NNLO QCD predictions for 2 to 3 processes

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Outline

- Precision vs. Multiplicity @ the LHC
- NNLO QCD for 2 to 3 processes without masses:

- \( pp \rightarrow \gamma \gamma \gamma + X \)
- \( pp \rightarrow \gamma \gamma j + X \)
- \( pp \rightarrow \gamma jj + X \)
- \( pp \rightarrow jjj + X \)

• Summary
Why are we interested in NNLO QCD for 2 → 3 processes? 

(8 talks this week)

Phenomenological aspects:

• For 2 → 2 NNLO QCD (+NLO EW) huge success for many measurements! In some cases N3LO on the wish list.
• Next phase of LHC → enough statistics to actually resolve 2→3 NNLO?!
  • Massless processes a clear case!
  • But also heavy processes H/V+2j,ttH, ttV, VVV, ... call for NNLO predictions!

Theory aspects:

• Development of NNLO QCD technology (amplitudes&subtraction) crucial work on the road towards NNLO event simulation.
• Crucial ingredient for differential 2 → 2 N3LO QCD
NNLO QCD prediction beyond $2 \to 2$

2 → 3 Two-loop amplitudes:
- (Non-) planar 5 point massless → talk by Vasily, Herschel, Federico
  fast progress in the last half of year
  → triggered by efficient MI representation [Chicherin,Sotnikov'20]
- 5 point with one external mass
  → talk by Nikolaos, Ben, Konstantinos, Boyu

Many leg, IR stable one-loop amplitudes → OpenLoops [Buccioni'19]

Cross sections → Combination with real radiation
- Various NNLO subtraction schemes are available:
  - qT-slicing [Catani'07], N-jettiness slicing [Gaunt'15/Boughezal'15], Antenna [Gehrmann'05-'08], Colorful [DelDuca'05-'15], Projection [Cacciari'15], Geometric [Herzog'18], Unsubtraction [Aguilera-Verdugo'19], Nested collinear [Caola'17],
  Sector-improved residue subtraction [Czakon'10-'14]
Phenomenological applications

Three photons

Two photons plus jet

Three jets

\[ pp \rightarrow \gamma\gamma\gamma + X \]

\[ pp \rightarrow \gamma\gamma j + X \]

\[ pp \rightarrow jjj + X \]
Three photon production

\[ pp \rightarrow \gamma\gamma\gamma + X \]

- First NNLO QCD 2 → 3 cross sections: [Chawdhry'19], [Kallweit'20]
- Simplest among the 2→3 massless cases: colour singlet
- Planar Two-loop virtuals:
  2*Re(M0*F2) with ‘original’ pentagon functions [Henn'18]
  → Fast helicity amplitudes: [Abreu'20], [Chawdhry'20]

- Large NNLO/NLO K-factors
- Similar behaviour as pp → γγ
- NNLO QCD corrections essential for theory/data comparison
- Contribution of 2l amps small ≈ 1%
Three photon production

Corrections to shape and normalization

Typical for colour singlets: Scale uncertainty stays large. Very different for $pp\to\gamma\gamma j$, $pp\to jjj$
Diphoton plus jet production

- Photon pair production @ LHC is of particular interest:
  - Main background to cleanest Higgs decay channel
- Inclusive diphoton shows large NNLO QCD corrections
  - Perturbative convergence @ N3LO?
    First steps: Talks by Xuan, Lorenzo
  - Diphoton plus jet @ NNLO QCD ($\rho T(AA) \rightarrow 0$ limit)
- $\rho T(\gamma\gamma)$ spectrum itself interesting for Higgs $\rightarrow \gamma\gamma$
  - Higgs – $\rho T$ measurements at large $\rho T$ resolves local Higgs couplings $\rightarrow$ BSM searches
  - Angular diphoton observables $\rightarrow$ spin measurements
Diphoton plus jet - setup

2105.06940: Inspired by Higgs → γγ measurement phase spaces

- Smooth photon isolation criteria (ET = 10 GeV, R = 0.4), dR(γγ) > 0.4 GeV
- $p_T(γ_1) > 30$ GeV, $p_T(γ_2) > 18$ GeV and $|y(γ)| < 2.4$
- $m(γγ) > 90$ GeV and $p_T(γγ) > 20$ GeV, below resummation important
- No further restrictions on jets (IR safety from $p_T(γγ)$ cut)

Technicalities:

