaking at EW scale $M_S$:

$$U_1 \times SU(3)^c$$
• Possible spectrum(?)

\[
\begin{align*}
m_{\tilde{g}} &\approx 1.5(10)\text{TeV} \\
m_{\tilde{C}} &\approx 1.5\text{TeV} \\
m_{\tilde{q}} &\approx 1.0\text{TeV} \\
m_{\tilde{\ell}} &\approx 0.5\text{TeV} \\
\end{align*}
\]

\[
m_{\chi_i^0} \approx \begin{cases} 
0.4\text{TeV}, & i = 1 \\
0.5\text{TeV}, & i = 2, 3, 4 
\end{cases}
\]

\[
m_{\chi_i^\pm} \approx 0.5\text{TeV}, \quad i = 1, 2
\]

\[
m_s = .5\text{TeV}, \quad S = A^0, \ H^\pm, \ H_2.
\]

\[
\Delta_{\text{GUT}} = \sum_{j \in \{\text{MSSM low energy susy partners}\}} \frac{(-1)^F n_j}{\rho_j^2} 
\approx 1.13(1.12) \times 10^{-2}
\]
Constraints on SUSY GUTS

\[ \Rightarrow \]

- Compensate by either (A) adding new susy families with scalars lighter than fermions or (B) allowing the gravitino mass to go to 
  \[ \sim 0.05 \ M_{\text{GUT}} \sim 2 \times 10^{15} \ \text{GeV} \].
- For approach (A), new quarks and leptons at 
  \[ M_{\text{High}} \sim 3.4(3.3) \times 10^3 \ \text{TeV}, \]
  scalar partners at \[ \sim 0.5 \ \text{TeV} = M_{\text{Low}} \].

100 TeV too small!

NEW LHC LIMITS??
Constraints on SUSY GUTS

- GUT and other breaking scales: small compared to $0.01M_{Pl}^4/64 \Rightarrow$ drop
- Covariance issues:

  Bianchi’s Identity,
  $$D^\nu(\Lambda g_{\nu\mu} + 8\pi G_N T_{\nu\mu}) = 0,$$
  allows
  $$\dot{\rho} + 3\frac{\dot{a}}{a}(1+\omega)\rho = -\frac{\Lambda + 8\pi \rho \dot{G}_N}{8\pi G_N},$$

  a more general form of the new Friedmann eqns: qualitatively the same but details differ -- see arXiv:0907.4555,1103.4632,1202.5097....

  Our estimate uses the more general form.
SUMMARY

- Precision Quantum Field Theory: EW, QCD, QG \equiv Control all limits:

  \[ \text{IR} \ (z \rightarrow 1) \]

  and

  \[ \text{Collinear} \ (p_T \rightarrow 0) \]

  \[ \text{UV limit} \]

- We now have control over all aspects of the QG corrections.

- Toward quantitative understanding of \( \rho^\Lambda \) along with other precision observables: possible tests in new GWP(d^{UV}_H)??