- LHC 13 TeV, PDF: NNPDF31, Scale: $μR^2 = μF^2 = ¼(m(γγ)^2 + p_T(γγ)^2)$
- 5 massless flavours and top-quarks (in all one-loop amps)
- Approximation of two-loop amps: $2Re(M0*F2) + F1*F1$ without top-quark loops and $2Re(M0*F2)$ in leading colour limit [Chawdhry’21]

→ Update to full colour planed [Agarwal’21]
Diphoton plus jet – $p_T$ spectrum

- Beautiful perturbative convergence
- Scale dependence:
  - NLO: ~10%
  - NNLO: ~1-2%
- Low $p_T$ region:
  - ? Resummation for $p_T(\gamma\gamma)/m(\gamma\gamma) \ll 1$
  - Strong effect from the loop induced!
Diphoton plus jet – Angular observables

Note: Normalization effected by low pT behaviour
Diphoton plus jet – two-loop contribution

- Two-loop contribution (green line) <~1%,
- Loop induced contribution:
  - sizeable effects for low $p_T$, vanishes for high $p_T$
  - flat effect in ‘bulk’ observables
  - Dominant source of scale dependence
- NLO QCD correction (formally N3LO) relevant, missing piece: $gg \to γγg$ two-loop
Three jet production

- Multi-jet rates provide an unique possibility to test (perturbative) QCD at the LHC
- Measurements of $\alpha_S$ from event shapes and jet rate ratios ($\sim \alpha_S$)
- Test of $\alpha_S$ running
- Multi-jet signatures are background for many LHC signatures.
- Allow to probe broad ranges of energy scales for heavy new physics
- Large cross sections $\rightarrow$ large statistics, in practice only limited by systematics!
Three jet production

Advances in perturbative QCD allow precision predictions for multi-jet rates

Here: NNLO QCD predictions for two and three jet rates

- NNLO QCD di-jet production known:
  - Gluons only [Gehrmann-De Ridder’13], partially leading colour [Currie’16]
  - Complete [Czakon’18] \(\rightarrow\) sub-leading colour effects < 1-2%

- NNLO QCD tri-jet production:
  - Bottleneck double virtual amplitudes: recently published in leading colour approximation [Abreu’21]
  - Handling of real radiation:
    - Sector-improved residue subtraction conceptually capable
      \(\rightarrow\) Tour-de-force (~4000 sectors for RR) \(\rightarrow\) preliminary results
Three jet production - Setup

Setup:
- LHC @ 13 TeV, NNPDF31
- Require at least three (two) jets with:
  - $p_T > 60$ GeV, $|y| < 4.4$
  - $HT_2 = p_T1 + p_T2 > 250$ GeV
- Scales: $\mu_R = \mu_F = Hhat = \Sigma p_T$ partons

R32 ratios:
- Two jet rate = $\sigma_2$
  Three jet rate = $\sigma_3$
- $R_{32} = \sigma_3/\sigma_2$
- Differentially in $X$:
  $R_{32}(X) = (d\sigma_3/dX)/(d\sigma_2/dX)$
- Scale dependence of $R_{32}(X)$ is determined by correlated variation in $\sigma_3$ and $\sigma_2$
Three jet production – leading $p_T$

Two jets:

Three jets:
Three jet production - $R_{3/2}(p_{T,1})$
Three jet production - R32(HT)

$$HT = \Sigma pT(\text{jet})$$

Scale dependence correlated in ratio

→ reduction of scale dependence

→ flat k-factor

→ scale bands in ratio barely overlap
Three jet production – azimuthal decorrelation

Kinematic constraints on the azimuthal separation between the two leading jets ($\phi_{12}$)

$\phi_{12}$ sensitive to the jet multiplicity:

2j: $\phi_{12} = \pi$

3j: $\phi_{12} > 2/3\pi$

4j: unconstrained

Study of the ratio

$R_{32}(HT,y^*,\phi_{Max}) = (d\sigma_3(\phi<\phi_{Max})/dHT/dy^*)/(d\sigma_2/dHT/dy^*)$

With $y^* = |y_1-y_2|/2$
Three jet production – $R_{32}(HT, y^*, \phi_{\text{Max}})$

NNLO/NLO K-factor smaller than NLO/LO
Scale dependence is reduced
Summary and Outlook

- NNLO QCD predictions for 2 to 3 processes will be essential part of precision phenomenology at the LHC.
- Results for:
  - Three photons
  - Diphoton plus jet
  - Three jet production
- Virtual matrix elements with high multiplicity and many scales are the bottleneck!
- Real radiation for 2→3 can be handled. But efficiency is a concern and needs some attention!
- Many interesting applications ahead! Stay tuned.

Thank you for your attention!